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PERTURBATIVE QCD AT FINITE TEMPERATURE †

T. Altherr

L.A.P.P. , BP 110, 74941 Annecy-le-Vieux Cedex, FRANCE

Abstract

We discuss an application of finite temperature QCD to lepton-pair production in a quark-gluon plasma. The perturbative calculation is performed within the "real-time" formalism. After cancellation of infrared and mass singularities, the corrections at $O(\alpha_S)$ are found to be very small in the region where the mass of the Drell-Yan pair is much larger than the temperature of the plasma. Interesting effects, however, appear at the annihilation threshold of the thermalized quarks.

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QCD predicts that at some critical temperature, there is a transition from hadron matter to a quark-gluon plasma (QGP) [1]. This interesting state of matter could have been seen in present ultra-relativistic heavy ions collisions performed at CERN [2].

Dileptons pairs have been claimed to be a good candidate for the signature of a plasma since electromagnetic signals should not be affected by strong interactions [3]. We are interested here in the differential rate in space and time and we do not discuss at all the integration over the history of the plasma. This has already been studied in detail but only in the pure electromagnetic sector [4]. We concentrate in these pages on the QCD corrections of order α_S to dilepton production in a QGP in equilibrium using the real-time formalism of finite temperature [5].

To our knowledge, extensive calculations of this type have been made by four groups of people [6-9] who all agree on the fact that both infrared and mass singularities cancel in the final result, at least at $O(\alpha_S)$. We have also made a detailed study of the finite terms [10] and more details can be found in this reference.

In terms of Feynman amplitudes, one has to compute processes such as the one shown in fig.1 where several scattering processes of gluons, quarks and antiquarks produce a virtual photon which does not thermalize and escapes from the plasma to create a dilepton pair.

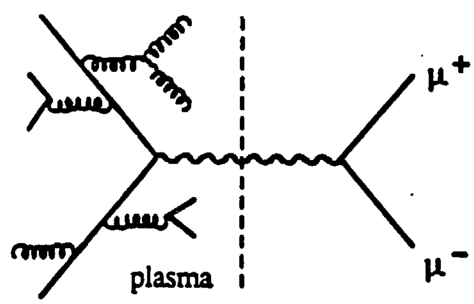

Fig.1 : Creation of a thermal $\mu^+\mu^-$ pair.

We are interested here in the left-hand side of this process, the formation of a virtual photon which rate is related to the imaginary part of the vacuum polarization tensor. For simplicity, the calculation is performed in the rest frame of the plasma, with the virtual photon of mass Q emitted at rest with respect to the heat bath.

First we need to generalize the computation of discontinuities of Green functions at finite temperature. This has recently been done by Kobes and Semenoff [11] in the "real-time" formalism. We just recall the 2x2 matrix propagators used in this formalism (in the case of a scalar particle)

$$D_{ab}(k) = \begin{pmatrix} D(k) & D^-(k) \\ D^+(k) & D^*(k) \end{pmatrix}$$
$$= \begin{pmatrix} \Delta(k) & \theta(-k_0)2\pi\delta(k^2) \\ \theta(k_0)2\pi\delta(k^2) & \Delta^*(k) \end{pmatrix} + 2\pi\delta(k^2)n_B(\omega)\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \quad (1)$$

where $\omega = |k_0|$ and $\Delta(k) = i/(k^2 + i\eta)$. The function $n_B(\omega) = 1/(\exp \beta\omega - 1)$ is the Bose-Einstein distribution ($\beta = 1/T$ is the inverse temperature).

The non-diagonal elements of the matrix, $D^{\pm}(k)$, are related to production/decay rates of the particle. The quantity of interest is

$$\Gamma = \Pi^{\mu}{}_{\mu}{}^{-}(q) \tag{4}$$

where Π^{ν}_{μ} is the vacuum polarization tensor of the photon which has the same matrix structure as eq.(1) at finite temperature. The production rate, per unit of four dimensional volume, of a lepton pair of mass Q is given by [3],

$$\frac{d(R/V)}{d^4Q} = \frac{1}{96\pi^5 Q^2} \sum e_q^2 \, \Gamma \tag{5}$$

At zero and first order in the strong coupling constant, Π is given by the diagrams shown in fig.2.

Fig.2 : Contributions to the vacuum polarization tensor of the photon.

At some intermediate steps of the calculation divergences appear: the Ultra-Violet ones which are contained only in the $T = 0$ part (the statistical factors in the $T \neq 0$ part prevent any U.V. divergences) and which are renormalized away; the Infra-Red singularities, both from $T = 0$ which we know are cancelled in such diagrams (fig.2), and from $T \neq 0$, where the n_B factor worsens the I.R. singularity structure; finally we have to deal with mass singularities. Cancellation of all these divergences is obtained in the final result. The rate, which was shown to be independent of the regularization method [12], can be expressed as (in the case of $T = 0$ massless quarks)

$$\Gamma = \Gamma^0 v_T \left(1 + \frac{\alpha_S}{\pi}\left(1 + \frac{4}{3}F(Q/T)\right)\right) \tag{6}$$

where Γ^0 is the Born rate,

$$\Gamma^0 = 6\alpha Q^2 n_F^2(\tfrac{Q}{2}) \tag{7}$$

and v_T is the kinematical threshold,

$$v_T = \sqrt{1 - \frac{4m_T^2}{Q^2}} \quad , \quad m_T^2 = \frac{4}{3}\pi\alpha_S T^2 \tag{8}$$

This threshold factor arises from the self-energy corrections on the fermion legs. The quarks in the plasma acquire an effective mass proportional to gT. This mass does not break chirality and appears only in the kinematics of the process. Contributions of this mass in the trace factors are found to disappear [10].

The behavior of the scaled thermal correction factor $(Q/2T)^2 F(Q/T)$ is shown in fig.3 as a function of $Q/2T$.

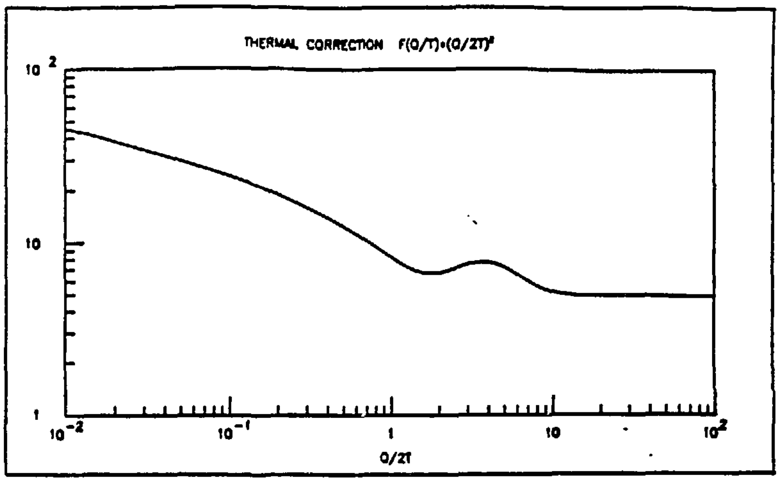

Fig.3 : Behavior of the scaled thermal correction over a large range of $Q/2T$.

One notes that $F(Q/T)$ becomes vanishingly small at large values of the variable ($Q/2T > 5.$). This is to be contrasted with Drell-Yan pair production in the hadron phase where the correction factor $(\alpha_S/\pi)K(Q/\sqrt{s})$ is about 2 to 2.5, in the energy $\sqrt{s}$ range available for CERN heavy ion experiments. For present data, this makes the hunt for the QGP more difficult in this region of mass (typically around the J/ψ resonance) since there is good reason to believe that the spectrum will be dominated by pure Drell-Yan [13] even if a large amount of plasma is created during the collision. Of course, higher temperatures could change this situation by increasing the Born rate for the thermal dileptons.

Concerning the validity of the perturbation series, this is shown in fig.4 where we compare, for different values of α_S, the Born term and the first order correction. We have written eq.(6) as

$$\Gamma = \Gamma_T^0 + \Gamma_T^1 \tag{9}$$

with the zero and first order contributions in α_S. From the pure theoretical point of view, it is amusing to note that one can never observe large corrections since these appear only for irrelevant values of α_S, say $\alpha_S < \alpha$, a region where the virtual photon becomes thermalized and then cannnot escape from the plasma. Still, the threshold factor v_T, affects very much the Born rate in the region of mass pairs of the order of the temperature of the plasma. Looking at dilepton pairs in this region could then become very interesting but needs high statistics [14].

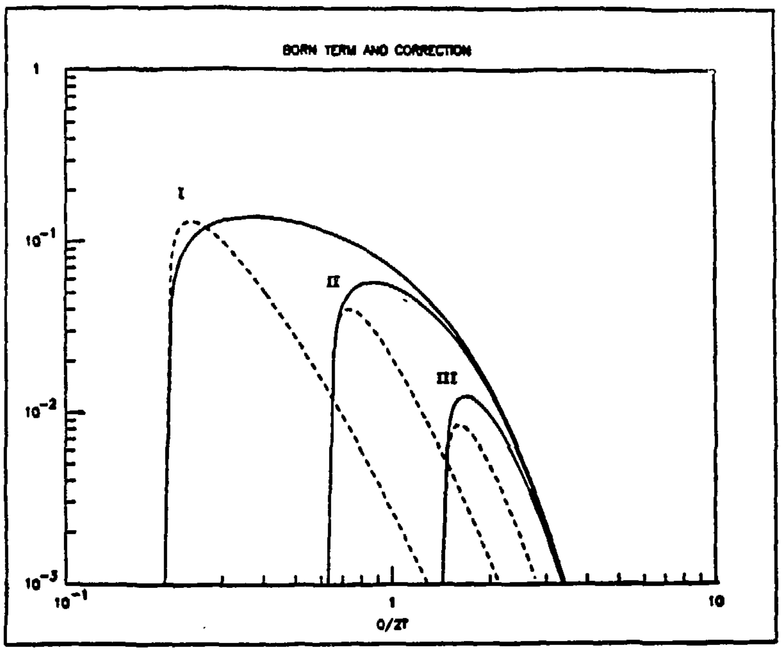

Fig.4 : The dilepton rate, up to a renormalization constant, $\alpha\sum e_q^2/(16\pi^5)$. Displayed are the Born rate (solid lines) and the correction (dashed lines) as functions of $Q/2T$ for different constant α_S values, I ($\alpha_S = 0.01$), II ($\alpha_S = 0.1$), III ($\alpha_S = 0.5$).

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References

[1] H. Satz, Ann.Rev.Nucl.Part.Sci. **35** (1985) 245.
[2] 'Quark Matter 87' Z.Phys.**C28** (1988).
[3] L.D. Mc Lerran and T. Toimela, Phys.Rev.**D31** (1985) 545.
[4] K. Kajantie, M. Kataja, L. McLerran and P. V. Ruuskanen, Phys.Rev.**D34** (1986) 811.
[5] N.P. Landsman and Ch.G. Van Weert, Phys.Rep. **145C** (1987) 141.
[6] T. Altherr, P. Aurenche and T. Becherrawy, Nucl.Phys.**B315** (1989) 436.
[7] R. Baier, B. Pire and D. Schiff, Phys.Rev.**D38** (1988) 2814.
[8] J. Cleymans and I. Dadic, Z.Phys.**C42** (1989) 133 and University of Cape Town preprint, UCT-TP 104/1988.
[9] T. Grandou, M. Le Bellac and J.-L. Meunier, Nice preprint NTH 88/9; Y. Gabellini, T. Grandou and D. Poizat, Nice preprint NTH 89/1.
[10] T. Altherr and P. Aurenche , Annecy preprint LAPP-TH-237/88.
[11] R.L. Kobes and G.W. Semenoff, Nucl.Phys.**B260** (1985) 714 and **B272** (1986) 329.
[12] T. Altherr and T. Becherrawy, Annecy preprint LAPP-TH-240/89.
[13] J.-L. Meunier, these proceedings.
[14] M. Vasseur, these proceedings.