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ABSTRACT

The classical ion heat conduction losses in Extrap discharges are calculated using polynomial magnetic field profiles and compared to the power input. For polynomials matched to magnetic field profiles measured in present experiments, these losses are small. By varying the coefficients of the polynomials, a region is found, where the power input can balance the classical heat conduction losses. Each set of coefficients corresponds to values of the parameters F and Θ used in RFP physics. The region determines a region in an F-Θ diagram, including the usual RFP region but extending to higher values of Θ and β_θ.

1. Introduction

The high values of β_θ obtained in Extrap discharges suggests that the classical cross-field ion heat conduction may be important for the energy balance and puts a limit on β_θ similar to that found for RFP with parabolic profiles [1]. The ion heat conduction is sensitive to the temperature and density profiles, however, and parabolic profiles are not appropriate, at least not for high β_θ. In the centre, where the density is high, the high heat conduction flattens the temperature profile, while it may become steeper near the boundary, where the density is low.

In the present work, a polynomial model [2] is used for the magnetic field. From the fields, current densities and, assuming radial equilibrium, pressure profiles are calculated. Using the Spitzer resistivity, also a resistive input power density profile as well as temperature and density profiles may be obtained. From this, the ion heat conduction can be obtained and its divergence compared to the input power density. Anomalous resistivity is not taken into account here, even if it is expected to influence both the power input and the loss. This is, however, not expected to change the qualitative behaviour of the balance between the two. For polynomial coefficients corresponding to profiles typical for present experiments, the ion heat conduction losses seem to be considerably smaller than the power input.

By varying the coefficients of the polynomials, it is possible to find a regime, where the divergence of the heat flux nowhere exceeds the input power density. Each set of coefficients corresponds to values of the parameters F and Θ used in RFP physics. The results can then be presented as a region in an F-Θ diagram, including the usual RFP region but extending to higher values of Θ and β_θ.

2. Classical ion heat conduction losses

Energy confinement scaling laws are often obtained by equating various loss terms to the power input. Here another approach is chosen. The classical cross-field ion heat conduction is evaluated and compared to the resistive power input. In order to simplify the problem, we choose a cylindrical pinch of length $2\pi R$ and a circular cross-section with radius r_p. The axial (corresponding to the toroidal) direction is denoted by ϕ and the azimuthal (poloidal) by θ. The radial heat flux then becomes

$$\Lambda_i = -\lambda_\perp \frac{dT}{dr}, \text{ where } \lambda_\perp = \frac{1.5\cdot 10^{-42} n^2 \sqrt{A}\ \ln\Lambda}{B^2 \sqrt{T_i}} \qquad (1)$$

In a stationary equilibrium, the heating power density must everywhere exceed the divergence of the heat flux, i.e.

$$P = \frac{V_\phi j_\phi}{2\pi R} > \frac{1}{r}\frac{d}{dr}(r\Lambda_i) \equiv P_i \quad (2)$$

3. Polynomial function model

The model [2] is based on Sprott's model for RFP discharges [3], but modified in order to be able to match the coefficients to Extrap experimental results. The model uses the following expressions for the magnetic fields and the current densities:

$$\frac{B_\phi}{B_{\phi 0}} = 1 - \alpha\rho^2 + \frac{\alpha}{2}\rho^4 \quad (3)$$

$$\frac{B_\theta}{B_{\phi 0}} = \Theta_0\rho\left(1 - \frac{\alpha}{2}\rho^2 - \frac{1-\alpha}{3}\rho^4\right) \quad (4)$$

$$\frac{\mu_0 r_p j_\phi}{B_{\phi 0}} = 2\Theta_0\left(1 - \alpha\rho^2 - (1-\alpha)\rho^4\right) \quad (5)$$

$$\frac{\mu_0 r_p j_\theta}{B_{\phi 0}} = 2\alpha\rho\left(1 - \rho^2\right) \quad (6)$$

where

$$\rho = \frac{r}{r_p} \text{ and } \Theta_0 = \frac{\mu_0 r_p j_{\phi 0}}{2B_{\phi 0}}$$

and F and Θ can be obtained as

$$F = \frac{B_\phi r_p}{\langle B_\phi \rangle} = \frac{3}{2}\,\frac{2-\alpha}{3-\alpha} \quad (7)$$

$$\Theta = \frac{B_\theta r_p}{\langle B_\phi \rangle} = \frac{\Theta_0}{2}\,\frac{4-\alpha}{3-\alpha} \quad (8)$$

The polynomial function model by Sprott is obtained for $\alpha=\Theta_0^2$, while Extrap discharges are best modelled by $\alpha=\kappa\Theta_0^2$, with $\kappa<1$.

4. Radial profiles

A typical Extrap discharge may be modelled by $\Theta_0=1$ and $\kappa=0.7$, i.e. $\alpha=0.7$. The corresponding radial profiles of the magnetic field are shown in Fig. 1. From these, current density and pressure profiles may be obtained by assuming radial momentum balance. The pressure at the boundary is here put $p(r_p)=0$.

An electron temperature profile normalized to the axial value may be obtained by assuming a constant axial (loop) voltage, evaluating the parallel component of the electric field and current density, and using the parallel Spitzer resistivity:

$$T^{3/2} = \frac{65.3 \ln \Lambda}{\eta_{//}} \qquad (9)$$

In the following, the ion and electron temperatures are assumed equal, $T_i=T_e=T$. Combination of pressure and temperature profiles now yield a normalized density profile, which together with the temperature profile is shown in Fig. 2. It is obvious, that the temperature profile is flat near the centre, where the density is high, and more steep at the boundary, where the density is lower.

Since the resistive input power density is given by $P = \frac{V_\phi j_\phi}{2\pi R}$, and the voltage is assumed constant, a normalized input power profile is given by

$$\frac{P}{P(0)} = \frac{j_\phi}{j_{\phi 0}} \qquad (10)$$

This profile is shown in Fig. 3 together with the profile of P_i /P(0) obtained using the profiles of temperature, density, and magnetic field. The assumption of zero pressure at the boundary results in zero ion heat conduction there and all the input power must leave the plasma via other processes. This is not exactly true, but a small finite pressure at the boundary would not change the picture significantly. The ion heat conduction here transfers heat from the central parts to the boundary region, where the divergence of the heat flux is negative. It is obvious, that classical ion heat conduction is not the dominant loss mechanism in the present Extrap T1 experiments.

By varying the coefficients Θ_0 and κ in the polynomial model we may change the importance of ion heat conduction and determine the limits where the P_i profile touches the P profile. The result is readily presented in an F-Θ diagram, as shown in Fig. 4, where also a collection of Extrap T1 experimental points are included for comparison. The RFP Bessel function model (BFM) is also included and is fully within the allowed region. In addition, a "corridor" seems to exist for positive F and extending to very large values of Θ. A curve Extrap Polynomial Function Model (EPFM) is drawn, connecting the region of the experimental points to the "corridor". The curve is defined by

$$\kappa = \frac{1}{\sqrt{1+\frac{1}{2}\Theta_0^4}} \tag{11}$$

The EPFM curve describes a path along which ion heat conduction losses are moderate. Of course it impossible to tell if this path can be followed without knowledge of the total energy balance, but it provides a means to evaluate the restrictions imposed on the beta value by classical ion heat conduction.

In Extrap, both the poloidal β_θ and the total β_{tot} beta values are of interest, as defined by

$$\beta_\theta = \frac{2\mu_0\langle p\rangle}{B_\theta(r_p)^2} \tag{12}$$

$$\beta_{tot} = \frac{2\mu_0\langle p\rangle}{\langle B\rangle^2} \tag{13}$$

These are calculated for profiles obeying Eq.(11) and shown in Fig. 5 as functions of Θ. It is clear, that significantly high values of beta are not prohibited by classical heat conduction alone.

Profiles for a point on the EPFM curve with Θ_0=3 and κ=0.155, i.e. F=0.7 and Θ=2.44, are shown in Figs. 6 (magnetic fields) and 7 (temperature and density).

5. Conclusions

The RFP beta limitation observed by Ortolani [1] using parabolic profiles is relaxed by flattening of the temperature profile in the centre for higher values of beta due to high ion heat conduction. Thus there are regions in the F-Θ diagram where beta may be high with moderate ion heat conduction losses. Whether the EPFM curve can be followed up to larger values of Θ_0 for Extrap discharges is not possible to judge without knowledge of the full energy balance.

6. Acknowledgements

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[3] J.C. Sprott, University of Wisconsin Laboratory Report DOE/ER/53198-117, Madison, Wisconsin (1988).

Figure Captions

Fig. 1 Radial profiles of the magnetic fields based on polynomials with $\Theta_0 = 1$ and $\kappa=0.7$, corresponding to a typical Extrap T1 discharge.

Fig. 2 Radial profiles of temperature and density calculated from the magnetic field of Fig. 1.

Fig. 3 Radial profiles of input power density and ion heat conduction loss calculated from the magnetic field of Fig. 1.

Fig. 4 F-Θ diagram with ion heat conduction loss limits, the Extrap Polynomial Function Model (EPFM), and a collection of Extrap T1 experimental points, marked by squares. The Bessel Function Model (BFM) is included for comparison.

Fig. 5 Poloidal and total beta values as functions of Θ along the EPFM curve.

Fig. 6 Radial profiles of the magnetic fields based on polynomials with $\Theta_0 = 3$ and $\kappa=0.155$.

Fig. 7 Radial profiles of temperature and density calculated from the magnetic field of Fig. 6.

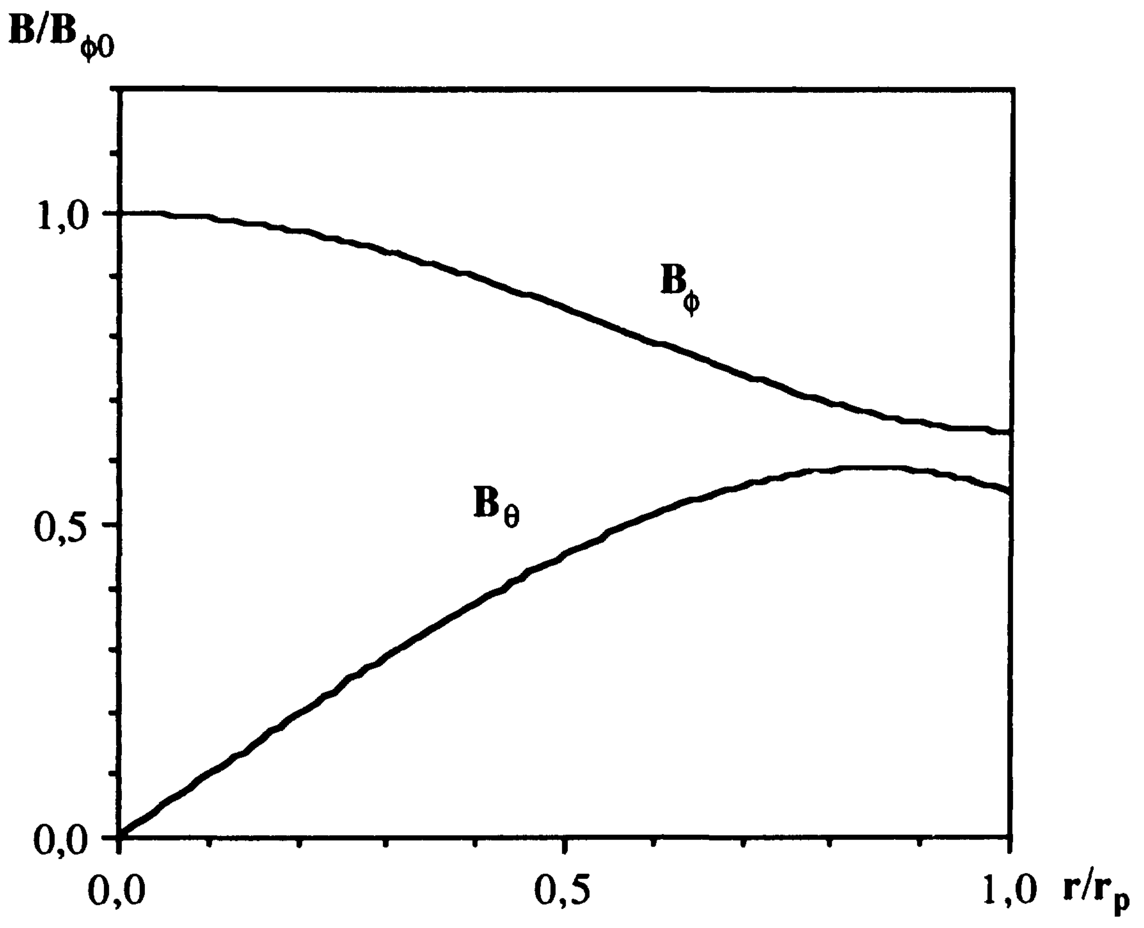


Fig. 1 Radial profiles of the magnetic fields based on polynomials with Θ_0=1 and κ=0.7, corresponding to a typical Extrap T1 discharge.

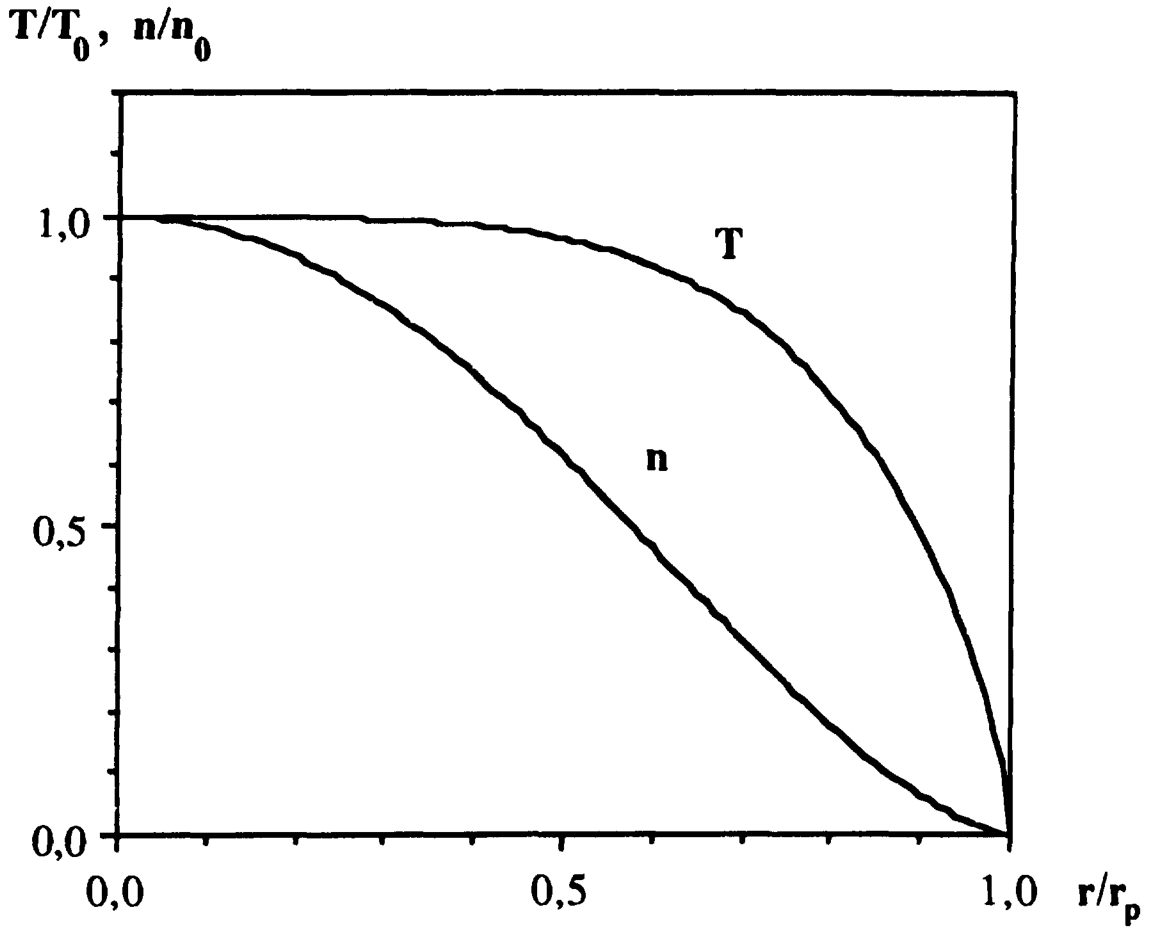


Fig. 2 Radial profiles of temperature and density calculated from the magnetic field of Fig. 1.

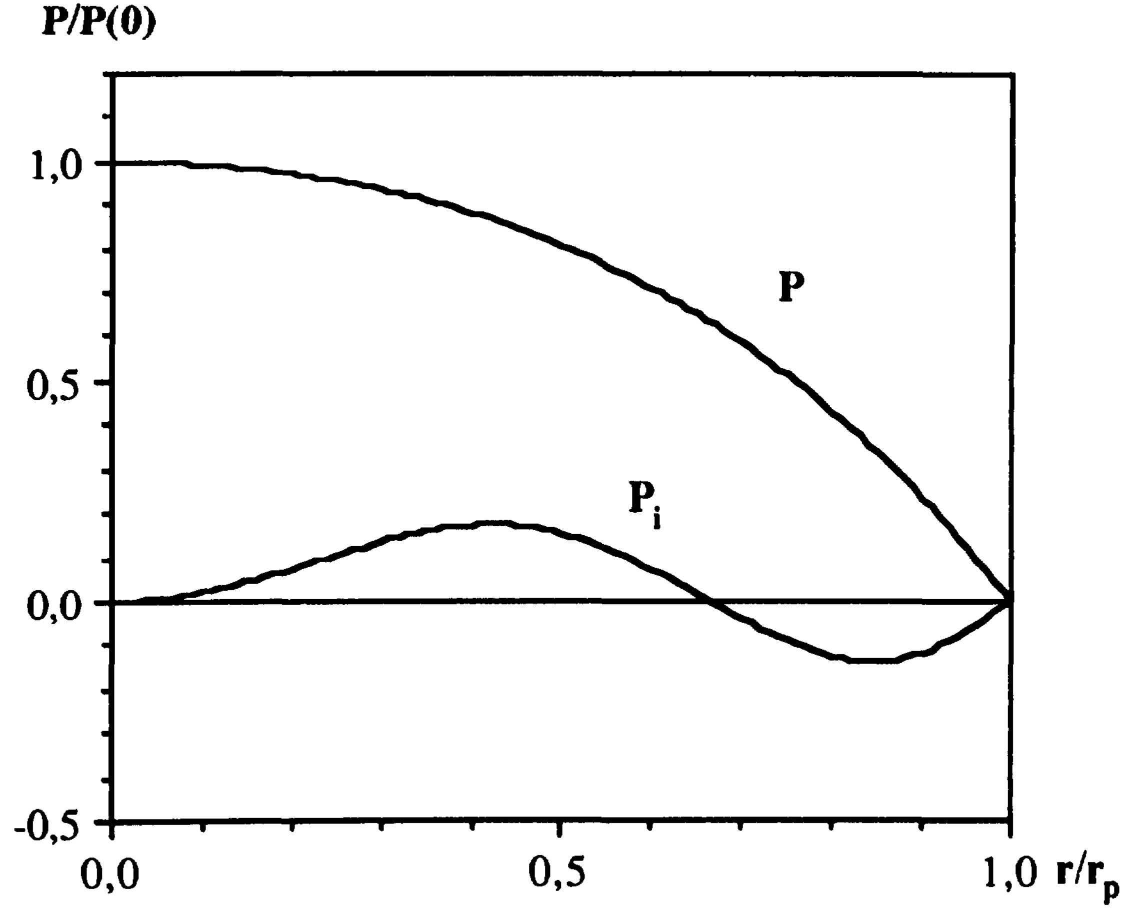
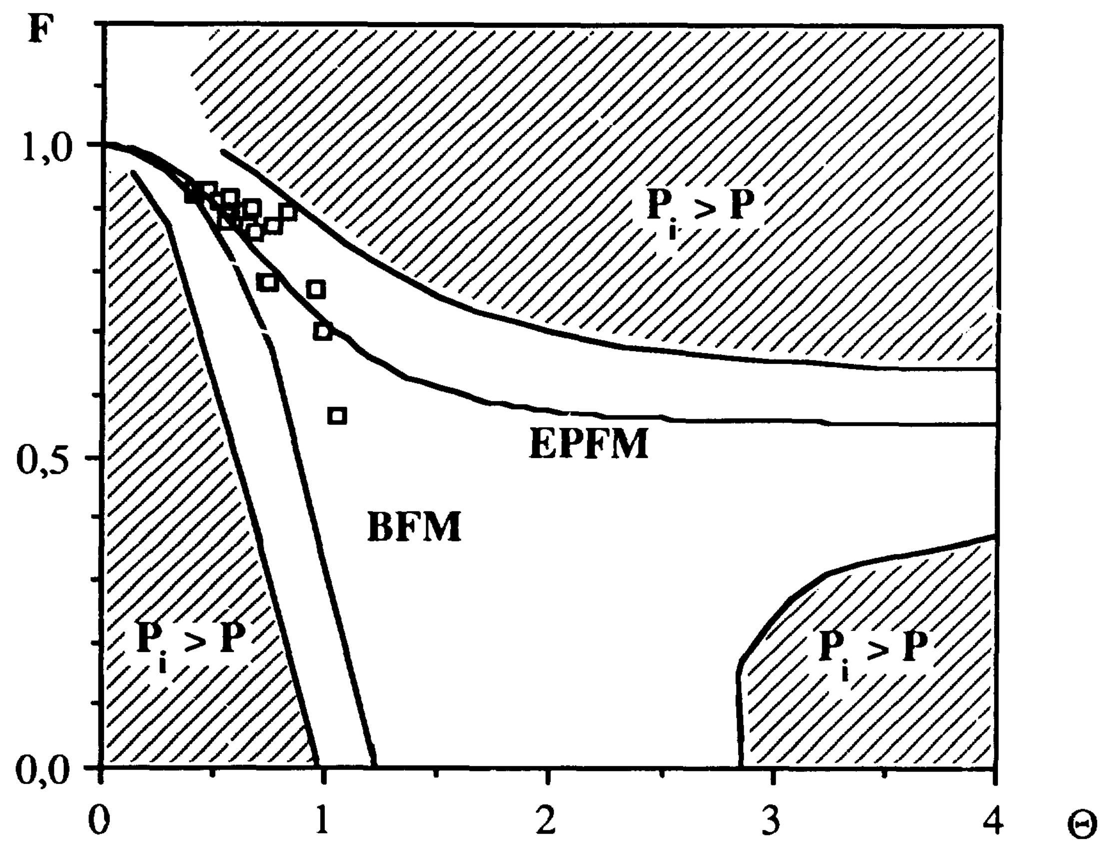


Fig. 3 Radial profiles of input power density and ion heat conduction loss calculated from the magnetic field of Fig. 1.



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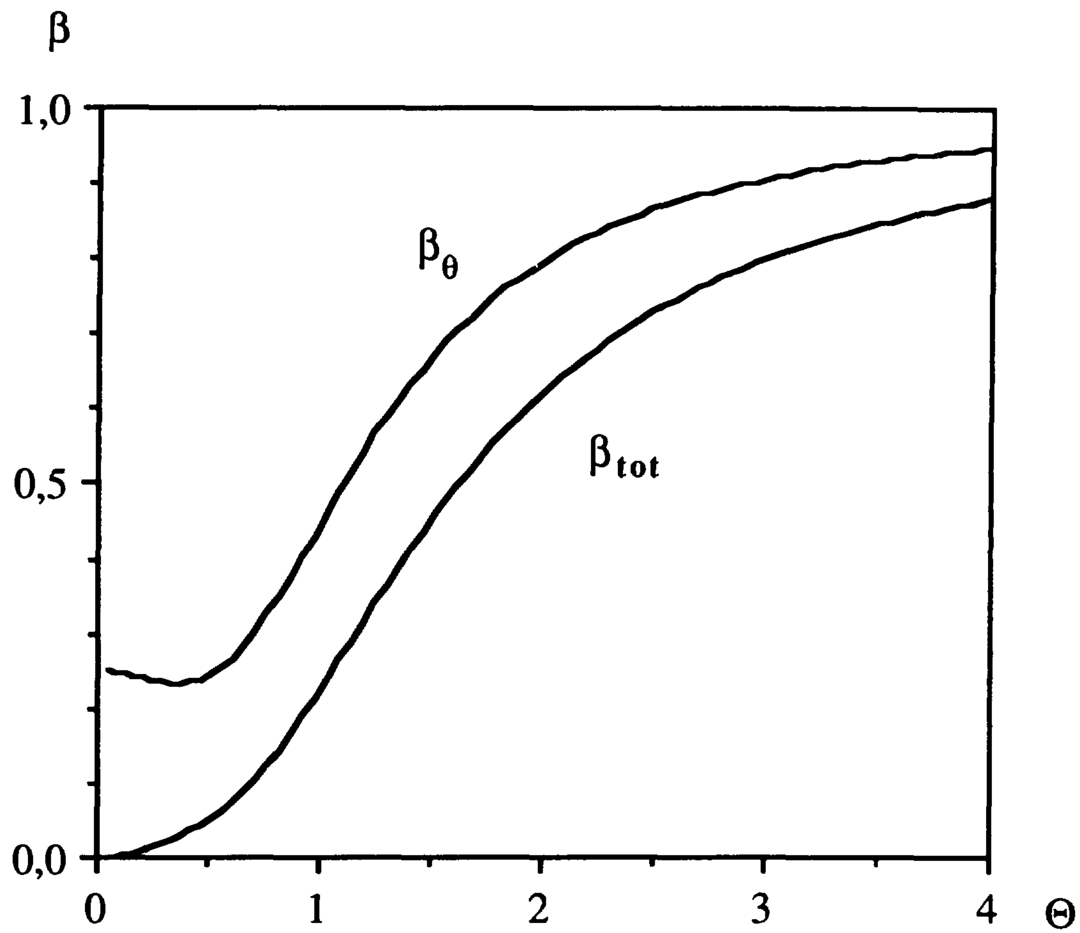

Fig. 5 Poloidal and total beta values as functions of Θ along the EPFM curve.

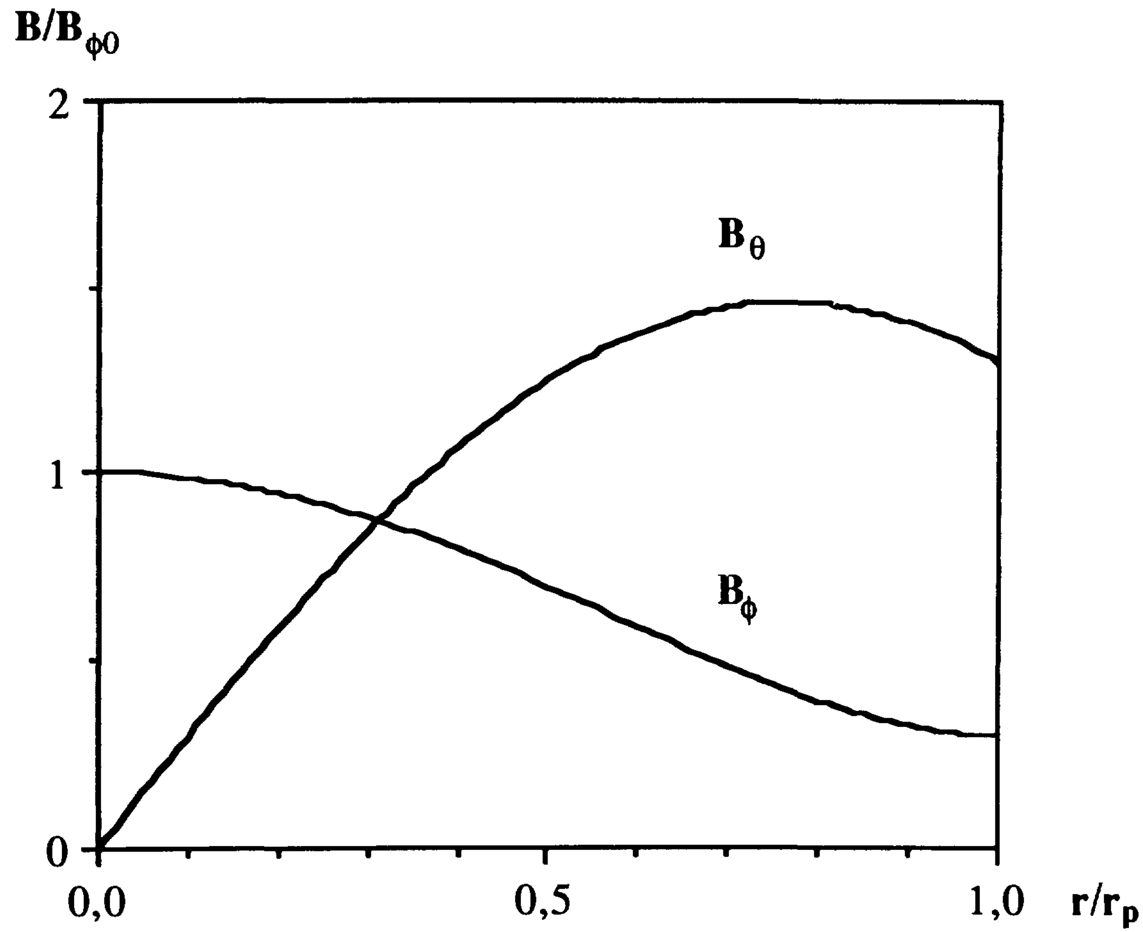

Fig. 6 Radial profiles of the magnetic fields based on polynomials with Θ_0=3 and κ=0.155.

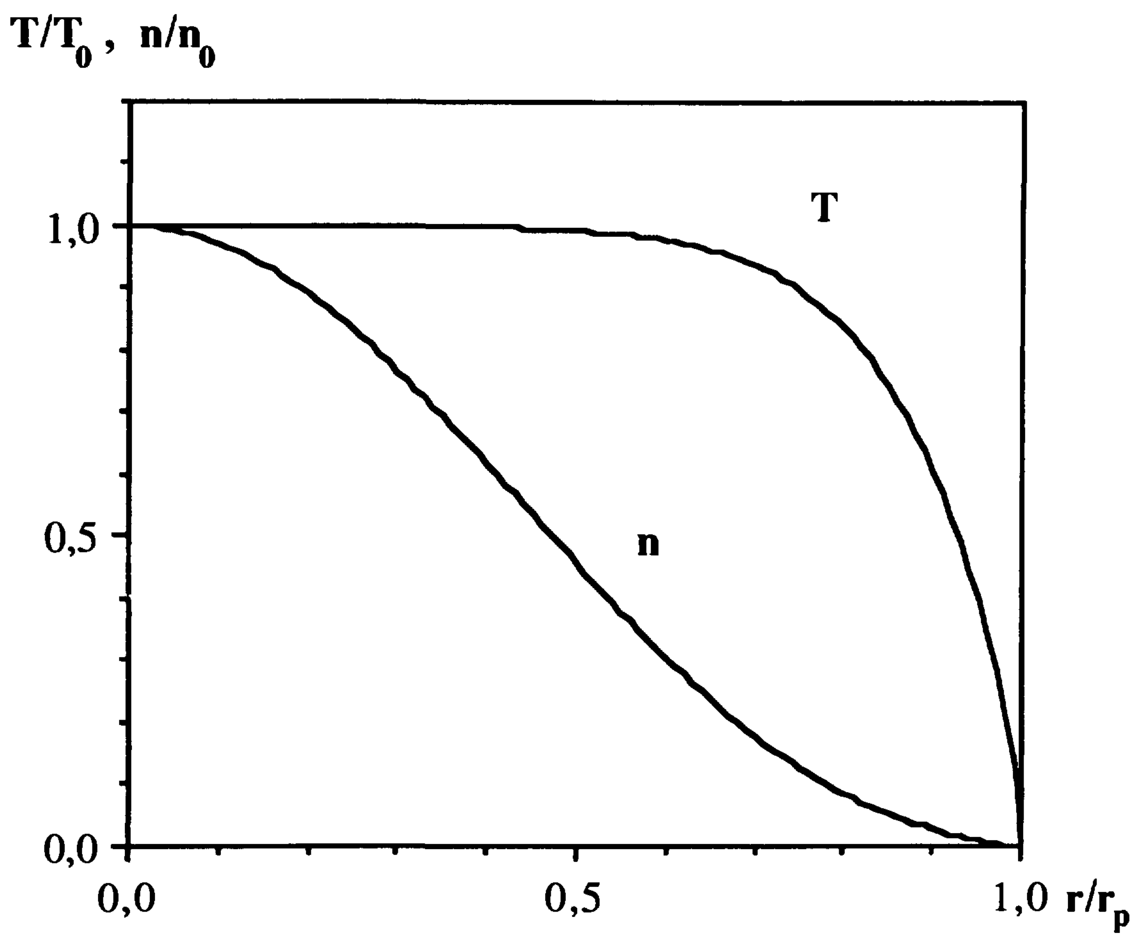

Fig. 7 Radial profiles of temperature and density calculated from the magnetic field of Fig. 6.

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Key words
Extrap, heat conduction, F-Θ diagram