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UPPER PINCH RADIUS LIMIT IN EXTRA

B. Lehnert

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Department of Plasma Physics and Fusion Research
Royal Institute of Technology
S-100 44 Stockholm, Sweden

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Royal Institute of Technology, S-10044 Stockholm, Sweden

ABSTRACT

A simple static equilibrium model of the Z-pinch is considered where a hot plasma core is surrounded by a cold-mantle (gas blanket). The pinch radius, defined as the radial extension of the fully ionized plasma core, is uniquely determined by the plasma particle, momentum and heat balance equations.

In Extrap configurations an octupole field is introduced which imposes a magnetic separatrix on Z-pinch geometry. This makes the conditions for Extrap equilibrium "overdetermined" when the characteristic pinch radius given by the plasma parameters tends to exceed the characteristic radius of the magnetic separatrix. In this case no conventional pinch equilibrium can exist, and part of the current which is forced into the plasma discharge by external sources must be channeled outside of the separatrix, i.e. into the surrounding support structure of the Extrap conductors and the vessel walls. A possibly existing bootstrap current in the plasma boundary layer is further expected to be "scraped off" in this case.

The present paper gives some illustrations of the marginal case of this upper pinch radius limit, in a state where the pinch current is antiparallel to the external rod currents which generate the octupole field.

1. Introduction

In this paper the static equilibrium of a Z-pinch is considered in which there is heat transport from the fully ionized core to a surrounding region of low temperature, such as that of a cold-mantle (gas blanket). The plasma profiles of a pinch with circular cross section are uniquely determined by the equations of particle, momentum and heat balance. Thus the pinch "radius" defined by the radial extension of the fully ionized plasma core becomes determined by these three moment equations, in combination with given values of external parameters such as the imposed axial electric field and the total pinch current.

In Extrap configurations there is a superimposed magnetic octupole field which forms a magnetic separatrix with the field generated by the pinch current. Under certain conditions the introduction of the separatrix makes the system "over-determined", i.e. when the pinch radius obtained from the balance equations becomes too large to "fit into" the space which is available inside the magnetic separatrix. This implies that there exists an upper limit for the pinch radius, and a corresponding limitation of the pinch parameters, as introduced by the imposed separatrix [1-2].

The present investigations on the upper pinch radius limit are extended to include the case of anisotropic resistivity. The analysis is restricted to a linear Extrap pinch for which the external conductor currents are antiparallel to the pinch current. Other types of Extrap systems can be treated in a similar way.

2. Equilibrium of a Linear Pinch with Circular Cross Section

This section treats the plasma equilibium of a Z-pinch with circular cross section. The Extrap configuration will later be considered.

2.1. Assumptions and Basic Equations

A linear Z-pinch is assumed to be sustained in a steady state, thereby being surrounded by a partially ionized boundary layer and a cold-mantle. A cylindrical frame (r,φ,z) is introduced with z along the axis of symmetry. The pinch should have a sufficiently large axial length for the axial particle and heat losses to be negligible as compared to the radial losses. These conditions correspond to a toroidal state of very large aspect ratio. Only the fully ionized plasma core is considered in this context, and for the core the analysis is restricted to static equilibrium where there is no macroscopic centre-of-mass velocity and no bootstrap effect of the plasma. Thus the core should be dense enough to be "impermeable" to neutral gas [1].

The relevant field quantities are defined as follows:

- The electric field $\underline{E} = (E_r, 0, E_z)$ consists of an axial component $E_z = E_c$ = const. given by external sources, and an ambipolar component E_r.

- The magnetic field $\underline{B} = (0, B_\varphi, B_z)$ consists of a transverse part B_φ generated by an axial current density j_z and an axial part B_z generated partly by a transverse current density j_φ, partly by the current in an external coil which surrounds the pinch and generates a contribution B_{za} = const. along z.

- The plasma has the particle density $n_i = n_e = n$, the temperature $T_i = T_e \equiv T$, and pressure $p = 2nkT$. The temperature $T_o \equiv T(r=0)$ at the axis is much higher than the temperature $T_a \equiv T(r=a)$ at the radius $r=a$ which defines the boundary between the fully ionized core and the relatively cool partially ionized boundary layer.

- The resistivity is anisotropic, having the values $\eta_{\parallel}$ and $\eta_{\perp}$ parallel and perpendicular to the magnetic field $\underline{B}$.

- There is heat transport in the radial direction only, being governed by a transverse heat conductivity $\lambda_{\perp}$.

The basic equations are

$$\mathrm{curl}\underline{B} = \mu_o \underline{j} \qquad (1)$$

$$\underline{\nabla} p = \underline{j} \times \underline{B} \qquad (2)$$

$$\underline{\underline{\eta}} \cdot \underline{j} = \underline{E} - (1/2\ en)\underline{\nabla} p \qquad (3)$$

$$P_{\eta} = -(d/dr)\left[r\lambda_{\perp}(dT/dr)\right] \qquad (4)$$

where

$$\underline{\underline{\eta}} \cdot \underline{j} = (\eta_{jk} j_k) \qquad \eta_{jk} = (\eta/2)\hat{B}_j\hat{B}_k + \eta(\delta_{jk} - \hat{B}_j\hat{B}_k) \qquad (5)$$

$$\eta \equiv \eta_{\perp} = 2\eta_{\parallel} = k_{\eta}/T^{3/2} \qquad k_{\eta} = 129(\ell n\Lambda) \qquad (6)$$

and $\hat{\underline{B}} = \underline{B}/B$ with the notation $B = |\underline{B}|$, Λ being the ratio between the Debye distance and the impact parameter, and P_{η} standing for the ohmic heating power.

2.2. General Properties of the Equilibrium State

The r-component of eq. (3) gives the ambipolar electric field in terms of dp/dr and need not be further discussed here.

Combination of the φ- and z-components of eq. (3) yields

$$j_\varphi = B_\varphi B_z E_c/\eta B^2 \qquad j_z = (B^2 + B_z^2)E_c/\eta B^2 \tag{7}$$

From the ratio of eqs. (7), in which the current density components are substituted from eq. (1), we obtain a relation between B_φ and B_z of the form

$$(B_\varphi^2 + 2B_z^2)(dB_z/dr) = -(B_\varphi B_z/r)(d/dr)(rB_\varphi) \tag{8}$$

The r-component of eq. (2) further combines with eq. (1) to

$$-\mu_o(dp/dr) = B_z(dB_z/dr) + (B_\varphi/r)(d/dr)(rB_\varphi) \tag{9}$$

From eqs. (8) and (9) we then obtain

$$dp/dr = -\beta_\eta j_z B_\varphi = -\beta_\eta(B_\varphi/\mu_o r)(d/dr)(rB_\varphi) \tag{10}$$

where

$$\beta_\eta = (B_\varphi^2 + B_z^2)/(B_\varphi^2 + 2B_z^2) \tag{11}$$

can be regarded as a kind of beta value due to the anisotropy of the resistivity. When $B_z^2 \ll B_\varphi^2$ we have $\beta_\eta \approx 1$ and the conventional Z-pinch balance is recovered. Finally, eqs. (1) and (4) yield

$$B_\varphi = (\mu_o/r) \int_o^r r j_z dr \tag{12}$$

$$dT/dr = -(1/r\lambda_\perp) \int_o^r P_\eta dr \tag{13}$$

The following properties should be noticed:

- With an imposed electric field E_c in the positive z-direction, say, we have $j_z > 0$ all over the plasma core as seen from eq. (7).

- For $j_z > 0$ also $B_\varphi > 0$ according to eq. (12).

- With $j_z > 0$, $B_\varphi > 0$ and $\beta_\eta > 0$ according to eq. (11), we must have $dp/dr < 0$ as given by eq. (10).

- Since $P_\eta > 0$ we also have $dT/dr < 0$ due to eq. (13).

- The value β_η is a rather slow function of r, varying from 1/2 at the pinch axis to somewhat larger values in the outer parts of the core. The current density j_z given by eq. (7) is therefore likely to decrease in the radial direction, nearly as rapidly as $T^{3/2}$.

The system (1)-(4) consists in the present case of 7 equations for 7 unknowns. This yields unique solutions for the corresponding radial profiles. In particular, since $dT/dr < 0$, the temperature has to decrease monotonically in the radial direction from T_o at the axis to the value T_a at the interface $r = a$ between the core and the partialy ionized boundary layer. We thus take the point $r = a$ where the profile $T(r)$ has dropped to the value T_a as the <u>definition</u> of the <u>pinch radius</u>. The value T_a is, in its turn, defined as the temperature at which the ionization rate just has a marginal effect on the plasma balance, i.e. $T_a \simeq 3\times10^4$ K according to the analysis of a fully developed cold-mantle [1]. For a sufficiently large heating power P_η we then have $T_o \gg T_a$. Likewise $dp/dr < 0$ leads to a pressure p_o at the axis which becomes much higher than the pressure $p_a = p(r=a)$ at the pinch radius. This holds, provided that there is not an excessively large density n_a at $r = a$, as compared to the density $n_o = n(r=0)$ at the axis. Likewise the axial current density $j_o \equiv j_z(r=0)$ has to become much larger than $j_a \equiv j_z(r=a)$.

2.3. Relations Based on a Simple Model of the Current Profile

As concluded in Section 2.2, the current density profile is expected to decrease monotonically in the radial direction under physically relevant steady-state conditions. In this paper a model is therefore adopted for which the profile is represented by

$$j_z(r) = j_o - (j_o - j_a)(r/a)^\alpha \tag{14}$$

where α is a positive constant greater than unity, i.e. for which dj/dr is finite at the axis $r = 0$. The correspon-ding transverse magnetic field becomes

$$B_\varphi(r) = \frac{1}{2}\mu_o r\left[j_o - 2(j_o - j_a)(\alpha+2)^{-1}(r/a)^\alpha\right] \tag{15}$$

It is now observed that the beta value defined by eq. (11) varies from 1/2 at the axis $r=0$ to unity at $r=a$, in the case where $B_{\varphi a} \equiv B_{\varphi}(r=a)$ becomes much stronger than the externally imposed axial field $B_{za} \equiv B_z(r=a)$ near the "edge" $r=a$ of the fully ionized core. In experiments where $B_{\varphi a}$ and B_{za} are comparable and $\beta_{\eta a} \equiv \beta_{\eta}(r=a) = 2/3$ this variation of β_{η} even turns out to be smaller. As a first approximation we therefore replace the slowly varying coefficient β_{η} in eq. (11) by a constant value

$$\bar{\beta}_{\eta} = (\beta_{\eta o} + \beta_{\eta a})/2 = (3B_{\varphi a}^2 + 4B_{za}^2)/(4B_{\varphi a}^2 + 8B_{za}^2) \tag{16}$$

With this approximation eq. (10) is easily integrated to

$$p = p_o - (\mu_o \bar{\beta}_{\eta} r^2/4)\Big[j_o^2 - 2(\alpha+4)j_o(j_o-j_a)(\alpha+2)^{-2}(r/a)^{\alpha} +$$

$$+ (j_o-j_a)^2(\alpha+1)^{-1} \cdot (\alpha+2)^{-1}(r/a)^{2\alpha}\Big] \tag{17}$$

The analysis is finally restricted to the conditions previously specified in Sec.2.2, i.e. where $p_a \ll p_o$ and $j_a \ll j_o$. Then the total pinch current flowing through the core becomes

$$J_a = \pi a^2 j_o \alpha(\alpha + 2)^{-1} \tag{18}$$

and eq. (17) yields a pinch radius

$$a = J_a\Big[\mu_o \bar{\beta}_{\eta}(\alpha + 3)/8\pi^2 k n_o T_o(\alpha + 1)\Big]^{1/2} \tag{19}$$

From eq. (19) is seen that the pinch radius becomes a slow function of the profile shape as represented by α . In particular, this applies to systems with a cold-mantle, where nearly square-shaped current profiles, corresponding to large values of α, are excluded.

The transverse field at $r=a$ is obtained from

$$B_{\varphi a}^2 = 2\mu_o n_o kT_o(\alpha + 1)/\bar{\beta}_\eta(\alpha + 3) \tag{20}$$

Combination of eqs. (16) and (20) then yields

$$\bar{\beta}_\eta = (1/4)(1 - \beta_o) + \left[(1/16)(1 - \beta_o)^2 + 3\beta_o/8\right]^{1/2} \tag{21}$$

where

$$\beta_o = 2\mu_o kn_o T_o(\alpha + 1)/(\alpha + 3)B_{za}^2 \tag{22}$$

2.4. Modifications due to an Anomalous Resistivity Ratio

In cases where the resistivity ratio $f = \eta_\perp/\eta_\parallel$ does not have the classical value $f = 2$, there is a corresponding modification of Sections 2.2 and 2.3. It is easily shown that eqs. (11) and (21) then are replaced by

$$\beta_\eta = (B_\varphi^2 + B_z^2)/(B_\varphi^2 + fB_z^2) \tag{23}$$

and

$$\bar{\beta}_\eta = \left[(1 + f)B_{\varphi a}^2 + 2fB_{za}^2\right]/2f(B_{\varphi a}^2 + fB_{za}^2) \tag{24}$$

3. Approximate Treatment of the Extrap Pinch

The imposed octupole field in Extrap creates a Z-pinch with a non-circular cross section. In an earlier analysis on the plasma balance, the deviation in the plasma confinement time was found to be of second order in the non-circularity of the pinch [3]. As a first approximation we shall therefore use the results of eqs. (16), (19) and (20) also in the case of the Extrap configuration, thereby replacing the circular radius at $r=a$ by an average $\bar{a}$ of the radial distance between the core edge and the axis.

3.1. The Magnetic Separatrix

With a current J_v flowing in each of four external conductors, and with an antiparallel plasma current J_a, the x-point distance from the axis of the corresponding magnetic separatrix becomes [4,5]

$$a_x = a_v\left[4(J_v/J_a) \quad -1\right]^{-1/4} \tag{25}$$

where a_v is the axial distance of a conductor. Since the cross section formed by the separatrix becomes nearly sqare-shaped, an equivalent average radius

$$\bar{a}_s \simeq \left[1 + (1/\sqrt{2})\right]a_x/2 \tag{26}$$

can be defined.

3.2. The Upper Pinch Radius Limit

For the plasma parameters to be reconcilable with a pinch radius $\bar{a}$ which "fits" into the space being available inside the magnetic separatrix, the condition

$$\bar{a} = \bar{a}(n_o, T_o, J_a, \bar{\beta}_\eta) \leqslant \bar{a}_s = \bar{a}_s(J_v, J_a) \tag{27}$$

has to be satisfied. Here we use expression (19) for $\bar{a}$ in the present approximation, where a current distribution across the non-circular magnetic surfaces is adopted, being similar to that defined by eq. (14) for a circular cross section. Condition (27) can be rewritten in the form

$$Q \leqslant U \tag{28}$$

where

$$Q = C_o J_a^2 \qquad C_o = \mu_o \bar{\beta}_\eta (\alpha+3)/2\pi^2 k n_o T_o a_v^2 (\alpha+1) \tag{29}$$

$$U = \left[1 + (1/\sqrt{2})\right]^2 / \left[4(J_v/J_a) - 1\right]^{1/2} \tag{30}$$

and $\bar{\beta}_\eta$ is given by eqs. (21) and (22). The marginal case $Q = U$ is illustrated by Fig.1.

3.3. Application to Device Extrap L1

Device Extrap L1 is run with antiparallel pinch and conductor currents [1]. The axial conductor distance is $a_v = 0.03$ m. Assuming a parabolic current density profile with $\alpha = 2$, we obtain

$$C_o = 8.54 \times 10^{18} \bar{\beta}_\eta / n_o T_o \tag{31}$$

and

$$\beta_o = 2.08 \times 10^{-29} n_o T_o / B_{za}^2 \qquad (32)$$

For given values of $(n_o, T_o, J_a, J_v, B_{za})$, the corresponding value of $\bar{\beta}_\eta$ can then be deduced from eqs. (32), (21) and (22), and the values of (C_o, Q, U) from eqs. (31), (29) and (30).

Some illustrations of the marginal case $Q = U$ are given in Fig.2, for the conductor-pinch current ratio J_v/J_a as a function of the axial density n_o when $B_{za} = 0.25$ T. The two curves represent the cases $J_a = 10^4$A, $T_o = 2\times10^5$ K and $J_a = 2\times10^4$ A; $T_o = 4\times10^5$ K. It is seen from the figure that the marginal ratio J_v/J_a corresponding to the upper pinch radius limit increases rather steeply, when varying the parameters from the asymptotic state at low densities n_o to states of high density.

4. Conclusions

The present simplified analysis indicates that there should exist an upper equilibrium limit of about unity for the ratio between the average pinch and separatrix radii. At given pinch parameters (n_o, T_o, J_a, α, B_{za}) this results in a corresponding limit of the conductor-pinch current ratio J_v/J_a. In this connection the following special points should be made:

- For parameters which lead to $Q > U$, the upper pinch radius limit would have to be exceeded. However, then part of the current which is forced into a linear Extrap system by external sources is expected to flow in the region outside of the magnetic separatrix, i.e. into the surrounding support structure of the conductors and to the vessel walls.

- It is likely that there exists an associated stability limit due to ballooning modes near the x-points, when the pinch radius $\bar{a}$ approaches the separatrix radius $\bar{a}_s$ from below [1,2]. This could somewhat decrease the range of stable equilibrium near the upper pinch radius limit.

- The present analysis does not take the possible existence of a bootstrap current into account. Such a current is expected to be particularly important in high-beta systems [6]. It is driven by a radial diffusion velocity in the present geometry. The bootstrap current therefore exists in the partially ionized boundary layer where there is a radial recirculation of matter, but not in a fully ionized static core which does not become subject to particle injection by pellets or by opther means. A bootstrap current can thus flow in the outer layers of Extrap, but only when the pinch radius $\bar{a}$ remains smaller than the separatrix radius $\bar{a}_s$, by a sufficient margin for such a current not to be "scraped off" by the separatrix.

- The heat balance equation has not been used explicitly in this analysis. However, heat transport does in an implicit way affect the results, by determining the axial values of temperature T_o and density n_o. Thus, n_o and T_o are considered as free parameters in this analysis, and can be given by experimental data when tests are made of the relations deduced in this context.

Stockholm, December 18, 1989

5. References

[1] B. Lehnert, Fusion Technology 16(1989)7.

[2] B. Lehnert, Proc. 15th European Conf. on Controlled Fusion and Plasma Heating, Dubrovnik, Yugoslavia, May 16-20, 1988, Vol. 12B, Part II, p. 605; TRITA-PFU-87-14, Royal Institute of Technology, Stockholm (1987).

[3] G. Eriksson, UPTEC-8741 R, Institute of Technology, Uppsala University (1987).

[4] J. Scheffel, TRITA-PFU-82-06, Royal Inst. of Technology, Stockholm (1982).

[5] B. Bonnevier, TRITA-PFU-82-10, Royal Inst. of Technology, Stockholm (1982).

[6] B. Lehnert, Fusion Technology 13 (1988)87.

Figure Captions

Fig.1. The full curve demonstrates the marginal case $Q = U$ where the average pinch radius $\bar{a}$ becomes equal to the average radius $\bar{a}_s$ of the separatrix. The quantity $Q = C_o J_a^2$ is given as a function of the conductor-pinch current ratio J_v/J_a. Here C_o depends on the axial plasma pressure $p_o = 2kn_oT_o$ and on the factor $\bar{\beta}_\eta$ resulting from anisotropic resistivity. Within the region to the left of the broken line there does not exist any closed magnetic separatrix defining the plasma confinement region at the upper pinch radius limit. This diagram refers to antiparallel pinch and conductor currents.

Fig.2. The conductor-pinch current ratio J_v/J_a as a function of the axial density n_o when $a_v = 0.03$ m, $\alpha = 2$ and $B_{za} = 0.25$ T in the marginal case $Q = U$. The two curves illustrate the special cases $J_a = 10^4$ A, $T_o = 2\times10^5$K and $J_a = 2\times10^4$ A, $T_o = 4\times10^5$ K.

Fig. 1

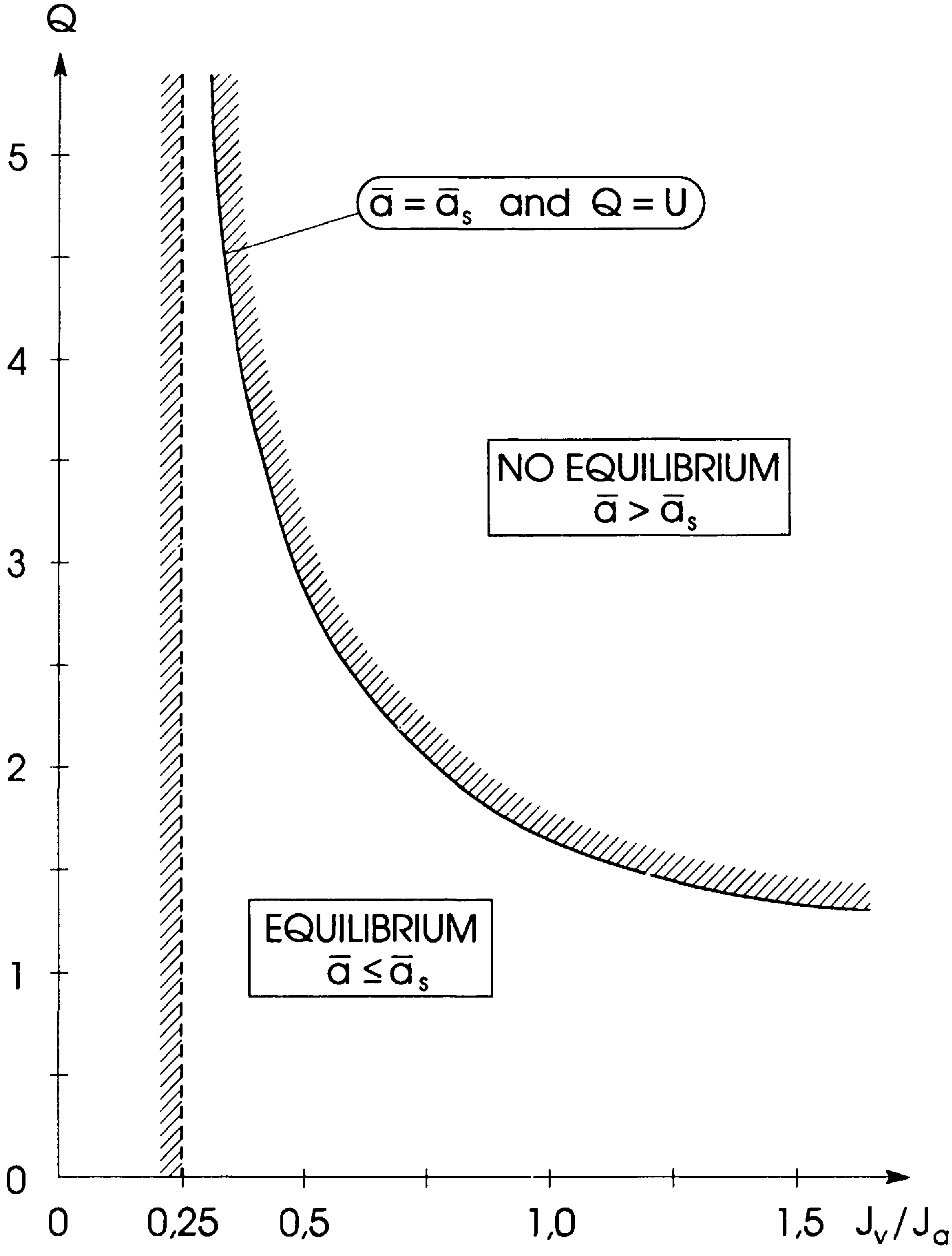
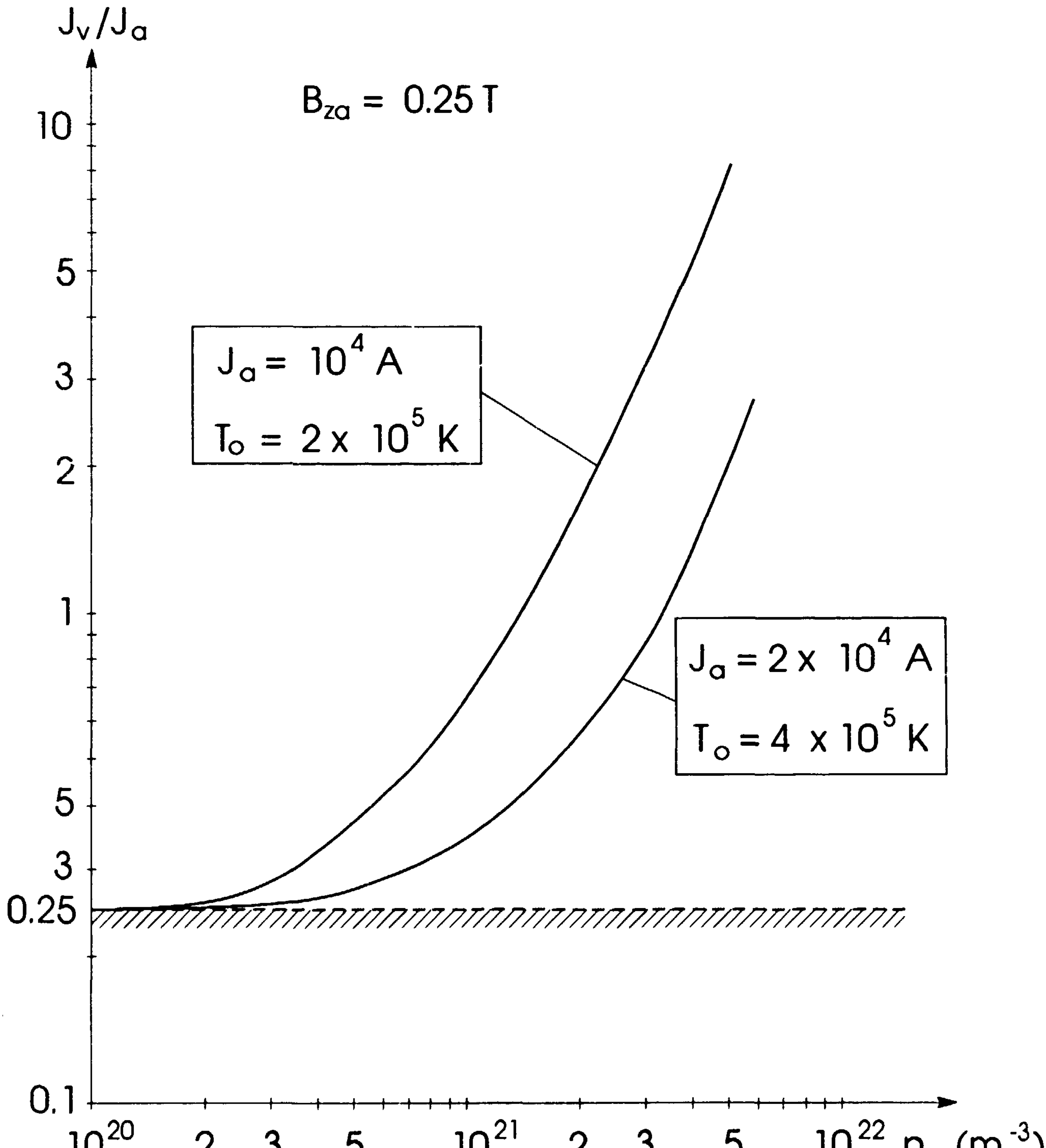

Q
5
4
3
2
1
0
$\bar{a} = \bar{a}_s$ and $Q = U$
NO EQUILIBRIUM
$\bar{a} > \bar{a}_s$
EQUILIBRIUM
$\bar{a} \leq \bar{a}_s$
0
0,25
0,5
1,0
1,5
J_v/J_a

Fig. 2



J_v/J_a
B_{za} = 0.25 T
10
5
3
2
1
5
3
0.25
0.1
$J_a = 10^4$ A
$T_o = 2 \times 10^5$ K
$J_a = 2 \times 10^4$ A
$T_o = 4 \times 10^5$ K
10^{20}
2
3
5
10^{21}
2
3
5
10^{22}
n_o (m^{-3})

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Royal Institute of Technology, Department of Plasma Physics and Fusion Research

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The present paper gives some illustrations of the marginal case of this upper pinch radius limit, in a state where the pinch current is antiparallel to the external rod currents which generate the octupole field.

Key words: Z-pinch, Extrap, pinch radius, magnetic separatrix, stability limits.