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LOWER PINCH RADIUS LIMIT IN EXTRA

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ABSTRACT

In an Extrap pinch there is a superimposed magnetic octupole field which forms a magnetic separatrix with the field generated by the pinch current. Earlier experiments have shown that the octupole field has a stabilizing influence on the plasma. Regardless of the details of this stabilizing mechanism, it is expected that the influence of the octupole field should become negligible for a sufficiently small ratio between the characteristic pinch and separatrix radii. In other words, there should exist a lower limit of this ratio below which the system approaches the state of an ordinary unstabilized Z-pinch.

The present paper presents an extended version of an earlier theoretical model of this lower limit, and its relation to the corresponding critical ratio between the external conductor and pinch currents. This ratio is found to vary substantially with the plasma parameters.

1. Introduction

The plasma profiles and the radius of a Z-pinch with a surrounding cold-mantle (gas blanket) are uniquely determined by the three moment equations of particle, momentum and heat balance [1,2].

In an Extrap pinch there is a superimposed magnetic octupole field which forms a magnetic separatrix with the field generated by the pinch current. This imposes an upper limit on the pinch radius, as determined by the radial extension of the separatrix [1-3]. Above this limit no conventional pinch equilibrium can exist.

Regardless of any detailed theoretical approach, there should also exist a lower pinch radius limit in Extrap below which the influence of the superimposed octupole field can be neglected [1,3]. At this lower limit the octupole field becomes too weak within the plasma body to be able to affect the plasma balance and stability. The Extrap pinch then behaves like an ordinary unstabilized Z-pinch.

In this paper an early simplified kink stability analysis on the lower limit [3] will be extended and formulated for direct comparison with experiments.

2. Starting Points

The present investigation concerns a linear Extrap pinch with a superimposed axial magnetic field. Previous theoretical kink stability analysis on the lower pinch radius limit [1,3,4] will be applied in this context, without repeating the details. Special attention should be given to the following points:

- At the lower limit the deviation from a circular plasma cross section usually becomes small. An approximation with a circular cross section is then justified when treating the pressure balance of the equilibrium state.

- The analysis is based on a static equilibrium state, for a plasma with anisotropic resistivity which couples the transverse and axial magnetic field components [2].

- As shown by Suydam for ordinary Z-pinches [5], kink stabilization by an axial field alone becomes difficult, because there are always spiral-shaped perturbations which match the helical magnetic field structure over a finite plasma region. Consequently, the stability conditions in presence of a weak superimposed axial (toroidal) field are here expected to be nearly the same as in absence of such a field, with the exception of its influence on the number of contained ion Larmor radii and associated kinetic effects [1,3].

- Dissipative effects are neglected in the stability analysis.

- A hybrid large Larmor radius (LLR)-MHD model is used in a first approach to kink stability [1,3].

3. The Lower Pinch Radius Limit

3.1. The Equilibrium State

For a nearly circular cross section with the average pinch radius $\bar{a}$ of the hot plasma core we introduce a cylindrical frame (r,φ,z) with z along the pinch axis. The current density $\underline{j}$ and the plasma pressure p are assumed to be negligible at the interface $r = \bar{a}$ between the fully ionized core and the surrounding cold-mantle, as compared to the values $\underline{j}_0$ and p_0 at the pinch axis. A model is adopted where there is a monotonically decreasing axial current density

$$j_z(r) = j_0[1-(r/a)^{\alpha}] \tag{1}$$

leading to the transverse magnetic field component

$$B_{\varphi}(r) = \tfrac{1}{2}\mu_0 j_0 r[1-2(\alpha+2)^{-1}(r/\bar{a})^{\alpha}] \tag{2}$$

From the plasma balance equations the pinch radius can be shown to have the approximate form [2]

$$\bar{a} = J_a[\mu_0\bar{\beta}_{\eta}(\alpha+3)/8\pi^2 k n_0 T_0(\alpha+1)]^{1/2} \tag{3}$$

where J_a is the total pinch current, n_0 and T_0 are the values of the plasma density n and temperature T at the axis $r = 0$, and

$$\bar{\beta}_{\eta} = (1/4)(1-\beta_0) + [(1/16)(1-\beta_0)^2 + 3\beta_0/8]^{1/2} \tag{4}$$

Here

$$\beta_o = 2\mu_o k n_o T_o(\alpha+1)/(\alpha+3)B_{za}^2 \qquad (5)$$

with $B_{za} = B_z(r=\bar{a})$ denoting the externally imposed axial magnetic field.

We further consider a linear Extrap configuration with an octupole field produced by a current J_v in each of four external conductor rods at $r = a_v$ and where J_v is antiparallel to J_a. The ratio M between the magnetic field strength $B_{\varphi a} = B_\varphi(r=\bar{a})$ due to the pinch current and the magnetic field strength $B_{va} = B_v(r=\bar{a})$ due to the conductor currents is then related to the average pinch radius by the expression

$$\bar{a}^2 \simeq [1 + (r_1/r_2)]^2 a_v^2/4\,[4M(J_v/J_a)-1]^{1/2} \qquad (6)$$

where $2r_1$ and $2r_2$ are the minor and major diameters of the non-circular plasma cross section. Approximate values of the ratio r_2/r_1 can be obtained from a simple model where J_a and J_v are substituted by line currents.

3.2. Kink Stability Limit

According to the previous analysis [3,4] there is a kink stability limit corresponding to a critical value of the ratio M as given for a factor $\alpha = 2$ in eq. (1) by

$$M_c = -(54/31) + [(3660/961) + (98304/31\theta_i^2)]^{1/2} \qquad (7)$$

Here the number of ion Larmor radii contained within the pinch radius is obtained from

$$\theta_i = [\mu_o e/(8\pi^2 m_i k)^{1/2}] \cdot [1 + (B_{za}/B_{\varphi a})^2]^{1/2} f_i J_a/\sqrt{T_o} \quad (8)$$

with the notation $B_{za} \equiv B_z(r=\bar{a})$ and $f_i = 0.75$ [3].

As seen from eq. (3) the pinch radius becomes a slow function of the profile shape as given by the constant α, and variations of this shape within physically relevant ranges would only have marginal effects on the deduced stability limit.

Combination of eqs. (6) and (3) now yields the stability condition

$$Q \geqslant L \quad (9)$$

where

$$Q = C_o J_a^2 \qquad C_o = 5\mu_o \bar{\beta}_\eta / 6\pi^2 k n_o T_o a_v^2 \quad (10)$$

$$L = [1 + (r_1/r_2)]^2 / [4M(J_v/J_a) - 1]^{1/2} \quad (11)$$

Marginal stability defined by $Q = L$ occurs for $M = M_c$ as shown in Fig.1 which also includes the case of the upper pinch radius limit at $M = 1$ according to earlier deductions [2]. Relation (11) holds for antiparallel pinch and conductor currents. For a given critical value $M = M_c$ the region below the corresponding curve in Fig.1 represents a kink unstable state as obtained from the present theoretical model.

3.3. Application to Device Extrap L1

Device Extrap L1 is run with antiparallel pinch and conductor currents, with an axial conductor distance $a_v = 0.03$ m. For a parabolic current density profile defined by $\alpha = 2$

$$C_o = 8.54 \times 10^{18}\bar{\beta}_\eta / n_o T_o \tag{19}$$

$$\beta_o = 2.08 \times 10^{-29} n_o T_o / B_{za}^2 \tag{20}$$

$$M_c = -1.74 + 1.95(1 + \lambda)^{1/2} \tag{21}$$

$$\lambda = 6.67 \times 10^4 A T_o / J_a^2 [1 + (B_{za}/B_{\varphi a})^2] \tag{22}$$

$$B_{\varphi a}^2 = 6\mu_o k n_o T_o / 5\bar{\beta}_\eta = 2.08 \times 10^{-29} n_o T_o / \bar{\beta}_\eta \tag{23}$$

where A is the ion mass number.

As an illustration, we put $J_a = 10^4$ A, $T_o = 2 \times 10^5$K and $B_{za} = 0.25$ T. The critical conductor-pinch current ratio $(J_v/J_a)_c$ then varies with the axial density n_o as demonstrated by Fig.2. From the figure is seen that relatively high densities, of the order of $n_o = 10^{22}$ m^{-3}, lead to critical current ratios $(J_v/J_a)_c \simeq 4$, i.e. similar to those observed in earlier experiments with Extrap L0 in the range $0 < B_{za} \lesssim 0.1$ T[1]. Substantially lower values of $(J_v/J_a)_c$ are expected to exist in Extrap L1, in a density range below $n_o = 2 \times 10^{21}$ m^{-3}.

4. Conclusions

There is experimental evidence for macroscopic stability of the Extrap pinch, due to the superimposed magnetic octupole field. Regardless of the detailed mechanisms being involved, this stabilizing effect of the octupole field should become negligible for sufficiently small ratios between the pinch and separatrix radii. In other words, the local octupole field within the plasma body then becomes weak enough for the plasma to behave like that in an ordinary unstabilized Z-pinch.

The present simple theoretical model thus predicts that there exists a lower limit of the pinch radius in Extrap. Whether this model also presents a relevant total picture of Extrap kink stability, and whether there are other instability modes of crucial importance, can at this stage only be judged from comparison with experiments.

Stockholm, December 11, 1989

5. References

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Figure Captions

Fig.1. The quantity $Q = C_o J_a^2$ as a function of the conductor-pinch current ratio J_v/J_a with the critical magnetic field ratio $M = M_c$ of eq. (7) as parameter. Here C_o depends on the axial pressure $p_o = 2kn_oT_o$ and on the factor $\bar{\beta}_\eta$ resulting from anisotropic resistivity. The curve for $M_c = 1$ represents the upper limit where the average pinch radius $\bar{a}$ approaches the average radius of the magnetic separatrix from below. For a given critical value M_c the region below the corresponding curve becomes kink unstable.

Fig.2. The critical value $(J_v/J_a)_c$ of the conductor-pinch current ratio, as a function of the axial density n_o at the special parameter values $J_a = 10^4$A, $T_o = 2 \times 10^5$K and $B_{za} = 0.25$ T.

Fig. 1

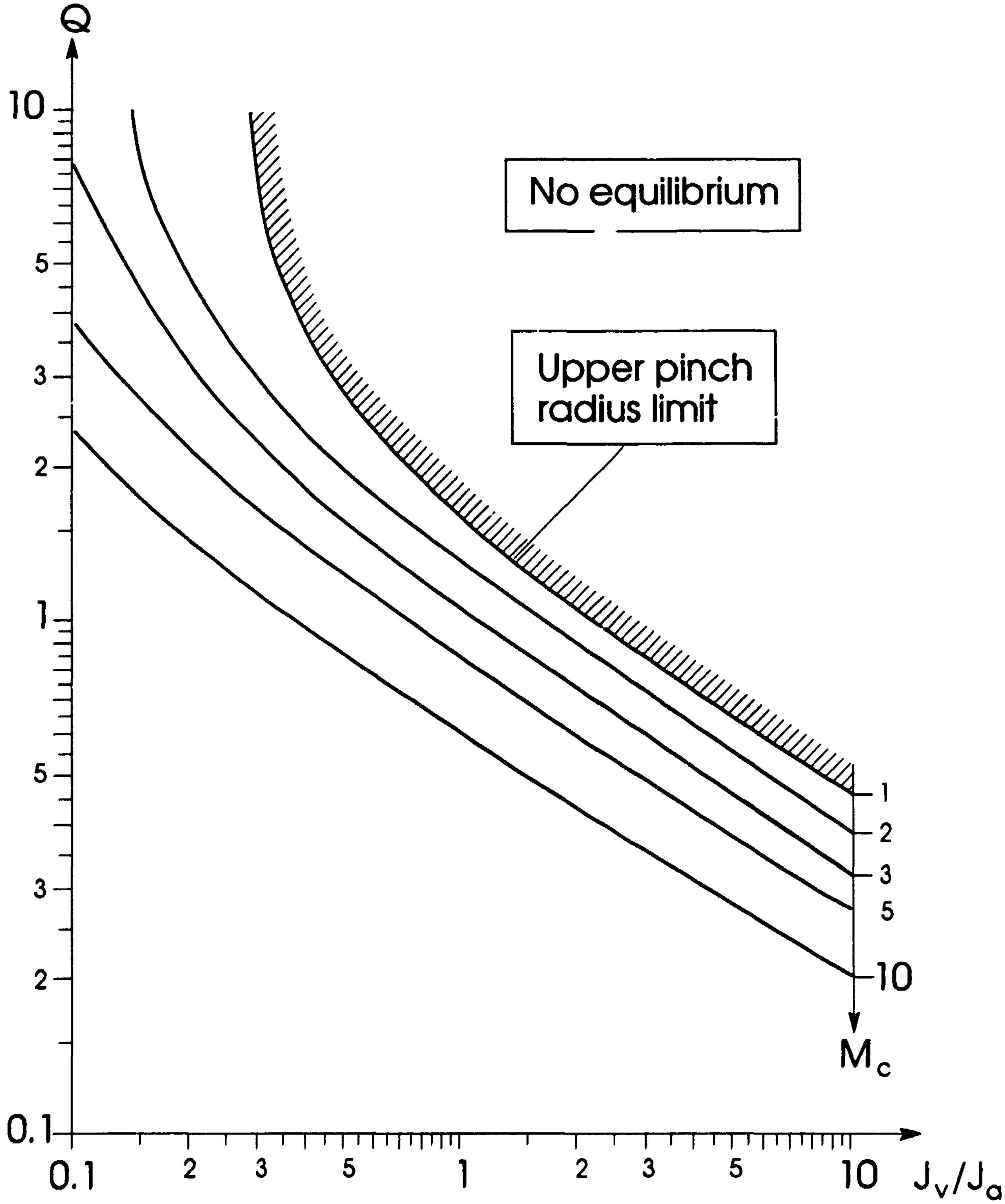
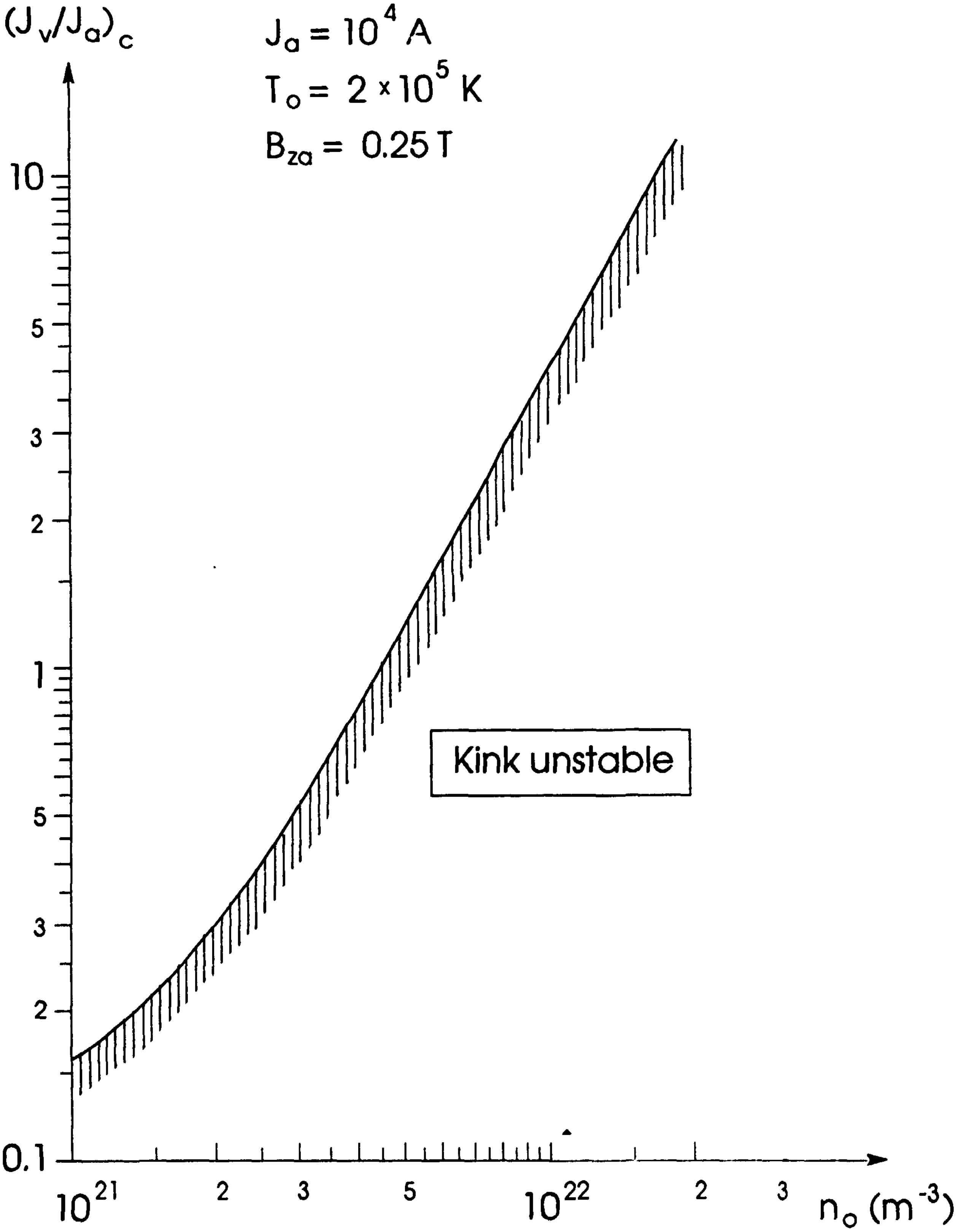

Q
10
5
3
2
1
5
3
2
0.1
0.1
2
3
5
1
2
3
5
10
J_v/J_a
No equilibrium
Upper pinch radius limit
1
2
3
5
10
M_c

Fig. 2


$(J_v/J_a)_c$
$J_a = 10^4$ A
$T_o = 2 \times 10^5$ K
$B_{za} = 0.25$ T
10
5
3
2
1
5
3
2
0.1
Kink unstable
10^{21}
2
3
5
10^{22}
2
3
n_o (m^{-3})

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Key words: Z-pinch, Extrap, pinch radius, magnetic separatrix, stability limits.