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BARYON REGGE TRAJECTORIES FROM THE AREA-LAW OF WILSON LOOP

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In the proper-time path integral representation of the three-quark Green function, baryon masses are calculated for large angular momenta L. Dynamics is given by vacuum background fields in the Wilson loop. Assuming an area law for large Wilson loops one obtains linear baryon Regge trajectories with the same slope as for mesons. For large L the baryon has an asymmetric structure of the quark- diquark type. Dynamic masses of the quark and diquark are generated, which grow with L.

Fig. - , ref. - 8

1. Introduction

In a recent paper [1] we have derived linear Regge trajectories from QCD taking as an input only the area-law behaviour of Wilson loops. Here we are using the same technic for the three-quark systems.

Baryon Regge trajectories possess some specific features which distinguish them from meson trajectories. First, baryons possess a new element, called string junction, which may have a dynamic meaning of its own.

Second, many phenomenological features prefer an interpretation of baryons as a quark-diquark system, e.g. the fact that the slopes of the baryon and meson trajectories are the same. Till now there is no dynamic explanation of a compact diquark system.

Third, the success of (relativistic) potential models in reproducing the spectrum of lowest states calls for explanation of their salient features, e.g. dynamic quark masses, pair potentials with the $\lambda_i \lambda_j$ color structure etc.

In this paper we derive the baryon Regge trajectories from the first principles of QCD without model assumptions. We follow the same strategy that was used for meson Regge trajectories in [1], i.e. we concentrate on large L heavy baryon states. For that reason we can use the large area limit of the Wilson loop, which can be obtained from the Confining Background Field (CBF) method [2,3] or directly from the Monte Carlo data.

Similarly to [3,4,1] we essentially exploit the Schwinger-Feynman representation of the quark Green function in the external (vacuum) background field which is inserted into the general expression of the three-quark Green function. For simplicity we disregard here the spins of quarks; in this way we obtain an averaged over spins combination of N and Δ trajectories. Spins can be included by the method which was used in [4] for heavy quarkonia; this topic will be discussed in a subsequent publication.

We also disregard in this paper the effects of the Chiral Symmetry Breaking (CSB). In this way we lose the mechanism which creates constituent quark mass at small momenta ($p < 0.8$ GeV [5]) and related phenomena of Goldstone degrees of freedom, like parity doubling. As in [1] we estimate the effects of CSB introducing chiral constituent mass in the final expression.

All these simplifications are done to make as clear as possible the main result of the paper: the proper treatment of confining dynamics of relativistic quarks leads to linear Regge trajectories for baryons with the same slope as for mesons, and this happens via dynamic formation of a diquark-quark configuration for the leading Regge trajectory. As a by-product there appears a new element: generation of a dynamic mass of the quark and diquark in the baryon. Similarly to the meson case, this mass grows with (angular) momentum and can be called as in [1] the "transverse mass". One should distinguish this mass from the constituent quark mass due to CSB [5]. The latter falls off with momentum very fast and is important for lowest baryonic states.

2. We consider scalar quarks and calculate the Green function

of a colorless system of three quarks evolving from a 4-point y to a 4-point x. The individual quark Green function G_q can be written in the Schwinger-Feynman representation as [6,3]:

$$G_q(x,y) = \int_0^\infty ds_i e^{-m_i^2 s_i} \int Dz \exp\left[-\frac{1}{4}\int_0^{s_i} (\dot{z}_\mu^{(i)})^2 d\sigma\right]\Phi^{(i)}(x,y) \equiv$$

$$\equiv \int d\mu(i)\Phi^{(i)}(x,y) \qquad (1)$$

where $i = 1,2,3$, $\Phi^{(i)}(x,y) = P \exp ig \int_y^x A_\mu dz_\mu^{(i)}$, $\dot{z}_\mu = \frac{\partial z_\mu(\sigma)}{\partial\sigma}$;

and the contour of integration in $\Phi^{(i)}(x,y)$ is the quark path $z_\mu^{(i)}(\sigma)$, $0 \le \sigma \le s$, to be integrated upon in (1). With the help of (1) G(x,y) can be written as

$$G(x,y) = \int d\mu(1)d\mu(2)d\mu(3) \langle W_3(x,y)\rangle \qquad (2)$$

where

$$\langle W_3(x,y)\rangle \equiv \langle \Phi_{ad}^{(1)}(x,y)\Phi_{bf}^{(2)}(x,y)\Phi_{ce}^{(3)}(x,y)\rangle e_{abc}e_{dfe} \qquad (3)$$

The contours of integration in $W_3(x,y)$ consist of three quark trajectories from the point y to x.

Now since we are interested in the large angular momenta of the baryon, we expect that the average interquark distances will be large and one can use asymptotics of $\langle W_3(x,y)\rangle$ for large contours. Just as in the meson case [1] we take as an asymptotic behaviour

$$\langle W_3(x,y)\rangle = \exp(-\sigma_0(S_1 + S_2 + S_3)) \qquad (4)$$

where S_i is the minimal surface bounded by the path of the i-th quark and the trajectory of the string junction $z_\mu^{(0)}(\sigma)$. The latter is determined by the requirement that the sum $S_1 + S_2 + S_3$ be minimal for given quark trajectories $z_\mu^{(1)}(\sigma_1)$, $z_\mu^{(2)}(\sigma_2)$, $z_\mu^{(3)}(\sigma_3)$.

It is convenient to introduce instead of σ_i parameters $\gamma_i = \sigma_i/s_i$, $0 \le \gamma_i \le 1$ and we can take only one of them. Then the Green function (2) becomes

$$G(x,y) = \int ds_1 ds_2 ds_3 Dz_\mu^{(1)} Dz_\mu^{(2)} Dz_\mu^{(3)} \exp(-B)$$

where the "action" B is

$$B = \int_0^1 d\gamma \left\{ m_1^2 s_1 + m_2^2 s_2 + m_3^2 s_3 + \frac{1}{4s_1}\left(\dot{z}_\mu^{(1)}\right)^2 + \frac{1}{4s_2}\left(\dot{z}_\mu^{(2)}\right)^2 + \frac{1}{4s_3}\left(\dot{z}_\mu^{(3)}\right)^2 + \right.$$
$$\left. + \sigma_0(S_1 + S_2 + S_3) \right\} \qquad (6)$$

We assume at this point that the minimal surface is formed by straight lines connecting a point on trajectory $z_\mu^{(i)}(\gamma)$ with the string junction taken at the same value of γ. These straight lines are parameterized as

$$w_\mu^{(k)} = z_\mu^{(k)}(\gamma)\cdot\beta_k + z_\mu^{(o)}(\gamma)\cdot(1-\beta_k),\ 0 \le \beta_k \le 1, \qquad (7)$$

and the minimal surface is given by a standard expression

$$\left[\sum_{k=1}^{3} S_k\right]_{min} = \sum_{i=1}^{3} \int_0^1 d\gamma \int_0^1 d\beta_k \left[\left(\frac{\partial w_\mu^{(k)}}{\partial\gamma}\right)^2 \left(\frac{\partial w_\mu^{(k)}}{\partial\beta_k}\right)^2 - \left(\frac{\partial w_\mu^{(k)}}{\partial\gamma}\cdot\frac{\partial w_\mu^{(k)}}{\partial\beta_k}\right)^2\right]^{1/2} \qquad (8)$$

For three quarks possessing a total angular momentum L there are two possible configurations: 1) all quarks share roughly an equal amount of angular momentum and correspondingly their distances to the string junction (and the center of mass in case of equal masses) are equal; 2) one of the quarks is much farther away from the string junction than the other two. The symmetric configuration is considered in the Appendix and we find in this case, that the slope of baryon trajectories is 1.5 times smaller, than that of meson trajectories. This is in contrast with experimental (phenomenological) Regge trajectories which have a universal slope. In the next section we therefore consider the asymmetric case.

3. Asymmetric case

Here we consider two quarks, say 1 and 2 close to each other and the third quark far away. In this case center of mass R_μ and string junction $z_\mu^{(0)}$ do not coincide. Indeed, one can show that the minimal sum of distances from quarks to $z^{(0)}$ occurs in the following situations: $z^{(0)}$ lies inside a triangle Δ_q with vertices at quark positions $z^{(k)}$ and all angles between the rays $\rho^{(k)} \equiv z^{(k)} - z^{(0)}$ are equal to $\pi/3$. If, however, one of the vertex angles of Δ_q is larger than $\pi/3$, then $z^{(0)}$ lies at the vertex (quark position) opposite to the largest side of Δ_q.

In our asymmetric case $z^{(0)}$ is near quarks 1 and 2 at a distance which never exceeds the distance between them. The center of mass position may be very far from $z^{(0)}$, if $z^{(3)}$ is far from $z^{(0)}$.

Let us now introduce instead of γ in (6) the "baryon time" τ as follows:

$$\gamma = \tau/T, \quad 0 \le \tau \le T. \tag{9}$$

Later on τ will be identified for heavy baryons with the fourth component of the center-of-mass coordinate R_4.

Moreover, we introduce instead of the proper times s_i the "proper mass" parameters μ_i:

$$\mu_i = \frac{T}{2s_i}, \quad i = 1,2,3. \tag{10}$$

The action (6) becomes:

$$B = \int_0^T d\tau \left\{ \frac{m_1^2}{2\mu_1} + \frac{m_2^2}{2\mu_2} + \frac{m_3^2}{2\mu_3} + \sum_{k=1}^{3} \frac{\mu_k^2}{2} \dot{z}^{(k)^2} \right\} + \sigma_0 (S_1 + S_2 + S_3) \tag{11}$$

where $S^{(k)}$ is:

$$S^{(k)} = \int_0^T d\tau \int_0^1 d\beta \left\{ \left[\dot{z}_\nu^{(0)} + \beta_k (\dot{\rho}_\nu^{(k)})^2 \right]^2 (\rho_\mu^{(k)})^2 - \left[(\dot{z}_\nu^{(0)} + \beta_k \dot{\rho}_\nu^{(k)}) \rho_\nu^{(k)} \right]^2 \right\}^{1/2} \tag{12}$$

where $\rho_\nu^{(k)} = z_\nu^{(k)} - z_\nu^{(0)}$.

We now separate the center-of-mass coordinate R_μ and relative coordinates ξ_μ, η_μ:

$$R_\nu = \frac{\sum_k \mu_k z_\nu^{(k)}}{\sum_k \mu_k} \tag{13}$$

$$z_\nu^{(3)} = R_\nu - \left[\frac{\mu(\mu_1+\mu_2)}{M\mu_3}\right]^{1/2} \xi_\nu \tag{14}$$

$$z_\nu^{(3)} = R_\nu + \left[\frac{\mu\mu_3}{M(\mu_1+\mu_2)}\right]^{1/2} \xi_\nu - \left[\frac{\mu\mu_1}{\mu_2(\mu_1+\mu_2)}\right]^{1/2} \eta_\nu \tag{15}$$

$$z_\nu^{(2)} = R_\nu + \left[\frac{\mu\mu_3}{M(\mu_1+\mu_2)}\right]^{1/2} \xi_\nu + \left[\frac{\mu\mu_2}{\mu_1(\mu_1+\mu_2)}\right]^{1/2} \eta_\nu \tag{16}$$

where μ is an arbitrary mass introduced for dimensional reasons, and $M = \mu_1 + \mu_2 + \mu_3$. The action in terms of R, ξ, η is

$$B = \int_0^T d\tau \left\{ \sum \frac{m_k^2}{2\mu_k} + \frac{M}{2}\dot{R}_\nu^2 + \frac{\mu}{2}(\dot{\xi}_\nu^2 + \dot{\eta}_\nu^2) \right\} + \sigma_o\{S_1 + S_2 + S_3\} \tag{17}$$

In principle, the problem is now exactly formulated and can be solved in the Hamiltonian or Lagrangian formalism. Leaving general analysis for subsequent publications we consider here only a limiting asymmetric case when quarks 1 and 2 are close together and 3 is far. As we argued above, in this case $z^{(0)} \sim \frac{1}{2}(z^{(1)} + z^{(2)})$ and therefore

$$z^{(1)} - z^{(0)} \sim \eta \,,\quad z^{(2)} - z^{(0)} \sim \eta \,,\quad z^{(3)} - z^{(0)} \sim \xi \,,\quad |\xi| \gg |\eta| \tag{18}$$

More explicitly

$$\rho_\nu^{(3)} \equiv z_\nu^{(3)} - z_\nu^{(0)} \cong z_\nu^{(3)} - \frac{1}{2}\left[z_\nu^{(1)} + z_\nu^{(2)}\right] = -b\xi_\nu \tag{19}$$

where $b = \left[\frac{\mu(\mu_1+\mu_2)}{M\mu_3}\right]^{1/2} + \left[\frac{\mu\mu_3}{M(\mu_1+\mu_2)}\right]^{1/2}$.

Inserting (19) and $z^{(0)} = \frac{1}{2}(z^{(1)} + z^{(2)})$ into (12) one obtains for $S^{(3)}$:

$$S^{(3)} = b\int_0^T\int_0^1 d\beta_3 \left\{ (\dot{R}_\nu + \lambda_3\dot{\xi}_\nu)^2\xi_\nu^2 - \left[(\dot{R}_\nu + \lambda_3\dot{\xi}_\nu)\xi_\nu\right]^2 \right\}^{1/2} d\tau \tag{20}$$

where we have defined

$$\lambda_3 = \left[\frac{\mu\mu_3}{M(\mu_1+\mu_2)}\right]^{1/2}(1-\beta_3) - \beta_3\left[\frac{\mu(\mu_1+\mu_2)}{M\mu_3}\right]^{1/2} \qquad (21)$$

In order to obtain more transparent results, one can expand (20) in powers of λ_3. For the equal mass case $m_1 = m_2 = m_3$ (or unequal but small masses) the first order term vanishes while quadratic in λ_3 contributes ~10% to the final result (a similar expansion has been used in case of meson Regge trajectories in [1]). As a result the action can be represented as

$$B = \int_0^T d\tau\left\{\sum_{k=1}^{3}\frac{m_k^2}{2\mu_k} + \frac{M}{2}\dot{R}^2\right\} + B_\xi + B_\eta \qquad (22)$$

where

$$B_\xi = \int_0^T\left\{\frac{\mu}{2}\dot{\xi}_\nu^2 + \sigma_0 b|\xi|\right\}d\tau, \quad |\xi| = (\xi_i\xi_i)^{1/2}, \; i = 1,2,3 \qquad (23)$$

$$B_\eta = \int_0^T\frac{\mu}{2}\dot{\eta}_\nu^2 d\tau + \sigma_0(S_1 + S_2) \qquad (24)$$

The Hamiltonian corresponding to B_ξ is

$$H_\xi = \frac{\pi_i}{2\mu} + \sigma_0 b|\xi| = \frac{1}{2\mu}\left[\frac{\partial}{\xi^2\partial\xi}\left(\xi^2\frac{\partial}{\partial\xi}\right) + \frac{l_\xi(l_\xi+1)}{\xi^2}\right] + \sigma b\xi \qquad (25)$$

The path integration in R_ν and ξ_4 in (5) and (22) can be easily done and in the limit of large T this amounts to the replacement in (22) $\dot{R}_4^2 = 1$, $\dot{R}_i^2 = 0$, $\dot{\xi}_4 = 0$.

As in [1], we scale out the mass dependence in ε_ξ: $H_\xi\psi = \varepsilon_\xi\psi$

$$\varepsilon_\xi = (2\mu)^{-1/3}(b\sigma_0)^{2/3}a(n) \qquad (26)$$

and a(n) is a solution of the reduced equation with the asymptotics which easily obtains from (25) (see also [1] and refs.quoted there)

$$a(n) = 3\left[\frac{n_r+l+1}{2}\right]^{2/3} \qquad (27)$$

An equivalent expression for the energy of quarks 1 and 2, ε_η, does not grow with l in contrast to ε_ξ and therefore can be disregarded. Substituting (26-27) into (22) and taking the limit of large T, we get the mass of the bound three quarks ($l_\xi = L$)

$$\mathcal{M}(\mu_1,\mu_2,\mu_3) = \sum_{k=1}^{3}\left[\frac{m_k^2}{2\mu_k} + \frac{1}{2}\mu_k\right] + \frac{3}{2}(\sigma_0 L)^{2/3}\left[\frac{M}{\mu_3(\mu_1+\mu_2)}\right]^{1/3} \tag{28}$$

The remaining integration over μ_i (or s_i) for the Green function (5) can be done using the steepest descent method. In this way the values of μ_k at the point of minimum are to be found from the equations (we put $m_1 = m_2$ and two equations for μ_1 and μ_2 coincide):

$$\frac{\partial \mathcal{M}}{\partial \mu_3} = -\frac{m_3^2}{2\mu_3^2} + \frac{1}{2} - \frac{(\sigma_0 L)^{2/3}}{2\mu_3^2}\left[\frac{\mu_3(\mu_1+\mu_2)}{M}\right]^{2/3} = 0 \tag{29}$$

$$\frac{\partial \mathcal{M}}{\partial(\mu_1+\mu_2)} = -\frac{2m_1^2}{(\mu_1+\mu_2)^2} + \frac{1}{2} - \frac{(\sigma_0 L)^{2/3}}{2(\mu_1+\mu_2)^2}\left[\frac{\mu_3(\mu_1+\mu_2)}{M}\right]^{2/3} = 0 \tag{30}$$

Neglecting for light quarks the current mass m_1, m_3 in (29-30), we obtain the following solutions

$$\mu_3^0 = \left[\frac{\sigma_0 L}{2}\right]^{1/2} = \mu_1^0 + \mu_2^0 \tag{31}$$

and the total baryon mass (28) is

$$\mathcal{M} = 4\mu_3^0 + \frac{m_3^2}{2\mu_3^0} + \frac{m_1^2+m_2^2}{\mu_3^0} + \varepsilon_\eta . \tag{32}$$

The asymptotics of the baryon Regge-trajectory can now be derived from (31-32)

$$L = \alpha'(0)\left[(\mathcal{M}-\varepsilon_\eta)^2 - 8\left[\frac{m_3^2}{2} + m_1^2 + m_2^2\right]\right] \tag{33}$$

where $\alpha'_B(0) = \frac{1}{8\sigma_0}$. This value exactly coincides with the slope of mesonic Regge trajectories found in [1]. Also the leading term in the baryon mass (32) - $4\mu_3^0$ - coincides with the asymptotics of the meson mass for large angular momenta.

4. Discussion and prospectives

We have shown that the leading baryon Regge trajectories correspond to the quark-diquark configuration and have the same slope as that of meson trajectories. This slope is different from the Nambu-Goto string where $\alpha'(0)=\frac{1}{2\pi\sigma_0}$. The daughter trajectories are obtained automatically through the radially excited solutions of the Hamiltonian (25). A comparison of baryon masses for symmetric and asymmetric cases, Eqs.(32) and (A.4), clearly demonstrates that the quark-diquark configuration is dynamically preferred over the symmetric one. The diquark has a "mass" $m_d \approx \mu_1^0 + \mu_2^0 + \varepsilon_\eta \approx \mu_3^0$, which grows with angular momentum and its transverse size is of the order of $\sigma_0^{-1/2}$.

The dynamic quark masses μ_1, μ_2, μ_3 which are nonzero even for zero masses m_k. Neglecting the latter, we have

$$\mu_3 \approx \mu_0, \ \mu_1 + \mu_2 = \mu_0 \equiv \left(\frac{\sigma L}{2}\right)^{1/2}, \ \mathcal{M} \approx 4\mu_0 . \qquad (34)$$

The same dynamic mass μ_0 appears in the quark-antiquark system and was called there a "transverse" mass, since it is equal to the momentum of the quark, perpendicular to the line ("string") connecting q and $\bar{q}$ (or q and qq). For large current masses m_k, $m_k \gg \mu_0$, the dynamic masses μ_k coincide with m_k, as can be seen in (28-29).

Now if one takes into account CSB, there appears a chiral constituent mass m(p) depending on momentum, which falls of fast for large p, so that m(0) ≈ 0.3 GeV and m(0.5 GeV) ≈ 0.5 m(0) [5]. As a first naive estimate one can identify m_k with $m(p=\mu)$ for u,d quarks. In this way we obtain "chirally renormalized" dynamic masses μ_k, however the CSB effects are not large ($\lesssim$25%) for lowest states and negligible for highest states. There are other effects, like approximate parity doubling, which are probably connected to CSB. These topics will be discussed in a subsequent publication, now we only mention that lowest baryon states are well reproduced in the relativized potential model [7] where CSB effects are absent.

The results of the present paper can be improved in two respects. First, one should consider any interquark distances which is important for lowest baryonic states. Second, one should introduce spin-dependent forces for light quarks. Both points can be done using the CSB method, as was explicitly demonstrated in [4]. Third, it is necessary include perturbative e.g. Coulomb-like effects at smaller distances. This topic will be treated in the next publication.

In conclusion, we have used as an input the minimal area law for the three-quark Wilson loop and have obtained linear baryonic Regge trajectories with the same slope as for mesonic trajectories. The configuration corresponding to the leading trajectories is of the quark-diquark type, while symmetric configurations yield trajectories with the 1.5 times smaller slope.

An effective dynamic mass appears due to the confining dynamics which grows with anguler and transverse momentum.

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Appendix

Symmetric configuration of three quarks

Let $m_1 = m_2 = m_3 = m$, and consider a symmetric configuration with $s_1 = s_2 = s_3 = s$ (we have seen in [1] that minimum points $s_i^{(0)}$ in the steepest descent method for equal masses are equal to each other, and we concentrate on this situation). We introduce relative coordinates ξ_μ, η_μ and center-of-mass coordinate R

$$z_\mu^{(1)} = R_\mu + \frac{1}{\sqrt{6}}\xi_\mu + \frac{1}{\sqrt{2}}\eta_\mu$$

$$z_\mu^{(2)} = R_\mu + \frac{1}{\sqrt{6}}\xi_\mu - \frac{1}{\sqrt{2}}\eta_\mu \tag{A.1}$$

$$z_\mu^{(3)} = R_\mu - \left(\frac{2}{3}\right)^{1/2}\xi_\mu, \quad Dz^{(1)}Dz^{(2)}Dz^{(3)} = DRD\xi D\eta$$

For a heavy baryon state we take only straight line trajectory for $R_\mu(\gamma)$ in the path integral (5), and if $x_i = y_i$, $i=1,2,3,$, then we can neglect $R_i(\gamma)$ and keep only $R_4(\gamma) = \gamma T \equiv \tau$, where $T = x_4 - y_4$. The action B becomes

$$B = \int_0^T d\tau \left\{ 3m^2 \frac{s}{T} + \frac{3T}{4s} + \frac{T}{4s}\left(\dot{\xi}_\mu^2 + \dot{\eta}_\mu^2\right) + \sigma_0 \sum_{k=1}^{3} |z^{(k)} - R| \right\} \tag{A.2}$$

where in the last term we have expanded in powers of β_k and kept only the leading term. We also have taken into account that in the symmetric configuration the center-of-mass coordinate R_μ coincides with that of the string junction $z_\mu^{(0)}$.

With the notation $\frac{T}{S} = 2\mu$ one can go over in (10) to the Hamiltonian

$$H = \frac{1}{2\mu}\left(-\frac{\partial^2}{\partial \xi_i^2} - \frac{\partial^2}{\partial \eta_i^2}\right) + \sigma_0 \sum_k |z^{(k)} - R| \tag{A.3}$$

so that the mass of the baryon is

$$\mathcal{M} = \frac{3m^2}{2\mu} + \frac{3\mu}{2} + E(\mu) \tag{A.4}$$

where $E(\mu)$ is the eigenvalue of H, and μ should be taken at the minimum of (12), when one takes the integral over s_k by the steepest-descent method:

$$\mathfrak{M}'(\mu=\mu_0) = -\frac{3m^2}{2\mu_0^2} + \frac{3}{2} + E'(\mu_0) = 0 \tag{A.5}$$

Dependence $E(\mu)$ can be scaled out as

$$E(\mu) = (2\mu)^{-1/3}\,\sigma_0^{2/3}\,\zeta \tag{A.6}$$

where ζ is an eigenvalue of the reduced equation to be obtained from (11) putting $2\mu = \sigma_0 = 1$. To calculate ζ it is convenient to use hyperspherical harmonics (K-harmonics [8]) with the hyperradius $\rho^2 = \xi^2 + \eta^2$ and O(6) angular momentum K, $K \geq L$, where O(3) angular momentum is

$$L_i = e_{ik\ell}\left(\xi_k \frac{\partial}{\partial \xi_\ell} + \eta_k \frac{\partial}{\partial \eta_\ell}\right) \tag{A.7}$$

The reduced equation for the lowest K-harmonics is

$$\left[-\frac{1}{\rho^5}\frac{\partial}{\partial\rho}\left(\rho^5\frac{\partial}{\partial\rho}\right) + \frac{\mathcal{K}(\mathcal{K}+4)}{\rho^2} + \langle \mathcal{K} | \sum_k |z^{(k)} - R| \, | \mathcal{K}\rangle\right]\psi = \zeta\psi \tag{A.8}$$

Since $\rho^2 = \sum_k \left[z_i^{(k)} - R_i\right]^2$, we can approximate the potential term in the symmetric situation in (15) by $g\sqrt{3}\rho$, and ζ can be found from (16), neglecting at large L the radial motion

$$\zeta = \frac{\hat{\mathcal{K}}^2}{\rho_0^2} + \sqrt{3}\rho_0, \quad \hat{\mathcal{K}}^2 = \mathcal{K}(\mathcal{K}+4) \tag{A.9}$$

and ρ_0 is found from

$$-\frac{2\hat{\mathcal{K}}^2}{\rho_0^3} + \sqrt{3} = 0; \quad \rho_0 \cong \left(\frac{2\hat{\mathcal{K}}^2}{\sqrt{3}}\right)^{1/3} \tag{A.10}$$

so that

$$\zeta \cong \frac{3}{2}\left(6\hat{\mathcal{K}}^2\right)^{1/3} \approx \frac{3}{2}\left(6L^2\right)^{1/3} \tag{A.11}$$

Inserting (19) into (12-14) immediately yields

$$\mathcal{M} = 3\left(\frac{4}{9}\right)^{3/4}\sqrt{\sigma_0}\, S^{3/4}\,, \quad L \approx \alpha'_{sym}(0)\,\mathcal{M}^2 \tag{A.12}$$

with

$$\alpha'_{sym}(0) = \frac{1}{12\sigma_0} \tag{A.13}$$

One can compare this value with the $\alpha'_{mes}(0)=\frac{1}{8\sigma_0}$ for meson trajectories found in [1] in the framework of the same approach as here. The ratio is

$$\frac{\alpha'_{sym}(0)}{\alpha'_{mes}(0)} = \frac{2}{3} \tag{A.14}$$

So we obtain linear baryon trajectories, but the slope is 1.5 times smaller than that of mesonic trajectories, in contradiction with phenomenology of Regge trajectories. Qualitatively the same phenomenon has been observed for higher resonances in the relativized quark potential model in [7]. To improve the situation the string tension for baryons has been taken smaller than that for mesons.

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