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Abstract

We study spinning self-dual particles in two dimensions. They are obtained from the chiral bosonic particle through the square root technique. We show that the resulting field theory can be either fermionic or bosonic and that the associated self-dual field reveals its Lorentz tensor structure which remains hidden in the usual formulations. We also calculate the spinning self-dual particle propagators using the BFV formalism.

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1. INTRODUCTION

Self-dual fields in two dimensions, also known as chiral bosons, play a fundamental role in the formulation of the heterotic string [1]. They can be understood, through bosonization, as solitonic excitations of a theory of Weyl fermions [2]. More generally, the bosonization procedure can be extended to show that chiral bosons give rise to a class of self-dual fields of higher spin [3,4]. They have also been used to provide a systematic construction of the Thirring field by using right and left moving fields [5].

The quantization of chiral bosons is plagued with difficulties. Siegel's formulation [6] has a local symmetry and when quantized there appears an anomaly [7]. It has been claimed [8] that Siegel's formulation is equivalent to the dimension zero field formulation of Floreanini and Jackiw [2]. However, as has been shown recently, this field is non-causal [4]. Another formulation proposed in ref.[2] makes use of a dimension one field with an unusual commutation relation and a non-local Lagrangian. This unusual commutation relation has its origin in a Dirac bracket structure because the non-local Lagrangian gives rise to a second-class constraint [9].

In order to gain some more insight into the quantization of self-dual fields it was proposed to reconsider the problem starting with a theory for a chiral bosonic particle, that is, a left moving massless relativistic particle [10]. It was shown that requiring reparametrization invariance (with respect to the proper-time) of the particle action it is possible to find a consistent formulation for the theory. The basic constraint for the chiral particle is that its energy and momentum are equal. $P_0 = P_1$. Since this constraint does not lead to a reparametrization invariant action [10] the following constraint was considered

$$T = (P_0 - P_1)P_1^{\gamma} \tag{1.1}$$

with $\gamma \neq 0$ a real parameter. The BFV quantization of this theory leads to the same propagators found in ref.[2] for the fields of dimension zero and one when $\gamma = 1$ and -1, respectively. For general γ it leads to the class of self-dual fields of reef.[3] but causality [4] restricts γ to be an integer lesser than 1. Since we also have $\gamma \neq 0$ we take from now on γ to be an integer lesser than 0, that is $\gamma = -1, -2, \dots$ †.

† Our γ is related to that of refs.[3,4] by $\gamma_{ours} = 1 - \frac{\gamma^2}{2\pi}$

On the other side one can consider the canonical quantization of (1.1). In this case the constraint (1.1) must be imposed on the wave function ψ

$$(P_0 - P_1)P_1^{\gamma}\psi = 0 \tag{1.2}$$

In a second quantized version we require that (1.2) should be obtained from an action

$$S = \int d^2x\, \psi T \psi \tag{1.3}$$

from which we find that the dimension (which has the same value as the spin) of the field ψ is $\frac{1}{2}(1-\gamma)$. At first sight it seems that ψ has only one component although it has spin equal or greater than one (since $\gamma = -1, -2, \ldots$). In fact its Lorentz tensorial structure is hidden in this formulation and, as we shall see later on, the spinning self-dual particle makes it manifest.

A procedure to find the supersymmetric version of a bosonic theory consists in taking the square root of the bosonic constraints [11]. In other words it means that we introduce a set of Grassmann variables $\theta_\mu (\mu = 0, \ldots, D-1$ where D is the dimension of the space-time), with Poisson brackets

$$\{\theta_\mu, \theta_\nu\} = 2i\eta_{\mu\nu} \tag{1.4}$$

and with them construct Grassmannian constraints S such that their Poisson brackets are proportional to the bosonic constraints. For the relativistic particle this procedure gives the spinning particle, a supersymmetric theory whose propagator is the Dirac propagator [12]. The appearence of an unsuspected supersymmetry associated to the resulting theory is characteristic of the square root technique.

In this paper we apply the square root technique to the constraint (1.1) to find the supersymmetric version of the chiral bosonic particle. In section 2 we study the classical spinning self-dual particle formulation, discussing the Grassmannian constraint, the Lagrangian formulation and its local symmetries. In section 3 we consider the canonical quantization showing how the Lorentz structure of the fields manifest themselves. We also show that the square root technique does not always lead to fermion fields since for $\gamma = 1 - 2n$, n an even positive integer we get fermionic fields while for n an odd positive integer we get bosonic fields. In section 4 we apply the Batalin-Fradkin-Vilkovisky (BFV) quantization formalism obtaining the propagators and showing its full structure. Final remarks are made in section 4.

2.CLASSICAL SPINNING SELF-DUAL PARTICLES

We start by considering the constraint (1.1) with $\gamma = -1, -2, \ldots$ and trying to find a Grassmannian constraint S such that it satisfies the Poisson bracket

$$\{S, S\} = \alpha T \tag{2.1}$$

with $\alpha \neq 0$ a real number. Using the Grassmann variables $\theta_\mu (\mu = 0, 1)$, satisfying (1.4), we choose S to be a linear combination of them with coefficients depending on the $P_\mu's$. If we also require that only integer powers of $P_\mu's$ are allowed in these coefficients † we conclude that γ must be odd, so that T has an even power of $P_\mu's$. We can then easily find the coefficients of the expansion of S in terms of $\theta_\mu's$ and with an appropriate normalization S can be written as

$$S = -\frac{i}{4}\theta_0(1 - 2\alpha - \frac{P_0}{P_1})P_1^{\frac{\gamma+1}{2}} - \frac{i}{4}\theta_1(1 + 2\alpha - \frac{P_0}{P_1})P_1^{\frac{\gamma+1}{2}}, \qquad \gamma = -1, -3, \ldots \tag{2.2}$$

The relative sign of the $\theta_\mu's$ terms is not fixed. It corresponds to a freedom allowed by the discrete symmetry $\theta_\mu \to -\theta_\mu$, for each θ_μ independently, which is present in (1.4). For definiteness we take the same sign for both terms. The real number α also is not fixed. It corresponds to a freedom allowed by the rigid continuous symmetry $\theta_+ \to \tilde{\alpha}\theta_+, \theta_- \to \tilde{\alpha}^{-1}\theta_-, \theta_\pm = \theta_0 \pm \theta_1$, which is manifest when we rewrite (1.4) in light-cone components. Since we did not find any natural value for α we shall keep it in all formulas. Of course, the results we obtain are independent of α and the only restriction on it is $\alpha \neq 0$.

We can now write the action

$$S = \int_{\tau_1}^{\tau_2} d\tau [P^\mu \dot{X}_\mu + \frac{i}{4}\theta^\mu \dot{\theta}_\mu + NT + \lambda S] - \frac{i}{4}\theta^\mu(\tau_2)\theta_\mu(\tau_1) \tag{2.3}$$

---

† Since otherwise the canonical quantization for example would be troublesome involving non-integer powers of $P_\mu's$.

where N and λ are the Lagrange multipliers for the constraints T and S, respectively, and τ is the proper time of the spinning self-dual particle. A dot denotes derivation with respec to τ. The boundary term in (2.2) is needed in order to have only one boundary condition on the Grassmann variables [12] $\theta_\mu(\tau_1) + \theta_\mu(\tau_2) = \gamma_\mu$, while for the X_μ we have two boundary conditions $X_\mu(\tau_1) = X_\mu(1), X_\mu(\tau_2) = X_\mu(2)$.

By eliminating P_μ through its equations of motion,

$$P_0 = \frac{1}{\gamma}\frac{\dot{X}_1}{N}\left(-\frac{\dot{X}_0}{N}\right)^{\frac{1-\gamma}{\gamma}} + \frac{1+\gamma}{\gamma}\left(-\frac{\dot{X}_0}{N}\right)^{\frac{1}{\gamma}}$$
$$+ \lambda(\theta_0 + \theta_1)[\frac{1-\gamma}{8\gamma^2}\frac{\dot{X}_1}{N^2}\left(-\frac{\dot{X}_0}{N}\right)^{\frac{1-3\gamma}{2\gamma}} + \frac{1+\gamma}{8\gamma^2}\frac{1}{N}\left(-\frac{\dot{X}_0}{N}\right)^{\frac{1-\gamma}{2\gamma}}]$$
$$+ \alpha\frac{1+\gamma}{4\gamma}\lambda(\theta_0 - \theta_1)\frac{1}{N}\left(-\frac{\dot{X}_0}{N}\right)^{\frac{1-\gamma}{2\gamma}}$$
$$P_1 = \left(-\frac{\dot{X}_0}{N}\right)^{\frac{1}{\gamma}}[1 + \frac{1}{4\gamma}\lambda(\theta_0 + \theta_1)\frac{1}{N}\left(-\frac{\dot{X}_0}{N}\right)^{-\frac{1+\gamma}{2\gamma}}] \tag{2.4}$$

we get after some algebra that the action (2.3) can be rewriten as

$$S = \int_{\tau_1}^{\tau_2} d\tau[\left(-\frac{\dot{X}_0}{N}\right)^{\frac{1}{\gamma}}(\dot{X}_0 - \dot{X}_1) + \frac{i}{4}\theta^\mu\dot{\theta}_\mu$$
$$+ \frac{1}{4\gamma}\lambda(\theta_0 + \theta_1)\frac{1}{N}(\dot{X}_0 - \dot{X}_1)\left(-\frac{\dot{X}_0}{N}\right)^{\frac{1-\gamma}{2\gamma}} - \frac{1}{2}\alpha\lambda(\theta_0 - \theta_1)\left(-\frac{\dot{X}_0}{N}\right)^{\frac{1+\gamma}{2\gamma}}] + b.t. \tag{2.5}$$

where b.t. are the boundary terms in θ in eq.(2.3). Under reparametrizations

$$\delta X_\mu = \epsilon\dot{X}_\mu, \qquad \delta N = (\epsilon N)^{\cdot}$$
$$\delta\theta_\mu = \epsilon\dot{\theta}_\mu, \qquad \delta\lambda = (\epsilon\lambda)^{\cdot} \tag{2.6}$$

with $\epsilon(\tau)$ the parameter for reparametrizations, we can easily show that for $S = \int d\tau\, L$

$$\delta L = (\epsilon L)^{\cdot} \tag{2.7}$$

In order for the action to be invariant under reparametrizations we must have $\epsilon(\tau_1) = \epsilon(\tau_2) = 0$ as usual.

We find the local supersymmetry transformations by taking the Poisson bracket between S and X_μ or θ_μ and we get

$$
\begin{aligned}
\delta X_0 &= \frac{1}{4}\xi(\theta_0+\theta_1)P_1^{\frac{\gamma-1}{\gamma}} \\
\delta X_1 &= \frac{1}{8}\xi(\theta_0+\theta_1)[\gamma+1-(\gamma-1)\frac{P_0}{P_1}]P_1^{\frac{\gamma-1}{\gamma}} - \alpha\frac{1+\gamma}{4}\xi(\theta_0-\theta_1)P_1^{\frac{\gamma-1}{\gamma}} \\
\delta\theta_0 &= \frac{1}{2}\xi(1-2\alpha-\frac{P_0}{P_1})P_1^{\frac{\gamma+1}{\gamma}} \\
\delta\theta_1 &= -\frac{1}{2}\xi(1+2\alpha-\frac{P_0}{P_1})P_1^{\frac{\gamma+1}{\gamma}}
\end{aligned}
\qquad (2.8)
$$

with P_μ given by (2.4), while for the Lagrange multipliers we have, according to the BFV formalism [13]

$$\delta N = \alpha\xi\lambda, \qquad \delta\lambda = \dot{\xi} \qquad (2.9)$$

where $\xi(\tau)$ is the parameter for local supersymmetry. The supersymmetry transformations so obtained do not have the standard form and look very complicated. While for the spinning particle case they can be seen as the transformations for one dimensional supergravity coupled to scalar fields here they do not have this interpretation or any other simple interpretation. After inserting P_μ in (2.8) we find for the variation of the Lagrangian

$$\delta L = \frac{d}{d\tau}[-\frac{1}{8}\xi(\theta_0+\theta_1)(\dot{X}_0-\dot{X}_1)\frac{1}{N}\left(-\frac{\dot{X}_0}{N}\right)^{\frac{1-\gamma}{2\gamma}} + \alpha\frac{\gamma}{4}\xi(\theta_0-\theta_1)\left(-\frac{\dot{X}_0}{N}\right)^{\frac{1+\gamma}{2\gamma}}] \qquad (2.10)$$

while the boundary terms give a contribution

$$
\begin{aligned}
-\frac{i}{4}\delta[\theta^\mu(2)\theta_\mu(1)] = &-\frac{i}{8}\xi(2)\{[\frac{1}{\gamma}[\dot{X}_0(2)-\dot{X}_1(2)]\frac{1}{N(2)}\left(-\frac{\dot{X}_0(2)}{N(2)}\right)^{\frac{1-\gamma}{2\gamma}} \\
&-\alpha\frac{1+\gamma}{4\gamma}\lambda(2)[\dot{\theta}_0(2)-\dot{\theta}_1(2)]\frac{1}{N(2)}][\theta_0(1)+\theta_1(1)] \\
&-2\alpha[\left(-\frac{\dot{X}_0(2)}{N(2)}\right)^{\frac{1+\gamma}{2\gamma}} + \frac{1+\gamma}{8\gamma}\lambda(2)[\theta_0(2)+\theta_1(2)]\frac{1}{N(2)}][\theta_0(1)-\theta_1(1)]\} \\
&-(\tau_1 \leftrightarrow \tau_2)
\end{aligned}
\qquad (2.11)
$$

Adding the two terms we find that in order for the action to be invariant we need $\xi(\tau_2) = \xi(\tau_1) = 0$ as in the spinning particle case [12].

The conditions on the parameters mean that when we quantize the theory we must have second order differential equations for them. This allows us to choose the proper time gauge $\dot{N} = 0$ for reparametrizations and $\dot{\lambda} = 0$ for supersymmetry.

3. CANONICAL QUANTIZATION

To perform the canonical quantization we promote P_μ and X_μ to operators obeying canonical commutation relations, θ_μ to operators γ_μ, that due to (1.4) obey the anticommutation relations

$$\{\gamma_\mu, \gamma_\nu\} = 2\eta_{\mu\nu} \tag{3.1}$$

and impose the constraint S (2.2) on the wave function ψ. Of course, we have a representation for γ_μ in (3.1) as 2×2 Dirac matrices, which for definiteness we take $\gamma_0 = \sigma_1, \gamma_1 = i\sigma_2$ and $\gamma_5 = \gamma_0\gamma_1$. In this representation we have for $S\psi = 0$

$$\begin{pmatrix} 0 & P_1^{\frac{\gamma+1}{2}} - P_0 P_1^{\frac{\gamma-1}{2}} \\ -2\alpha P_1^{\frac{\gamma+1}{2}} & 0 \end{pmatrix} \psi = 0 \tag{3.2}$$

We now require that this equation should be obtained from an action

$$S = \int d^2x\, \bar{\psi} S \psi \tag{3.3}$$

and from this we conclude that the dimension of ψ is $1 - \frac{1}{4}(\gamma + 1)$. Since γ is odd and negative we can write it as $\gamma = 1 - 2n, n = 1, 2, \ldots$ and we find that for n odd ψ is a bosonic field while for n even ψ is a fermionic field. This is quite remarkable since in general the square root of a bosonic theory produces a fermionic theory.

For $n = 1, \gamma = -1$, ψ has dimension 1 and it is a boson. We can not get much information about the Lorentz tensorial structure of ψ from (3.2) or (3.3) besides that it must have at least one spinor index. Since in this case ψ has spin 1 we can take it to be in the $(\frac{1}{2}, \frac{1}{2})$ symmetric representation of the Lorentz group $\psi_{\alpha\beta} = \psi_{\beta\alpha}$. If we now introduce the charge conjugation matrix $C = i\sigma_2$ in order to upper and lower spinor indices we can write

$$\psi_{\alpha\beta} = A^\mu (\gamma_\mu C)_{\alpha\beta} + B(\gamma_5 C)_{\alpha\beta} \tag{3.4}$$

since only $\gamma_\mu C$ and $\gamma_5 C$ are symmetric. We do not need the pseudo-scalar field B since we are dealing with a spin one field and as we shall see its field equation is actually trivial. Using explicitly the representation for the Dirac matrices in (3.4) we find that (3.2) yelds

$$B(x) - \frac{1}{2}\int dy_1\, \epsilon(x_1 - y_1)\dot{B}(x_0, y_1) = B(x) = 0$$
$$A^-(x) - \frac{1}{2}\int dy_1\, \epsilon(x_1 - y_1)\dot{A}^-(x_0, y_1) = A^+(x) = 0 \tag{3.5}$$

where $A^\pm = A^0 \pm A^1$. Then $B = A^+ = 0$ and A^- is the minus light-cone component of a vector self-dual field. Notice that the addition of a further index in ψ does not invalidate (3.3) so we still have it as an action for the vector field.

If we now take $n = 2, \gamma = -3$ and we have a fermionic field of spin $\frac{3}{2}$. We take ψ in the $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ totally symmetric representation of the Lorentz group

$$\psi_{\alpha\beta\gamma} = \psi^\mu_\alpha(\gamma_\mu C)_{\beta\gamma} + \psi^\mu_\beta(\gamma_\mu C)_{\alpha\gamma} + \psi^\mu_\gamma(\gamma_\mu C)_{\alpha\beta} \tag{3.6}$$

and (3.2) reduces to

$$\begin{pmatrix} 0 & P_1^{-1} - P_0 P_1^{-2} \\ -2\alpha P_1^{-1} & 0 \end{pmatrix} \begin{pmatrix} \psi_{1\beta\gamma} \\ \psi_{2\beta\gamma} \end{pmatrix} = 0 \tag{3.7}$$

We then find the two equations

$$\frac{1}{2}\int dy_1\, \epsilon(x_1 - y_1)\psi_{2\beta\gamma}(x_0, y_1) - \frac{1}{2}\int dy_1\, \epsilon(x_1 - y_1)\frac{1}{2}\int dz_1\, \epsilon(y_1 - z_1)\dot{\psi}_{2\beta\gamma}(x_0, z_1) = 0 \tag{3.8}$$

$$\int dy\, \epsilon(x_1 - y_1)\dot{\psi}_{1\beta\gamma}(x_0, y_1) = 0 \tag{3.9}$$

From (3.9) we have $\psi_{1\beta\gamma} = 0$ and by using (3.6) we find $\psi^\mu_1 = \psi^+_2 = 0$. And from (3.8) we learn that ψ^-_2 is a self-dual field. Therefore ψ_μ is a Weyl fermion with the plus light-cone component zero and the minus light-cone component self-dual.

This procedure can be extended straightfowardly. For general n we have a field $\psi_{\alpha_1 \ldots \alpha_{n+1}}$ totally symmetric describing spin $\frac{1}{2}(n+1)$ with only one of its components nonvanishing.

Notice that the case $n = 0$ can also be considered. It would give us a spinor field satisfying

$$(\partial_1 - \partial_0)\psi_2 = 0$$
$$\partial_1 \psi_1 = 0 \tag{3.10}$$

which with appropriate boundary conditions for ψ_1 describes a Weyl fermion. We only disregard it because the bosonic theory which it came from involves a non-causal field [4].

4. BFV QUANTIZATION

The BFV quantization [13] is straightfoward. We introduce for the constraint T two pairs of fermionic ghosts $\eta, \bar{\eta}, \mathcal{P}, \bar{\mathcal{P}}$ satisfying

$$\{\eta, \bar{\mathcal{P}}\} = \{\bar{\eta}, \mathcal{P}\} = -1 \tag{4.1}$$

and for the constraint S two pairs of bosonic ghosts $b, \bar{b}, c, \bar{c}$ satisfying

$$\{\bar{b}, c\} = \{b, \bar{c}\} = -1 \tag{4.2}$$

Taking into account (2.1) the BRST charge is given by

$$Q = \eta T + cS + \Pi_N \mathcal{P} + \Pi_\lambda b + \frac{i}{2}\alpha \bar{\mathcal{P}} c^2 \tag{4.3}$$

where Π_N and Π_λ are the canonical momenta of the Lagrange multipliers N and λ, respectively. The propagator is defined as

$$K(X(1), X(2), \gamma_\mu) = \int DX_\mu\, DP_\mu\, D\theta_\mu\, DN\, D\Pi_N\, D\lambda\, D\Pi_\lambda\, D\mathcal{P}\, D\bar{\eta}\, D\bar{\mathcal{P}}\, D\eta\, Db\, D\bar{c}\, D\bar{b}\, Dc \exp iS_{eff} \tag{4.4}$$

where the effective action S_{eff} is given by

$$S_{eff} = \int_{\tau_1}^{\tau_2} d\tau [P^\mu \dot{X}_\mu + \frac{i}{4}\theta^\mu \dot{\theta}_\mu + \Pi_N \dot{N} + \Pi_\lambda \dot{\lambda} - \mathcal{P}\dot{\bar{\eta}} - \bar{\mathcal{P}}\dot{\eta} + b\dot{\bar{c}} + \bar{b}\dot{c} - \{Q, \Psi\}] \tag{4.5}$$

the propagator being independent of Ψ [13]. According to the end of section 2 we can choose the proper-time gauge $\dot{N}=0$ and $\dot{\lambda}=0$. We can implement this by choosing the gauge fermion Ψ as

$$\Psi = N\overline{\mathcal{P}} + \lambda\bar{\mathcal{S}} \tag{4.6}$$

which implies

$$\{Q,\Psi\} = -NT + S\lambda - \mathcal{P}\overline{\mathcal{P}} - \mathcal{S}\bar{\mathcal{S}} + i\alpha\overline{\mathcal{P}}c\lambda \tag{4.7}$$

The functional integral over the ghost fields gives rise to harmless determinant factors that can be absorbed in an overall normalization constant. The integration over Π_N means that only the zero mode of N survives, call it c, and the functional integral over N reduces to an ordinary integral over c. The range of integration for c is taken to be from 0 to ∞, as usual, in order to integrate over only one classical trajectory †. The functional integration over Π_λ means that only the zero mode of λ survives, call it λ_0, and the functional integral over λ reduces to an ordinary Berezin integral over λ_0.

Then (4.4) reduces to

$$K(X(1),X(2),\gamma_\mu) = \int_0^\infty dc \int d\lambda_0 \int DX_\mu DP_\mu D\theta_\mu \exp i\int d\tau(P\dot{X} + \frac{i}{4}\theta\dot{\theta} + cT - S\lambda_0) \tag{4.8}$$

We now perform the change of variables $X_\mu = X_\mu(1) + \frac{\Delta X_\mu}{\Delta\tau}(\tau-\tau_1) + Y_\mu$ where $\Delta X_\mu = X_\mu(2) - X_\mu(1), \Delta\tau = \tau_2 - \tau_1$ and with Y_μ satisfying the boundary conditions $Y_\mu(\tau_1) = Y_\mu(\tau_2) = 0$, and also $\theta_\mu = \frac{1}{2}\gamma_\mu + \tilde{\theta}_\mu$ with $\tilde{\theta}_\mu$ satisfying the boundary conditions $\tilde{\theta}_\mu(\tau_1) = \tilde{\theta}_\mu(\tau_2) = 0$. Then, integrating over the remaining variables we get (up to numerical factors)

$$K(X(1),X(2),\gamma_\mu) = \int \frac{d^2p}{(2\pi)^2} \frac{[\gamma_0(1-2\alpha-p_0/p_1) + \gamma_1(1+2\alpha-p_0/p_1)]p_1^{\frac{2\alpha-1}{2}}}{p_0 - p_1 + i\epsilon} e^{ip\cdot\Delta X} \tag{4.9}$$

Using the representation for the Dirac matrices given before, (4.9) becomes in momentum space (dropping out the $i\epsilon$ factors)

$$K(X(1),X(2),\gamma_\mu) = \int \frac{d^2p}{(2\pi)^2} e^{ip\cdot\Delta X} K(p)$$

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† See ref.[14] for a detailed exposition on the BFV formalism applied to relativistic particles.

$$K^{-}(p) = \begin{pmatrix} 0 & p_1^{\frac{\gamma-3}{2}} \\ \frac{2\alpha p_1^{\frac{\gamma+1}{2}}}{p_0-p_1} & 0 \end{pmatrix} \tag{4.10}$$

In view of the results of section 3 we interpret this propagator as the propagator for the field $\psi_{a,a_1 \ldots a_n}$ so that the propagator K^{-} has the general structure

$$K(X(1), X(2), \gamma_\mu) = \begin{pmatrix} < \psi^{\dagger}_{2a_1 \ldots a_n}(x_1)\psi_{1a'_1 \ldots a'_n}(x_2) > & < \psi^{\dagger}_{1a_1 \ldots a_n}(x_1)\psi_{1a'_1 \ldots a'_n}(x_2) > \\ < \psi^{\dagger}_{2a_1 \ldots a_n}(x_1)\psi_{2a'_1 \ldots a'_n}(x_2) > & < \psi^{\dagger}_{1a_1 \ldots a_n}(x_1)\psi_{2a'_1 \ldots a'_n}(x_2) > \end{pmatrix} \tag{4.11}$$

Thus, e.g., for $\gamma = -1$ we have in momentum space, from (4.10) and (4.11),

$$\begin{pmatrix} - < A^- A^+ > & < A^+ A^+ > \\ < A^- A^- > & - < A^+ A^- > \end{pmatrix} = \frac{\alpha}{2} \begin{pmatrix} 0 & -\frac{1}{\alpha} \\ \frac{-2p_1^{0}}{p_0-p_1} & 0 \end{pmatrix} \tag{4.12}$$

which is precisely the propagator (up to numerical factors) for the light-cone components of the A_μ field satisfying the field equations (3.5).

5. CONCLUSIONS

We have presented a theory for spinning self-dual particles which has the property of describing either bosons or fermions. In fact, it reproduces all the propagators for the bosonic chiral particle since from (4.10) the non-trivial components of the propagator is $\frac{p_1^{\frac{\gamma+1}{2}}}{p_0-p_1}$ which for odd γ gives p_1 to an integer power in the numerator. This is the same result as for the bosonic chiral particle [10]. The advantadge of this formulation however is that it makes the Lorentz tensor structure more transparent and it opens the possibility of coupling covariantly chiral bosons to other fields, mainly to the gravitational field. Work in this direction is in progress.

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REFERENCES

[1] D.J.Gross, J.A.Harvey, E.Martinec and R.Rohm, Phys.Rev.Lett. 54 (1985) 502, Nucl.Phys.B 256 (1985) 253 and B 267 (1986) 75

[2] R.Floreanini and R.Jackiw, Phys.Rev.Lett. 59 (1987) 1873

[3] H.O.Girotti, M.Gomes, V.Kurak, V.O.Rivelles and A.J.da Silva, Phys. Rev. Lett. 60 (1988) 1913

[4] H.O.Girotti, M.Gomes, V.O.Rivelles and A.J.da Silva, preprint IFUSP/P-765 (1989)

[5] M.Gomes, V.Kurak, V.O.Rivelles and A.J.da Silva, Phys.Rev.D 38 (1988) 1344

[6] W.Siegel, Nucl.Phys.B 238 (1984) 307

[7] C.Imbimbo and A.Schwimmer, Phys.Lett. 193B (1987) 435;
J.M.F.Labastida and M.Pernici, Nucl.Phys.B 297 (1988) 557

[8] J.Sonnenschein, Phys.Rev.Lett. 60 (1988) 1772

[9] M.E.V.Costa and H.Girotti, Phys.Rev.Lett. 60 (1988) 1771

[10] M.Gomes, V.O.Rivelles and A.J.da Silva, Phys.Lett.B, to appear

[11] C.Teitelboim, Phys.Rev.Lett. 38 (1977) 1106;
R.Tabensky and C.Teitelboim, Phys.Lett.B 69 (1977) 453

[12] M.Henneaux and C.Teitelboim, Ann.Phys.(N.Y.) 143 (1982) 143

[13] E.S.Fradkin and G.Vilkovisky, Phys.Lett. 55B (1975) 224;
I.A.Batalin and G.Vilkovisky, Phys.Lett. 69B (1977) 309

[14] J.Gamboa and V.O.Rivelles, in preparation