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The exact analytical form for the box diagram with one heavy external quark

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Abstract

The exact analytical form for the box-diagram amplitude with one non-vanishing external quark mass is presented. The consequences of this work for meson mixing and the CP violating charge asymmetry are investigated in heavy quark systems. In the T^0 - $\bar{T}^0$ system with the three generation model the charge asymmetry is typically of order ($\sim 10^{-4}$) but the mixing is extremely small ($<10^{-18}$). In the four generation case, the mixing can be greatly enhanced. With a normal KM matrix, the mixing is still very small ($< 10^{-10}$) but with a flavor-flipped KM matrix, the mixing can be as large as a few percent while the charge asymmetry is still small ($<10^{-4}$) . In the B'^0 - $\bar{B}'^0$ system, the mixing and charge asymmetry can be of order a few percent whereas in the T'^0 - $\bar{T}'^0$ system the mixing is small.

In the standard model of electroweak interactions the mixing present in a general meson-antimeson, P^0-$\bar{P}^0$, system occurs at second order via the well known box-diagram.[1] The box-diagram plays an important role in the standard model in explaining the observed K^0 - $\bar{K}^0$ and B^0 - $\bar{B}^0$ mixings and CP violation in the K^0 - $\bar{K}^0$ system. The theoretical calculations are consistent with the experimental data provided that the top quark is heavy enough. In K^0 - $\bar{K}^0$ and B^0 - $\bar{B}^0$ systems the external quarks in the box-diagram are relatively light compared with the internal top quark and hence the external quark momenta can be neglected to a good approximation and this results in a simple expression for the box-diagram amplitude.[2] However, if one wants to study mixing and CP violation for a heavy P^0 - $\bar{P}^0$ system such as the T^0 - $\bar{T}^0$ system where a top quark and a light quark, ℓ, constitute T^0, the zero external momenta approximation is no longer valid. One has to keep at least the top quark momentum non zero. In Ref.[3] this was done for the case where the mass of the external quark is small compared with m_W. The more general calculations for the box-diagram with one arbitrary external quark momentum have been considered in Refs.[4] and [5]. The results are given in integral form. Due to the complexity of the integral, it is not convenient to use in practice. In this paper, we will analytically evaluate the box-diagram amplitude with one arbitrary external quark momentum and use it to study mixing and CP violation in heavy P^0 - $\bar{P}^0$ systems with P^0 made of a heavy quark, h and a light one, ℓ.

We have done a detailed check of the calculations in Refs.[4] and [5] for the dispersive component of the box-diagram amplitude, and we confirm the results of Ref.[5], obtaining

$$M_{12} = \frac{G_F^2 m_W^2 B_P f_P^2 M_P}{12\pi^2} \sum_{ij} \lambda_i \lambda_j D(\alpha_i, \alpha_j) \qquad (1)$$

where

$$D(\alpha_i, \alpha_j) = B(\alpha_i, \alpha_j) - \frac{5}{8} C(\alpha_i, \alpha_j),$$

$$B(\alpha_i, \alpha_j) = \frac{1}{(1-\alpha_i)(1-\alpha_j)} \sum_k{}' \int_0^1 dx\ \{(2+\frac{1}{2}\alpha_i\alpha_j)\Lambda_k(x) - 2\alpha_i\alpha_j$$

$$+ \alpha_h[x\alpha_i + (1-x)\alpha_j]\} \ln|\Lambda_k(x)|,$$

$$(2)$$

$$C(\alpha_i, \alpha_j) = \frac{\alpha_h(4 + \alpha_i\alpha_j)}{(1-\alpha_i)(1-\alpha_j)} \sum_k{}' \int_0^1 dx\ x(1-x)\ln|\Lambda_k(x)|$$

with

$$\alpha_i = (\frac{m_i}{m_W})^2, \quad \sum_k{}' = \sum_{k=1,2} - \sum_{k=3,4}, \quad \lambda_i = V_{ih}^* V_{i\ell}$$

$$\Lambda_1(x) = x\alpha_i+(1-x)\alpha_j-x(1-x)\alpha_h, \qquad \Lambda_2(x) = 1-x(1-x)\alpha_h,$$

$$\Lambda_3(x) = x+(1-x)\alpha_j-x(1-x)\alpha_h, \qquad \Lambda_4(x) = x\alpha_i+1-x-x(1-x)\alpha_h.$$

where h and ℓ are the heavy and light external quarks respectively and i and j are all possible internal quarks; V is the KM quark-mixing matrix. The k pseudo-summation is due to splitting the four propagator product from the box-diagram amplitude into four terms, each with two propagator products.

When the Λ_k in eq(2) are negative an absorptive component of the amplitude is generated. It may be evaluated by integrating over the region for which the Λ_k are negative, with the logarithms replaced by 2π [5]

$$\Gamma_{12} = \frac{G_F^2 m_W^2 B_P f_P^2 M_P}{12\pi^2} \sum_{ij} A(\alpha_i, \alpha_j)\lambda_i\lambda_j \tag{3}$$

where

$$A(\alpha_i, \alpha_j) = -\frac{\pi}{2\alpha_h^2}\frac{1}{(1-\alpha_i)(1-\alpha_j)}\sqrt{(\alpha_i-\alpha_j)^2+\alpha_h^2-2\alpha_h(\alpha_i+\alpha_j)}$$

$$\cdot \{(1+\frac{\alpha_i\alpha_j}{4})[3\alpha_h^2 - \alpha_h(\alpha_i+\alpha_j) - 2(\alpha_i-\alpha_j)^2] \tag{4}$$

$$+ 2\alpha_h(\alpha_i+\alpha_j)(\alpha_i+\alpha_j-\alpha_h)\}$$

and $A(\alpha_i, \alpha_j)$ is zero if $m_h < m_i + m_j$.

We have performed a careful calculation to obtain the exact analytical form for the dispersive component of the amplitude in eq(2). Writing $D(\alpha_i, \alpha_j)$ as

$$D(\alpha_i, \alpha_j) = \frac{1}{(1-\alpha_i)(1-\alpha_j)}\sum_k{}' D_k(\alpha_i,\alpha_j), \tag{5}$$

we have the analytical expression for $D_k(\alpha_i,\alpha_j)$

$$D_k(\alpha_i,\alpha_j)=[\alpha_j(\frac{(p_{ki}-q_{kj})^2}{4\alpha_h}-\frac{q_{kj}}{2})-\alpha_i(\frac{(p_{ki}-q_{kj})^2}{4\alpha_h}-\frac{p_{ki}}{2})-\frac{\alpha_i\alpha_j}{\alpha_h}(p_{ki}-q_{kj})$$

$$+(1+\frac{\alpha_i\alpha_j}{4})(\frac{(p_{ki}-q_{kj})^3}{4\alpha_h^2}-\frac{p_{ki}^2-q_{kj}^2}{8\alpha_h}-\frac{p_{ki}+q_{kj}}{2})]\ln(\frac{p_{ki}}{q_{kj}})$$

$$+ \left(1+\frac{\alpha_i\alpha_j}{4}\right)q\ln(p_{ki})+\left(1 + \frac{\alpha_i\alpha_j}{4}\right)\frac{p_{ki}q_{kj}}{\alpha_h}$$

$$+\left[\ln|\Xi_k|\sqrt{\frac{|\Omega_k|+\Omega_k}{2}} - \Theta_k\sqrt{\frac{|\Omega_k|-\Omega_k}{2}}\right]\left[\left(1+\frac{\alpha_i\alpha_j}{4}\right)\left(\frac{(p_{ki}-q_{kj})^2}{2\alpha_h}+\frac{p_{ki}+q_{kj}}{4}\right.\right.$$

$$\left.\left.-\frac{3}{4}\alpha_h\right) + \frac{\alpha_j}{2}(p_{ki} - q_{kj} + \alpha_h) - \frac{\alpha_i}{2}(p_{ki} - q_{kj} - \alpha_h) - 2\alpha_i\alpha_j\right], \qquad (6)$$

where

$$\Omega_k = \frac{1}{4\alpha_h^2}\left((p_{ki} - q_{kj})^2 + \alpha_h^2 - 2\alpha_h(p_{ki} + q_{kj})\right),$$

$$\Xi_k = \frac{p_{ki}+q_{kj}-\alpha_h-2\alpha_h\sqrt{|\Omega_k|}}{p_{ki}+q_{kj}-\alpha_h+2\alpha_h\sqrt{|\Omega_k|}},$$

$$\Theta_k = 2\left(\frac{p_{ki}-q_{kj}-\alpha_h}{|p_{ki}-q_{kj}-\alpha_h|}\tan^{-1}\frac{|p_{ki}-q_{kj}-\alpha_h|}{2\alpha_h\sqrt{|\Omega_k|}}\right.$$

$$\left.-\frac{p_{ki}-q_{kj}+\alpha_h}{|p_{ki}-q_{kj}+\alpha_h|}\tan^{-1}\frac{|p_{ki}-q_{kj}+\alpha_h|}{2\alpha_h\sqrt{|\Omega_k|}}\right),$$

$$p_{1i} = p_{4i} = q_{1i} = q_{3i} = \alpha_i,\ p_{2i} = p_{3i} = q_{2i} = q_{4i} = 1 .$$

Note that Ξ_k must be carefully handled when using eq.(6) numerically.

Using the unitarity of the KM matrix, we write M_{12} as

$$M_{12} = \frac{G_F^2 m_W^2 B_P f_P^2 M_P}{12\pi^2}\sum\lambda_i\lambda_j D_{ij}, \qquad (7)$$

$$D_{ij} = D(\alpha_l, \alpha_l) + D(\alpha_i, \alpha_j) - D(\alpha_i,\alpha_l) - D(\alpha_j,\alpha_l),$$

where l is the lightest internal quark and the summations over i and j are over all internal quarks except l and similarly for Γ_{12} and A_{ij}.

In the rest of the paper, we will use the exact analytical form for the box-diagram to study mixings and CP violation in $P^0 - \bar{P}^0$ systems. The mixing parameter r and the CP violating parameter a, the charge asymmetry, are defined as follows

$$r = \frac{N^{++} + N^{--}}{N^{+-}}, \qquad a = \frac{N^{--} - N^{++}}{N^{--} + N^{++}} \tag{8}$$

where $N^{\alpha\beta}$ (α, β = +, -) is the number of events with one α and one β charged lepton in the semi-leptonic decay of the $P^0 - \bar{P}^0$ system. In the case where $P^0\bar{P}^0$ are coherently produced with the angular momentum L=1 (e.g. $e^+e^- \to \Upsilon(4s) \to B^0\bar{B}^0$), in terms of the parameters, M_{12}, Γ_{12} and the P^0 lifetime Γ_P, we have

$$r = \frac{x^2 + y^2}{2 + x^2 - y^2},$$

$$a = \frac{|\Gamma_{12}||M_{12}|\sin(\phi_\Gamma - \phi_M)}{|M_{12}|^2 + |\Gamma_{12}|^2/4} \tag{9}$$

where $x^2 = (\Delta M/\Gamma_P)^2$, $y^2 = (\Delta\Gamma/2\Gamma_P)^2$ with the mass difference ΔM and lifetime difference $\Delta\Gamma$ given by

$$\Delta M^2 = 2\{[(|M_{12}|^2 - |\Gamma_{12}|^2/4)^2 + (\mathrm{Re}\Gamma_{12}\mathrm{Re}M_{12} + \mathrm{Im}\Gamma_{12}\mathrm{Im}M_{12})^2]^{1/2} + |M_{12}|^2 - |\Gamma_{12}|^2/4\} \tag{10}$$

$$\Delta\Gamma^2 = 8\{[(|M_{12}|^2-|\Gamma_{12}|^2/4)^2 + (\mathrm{Re}\Gamma_{12}\mathrm{Re}M_{12} + \mathrm{Im}\Gamma_{12}\mathrm{Im}M_{12})^2]^{1/2} - |M_{12}|^2 + |\Gamma_{12}|^2/4\}$$

and ϕ_Γ, ϕ_M are the phases of Γ_{12}, M_{12} respectively:

$$\sin(\phi_\Gamma - \phi_M) = \frac{\mathrm{Im}\Gamma_{12}\mathrm{Re}M_{12} - \mathrm{Re}\Gamma_{12}\mathrm{Im}M_{12}}{|M_{12}||\Gamma_{12}|} \qquad (12)$$

In the spectator model, the width, Γ_P is given by[6]

$$\Gamma_P = \frac{G_F{}^2 m_P{}^5}{192\pi^3} \sum_{q,i,j} |V_{Pq}|^2 |V_{ij}|^2 f(x_W, \gamma, x_i, x_j, x_q) \qquad (13)$$

where $x_n = (m_n/m_P)^2, \gamma = (\Gamma_W/m_P)^2$, the q, i, j summation is over all possible final states and

$$f(\omega, \gamma, \alpha, \beta, \delta) = \int dx \frac{\omega^2}{(x-\omega)^2+\omega\gamma} \lambda^{1/2}(1,\delta,x)\, \lambda^{1/2}(x,\alpha,\beta)$$

$$\cdot\Big\{\frac{2}{x^2}\lambda(x,\alpha,\beta)(1+\delta-x) + \frac{2}{x^3}[x(x+\alpha+\beta)-2(\alpha-\beta)^2](1-\delta-x)(1+x-\delta)$$

$$+\frac{3}{x^2\omega^2}(x-2\omega)[x(\alpha+\beta)-(\alpha-\beta)^2][(1-\delta)^2-x(1+\delta)]\Big\} \qquad (14)$$

and

$$(\sqrt{\alpha} + \sqrt{\beta})^2 < x < (1 - \sqrt{\delta})^2,$$

where $\lambda(a,b,c) = a^2+b^2+c^2-2ab-2ac-2bc$ and we will use $\Gamma_W = 2.25$ GeV.

It is clear that when $m_h > m_W + m_q$, the decay channel $h \to W + q$ is opened. This increases Γ_P dramatically. Since the mixing r is proportional to $1/\Gamma_P^2$, an increased Γ_P will decrease r. Hence when m_h is greater than $m_W + m_q$, r will be much smaller than in the case where $m_h < m_W$.

To calculate r, we need to know the parameters B_P and f_P. For B_P we will use the vacuum insertion approximation $B_P = 1$ which is expected to be good for heavy quark systems and for f_P we compare the effects of using $f_P = f_B$ and the non-relativistic quark model expression for f_P[7]. In the later case we assume that the wave function at the origin is independent of the quark masses and hence $f_P = f_B\sqrt{m_B/m_P}$.

We will consider the mixings and charge asymmetries in three and four generation models. In the three generation model, the possible heavy P^0 - $\bar{P}^0$ system is T^0 - $\bar{T}^0$ ($T^0 = \bar{\ell}t$). With four generations, two more heavy P^0 - $\bar{P}^0$ systems become possible, B'^0 - $\bar{B}'^0$ ($B'^0 = \bar{\ell}b'$) and T'^0 - $\bar{T}'^0$ ($T'^0 = \bar{\ell}t'$). We study the three generation case first.

Before we study the T^0 $\bar{T}^0$ system, we comment on the B^0 - $\bar{B}^0$ system. The issue we want to address here is whether the zero external momenta approximation for the B^0 - $\bar{B}^0$ system is valid. We find that the difference between using the exact formula and the zero external momenta approximation is only a few percent; for example, in the calculation for r, using the zero-external momenta approximation results in an underestimation of the amplitude by 4% (1%) for m_t = 60 GeV (120 GeV). Hence the previous calculations[8] using the zero external momenta approximation for B^0 - $\bar{B}^0$ are valid.

For the T^0 - $\bar{T}^0$ system, the exact formulae have to be used. The dominant contribution to the mixing is from the D_{bb} and A_{bb} terms in M_{12} and Γ_{12} respectively. We have

$$|x_T| = \frac{G_F^2 m_W^2 B_T f_T^2 M_T}{6\pi^2 \Gamma_T} |V_{tb}^* V_{lb}|^2 |D_{bb}|$$

$$|y_T| = \frac{G_F^2 m_W^2 B_T f_T^2 M_T}{12\pi^2 \Gamma_T} |V_{tb}^* V_{lb}|^2 |A_{bb}| \tag{15}$$

In figure 1, we show D_{bb} and A_{bb} as a function of m_t for m_b = 5 GeV. For m_t = 0, we have $D_{bb} = 3.7 \times 10^{-3}$ and $A_{bb} = 0$. From Figure 1, it is clear that neglecting the external momentum is far from justified. Using $|V_{tb}^* V_{ub}|$ =0.005 to 0.01 and $f_T = f_B$ = 200 MeV ($f_T = f_B\sqrt{m_B/m_T}$) we find $r_T < 10^{-23}$ (10^{-24}) for T^0 ($\bar{u}t$) with m_t in the range 30 - 200 GeV. This is extremely small and hence is practically impossible for experimental detection. For the T^0 ($\bar{c}t$) system, r is enhanced by a factor $|V_{cb}/V_{ub}|^4$. However, r is still very small, $< 10^{-18}$.

For the charge asymmetry, we have

$$a_T = 2\frac{\mathrm{Im}\lambda_b^*\lambda_s}{|\lambda_b|^2}\left(\frac{A_{bs}}{A_{bb}} - \frac{D_{bs}}{D_{bb}}\right)\frac{D_{bb}A_{bb}}{|D_{bb}|^2 + |A_{bb}|^2/4} \tag{16}$$

For ($\bar{u}t$) and ($\bar{c}t$) systems, we have $\mathrm{Im}\lambda_b^*\lambda_s$ equal to $-J_{CP}$ and $+J_{CP}$ respectively. Here J_{CP} is the CP violation measure[9] which is given by $J_{CP} = s_1^2 s_2 s_3 c_1 c_2 c_3 \sin\delta$ in the KM notation. From CP violation in the K^0 - $\bar{K}^0$ system, J_{CP} is constrained to be in the range of (10^{-5} to 10^{-4}). We find a_T is of order 10^{-3} for the $T^0(\bar{u}t)$ system and 10^{-7} for the $T^0(\bar{c}t)$ system. CP violation is comparatively large in the

$T^0(\bar{u}t)$ system. However, due to the smallness of r, experimental observation is again not possible.

A similar analysis has been done in Ref.[10]. Since Ref.[10] is based on Ref.[4], we do not agree on details of the numerical values but reach the same qualitative results.

We now turn to the four generation model. In this case, the mixing in the $T^0 - \bar{T}^0$ system can be greatly enhanced for two main reasons:

(i) The fourth down quark b' can be heavy so that the dominant term in M_{12} is proportional to $|V_{tb'}{}^* V_{lb'}|^2 |D_{b'b'}|$. It is found that for $m_t > m_i$, m_j, D_{ij} is proportional to $m_i^2 m_j^2$ rather than $m_i m_j$ when the external momenta vanish and hence $|D_{b'b'}|$ can be several orders of magnitude larger than $|D_{bb}|$. If $|V_{tb'}{}^* V_{lb'}|$ is not too small, M_{12} is enhanced by several orders.

(ii) With four generations, it is possible to have a flavor-flipped KM matrix, that is, one in which $V_{tb} = \varepsilon$ is small while $|V_{tb'}|$ is close to 1.[11] In this case, Γ_P is reduced by a factor of $|\varepsilon|^2$ if $m_t < m_{b'}$.

In total, the enhancement factor for $x_T{}^2$ is $|D_{b'b'}/D_{bb}|^2/|\varepsilon|^4$. In figure 2 we show $D_{b'b'}$ and $A_{b'b'}$ as functions of m_t and $m_{b'}$. To have a specific feeling about r and a in the four generation model, we use two numerical KM matrices which are consistent with the experimental data of the $K^0 - \bar{K}^0$ and $B^0 - \bar{B}^0$ systems, the matrices (b) and (d), obtained in Ref.[11]. Using the normal KM matrix (matrix (b)), we find that the mixings are small $< 10^{-10}$ for $T^0(\bar{u}t)$ and $T^0(\bar{c}t)$. The charge asymmetries can be of order 10^{-2} for $T^0(\bar{u}t)$ and 10^{-1} for $T^0(\bar{c}t)$. For the flavor-flipped KM matrix (matrix (d)), the life time of the top quark is prolonged in the case that $m_t < m_{b'}$.

The mixing in this case is substantially enhanced and a measurable effect $r_T(\bar{c}t) = 0.1$ is possible if the top quark mass is not too heavy, ~30 GeV. The results are shown in figure 3 (solid lines). For $T^0(\bar{u}t)$, mixing is suppressed by $|V_{ub'}|^4/|V_{cb'}|^4$ so that $r_T(\bar{u}t) < 10^{-4}$. Here we have assumed that f_T = 200 MeV. In the case of the non-relativistic quark model r_T is suppressed by a factor of $(m_b/m_t)^2$. The charge asymmetry is small, of order 10^{-7} and 10^{-4} for $T^0(\bar{u}t)$ and $T^0(\bar{c}t)$ respectively.

We finally calculate r and a in the B'^0 - $\bar{B}'^0$ and T'^0 - $\bar{T}'^0$ systems. For the B'^0 -$\bar{B}'^0$ system, the most interesting case is where the mass hierarchy pattern is $m_{b'} < m_t$, $m_{t'}$.[12] In this case the life time of b' is long because it can decay only through mixing. For the normal KM matrix the largest mixing occurs for the B'^0 ($\bar{b}b'$) system. The mixing can be as large as a few times 10^{-2}. The mixings in the ($\bar{d}b'$) and ($\bar{s}b'$) systems are suppressed relative to the ($\bar{b}b'$) system by ~ 10^{-1} and ~ 10^{-4} respectively. For the ($\bar{b}b'$) system, where mixing is large, it turns out that the charge asymmetry is also typically large, of order 10^{-2}, as in fig.4. This is to be expected because the internal t't', tt' and tt quark channels all contribute comparably in magnitude to M_{12} and Γ_{12} thus yielding a possibly large phase difference between M_{12} and Γ_{12}. In the flavor-flipped case the system is less interesting because the mixing is generally suppressed by a factor $<10^{-2}$.

Mixing in the T'^0 - $\bar{T}'^0$ systems is always small and is maximized in the case of $m_{t'} < m_{b'}$ so that the decay of t' occurs through mixing to b. In the normal KM matrix case (matrix (b)), r_T $<10^{-8}$ for ($\bar{c}t'$) and ($\bar{u}t'$). The mixing could be enhanced by several orders with a smaller choice of $V_{t'b}$. For these systems the charge asymmetry ranges between 10^{-4} and 10^{-2}. In the flavour-flipped KM case (matrix (d)) the charge asymmetry can be 10^{-1} ($\bar{c}t'$) and 10^{-5} ($\bar{u}t'$) but the mixings are $<10^{-13}$.

We emphasize that our numerical results in the four generation model depend on the unknown parameters of the fourth generation and hence should be taken as order of magnitude estimates. However, as has been shown above, allowing the existence of a fourth generation can introduce interesting phenomena.

While we were writing up this paper, we received a preprint[13] by Nandi and Zhang+ who have also studied this problem. Their work and ours differ in a number of ways. Our expression for the box-diagram is more compact than theirs. In particular CP violation in heavy P^0 - $\bar{P}^0$ systems is not studied in Ref.[13]. Their larger mixing in the B'^0 - $\bar{B}'^0$ system than ours is due to their use of a smaller $V_{cb'}$ value and hence a longer B' lifetime.

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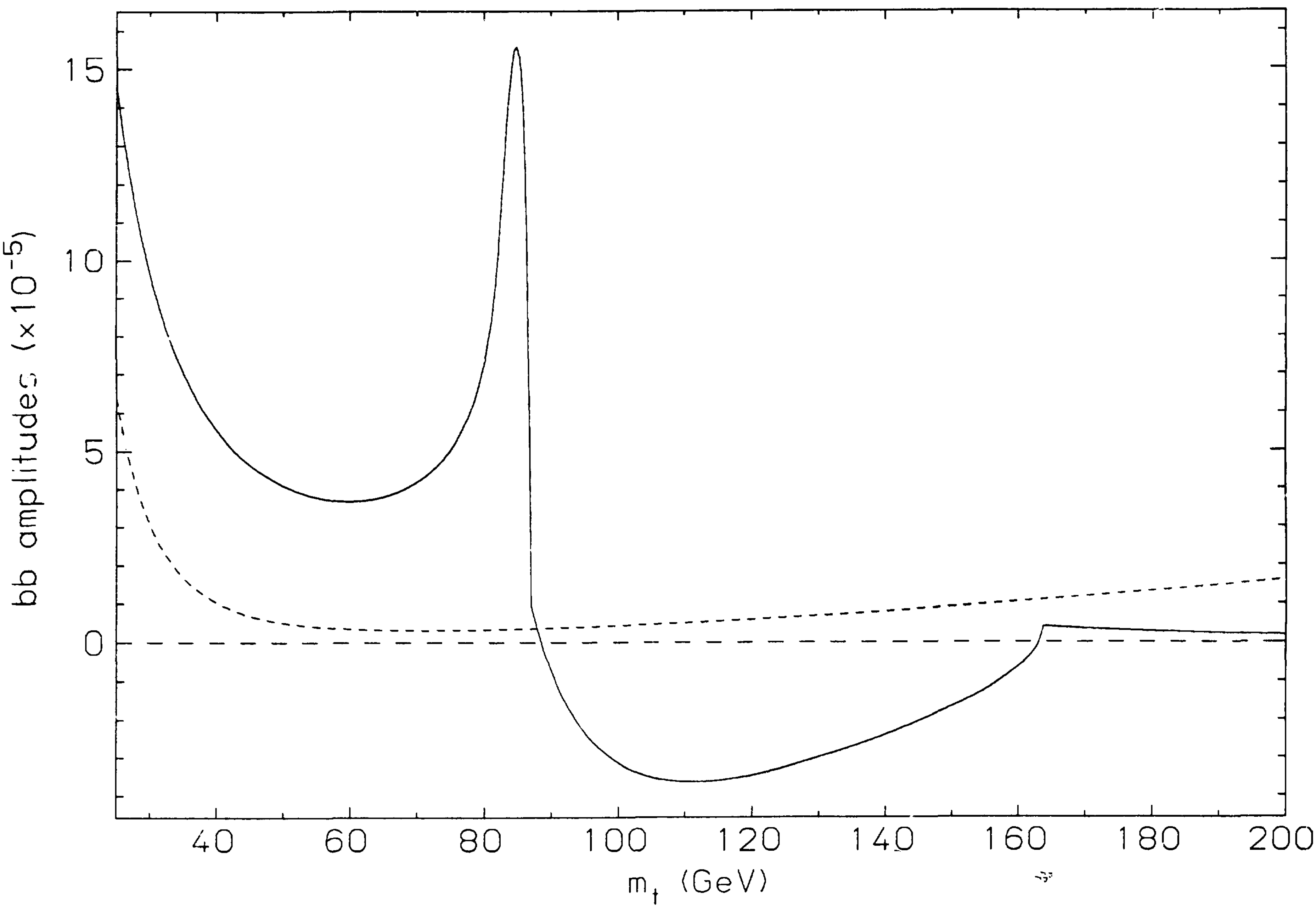
Figure Captions

Fig.1. D_{bb} (solid line) and $A_{bb}/2$ (dashed line) vs. m_t with m_b = 5.0 GeV.

Fig.2. $D_{b'b'}$ (solid lines) and $A_{b'b'}/2$ (dashed lines) vs. m_t with $m_{b'}$ = 50 GeV (lines 1 and 2) and $m_{b'}$ = 150 GeV (lines 3 and 4).

Fig.3. Mixing, r, for $T^0(\bar{c}t)$ (solid lines) and $B'^0(\bar{b}b')$ (dashed lines) vs. mass of external quark: m_t and $m_{b'}$ respectively. The internal quark masses are: $m_{b'}$ = 150 GeV (line 1); m_t = 100 GeV, $m_{t'}$ = 200 GeV (line 2); $m_{b'}$ = 50 GeV (line 3) and m_t = 150 GeV, $m_{t'}$ = 200 GeV (line 4). A flavor-flipped and normal KM matrix have been used for $T^0(\bar{c}t)$ and $B'^0(\bar{b}b')$ respectively.

Fig.4. Charge asymmetry, a, vs. $m_{b'}$ for the $B'^0(\bar{b}b')$ system. The internal quark masses are: m_t = 150 GeV, $m_{t'}$ = 200 GeV (solid line) and m_t = 100 GeV, $m_{t'}$ = 200 GeV (dashed line). A normal KM matrix has been used.



bb amplitudes (×10⁻⁵)
15
10
5
0
40
60
80
100
120
140
160
180
200
m_t (GeV)

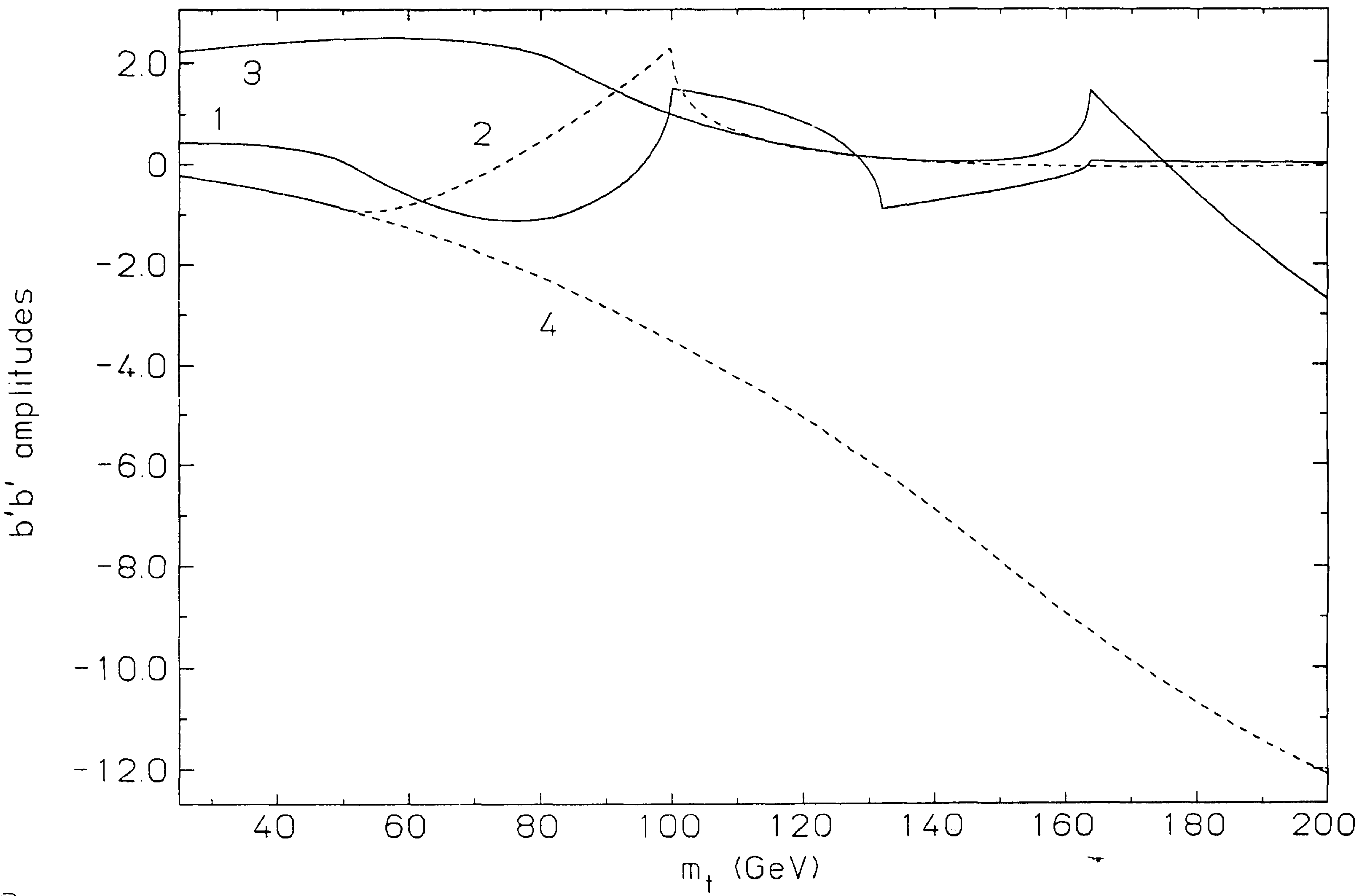
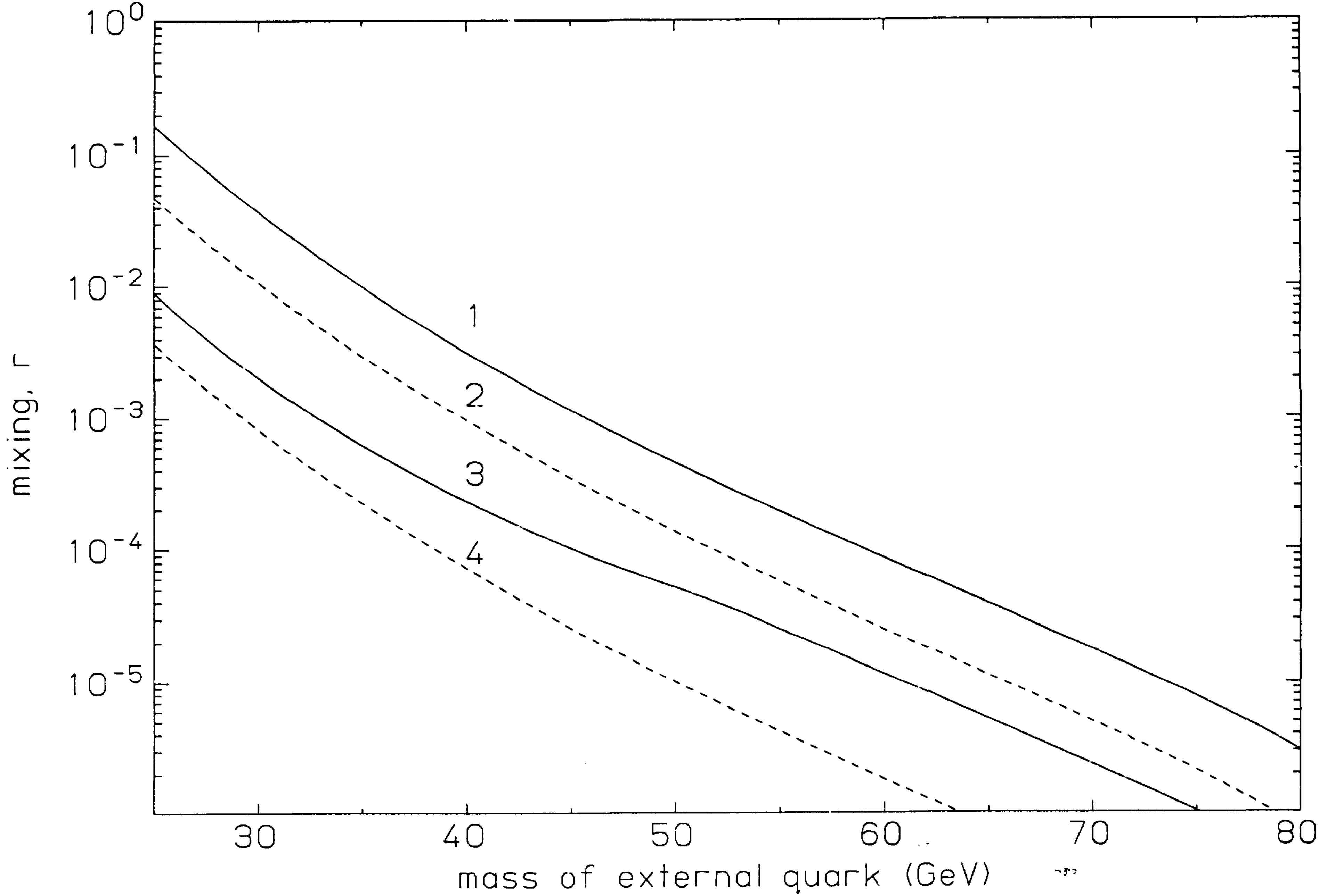
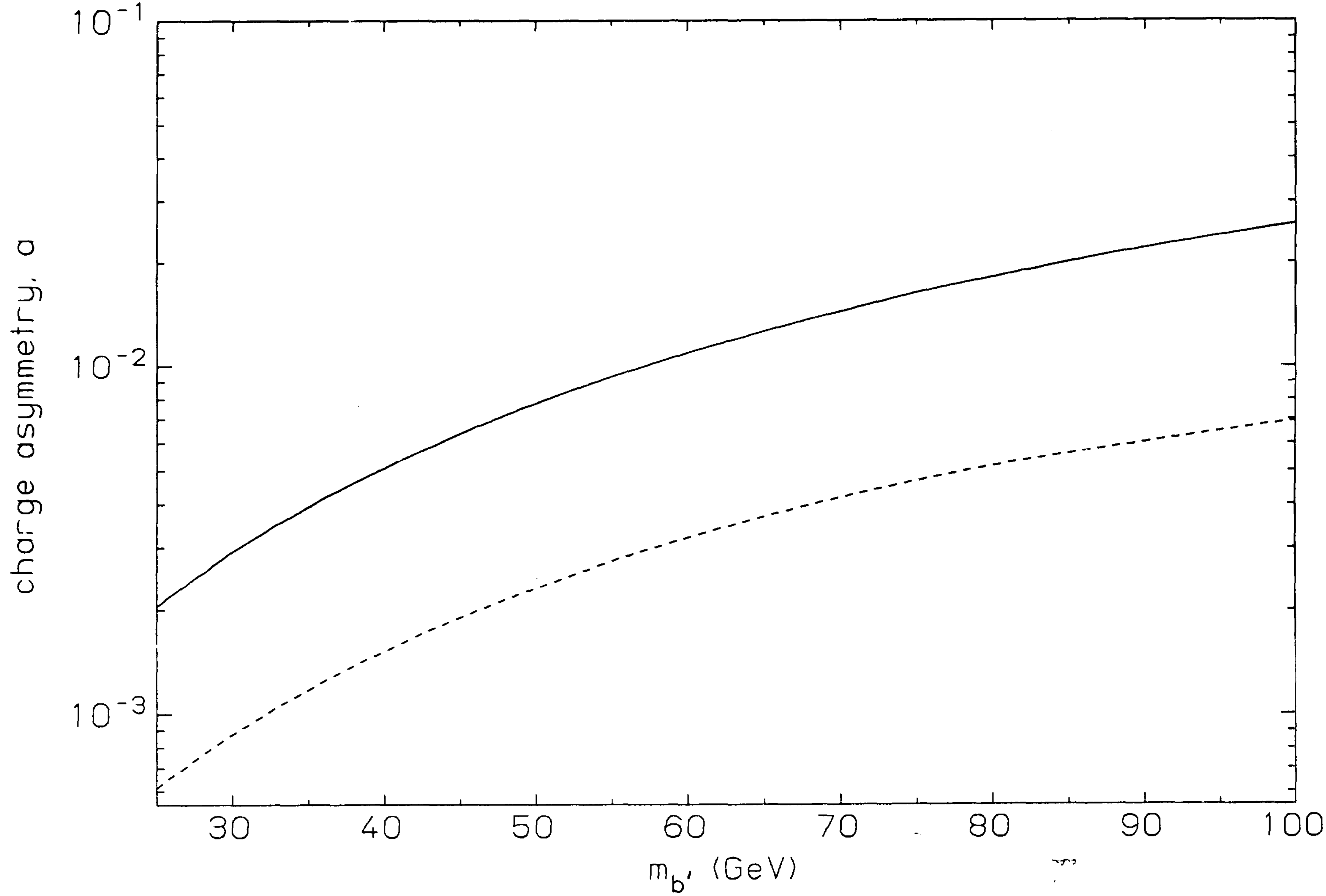

b'b' amplitudes
2.0
0
-2.0
-4.0
-6.0
-8.0
-10.0
-12.0
40
60
80
100
120
140
160
180
200
m_t (GeV)
1
2
3
4

fig 2



10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
mixing, r
1
2
3
4
30
40
50
60
70
80
mass of external quark (GeV)



charge asymmetry, a
10^{-1}
10^{-2}
10^{-3}
30
40
50
60
70
80
90
100
$m_{b'}$ (GeV)