APPLICATION OF THE RESULTS OF CARBON PELLET MODELING TO THE PROBLEM OF PLASMA PENETRATION.*

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ABSTRACT

The assumptions of the evaporation model for low-Z pellets interacting with magnetic fusion plasmas developed by P.B. Parks are tested. These assumptions are that the vapor density profile in the region adjacent to the pellet surface, falls off with radial distance as $r^{-\alpha}$, where $5 < \alpha < 6$, and that the ionization time for the transition between charge states $\tau_{i,j}$ is much less than a flow time for the vapor in this same region $\tau_r$ (i.e. for $r < -3$ sonic radii). The first assumption is tested by solving a two-parameter eigenvalue problem for the evaporation cloud in the region interior to the sonic radius; the results are found to be consistent with the low-Z evaporation model. The second assumption, that $\tau_{i,j} \ll \tau_r$ is tested at the sonic radius using the results from atomic physics and the low-Z evaporation model. It is found that indeed $\tau_{i,j} \ll \tau_r$ for plasmas with parameters close to thermonuclear conditions (e.g. TFTR and CIT), but not for those of smaller Tokamaks such as TEXT. The results of pellet penetration calculations for the conditions of the TEXT carbon-pellet injection experiments are presented which show better agreement with experiment if the shielding fraction is calculated at each step of the pellet-penetration calculation, the effect of ionization is ignored, and if the effect of possible uncertainties in the background plasma parameters is included.

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I. Introduction.

The modeling of the interaction of low-Z pellets with magnetic fusion plasmas is important for several reasons. The first is that in the production of the high densities required in Tokamaks for achieving high energy confinement times, pellet injection appears to play a key role [1]. Since hydrogen-like refueling pellets cannot penetrate the near-thermonuclear type plasmas such as TFTR and since carbon pellets can penetrate such plasmas with apparently no adverse effects[2], understanding the physics of penetration of such pellets would be important in the design of pellet injectors for more advanced Tokamak experiments such as the proposed Compact Ignition Tokamak Experiment [3] or CIT. Secondly, the evaporation cloud could reveal information about the current distribution in such Tokamaks, if the cloud streams along the magnetic field once ionized sufficiently, or through Zeeman splitting of spectral lines being emitted by some of the charge states of carbon existing in the cloud [4]. Finally, the interaction of the thermonuclear alpha particles and the cloud could yield information about the energy distribution of the alpha particles through the processes of neutralization and escape via double charge exchange with the evaporation species $C^{+Z}$, where $Z < +5$ [5], or of single charge exchange of the alpha particles with the evaporation species resulting in Doppler-shifted charge-capture radiation [6].

An important model for the interaction of plasmas with low-Z pellets was developed by P.B. Parks [7] based on the vapor shielding phenomenon. Although this model has been fairly successful in predicting the penetration of carbon pellets into TFTR [2] and Alcator and T-10 [7], there are some discrepancies with similar experiments on the TEXT device, namely that the model overestimates the range of pellets in the TEXT plasmas [8,9]. This discrepancy has been believed to be the result of a non-thermal electron distribution in that device causing a higher evaporation rate than that predicted by the model which is based on the assumption of a Maxwellian electron distribution.

In the work reported here, several assumptions in the low-Z model were tested by solving the steady flow problem by means of a two-parameter shooting code to match external plasma electron heat flux on a singular surface near the sonic surface of the evaporation cloud, and the evaporation rate on the pellet surface with the energy loss by these electrons with the evaporation cloud being treated self-consistently. The solutions resulting from this procedure are then compared with the assumptions made by Parks in deriving this relationship analytically.

One basic assumption of both models is that the ionization time for a given Z to $Z+1$ transition is zero. Hence the evaporation cloud is assumed to be in charge-state equilibrium predicted by a collisional-radiative model so that the charge-state fractions at a given location in the cloud depend only on the local densities and temperature of the cloud and not on the ratios of the charge states in the interior regions. By calculating the ionization times on the sonic surface, where the percentage of the $C^{+1}$ state is around 10%, one can determine the validity of this assumption for the interior regions which contribute most strongly to the shielding of the pellet surface from the external electron heat flux [7]. Since this partial shielding contributes directly to the evaporation rate [7], the predictions of the model for pellet penetration are also dependent on this assumption.
In the following sections, the basic assumptions in the low-Z evaporation model are reviewed and the two-parameter shooting calculation is outlined in section II; the results of the two-parameter shooting calculation for various cases are compared with the assumptions in section III. In section IV, the ionization lengths on the sonic surfaces are estimated for TEXT, TFTR, and CIT plasmas and some modifications in the low-Z model for TEXT are suggested. In section V, the results of the normal and modified low-Z models are compared with the results from the TFTR and TEXT experiments.

II. The Assumptions of the Low-Z Evaporation Model and the Two-Parameter Shooting Calculation.

In his low-Z evaporation model [7], Parks couples the sonic and pellet surface values of the electron heat flux using thermodynamical arguments to determine the flow parameters of the vapor. From these results, an analytical self-consistent model is derived between the evaporation rate at the pellet surface and the external plasma electron heat flux at the sonic surface (i.e. the transition between the interior subsonic and the exterior supersonic regions). The major assumptions he made were as follows [7]:

1. The vapor density in the subsonic region falls as \( r^{-\alpha} \), where \( 5 < \alpha < 6 \).
2. The ratio of the radius of the sonic surface to that of the pellet \( r_s/r_p \) is 1.33.
3. The temperature of the vapor at the pellet surface is 0.6 eV.
4. The Mach number of the vapor at the pellet surface is 0.5.
5. The conditions of local thermodynamic equilibrium prevail.

In the rest of this section, a two-parameter shooting calculation is described to test the first four assumptions. The fifth assumption will be treated in sections IV and V.

The two-parameter shooting calculation is very similar to those previously used for hydrogen pellets [10,11], except the boundary conditions on the pellet surface are now quite different due to the much larger heat of vaporization for carbon pellets. These boundary conditions, which are based on the conservation of mass, momentum and energy across the solid/gas transition are given as follows [7]:

\[
\frac{T_{p}^{1/2}}{p_{s}(T_{p})} = \frac{(\gamma \, m)^{1/2} \, M_p}{G \left[ 1 + (2\pi\gamma)^{1/2} \, M_p + M_p^2 \right]} \tag{1}
\]

\[
q_p = \frac{G}{4\pi m \, r_s^2 \, T_p} \left[ \Delta H + T_p \left\{ \frac{\gamma}{(\gamma - 1)} + \gamma \, M_p^2/2 \right\} \right] \tag{2}
\]
where $q_p$, $T_p$, $M_p$, $\Delta H$, and $G$ are the electron heat flux, the temperature of the vapor, the Mach number of the vapor, the heat of sublimation, and the ablation rate, all evaluated on the pellet surface.

The two-parameter shooting calculation involves the choice of the singular surface (s sonic surface) parameters of external plasma electron energy $E_e$ (a one energy group model is being used at present), external plasma electron heat flux $q_e$, and sonic radius $r_s$, and then using a shooting code to locate the plasma surface. The shooting code involves the propagation of $q(r)$ and $E(r)$ from the singular surface to the pellet surface based on the attenuation of $q(r)$ and $E(r)$ by the vapor which is a function of the mass density of the vapor $\rho(r)$. Thus the attenuation equations for $q$ and $E$ must be propagated inward as well as the fluid equations for the vapor. These fluid equations are the conservation of mass, momentum, and energy, where the latter includes the heat being deposited by the external plasma electrons self-consistently through $dq(r)/dr$. These equations are then propagated inward iterating on two parameters of the properties of the fluid on the singular surface: number density $n_e$ and temperature $T_e$, until a solution is found where equations (1) and (2) are satisfied. Since this two-parameter technique requires several iterations, only one solution has been found at present.

III. The Results of the Shooting Code in Comparison with Parks' Assumptions.

The results of the shooting code solution are shown in Figures 1 and 2 and in the following table:

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>PARKS' MODEL</th>
<th>SELF-CONSISTENT SOLUTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_e$</td>
<td>--------------</td>
<td>$1.81 \times 10^{14}$cm$^{-3}$</td>
</tr>
<tr>
<td>$T_e$</td>
<td>--------------</td>
<td>2.68 keV</td>
</tr>
<tr>
<td>$r_p$</td>
<td>--------------</td>
<td>0.418 mm</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>5 to 6</td>
<td>4.536</td>
</tr>
<tr>
<td>$r/r_p$</td>
<td>1.33</td>
<td>1.68</td>
</tr>
<tr>
<td>$r_p^*$</td>
<td>--------------</td>
<td>1.32</td>
</tr>
<tr>
<td>$T_p$</td>
<td>0.6 eV</td>
<td>0.55 eV</td>
</tr>
<tr>
<td>$M_p$</td>
<td>0.5</td>
<td>0.30</td>
</tr>
<tr>
<td>$\delta_0/r_p$</td>
<td>0.0</td>
<td>0.021</td>
</tr>
</tbody>
</table>
where \( r^* \) is the radius of the singular surface which lies just inside the sonic surface in this model and \( \zeta_0^* \) is the ionization length at \( r^* \) for 0 to +1 charge transition (see below). \( T_{\infty} \) and \( n_{\infty} \) are the external plasma electron temperature and density respectively.

Figure 1 illustrates that the vapor density dependency with radius does indeed follow a power law in the region inside \( r^* \). Figure 2 shows how the heat flux and the mean energy of the external electrons behave in the inner region; the behavior of the temperature and density of the vapor in the inner region are also shown in Figure 2. These results are summarized in Table I.

As can be seen from Table I, the assumptions made by Parks are in reasonable agreement with the solution. Thus one would expect reasonable agreement between the predictions for the penetration of carbon pellets into magnetic fusion plasmas as long as Parks fifth assumption of local thermodynamic equilibrium is valid on the singular surface. It is this assumption, which we now turn.

**IV. Ionization Lengths on the Sonic Surface.**

While the temperatures and densities in the interior region in the figures are sufficient for local-thermodynamic equilibrium (LTE) to exist [12], this is not a sufficient condition for LTE. That is, there must be sufficient time for the collisional processes to establish this condition, and this time will be called the ionization time \( r_{ij}^* \). Since in the present problem, the vapor is flowing radially outward with a velocity \( v \) (at least in the weakly ionized inner region), this ionization time may be converted to an ionization length, \( \zeta_z = v \times r_{ij}^* \). Here, the subscript \( z \) refers to the transition from the \( Z \) to \( Z+1 \) charge state. To be meaningful, this length is then compared with a characteristic scale length for the problem such as the pellet radius. For these inner regions, the vapor densities are so high that the collisional processes involving the thermodynamically produced electrons in the vapor dominate the ionization processes of the external plasma electrons by at least an order of magnitude. Therefore, the ionization length calculated on the basis of the vapor temperature and density is sufficient.

From the table, the assumption of LTE at the singular surface appears to be reasonable, since the ratio of \( \zeta_0^*/r_p \) is small. To see how this parameter might change for other conditions, one can calculate this ratio on the sonic surface, using the results from Parks low-Z model[7] and atomic physics[12]. For typical conditions and pellet radii for the TEXT, TFTR, and proposed for CIT devices, the results are summarized in Table II.

The data in Table II reveal that, while LTE is a reasonable assumption for TFTR and CIT, it breaks down for TEXT, since \( \zeta_0^*/r_p \) is very small for the TFTR and CIT and is very large for TEXT. \( \zeta_0^* \) is the ionization length for vapor ionization by the external plasma electrons, based on the cross sections for electron impact ionization of C\( ^0 \) [13]. For the cases presented, it can be seen that these electrons play a negligible role for TFTR and CIT in determining the charge-state distribution on the sonic surface (i.e. \( \zeta_0^* << \zeta_{ox}^* \) for these cases), whereas for TEXT they are more important than the vapor electrons. But even here, the ionization length is so long with respect to the distance travelled from the pellet surface (=0.33 \( r_p \) in the model), that a better assumption would be that the vapor
is not ionized at all at this distance. This new model will be used in the comparisons with the results of the experiments on TEXT discussed below. The row labeled SCS in Table II represents the self-consistent solution (SCS) presented in section III, which has been included for comparison.

<table>
<thead>
<tr>
<th>TABLE II.</th>
</tr>
</thead>
<tbody>
<tr>
<td>DEVICE</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>CIT</td>
</tr>
<tr>
<td>TFTR</td>
</tr>
<tr>
<td>TEXT</td>
</tr>
<tr>
<td>TEXT</td>
</tr>
<tr>
<td>SCS</td>
</tr>
</tbody>
</table>

V. Comparison of the Model with Pellet Penetration Experiment Results.

As might be expected, since the ionization lengths are small for TFTR, the low-Z model is in fairly good agreement with the experimental results for carbon pellet penetration into TFTR [2]. The results are shown in Figure 3, where the experimental results are compared with the results from two versions of the low-Z model for pellet penetration. The vertical axis for the predictions of the model is the ablation rate in relative units, whereas for the experiment, it is the plasma light output. In the one labeled AVE \( \eta \), an average of \( \eta \), the fraction of the incident external electron heat flux that is shielded from the pellet surface by the cloud, is calculated and held constant for the entire penetration calculation, whereas in the version labeled \( \eta(\xi) \), \( \eta \) is calculated at each radial step inward. As can be seen from the figure, the model and experiment are in good agreement especially if \( \eta \) is calculated at each radial step.

Next the results of a series of injection experiments on TEXT [8,9] will be compared with the low-Z model; this is done in Table III. \( r_{\text{pinit}}, v_p, T_{e\infty}, r_p, \) and \( r_{\text{pf}} \) are the initial pellet radius, its speed, the external electron temperature on the device axis, the minor radius where the pellet extinguished, and the final pellet radius if it exited the plasma, respectively. \( \eta(\xi) \) is the model where \( \eta \) is calculated at each step and LTE is assumed out to the sonic radius; \( \eta(\xi) \) is the model where \( \eta \) is calculated at each radial step and the vapor is assumed to be un-ionized. This latter model appears to be the more reasonable for TEXT in view of the large ionization lengths given in Table II. For all the cases listed
in Table III, the density of the plasma electrons on the device axis is assumed to be $4.0 \times 10^{13}$ cm$^{-3}$, the electron density profile scales as $[1 - (r/a)^2]^{0.87}$, and the electron temperature profile scales as $[1 - (r/a)^2]^{1.42}$ [8]. For a more thorough presentation of the impurity pellet injection experiments on TEXT, see reference 9.

TABLE III.

<table>
<thead>
<tr>
<th>$r_{\infty}$</th>
<th>$v_{\infty}$</th>
<th>$T_{\infty}$</th>
<th>EXP. $r_b(\mu m)$</th>
<th>AVE. $\eta(r)$</th>
<th>$\eta(r)I$</th>
<th>$\eta(r)II$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\mu m)</td>
<td>(m/s)</td>
<td>(eV)</td>
<td>$r_b(\mu m)$</td>
<td>$r_b(\mu m)$</td>
<td>$r_b(\mu m)$</td>
<td>$r_b(\mu m)$</td>
</tr>
<tr>
<td>137</td>
<td>326</td>
<td>1595</td>
<td>-5.9</td>
<td>0.6</td>
<td>-3.38</td>
<td>-3.9</td>
</tr>
<tr>
<td>158</td>
<td>349</td>
<td>1679</td>
<td>-4.1</td>
<td>2.5</td>
<td>-1.82</td>
<td>-2.35</td>
</tr>
<tr>
<td>186</td>
<td>396</td>
<td>1455</td>
<td>-2.0</td>
<td>$[3\mu m]$</td>
<td>5.7</td>
<td>4.4</td>
</tr>
<tr>
<td>198</td>
<td>390</td>
<td>1598</td>
<td>-1.0</td>
<td>$[12\mu m]$</td>
<td>4.2</td>
<td>2.9</td>
</tr>
<tr>
<td>199</td>
<td>358</td>
<td>1421</td>
<td>3.5</td>
<td>$[23\mu m]$</td>
<td>6.0</td>
<td>4.7</td>
</tr>
</tbody>
</table>

As can be seen in the table, the model is in reasonable agreement with the data especially if model $\eta(r)II$ is used. That is, a 10% change in the $T_{\infty}$, results in a $\pm 2$ cm change in $r_b$ for this model. Uncertainties in the profiles such as a 50% change in the exponents of the profiles given above results in a $\pm 2$ cm change in $r_b$. Considering by how much these profiles are changing during the injection process [8,9], effects which are well beyond the scope of the present model, the agreement between theory and experiment is quite good.

VI. Conclusions.

The low-Z model of Parks[7], appears to be in reasonable agreement with the results of a self-consistent two-parameter shooting-code solution. The assumption that LTE prevails in the inner vapor region, appears to be fine for predicting the penetration of carbon pellets into the higher temperature experiments such as TFTR and CIT, but it appears to break down for the conditions on the TEXT experiment. Assuming that no ionization occurs in the inner region of the carbon vapor, gives better agreement with the TEXT results. Thus in determining the design requirements for a carbon pellet injector for CIT, which is designed to have even higher temperatures than TFTR, the low-Z model [7] should do a good job in predicting the penetration on that device. That is, unless effects not considered here, such as magnetic shielding [14], play a role.
VII. Acknowledgements.

The authors are indebted to R.K. Fisher of General Atomics Inc. and S.C. McCool and M.L. Walker of the University of Texas for helpful discussions.

REFERENCES

Cloud Density Scaling \([r^*=0.55 \text{ mm}]\)

\[ \ln (N) = -4.5361 \ln r + \text{const.} \]

Figure 1. Scaling of the vapor density with normalized radius inside the singular surface (normalized to the singular surface) for the self-consistent solution.
Figure 2. Results of the two-parameter self-consistent shooting calculation. Scaling of the normalized external electron heat flux $q$, mean electron energy $E$, and the normalized vapor velocity $v$, and temperature $T$ with normalized radius.
Figure 3. Comparison of the experimental injection experiment on TFTR (EXP) and the average $\eta$ model (AVE $\eta$), and the $\eta(r)$ model. The pellet was initially 0.8 mm in diameter and 1.2 mm long. Prior to injection of the carbon pellet, $n_{\infty}$ and $T_{\infty}$ were $5 \times 10^{13}$ cm$^{-3}$ and 4.9 keV respectively and the minor radius was 80 cm.