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A MICROSCOPIC DESCRIPTION OF THE S-WAVE πN-SCATTERING LENGTHS AND THE $(p\pi^-)$-ATOM LIFETIME IN THE QUARK CONFINEMENT MODEL

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INTRODUCTION

The fundamental low-energy $\pi\pi$ -, πN - and NN - scattering processes are of the present-day interest due to their important role in the investigation of the structure of strong interactions. The experimental information on the πN - and NN - interactions is provided by the measurements of the total and differential cross sections as well as different polarization effects [1-4]. The $\pi\pi$ - scattering is studied indirectly from the multiparticle reactions $\pi N \to \pi\pi N$ and $\pi N \to \pi\pi\Delta$ on the basis of the hypothesis about the dominating role of the one-pion exchange [5]. The main characteristics of the processes involvind pions and nucleons are the scattering phase shifts and scattering length. At present, the precise measurements of the phase shifts have been carried out up to the energy $E_{lab} \lesssim 400$ MeV. The scattering lengths cannot be directly measured in experiment and are determined by extrapolation of empirical data of the phase-shift analysis to the reaction threshold. This procedure suffers from model ambiguities and usually leads to large errors. In this connection, future experiments conducted at the "Positronium" (HEPI-JINR) facility aimed at generating and carrying out measurements of the main characteristics of the ($\pi\pi$)-, (πK)-, (KK)- and ($p\pi^-$)- atoms [6] are of interest. The lifetime of atoms, the wave-function at origin and the Lamb shift are intended to be measured. Since the life-time and wave-function at origin are related to the difference of the S -wave scattering lengths, the suggested experiments will allow one to determine the latter in a model-independent way.

From the theoretical point of view the study of the $\pi\pi$ -, πN- and NN -interactions can serve as a probe for the hypothesis of various models aimed at description of the low-energy physics. It is known that QCD - the theory of strong interactions cannot be directly applied to the investigation of the low-energy region due to the large coupling constant. The processes of $\pi\pi$ -, πN - and NN - scattering have been treated, for the most part, within the current-algebra approach [7] and the dispersion relation method [8]. ($\pi\pi$ -

and πN -scattering) as well as within the framework of Lippmann - Shwinger[9-10] (πN - and NN - scattering) and Low-type[11] (πN--scattering) equations. A detailed investigation of the NN -scattering process has been carried out within the meson-exchange model[12]. In this model, based on the one- and two-boson exchange picture with the allowance for the virtual Δ (1230)-isobar the fair description of the NN -scattering phase shifts has been obtained up to the energy $E_{lab} \leqslant$ 300 MeV. The meson exchange in this model is described by the local propagators while the internal structure of nucleons caused by quarks and other heavy states is taken into consideration by using strong meson-nucleon form factors.

At present, a great amount of experimental data exists corroborating the quark structure of hadrons. In this connection interest arises to investigate the $\pi\pi$ -, πN - and NN -scattering processes with the QCD- based quark models. In the existing bag models the πN-scattering lengths and scattering phase shifts can be calculated; however, the full analysis of these three types of interaction is not available.

In refs.[17,18], the Quark Confinement Model (QCM) based on a certain hypothesis about the hadronization and quark confinement has been proposed.

In QCM, the hadrons are considered to be colourless collective variables occurring as a result of quark-gluon interactions. The confinement mechanism is based on the averaging of intermediate quark states over the nontrivial QCD-vacuum. The QCM allows a unified description of strong, electromagnetic and weak interactions of hadrons. The internal structure of hadrons (mesons as well as baryons) in the QCM is fully determined by the behaviour of constituent quarks at large distances.

A number of low-energy effects of light-mesons, nucleon and Δ -isobar physics have been considered within the QCM[17-20]. The statical characteristics of the low-energy hadron interactions (such as decay widths, magnetic moments, etc.) have been calculated and the behaviour of the formfactors and scattering phase shifts reflecting the internal structure of hadrons has been derived. The results obtained agree well with the experimental data.

The QCM allows a unique description of $\pi\pi$ -, πN - and NN-scattering processes. Refs.[20,21,25,26] deal with the pion-pion and nucleon-nucleon interactions. The S -wave $\pi\pi$ -scattering lengths a_0^0 and a_0^2 were calculated in ref.[20] while in ref.[21] the δ_1^1 $\pi\pi$ -scattering phase shift was obtained. In refs.[25,26] the main low-energy nucleon characteristics were studied on the basis of the quark-diquark approximation to the three-quark structure of nucleons. The basic idea of the quark-diquark approximation consists in approximating the quark pair interactions by the diquark fields. The magnetic moments, electromagnetic radii, G_A/G_V ratio in the neutron β - decay and the meson-nucleon coupling constants were calculated. The behaviour of the nucleon electromagnetic and strong formfactors in a low-energy region was obtained. Making use of these strong meson-nucleon formfactors within the framework of the One-Boson Exchange (OBE) model[12] the NN -scattering phase shifts were calculated up to the energy $E_{lab} \lesssim$ 300 MeV. The results agree with the experimental data and with the results obtained in the Bonn potential model.

The present work is aimed at the description of the S -wave -scattering lengths and calculation of the ($p\pi^-$)-atom lifetime within the framework of the quark-diquark approximation to the nucleon structure proposed in ref.[26].

The paper is organized as follows. Section 1 deals with the quark-diquark approximation to the three-quark diagrams which is used in the QCM for the calculation of the effects of nucleon physics. In section 2 the S -wave πN -scattering lengths and ($p\pi^-$)-atom lifetime are calculated. The agreement with the experimental data and with the results obtained within other theoretical approachss is achieved.

1. NUCLEON STRUCTURE IN QCM

In the QCM the hadron interactions result from the quark exchange. Therefore, the interaction Lagrangians are basic objects for the dynamical description of the low-energy processes. They should be ultimately obtained from QCD. However, due to the mathematical problems raised by the nonperturbative character of QCD in the low-energy region we are still far from answering this question. So we will construct the interaction Lagrangians under the following requirements. The Lagrangians should be chosen in the simplest form without the derivatives.

The Lagrangian describing the interactions of hadrons with quarks can be written in the form[17-25]

$$\mathcal{L}_H(x) = H(x) \sum_A g_H^A J_H^A(x), \tag{1.1}$$

where $J_H^A(x)$ is the quark current with the quantum numbers of a hadron H.

The explicit form of the Lagrangians $\mathcal{L}_H$ for π, ρ, ε -mesons as two-quark[17-22] and nucleons as three-quark[23,24,25] systems are given below.

1) <u>The mesons</u>:

$$\begin{aligned} &\mathcal{L}_M = \frac{g_M}{\sqrt{2}} \bar{q}_a \Gamma_M \hat{M} q_a \\ &\hat{P} = \vec{\pi} \cdot \vec{\tau}, \ \Gamma_P = i\gamma^5 \ ; \ \hat{V} = \vec{\rho} \cdot \vec{\tau}, \ \Gamma_V = \gamma^\mu ; \\ &\hat{S} = \varepsilon \cdot I \cdot \cos\delta_S, \ \Gamma_S = I - iH_S \overleftrightarrow{\hat{\partial}}/\Lambda . \end{aligned} \tag{1.2}$$

The choice of the parameters characterizing ε -meson, the mass m_ε, the mixing angle δ_S and the parameter H_S, has been discussed in detail in ref.[20]. It was shown that the best description of the low-energy meson physics is achieved when these parameters are restricted to the following values:

$$m_\varepsilon = 600 \text{ MeV}, \qquad H_S = 0.545, \qquad \delta_S = 17^\circ . \tag{1.3}$$

2. <u>Nucleon as a three-quark system</u>

$$\begin{aligned} &\mathcal{L}_N^{(3q)} = \sum_{A=V,T} g_{NA} \bar{N}_j J_j^A + h.c. \\ &J_j^A = R^A_{j;j_1j_2j_3}\, q^{a_1}_{j_1} q^{a_2}_{j_2} q^{a_3}_{j_3}\, \varepsilon^{a_1a_2a_3} \end{aligned} \tag{1.4}$$

Here $j = (\alpha, m)$, α, m, a being the spinor, isotopic and colour indices, respectively. The special choice for the R^A matrices provides the symmetry of the currents J_j^A with respect to the transpposition of the quark fields. There exist two different forms of quark currents carrying the nucleon quantum numbers that satisfy the requirement – the tensor current J_j^T and the vector current J_j^V. The corresponding matrices $R^A_{\{j\}}$ have the following form:

$$\begin{aligned} R^T_{j;j_1j_2j_3} = &\, 6 g^{\alpha\alpha_1} \delta^{mm_1} \tau_2^{m_2m_3} (C\gamma^5)^{\alpha_2\alpha_3} + 6 (\gamma^5)^{\alpha\alpha_1} \delta^{mm_1} \tau_2^{m_2m_3} C^{\alpha_2\alpha_3} + \\ &+ (\sigma^{\mu\nu}\gamma^5)^{\alpha\alpha_1} (\vec{\tau})^{mm_1} (\tau_2 \vec{\tau})^{m_2m_3} (C\sigma^{\mu\nu})^{\alpha_2\alpha_3} , \end{aligned} \tag{1.5}$$

$$R^{V}_{j_1 j_1 j_2 j_3} = 2\, g^{\alpha\alpha_1}\, \delta^{mm_1}\, \tau_2^{m_2 m_3} (C\gamma^5)^{\alpha_2\alpha_3} - 2 (\gamma^5)^{\alpha\alpha_1}\, \delta^{mm_1}\, \tau_2^{m_2 m_3}\, C^{\alpha_2\alpha_3} -$$
$$- (\gamma^\mu)^{\alpha\alpha_1} \delta^{mm_1}\, \tau_2^{m_2 m_3} (C\gamma^\mu\gamma^5)^{\alpha_2\alpha_3} + (\gamma^\mu\gamma^5)^{\alpha\alpha_1} (\vec{\tau})^{mm_1} (\tau_2\vec{\tau})^{m_2 m_3} (C\gamma^\mu)^{\alpha_2\alpha_3}.$$

In terms of the isotopic fields (u - and d - quarks) the three-quark currents corresponding to the proton and neutron have the same form as in the QCD sum rules/28/

$$J^A_p = Q^A(u,d) \qquad (1.6)$$
$$J^A_n = Q^A(d,u).$$

where

$$Q^T(u,d) = -6i\,\sigma^{\mu\nu}\gamma^5 d^{a_1} (u^{a_2} C \sigma^{\mu\nu} u^{a_3})\, \varepsilon^{a_1 a_2 a_3} \qquad (1.7)$$
$$Q^V(u,d) = -6i\,\gamma^\mu\gamma^5 d^{a_1} (u^{a_2} C\gamma^\mu u^{a_3})\, \varepsilon^{a_1 a_2 a_3}.$$

The hadron interactions are described by the S -matrix

$$S = \int d\sigma_{vac}\, T\exp\left(i\int dx\, \mathcal{L}_H(x)\right). \qquad (1.8)$$

The time-ordering in (1.8) is the familiar with T -product for the hadron and quark fields. The quark propagator has the following form:

$$S(x_1, x_2|B_{vac}) = \langle 0|T(q(x_1)\bar{q}(x_2))|0\rangle = -(\hat{p}+\hat{B}_{vac})^{-1}\cdot\delta(x_1-x_2). \qquad (1.9)$$

Here $B_{vac}(x,\sigma_{vac})$ denotes the vacuum gluon field characterized by the set of parameters $\{\sigma_{vac}\}$. The procedure of averaging the S - matrix (1.8) induced quark diagrams over the vacuum gluon fields $d\sigma_{vac}$ is supposed, on the one hand, to provide the quark confinement, and on the other hand, to lead to the results free from ultraviolet divergences.

The confinement ansatz in the case of one-loop quark diagrams describing meson-meson interactions is defined in the following manner:

$$\int d\sigma_{vac}\; tr\left[M(x_1)S(x_1,x_2|B_{vac})\ldots M(x_n)S(x_n,x_1|B_{vac})\right] \longrightarrow \qquad (1.10)$$
$$\longrightarrow \int d\sigma_v\; tr\left[M(x_1)S_v(x_1-x_2)\ldots M(x_n)S(x_n-x_1)\right],$$

where

$$S_v(x) = \int \frac{d^4p}{(2\pi)^4 i}\cdot e^{-ipx}\cdot\frac{1}{v\Lambda-\hat{p}} \qquad (1.11)$$

The dimensional parameter Λ characterizes the quark confinement region. The measure $d\sigma_v$ is defined in the following way:

$$\int d\sigma_v \cdot \frac{1}{v-z} = G(z) = a(-z^2) + z \cdot b(-z^2). \tag{1.12}$$

The universal function $G(z)$ is the same for all hadron processes in the low-energy region; $G(z)$ is assumed to be the entire analytical function and to rapidly decrease in the Euclidean direction.

In the calculations of the effects of meson physics [17-22] the following expression for the function $G(z)$ was used:

$$\begin{aligned} a(u) &= 2\exp(-u^2-u) \\ b(u) &= 2\exp(-u^2+0.4u). \end{aligned} \tag{1.13}$$

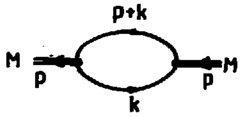
The parameter Λ was determined by fitting the main constants of meson physics to their experimental values and turned out to be:

$$\Lambda = 460 \text{ MeV}. \tag{1.14}$$

The wave-function renormalization constant for hadrons vanishes due to the hadronization condition accepted in QCM [17,18]

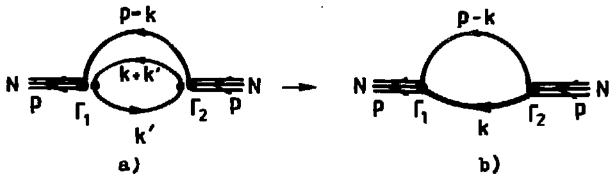
$$Z_H = 1 + g_H^2 \widetilde{\Pi}'_H(m_H) = 0. \tag{1.15}$$

Here $\widetilde{\Pi}_H(\hat{p})$ denotes the self-energy operator for the hadron H ; $\widetilde{\Pi}_H(\hat{p})$ is determined by the diagrams in fig.1 and fig. 2a in the case of mesons and nucleons respectively.



F i g. 1

In the case of pseudoscalar and vector mesons the hadronization condition specifies the quark-hadron coupling constant g_H in terms of the hadron mass m_H and quark parameters (1.13) and (1.14). In the case of scalar mesons and nucleons several forms of the quark-hadron interactions are possible and the hadronization condition imposes a restriction on the coupling constants g_H^{Λ}. The approximations to the S -matrix (1.8) elements are constructed in accordance with the $1/N_c$ expansion [17,18]. The lowest-order approximation corresponds to the tree diagrams of the chiral theory; however, an essential difference occurs: in the chiral theory the point-like hadron-hadron interaction vertices without any internal struc-



F i g. 2

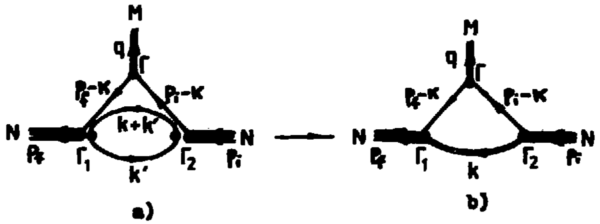
ture correspond to the local interaction whereas in QCM the hadron-hadron vertices are described by the quark loops which determine the hadron structure - the formfactors, the slope parameters, etc.

The symmetry of the three-quark nucleon currents (1.4) with respect to the quark field transposition provide the basis for the quark-diquark approximation. Due to this symmetry in the diagrams describing the nucleon self-energy operator (fig. 2a) and the vertex part (fig. 3a), the subdiagram corresponding to the independent quark loop.

$$\Pi^{\Gamma_1\Gamma_2}(k)=\int\frac{d^4k'}{4\pi^2 i}\cdot tr\left[\Gamma_1 G(\hat{k}+\hat{k}')\Gamma_2 G(\hat{k}')\right] \qquad (1.16)$$

is singled out. Furthermore, from the ward identity for the nucleon electromagnetic vertex and from the hadronization condition (1.15) the following identity can be derived

$$g_{NT}^2 F_{TT}(p)+g_{NV}g_{NT}F_{VT}(p)+g_{NV}^2 F_{VV}(p)=0 \qquad (1.17)$$



F i g. 3

This relation holds for the arbitrary value of momentum p . Thus, the following relations should be fulfilled simultaneously:

$$\Gamma_{TT}(p)=\int\frac{d^4k}{\pi^2 i}\cdot\Big\{36\,G(\hat{p}-\hat{k})\Pi^{PP}(k)+36\gamma^5 G(\hat{p}-\hat{k})\gamma^5\Pi^{SS}(k)-$$
$$-3\,\sigma^{\mu\nu}\gamma^5 G(\hat{p}-\hat{k})\,\sigma^{\alpha\beta}\gamma^5\,\Pi^{TT}_{\mu\nu,\alpha\beta}(k)\Big\}=0,$$
$$\Gamma_{VT}(p)=\int\frac{d^4k}{\pi^2 i}\cdot\Big\{24\,G(\hat{p}-\hat{k})\Pi^{PP}(k)-24\gamma^5 G(\hat{p}-\hat{k})\gamma^5\,\Pi^{SS}(k)+$$
$$+6\,G(\hat{p}-\hat{k})\gamma^\mu\Pi^{PA}_{\mu}(k)-6\gamma^\mu G(\hat{p}-\hat{k})\Pi^{AP}_{\mu}(k)+ \tag{1.18}$$
$$+3\sigma^{\alpha\beta}\gamma^5\,G(\hat{p}-\hat{k})\gamma^\mu\gamma^5\,\Pi^{TV}_{\alpha\beta,\mu}(k)-3\gamma^\mu\gamma^5 G(\hat{p}-\hat{k})\sigma^{\alpha\beta}\gamma^5\,\Pi^{VT}_{\mu,\alpha\beta}(k)\Big\}=0$$
$$\Gamma_{VV}(p)=\int\frac{d^4k}{\pi^2 i}\cdot\Big\{4\,G(\hat{p}-\hat{k})\Pi^{PP}(k)+4\gamma^5 G(\hat{p}-\hat{k})\gamma^5\Pi^{SS}(k)-$$
$$-\gamma^\mu G(\hat{p}-\hat{k})\gamma^\nu\,\Pi^{AA}_{\mu\nu}(k)+2\,G(\hat{p}-\hat{k})\gamma^\mu\Pi^{PA}_{\mu}(k)-$$
$$-2\gamma^\mu G(\hat{p}-\hat{k})\,\Pi^{AP}_{\mu}(k)+3\gamma^\mu\gamma^5 G(\hat{p}-\hat{k})\gamma^\nu\gamma^5\,\Pi^{VV}_{\mu\nu}(k)\Big\}=0\,.$$

The basic assumption consists in the substitution of the one-loop diagrams for the diagrams in fig. 2a and 3a in accordance with the following prescription

$$\int\frac{d^4k}{\pi^2 i}\int d\sigma_\nu\;\Gamma_1 S_\nu(\hat{p}-\hat{k})\Gamma_2\,\Pi^{\Gamma_1\Gamma_2}(k)\longrightarrow$$
$$\longrightarrow\int\frac{d^4k}{\pi^2 i}\int d\sigma_\nu\;\Gamma_1 S_\nu(\hat{p}-\hat{k})\Gamma_2\,D_\nu^{\Gamma_1\Gamma_2}(k)\quad, \tag{1.19}$$
$$\int\frac{d^4k}{\pi^2 i}\int d\sigma_\nu\;\Gamma_1 S_\nu(\hat{p}_f-\hat{k})\,\Gamma\, S_\nu(\hat{p}_i-\hat{k})\Gamma_2\,\Pi^{\Gamma_1\Gamma_2}(k)\longrightarrow$$
$$\longrightarrow\int\frac{d^4k}{\pi^2 i}\int d\sigma_\nu\;\Gamma_1 S_\nu(\hat{p}_f-\hat{k})\,\Gamma\, S_\nu(\hat{p}_i-\hat{k})\Gamma_2\,D_\nu^{\Gamma_1\Gamma_2}(k)\quad,$$

where

$$D_\nu^{\Gamma_1\Gamma_2}(k)=\frac{d^{\Gamma_1\Gamma_2}}{\nu^2\Lambda^2+\Lambda_d^2-k^2}\,. \tag{1.20}$$

The coefficients $d^{\Gamma_1\Gamma_2}$ in (1.20) are determined from the identity (1.17). It turns out that for arbitrary values of g_{NT} and g_{NV} the solution to (1.17) does not exist since the identity $\Gamma_{VT}(p)=0$ cannot be fulfilled with any choice of $d^{\Gamma_1\Gamma_2}$. For the special cases when either $g_{NT}=0$, $g_{NV}\neq 0$ or $g_{NT}\neq 0$, $g_{NV}=0$ the coefficients $d^{\Gamma_1\Gamma_2}$ can be uniquely determined

$$
d^{\Gamma_1\Gamma_2} = \begin{cases} 1, & \Gamma_1=\Gamma_2=S,P \\ g_{\mu\nu}/3, & \Gamma_1=\Gamma_2=V \\ g_{\mu\nu}, & \Gamma_1=\Gamma_2=A \\ 2g_{\mu\lambda}g_{\nu\rho}, & \Gamma_1=\Gamma_2=T \\ 0, & \Gamma_1\neq\Gamma_2 . \end{cases} \tag{1.21}
$$

In other words, the quark-diquark approximation to the three-quark structure of nucleons can be carried out separately for each nucleon current. Below, the quark-diquark approximation in the care of the tensor current is referred as a T -case, and in the case of the vector current as a V -case.

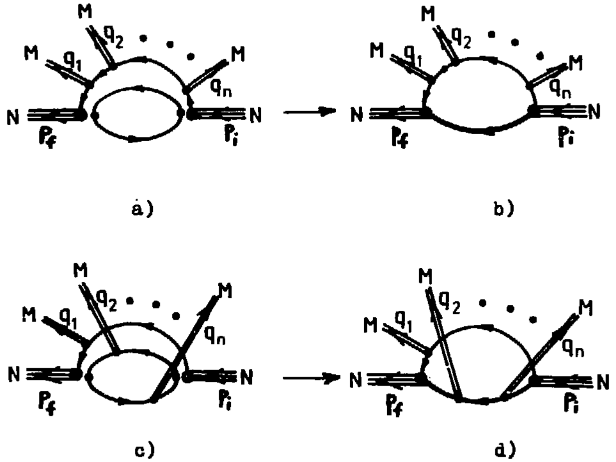
The interaction of nucleons with several mesons is described by two-different types of diagrams. The diagrams in the figs. 4a and 4c correspond to the processes where all mesons are emitted from the single quark line and different quark lines, respectively. In the quark-diquark picture of nucleons, the diagram describing the meson-diquark interactions (fig. 4d) should be substituted for the diagram in fig. 4c. In the present paper, in the first approximation diquarks are assumed to be the rigid formations like the nucleon "core" which do not interact with other particles (mesons, leptons and photons). Consequently, in this approximation the diagrams in fig. 4 c are neglected.

Λ_d corresponding to the diquark "mass" is supposed to be an adjustable parameter and is determined by fitting the low-energy properties of a nucleon (proton and neutron): the magnetic moments μ_p and μ_n, the electromagnetic radii r_p^E, r_p^M, r_n^E and r_n^M ratio in the neutron β -decay and the strong coupling constant $G_{\pi NN}(0)$ to their experimental values[26]

a) T - case $\quad \Lambda_d$ = 680 MeV

b) V - case $\quad \Lambda_d$ = 419 MeV.

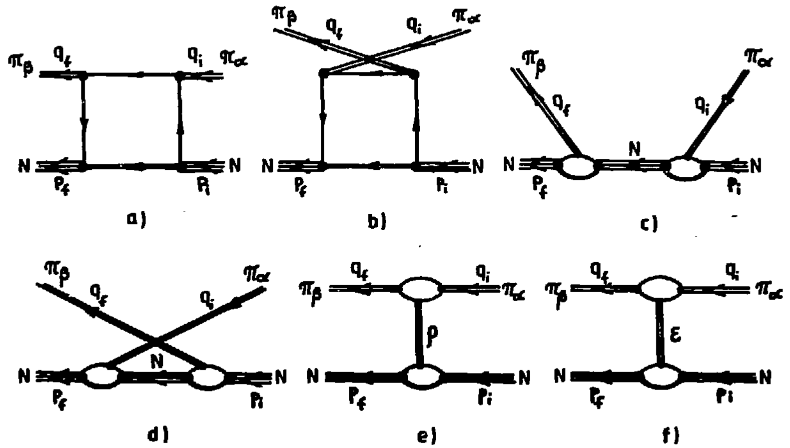
In addition, this choice for Λ_d allows the description of the NN - scattering phase shifts in the T -case[25,26]. In the present work, the parameter Λ_d will be restricted to these values.



F i g. 4

2. THE S - SCATTERING LENGTHS AND THE ($p\pi^-$)- ATOM LIFETIME

Diagrams describing the πN - scattering to the leading order of the $1/N_c$ expansion are given in fig. 5.



F i g. 5

In figs. 5a and 5b the s - and u - channel quark box diagrams are depicted and in figs. 5c and 5d the nucleon pole diagrams are given. Diagrams in figs. 5e and 5f describe the ρ - and ε - meson exchange, respectively. In QCM, the contribution of the diagram 5e is supposed to imitate the contribution of the strong nonresonant interaction of pion pairs in the 0^{++} channel in the πN -scattering process.

The invariant amplitude has the following form:

$$\begin{aligned} t^{\beta\alpha}(p_f,q_f;p_i,q_i) &= t_{\Box}^{\beta\alpha}(p_f,q_f;p_i,q_i)+t_{\Box}^{\alpha\beta}(p_f,-q_i;p_i,-q_f)+ \\ &+g_{\pi NN}(p_f,p_f+q_f)\tau^{\beta}\frac{1}{m_N-\hat{p}_f-\hat{q}_f}\tau^{\alpha}g_{\pi NN}(p_i+q_i,p_i)+ \\ &+g_{\pi NN}(p_f,p_f-q_i)\tau^{\alpha}\frac{1}{m_N-\hat{p}_f+\hat{q}_i}\tau^{\beta}g_{\pi NN}(p_i-q_f,p_i)+ \\ &+i\varepsilon^{\beta\alpha\gamma}\tau_{\gamma}\,g_{\rho NN}(p_f,p_i)D_{\rho}(p_f-p_i)g_{\rho\pi\pi}(q_f,q_i)+\delta^{\beta\alpha}g_{\varepsilon NN}(p_f,p_i)D_{\varepsilon}(p_f-p_i)g_{\varepsilon\pi\pi}(q_f,q_i). \end{aligned} \tag{2.1}$$

Here p_f and p_i , q_f and q_i denote the momenta of nucleons and π -mesons in the final (f) and initial (i) states, β and α are the isotopic indices of the final and initial pions, respectively; $t_{\Box}$ correspond to the box diagrams; D_{ρ} and D_{ε} denote the renormalized propagators of ρ - and ε -mesons and $g_{\pi NN}$, $g_{\rho NN}$, $g_{\varepsilon NN}$ and $g_{\rho\pi\pi}$, $g_{\varepsilon\pi\pi}$ stand for different meson-nucleon and meson-meson vertex functions. These vertex functions were obtained in refs. /17,20,26/.

The expression for the matrix element corresponding to the s - channel "box" diagram has the following form:

$$\begin{aligned} t_{\Box}^{\beta\alpha(A)}(p_f,q_f;p_i,q_i) &= (3!)^3\,\frac{g_{NA}^2}{16\pi^2}\left(\frac{g_{\pi}}{\sqrt{2}}\right)^2\int\frac{d^4k}{\pi^2 i}\int d\sigma_{v}\cdot \\ &\cdot J_{v\Box}^{\beta\alpha(A)}\cdot\frac{1}{v^2\Lambda^2+\Lambda_d^2-k^2}\;;\quad A=V,T\,; \end{aligned} \tag{2.2}$$

where

$$\begin{aligned} J_{v\Box}^{\beta\alpha(T)} &= \tau^{\beta}\tau^{\alpha}[36Z_v+36\gamma^5 Z_v\gamma^5]+2\vec{\tau}\tau^{\beta}\tau^{\alpha}\vec{\tau}\,[\sigma^{\mu\nu}\gamma^5 Z_v\sigma^{\mu\nu}\gamma^5], \\ J_{v\Box}^{\beta\alpha(V)} &= \tau^{\beta}\tau^{\alpha}[4Z_v+4\gamma^5 Z_v\gamma^5-\gamma^{\mu}Z_v\gamma^{\mu}]+ \\ &\quad+\frac{1}{3}\vec{\tau}\tau_{\alpha}\tau_{\beta}\vec{\tau}[-\gamma^{\mu}\gamma^5 Z_v\gamma^{\mu}\gamma^5], \\ Z_v &= S_v(\hat{p}_f-\hat{k})i\gamma^5 S_v(\hat{p}_f+\hat{q}_f-\hat{k})i\gamma^5 S_v(\hat{p}_i-\hat{k})\,. \end{aligned} \tag{2.3}$$

The expression for the universal matrix element is written in the following manner:

$$\int\frac{d^4k}{\pi^2 i}\int d\sigma_\nu\,\Gamma_1 Z_\nu \Gamma_2\,\frac{1}{\nu^2\Lambda^2+\Lambda_1^2-k^2}=\Lambda^{-1}\Big[\Gamma_1\Gamma_2\cdot I_1+\Gamma_1(\hat{p}_f+\hat{p}_i)\Gamma_2\cdot I_2+$$
$$+\Gamma_1(\hat{q}_f+\hat{q}_i)\Gamma_2\cdot I_3+\Gamma_1\hat{q}_f\hat{q}_i\Gamma_2\cdot I_4+\Gamma_1\hat{q}_f(\hat{p}_f+\hat{p}_i)\hat{q}_i\Gamma_2\cdot I_5\Big]\cdot \tag{2.4}$$

The explicit form of the functions I_i is given in the Appendix. The expression for the matrix element corresponding to the u -channel "box" diagram is obtained from (2.2)-(2.4) by substituting $\beta\leftrightarrow\alpha$, $q_f\leftrightarrow -q_i$.

The on — mass — shell invariant amplitude has the following form:

$$t^{\beta\alpha}(p_f,q_f;p_i,q_i)=\delta^{\beta\alpha}t^+(p_f,q_f;p_i,q_i)+\frac{1}{2}[\tau^\beta,\tau^\alpha]t^-(p_f,q_f;p_i;q_i) \tag{2.5}$$
$$t^\pm(p_f,q_f;p_i,q_i)=A^\pm(s,t)+\frac{1}{2}(\hat{q}_i+\hat{q}_f)B^\pm(s,t)\cdot$$

The S -wave πN -scattering lengths are related to the scattering amplitude at the threshold

$$a^\pm=\frac{2m_N}{8\pi(m_N+m_\pi)}\cdot\left(A^\pm(s_0,t_0)+m_\pi B^\pm(s_0,t_0)\right), \tag{2.6}$$

where $s_0=(m_N+m_\pi)^2$, $t_0=0$. The scattering lengths corresponding to the amplitudes with the definite value of the total isotopic spin are expressed in the following way:

$$a_0^{1/2}=a^+ + 2a^-$$
$$a_0^{3/2}=a^+ - a^-\,. \tag{2.7}$$

In the present work, in the calculations of diagram 5e corresponding to the $\mathcal{E}$ -meson exchange the parameters $m_\mathcal{E}$ and δ_S were set equal to the values used in the description of meson physics (1.3) /20/. H_S was assumed to be an adjustable parameter. The $\mathcal{E}$ - meson exchange contribution was required to cancel the "box" and nucleon pole diagram contributions to the even amplitude $t^+(p_f,q_f;p_i,q_i)$ in accordance with the "pair suppression" mechanizm providing the basis for fitting the parameter H_S . It turned out that the cancellation occurs when the parameter H_S is set equal to 0.453 in the T-case and to 0.438 in the V - case. Both the values are close to the value 0.545 used in the calculations of $\pi\pi$ - scattering lengths indicating that the contribution of the strong interactions in the 0^{++} channel which is approximated by the $\mathcal{E}$ -meson exchange contribution is likely to be nearly equal for the $\pi\pi$ - and πN - scattering processes. The results of calculation of the S -wave πN -scattering lengths are listed in table 1. For comparison, the experimental data[1-3] and the results obtained within the bag model[14] and the Skyrme model[29] are given. It can be observed that the calculated values for the scattering lengths agree satisfactorily with the experiment and the results obtained within other approaches both for the T - and V -cases.

T a b l e 1

Approach	$a_0^{1/2}$, fm	$a_0^{3/2}$, fm	τ, 10^{-15} sec
Experiment[1]	0.241±0.007	-0.124±0.006	0.82±0.06
Experiment[2]	0.286±0.010	-0.153±0.0024	0.57±0.02
Experiment[3]	0.23	-0.11	0.95
Current algebra[7]	0.22	-0.11	1.00
Skyrme model[29]	0.23	-0.14	0.80
Bag model[14]	0.22	-0.11	1.00
	0.33*	-0.08*	0.65*
QCM: T- case, Λ_d=680 MeV, H_s=0.453	0.18	-0.09	1.57
QCM: V- case, Λ_d=419 MeV, H_s=0.438	0.22	-0.11	1.00

* The rescattering effects are taken into account.

In table 2 the contributions of the individual diagrams to the a^+ and a^- πN-scattering lengths are given. Both for the T - and V - cases a^-- receives leading contribution from the ρ -meson exchange diagram in accordance with the ρ -meson dominance hypothesis at low energies[7]. The expression for this contribution has the following form:

$$a_\rho^- = 2\pi \frac{m_N m_\pi}{m_N + m_\pi} \cdot \frac{1}{m_\rho^2} \cdot D_V^{-1}(0) \left[-\tilde{\Pi}_V'(m_\rho^2)\, g_{\rho NN}(0)\, g_{\rho\pi\pi}(0) \right], \quad (2.8)$$

where

$$\mathcal{D}_V(0)=\int_0^\infty du\cdot b(u)+\frac{m_\rho^2}{4\Lambda^2}\int_0^1 dt\cdot(1-t)^{1/2}\left(1+\frac{t}{2}\right)\cdot b\left(-\frac{t m_\rho^2}{4\Lambda^2}\right). \qquad (2.9)$$

Table 2

Contribution of the individual diagrams	QCM T- case		QCM V- case	
	a^+, fm	a^-, fm	a^+, fm	a^-, fm
"Box" diagrams	-0.95	$8.7\cdot10^{-4}$	-0.51	0.012
Nucleon pole diagrams	-0.22	-0.026	-0.13	-0.013
ε -meson exchange	1.17	-	0.64	-
ρ -meson exchange	-	0.114	-	0.114

Due to the Ward identity and the hadronization condition (1.15) the expression in the square brackets in (2.8) equals to unity. Thus, the quantity a_ρ^- - does not depend on the diquark "mass" Λ_d being determined solely by the shape of confinement functions and the value of the parameter Λ .

It is of interest to compare the scattering lengths obtained within QCM with the current algebra prediction. Chiral symmetry leads to the well-known Weinberg - Tomozawa relations for the S - wave πN - scattering lengths.

$$a^-=\frac{1}{3}\left(a_c^{1/2}-a_o^{3/2}\right)\approx\frac{1}{8\pi(m_N+m_\pi)}\cdot\frac{m_N m_\pi}{f_\pi^2}\approx 0.11\,fm$$
$$a^+=\frac{1}{3}\left(a_o^{1/2}+2a_c^{3/2}\right)\approx 0. \qquad (2.10)$$

For these scattering lengths the following values in QCM are obtained a^- = 0.09 fm, a^+ = 0(T - case) and a^- = 0.11 fm, a^+ =0 (V -case). Comparing these values with the result obtained with the use of the current algebra approach the conclusion can be drawn that the Weinberg - Tomazawa relations are approximately fulfilled both for the T - and V -cases. The fulfillment of the Weinberg - Tomozawa relations in QCM is ensured by the ρ -meson dominance mechanism.

Thus, in the QCM the πN - scattering lengths were obtained within the framework of the quark-diquark picture of nucleon S -wave. The model parameters were set equal to the values used in the description of the $\pi\pi$ - scattering process as well as the statical properties of mesons and nucleons. The calculated scattering lengths agree satisfactorily with the experiment and the results obtained within other approaches.

Making use of the πN - scattering lengths calculated in QCM the ($p\pi^-$)- atom lifetime is evaluated. The expression for the quantity τ_{1S} has the following form[6]:

$$1/\tau_{1S} = \frac{8\pi}{9}\left(\frac{2\Delta m}{\mu}\right)^{1/2} \left|a_0^{1/2} - a_0^{3/2}\right|^2 \left|\Psi_{1S}(0)\right|^2 \left(1 + P^{-1}\right), \qquad (2.11)$$

where $\Delta m = m_p + m_{\pi^-} - m_n - m_{\pi^0} =$ 3.31 MeV is the missing mass, $\mu = \frac{m_p m_{\pi^-}}{m_p + m_{\pi^-}}$ = 121.5 MeV is the reduced mass, $P^{-1} = \frac{W(p+\pi^- \to n+\gamma)}{W(p+\pi^- \to n+\pi^0)} = 0.61$.[27]

The quantity $\Psi_{1S}(0)$ is determined by the Coulomb wave function and the electromagnetic and strong corrections. It has been demonstrated[31] that the electromagnetic corrections turn out to make negligible contribution (less than 1%) to the Coulomb wave function. The strong corrections can be taken into account with the use of the Deser formula[32]

$$\Delta\Psi_{1S}^{S}(0) \sim \left(a_0^{1/2} + 2a_0^{3/2}\right) \frac{\left|\Psi_{2S}^{coul}(0)\right|^2}{E_{1S}^{coul} - E_{2S}^{coul}} \Psi_{1S}^{coul}(0) \cdot \qquad (2.12)$$

Since $a_0^{1/2} + 2a_0^{3/2} \approx 0$ is implicit in our calculations, $\Delta\Psi_{1S}^{S}(0) \approx 0$. Thus, the ($p\pi^-$)- atom lifetime is solely determined by the Coulomb wave function

$$\Psi_{1S}^{coul}(0) = \left(\frac{\mu^3 \alpha^3}{\pi}\right)^{1/2}, \quad \alpha = \frac{1}{137}. \qquad (2.13)$$

The results of the ($p\pi^-$)- atom lifetime calculation in QCM are given in table 1. For comparison we give the results of calculations by formula (2.11) with the use of the values for the scattering lengths taken from the experiment and other approaches.

Future measurements of the ($p\pi^-$)- atom lifetime are supposed to resolve some ambiguities in the description of the nucleon structure within various models.

In future the πN- scattering phase shifts and the NN-scattering lengths are intended to be obtained within the QCM.

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APPENDIX

The structure functions $I_i(s,t)$ (2.4) for the "box" diagrams are expressed in the following form:

$$
\begin{aligned}
I_1(s,t) &= \int d\mu_\alpha \cdot \left[3a(z) + a'(z)(z+y)\right] \\
I_2(s,t) &= \int d\mu_\alpha \cdot \left[2b(z)\cdot\alpha_1 + b'(z)\,(z+y)\cdot\frac{\alpha_1}{2}\right] \\
I_3(s,t) &= \int d\mu_\alpha \left[-b(z)(1+2\alpha_2) - \frac{1}{2}b'(z)\left((1+\alpha_3)(z+y) - (1-\alpha_2)m_\pi^2\right)\right] \qquad \text{(A.1)} \\
I_4(s,t) &= \int d\mu_\alpha \cdot \left[-a'(z)\right] \\
I_5(s,t) &= \int d\mu_\alpha \cdot \left[\frac{b(z)}{2}\,\alpha_1\right],
\end{aligned}
$$

where

$$
\begin{aligned}
d\mu_\alpha &= \prod_{i=1}^{4} d\alpha_i\,\delta(1-\alpha_1-\alpha_2-\alpha_3-\alpha_4) \\
y &= \left[-\alpha_1(\alpha_3+\alpha_4)m_N^2 + \alpha_1(1-\alpha_2)s - \alpha_3\alpha_4 t + (1-\alpha_2)(\alpha_3+\alpha_4)m_\pi^2\right]/\Lambda^2 \\
z &= \left[\alpha_1\Lambda_d^2 - \alpha_1(\alpha_3+\alpha_4)m_N^2 - \alpha_2(\alpha_3+\alpha_4)m_\pi^2 - \alpha_1\alpha_2 s - \alpha_3\alpha_4 t\right]/\Lambda^2 \qquad \text{(A.2)} \\
s &= (p_i+q_i)^2 = (q_f+p_f)^2;\ t = (p_i-p_f)^2 = (q_f-q_i)^2;\ u = (p_i-q_f)^2 = (p_f-q_i)^2
\end{aligned}
$$

$a(z)$ and $b(z)$ are the confinement functions.

The structure functions for the u -channel box diagrams are obtained from (A.1) and (A.2) by substituting $s \to u$.

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