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ПОЛТВЁРДАЯ КХД

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Abstract.

Recent results concerning the small x limit of parton distributions in perturbative QCD are reviewed. This includes in particular discussion of the bare Pomeron in perturbative QCD and of shadowing corrections. The minijet production processes and possible manifestation of semihard interactions in high energy $p\bar{p}$ elastic scattering are also discussed.

Streszczenie.

Przedstawiono przegląd współczesnych wyników dotyczących własności funkcji rozkładu partonów w granicy małych wartości zmiennej x. Przedyskutowano m.in. problem pierwotnego Pomeronu w perturbacyjnej chromodynamice kwantowej oraz poprawki cieniujące. Omówiono również procesy produkcji minidżetów oraz wpływ oddziaływań półtwardych na rozpraszanie elastyczne $p\bar{p}$ przy wysokich energiach.

Резюме.

Представлены современные результаты касающиеся функций рас_ пределения партонов в границы малых значении переменной x. Продискутировано проблему затравочного Померона и экрапировку. Рассмотрены тоже процессы рождения струи и влияние полтвердых взаимодействий на поведение упругого рассеяния в области высоких энергия.

1. Introduction.

The high energy hadronic processes are conventionally divided on the soft and the hard ones depending upon the values of transverse momenta involved. For the moderately large energies (i.e. up to top ISR energies or so) the bulk of the cross section comes from the soft processes which are beyond the region of applicability of perturbative QCD. It has however been realised recently [1] that at sufficently high energy (i.e.above 1 TeV or so) important if not the dominant role can be played by the so called semi-hard processes [2] like the (mini) jet production [3] etc. Those processes are still "hard" in the sense that they involve relatively large transverse momenta (i.e.> 2 GeV/c or so) yet they correspond to a collision of very soft partons (predominantly gluons) which carry very small momentum fraction x of their parent hadrons ($x<10^{-2}$). Perturbative QCD predicts very rapid increase of parton distributions in the small x limit and it is this increase which leads in particular to the very strong increase of the mini-jet cross section with energy [4-8]. The semihard interactions are therefore closely linked with the small x limit of parton distributions. This "small x physics" quite apart from its straightforward phenomenological implications contains several very interesting theoretical aspects due to the fact that it is related to the so called Regge limit in the region of applicability of perturbative QCD [2,9]. The study of the small x limit of parton distributions in perturbative QCD therefore follows the conventional track adopted in the analysis of high energy limit in field theory [10,11]. Thus in the first step which corresponds to the "leading logarithmic approximation" one obtains the so called "bare Pomeron" [2,11-14]. In gauge theories this "bare Pomeron" violates unitarity since it gives cross-sections that increase as a power of energy. As the next step one therefore considers various rescattering and absorptive "corrections" which restore unitarity. In the case of the parton distributions in the small x limit the latter correspond to the shadowing corrections which tame the parton overcrowding implied by perturbative QCD [2,15-17].

The main aim of those lectures is to review various problems of this "small x physics" and their possible phenomenological implications. In the next Section we recall the classical QCD improved parton model for deep inelastic lepton hadron scattering, remind the connection between the high energy limit of the virtual Compton scattering cross-section and the small x limit of parton distributions , recall the leading order Altarelli-Parisi evolution equations for parton distributions and disuss the small x-behaviour of their solution. The Section 3 is related to the "bare Pomeron" in perturbative QCD. In this Section the Balitzkij-Lipatov equation [12-14] is recalled and its solution discussed. In Section 4 the problem of "parton overcrowding" and the shadowing corrections to parton distributions are discussed. The Section 5 is devoted to the discussion of the minijet production in hadronic collisions including its effects for elastic $p\bar{p}$ scattering in high energy.

2. The QCD improved parton model for deep inelastic lepton-nucleon scattering in the small x region and small x behaviour of parton distributions in the leading order QCD.

The one photon exchange mechanism (see Fig.1) relates the inelastic muon-nucleon scattering cross-section to the total virtual Compton scattering total cross-section which by the optical theorem is related to the imaginary part of the forward virtual Compton scattering amplitude. This imaginary part is described by the tensor $W_{\mu\nu}(p,q)$ which is expressed in the following way in terms of the dimensionless structure functions $F_{1,2}(x,Q^2)$ [18]:

$$W_{\mu\nu}(p,q) = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) \frac{F_1(x,Q^2)}{M}$$

$$+ \left(p_\mu - \frac{pq}{q^2} q_\mu \right)\left(p_\nu - \frac{pq}{q^2} q_\nu\right) \frac{F_2(x,Q^2)}{M^2} \qquad (2.1)$$

The definition of various variables is standard i.e.:

$$\nu = pq/M, \quad Q^2 = -q^2, \quad x = Q^2/2M\nu \tag{2.2}$$

where p and q are the four momenta of the nucleon and of the virtual photon respectively and M is the nucleon mass. The deep inelastic scattering corresponds to the Bjorken limit [18]:

$$Q^2 \to \infty, \ \nu \to \infty \ ; \ x \text{ fixed} \tag{2.3}$$

The small x limit of deep inelastic scattering is obviously equivalent to the limit:

$$Q^2 \ll \nu \to \infty \tag{2.4}$$

i.e. to the limit when the mass squared $s = W^2$ of the electroproduced system (notice that $s = 2M\nu - Q^2 + M^2$) is much larger than the virtuality Q^2 of the photon. The limit (2.4) is by definition the Regge limit of deep inelastic scattering [19] (i.e. Q^2 is still assumed to be large if compared to some typical hadronic scale), and it is presence of this large scale Q^2 which will eventually make it possible to use the perturbative QCD. Thus the study of the deep inelastic scattering in the small x region offers a possibility to probe the high energy limit of perturbative QCD.

In the Bjorken limit (2.3) the virtual Compton scattering is described by the QCD improved parton model (Fig.2) which in the leading order of perturbative QCD relates the structure functions $F_{1,2}(x,Q^2)$ to the quark and antiquark distribution functions:

$$F_2(x,Q^2) = x\sum_i e_i^2 [q_i(x,Q^2) + \bar{q}_i(x,Q^2)] \tag{2.5}$$

$$F_1(x,Q^2) = F_2(x,Q^2)/(2x) \tag{2.6}$$

where $q_i(x,Q^2)$ and $\bar{q}_i(x,Q^2)$ are the quark and antiquark distributions in a nucleon and e_i is the charge of the i-the quark. The sum is over the quark flavours. The variable x equals to the momentum fraction of the parent nucleon carried by the quark (antiquark). Q^2 evolution of parton densities is at large Q^2 governed by perturbative QCD [20-23]. The formulas like (2.5) correspond to the factorisation (i.e. convolution) of the hard hadronic cross section into a product of the universal parton distribution and hard cross section on the partonic level [23-24]. In (2.5) the latter is evaluated in the leading (i.e. in zero) order with respect to the QCD coupling constant. It has to be remembered however that beyond the leading logarithmic approximation the parton distributions and hard partonic cross sections are in general renormalisation scheme dependent while the measurable quantities like $F_2(x,Q^2)$ are obviously not. (Also the relation (2.6) does not hold beyond the leading logarithmic approximation).

The quark distributions are the sums of the valence and sea quark contributions i.e.:

$$q_i(x,Q^2)=q_i^v(x,Q^2) + q_i^s(x,Q^2)$$

$$\bar{q}_i(x,Q^2) = q_i^s(x,Q^2) \tag{2.7}$$

Let us also introduce the distribution $\Sigma(x,Q^2)$:

$$\Sigma(x,Q^2) = \sum_i [q_i(x,Q^2) + \bar{q}_i(x,Q^2)] \tag{2.8}$$

The QCD evolution can be most simply formulated for the moments of the quark and gluon distributions. The moment $p(n,t)$ of the distribution $p(x,t)$ is defined as below:

$$p(n,t) = \int_0^1 dx\, x^{n-1}\, p(x,Q^2) \tag{2.10}$$

where $t=\ln(Q^2/\Lambda^2)$ and n is the moment index. Knowing the moments as the analytic functions of the index n one gets the distributions $p(x,Q^2)$ from the inverse Mellin transform i.e.:

$$p(x,Q^2) = \frac{1}{2\pi i}\int_{n_0-i\infty}^{n_0+i\infty} dn\, x^{-n}\, p(n,t)$$

(2.11)

where the integration contour should be located to the right of the singularities of $p(n,t)$ in n. It follows from (2.11) that the leading small x behaviour of $p(x,Q^2)$ is controlled by the rightmost singularity of $p(n,t)$ in the n plane. In particular the singularity of the type $1/(n-a)^m$ of the moment leads to a behaviour $x^{-a}[\log(1/x)]^{m-1}/(m-1)!$ etc..

The QCD evolution equations in the leading log Q^2 approximation have the following form [20-23]:

$$\frac{dq_i^v(n,t)}{dt} = \frac{\alpha(t)}{2\pi}\,\gamma_{qq}(n)\, q_i^v(n,t)$$

$$\frac{d\Sigma(n,t)}{dt} = \frac{\alpha(t)}{2\pi}\left[\gamma_{qq}(n)\,\Sigma(n,t) + 2n_f\gamma_{qg}(n)\, g(n,t)\right]$$

$$\frac{dg(n,t)}{dt} = \frac{\alpha(t)}{2\pi}\left[\gamma_{gq}(n)\,\Sigma(n,t) + \gamma_{gg}(n)\, g(n,t)\right]$$

(2.12)

where $\alpha(t)$ is the QCD running coupling :

$$\alpha(t) = \frac{12\pi}{(33-2n_f)}\frac{1}{t}$$

(2.13)

n_f denotes the number of flavours and $g(n,t)$ is the moment of the gluon distribution in a nucleon. The evolution equations (2.12) correspond in the axial gauge to the ladder diagrams. In the case of valence quarks those diagrams contain only exchange of quarks (antiquarks) while the coupled equations for the sea quark and gluon distributions correspond to the ladders with gluon and/or quark (antiquark) exchange. The functions $\gamma_{ij}(n)$ are the moments of the corresponding splitting functions $P_{ij}(z)$ [20-23] and depend upon n as below:

$$\gamma_{qq}(n) = \frac{4}{3}\left[-\frac{1}{2} + \frac{1}{n(n+1)} - 2\,S(n)\right]$$

$$\gamma_{qg}(n) = \frac{(2+n+n^2)}{2n(n+1)(n+2)}$$

$$\gamma_{gq}(n) = \frac{4}{3}\,\frac{(2+n+n^2)}{n(n-1)(n+1)}$$

$$\gamma_{gg}(n) = 6\left[-\frac{1}{12} + \frac{1}{n(n-1)} + \frac{1}{(n+1)(n+2)} - S(n)\right] - \frac{n_f}{3} \tag{2.14}$$

where $S(n)$ denotes the analytic continuation of the sum $\sum_{i=2}^{n}\frac{1}{i}$. The leading poles of the functions $\gamma_{ij}(n)$ at n=0 for $\gamma_{qq}(n)$ and $\gamma_{qg}(n)$ and at n=1 for $\gamma_{gg}(n)$ and $\gamma_{gq}(n)$ which will be relevant for the small x behaviour have a simple physical origin - they are the consequence of the fact that the spin of a quark equals 1/2 and the spin of the gluon is equal to one.

The solution of the evolution equations is:

$$q_i^v(n,t) = q_i^v(n,t_0)\exp[\gamma_{qq}(n)\xi(t,t_0)]$$

$$\Sigma(n,t) = \Sigma^+(n,t_0)\exp[\gamma^+(n)\xi(t,t_0)] + \Sigma^-(n,t_0)\exp[\gamma^-(n)\xi(t,t_0)]$$

$$g(n,t) = g^+(n,t_0)\exp[\gamma^+(n)\xi(t,t_0)] + g^-(n,t_0)\exp[\gamma^-(n)\xi(t,t_0)] \tag{2.15}$$

where

$$\xi(t,t_o)=\int_{t_o}^{t'}dt'\,\frac{\alpha(t')}{2\pi}=\frac{6}{(33-2n_f)}\ln(t/t_o) \tag{2.16}$$

and

$$\gamma^{\mp}(n)=\frac{1}{2}\left\{\gamma_{qq}(n)+\gamma_{gg}(n)\mp\sqrt{[\gamma_{qq}(n)-\gamma_{gg}(n)]^2+8n_f\gamma_{qg}(n)\gamma_{gq}(n)}\right\} \tag{2.17}$$

The functions $q_i^v(n,t_o)$ are the moments of the input valence quark distributions at $t=t_o$ while the functions $\Sigma^{\mp}(n,t_o)$ and $g^{\mp}(n,t_o)$ can be expressed in terms of the input moments $\Sigma(n,t_o)$ and $g(n,t_o)$. As it has been mentioned above the small x behaviour of parton distributions is controlled by the singularities of the moments in the n plane. They can be present both in the input moments at $t=t_o$ as well as in the exponents in the rhs. of (2.15) -the latter being controlled by the singularities of $\gamma_{ij}(n)$. The leading singularity of $\gamma_{qq}(n)$ which governs the evolution of the valence quark distributions is a pole at n=0 [see the formula (2.14)] while the function $\gamma^{+}(n)$ which contributes to the sea quark and gluon distributions [see the formulas (2.14)-(2.17)] has a pole at n=1. The standard Regge phenomenology when applied to the virtual Compton scattering implies that the valence quark distributions which correspond to flavour non-singlet should have their small x behaviour controlled by the non-vacuum Regge poles i.e. $q_i^v(x,Q^2)\approx x^{-\alpha_R}$ where α_R is the intercept of the Regge pole trajectory $(\alpha_R\cong 1/2)$. This means that the input distributions $q_i^v(n,t_o)$ should have a pole at $n=\alpha_R$. Since $\alpha_R>0$ [i.e. the Regge pole is located to the right of the singularity of $\gamma_{qq}(n)$] the QCD evolution leaves the $x^{-\alpha_R}$ behaviour of the valence quark distributions as the leading one in the small x limit affecting only its Q^2 evolution

$$q_i^v(x,Q^2)\cong x^{-\alpha_R}\exp[\gamma(\alpha_R)\xi(t,t_o)] \tag{2.18}$$

Since $\alpha_R \cong 1/2$ the valence quarks give vanishing contribution ($\cong x^{1/2}$) to the structure function $F_2(x,Q^2)$ in the small x limit.

The singlet distributions (i.e. sea quark and gluon distributions) should in the small x region reveal the properties of the Pomeron and so they will dominate at small x. In what follows we shall be therefore only concerned with the singlet distributions. The usual assumption concerning the small limit of the sea quark distributions is the 1/x behaviour which corresponds to the virtual Compton scattering cross section being independent of ν . The 1/x behaviour of gluon distributions follows from the fact that the spin of a gluon is equal to one. The 1/x behaviour corresponds to a simple pole at n=1. Inspection of the solution (2.17) of the evolution equation shows that the pole of $\gamma^{+}(n)$ at n=1 leads to essential singularity of the moments of sea quark and gluon distributions at n=1 and to the following behaviour at small x [2,20]:

$$g(x,Q^2) \sim \frac{1}{x} \exp(2[6\xi(t,t_0)\ln(1/x)]^{1/2}) \tag{2.19}$$

with the similar behaviour for the sea quark distributions. This behaviour of sea quark distributions leads in particular to the increase of total virtual Compton scattering cross-section with energy which is faster than any power of its logarithm.
This will be the leading behaviour at small x provided the input moments are free from singularities for n>1. If however the input distributions behave as $x^{-\alpha_P^B}$ with the "bare Pomeron intercept" α_P^B above unity then this behaviour will not be changed by the QCD evolution being, of course, more singular than (2.19). In the next Section we shall argue that this behaviour with α_P^B substantially above unity can be expected in perturbative QCD in the leading log1/x approximation. In either case this very singular behaviour cannot hold down to the arbitrary small values of x since this would lead to conflict with unitarity sometimes also called the problem of parton overcrowding. This is one of the most important issues of the small x physics and we shall discuss it with some detail in Sect. 4. We conclude this Section with the

illustration of the solution of the evolution equations in the small x region. In Fig.3 we plot the Q^2 evolution of the gluon distributions in a nucleon which was assumed to have the 1/x behaviour at the reference scale $Q_0^2=5GeV^2$ [17]. It can be seen that the effect of the QCD evolution in the small x region is strong. It can lead to an increase of the gluon distribution multiplied by x by a factor as large as equal to three or so when x decreases from $x=10^{-2}$ to $x=10^{-4}$ and for $Q^2>10GeV^2$ or so (i.e. for Q^2 in the region located some distance above the reference scale Q_0^2 where the gluon distribution is contrived to have the 1/x behaviour). It has to be emphasised that this increase can be still much bigger if more singular input behaviour than 1/x is assumed [25].

3. Bare Pomeron in perturbative QCD.

In the preceeding Section we found that in the leading $\log(Q^2)$ approximation of perturbative QCD the small x behaviour of singlet distributions is more singular than 1/x. This is a simple consequence of the fact that QCD is the gauge theory and that its gauge bosons (i.e.gluons) have their spin equal to one.
The leading $\log(Q^2)$ approximation is still incomplete at small x since it neglects terms containing leading powers of log(1/x) if not accompanied by the leading power of $\log(Q^2)$. The appropriate procedure to account for those terms is provided by the leading log(1/x) approximation. For the fixed (i.e. non running coupling)α one gets the following equation derived by Balitzkij and Lipatov [12] (see also the refs [11,14]):

$$f(n,k^2) = \frac{1}{(n-1)} f^0(n,k^2)$$

$$+ \frac{3\alpha k^2}{\pi(n-1)}\int_0^\infty \frac{dk'^2}{k'^2}\left[\frac{f(n,k'^2) - f(n,k^2)}{|k'^2 - k^2|} + \frac{f(n,k^2)}{(k^4+4k'^4)^{1/2}}\right] \tag{3.1}$$

where $f(n,k^2)$ is the n-th *moment* of the gluon-hadron scattering amplitude and k and k' are transverse momentum variables. Up to corrections which are smaller by order α the function $f(n,k^2)$ equals to the derivative with respect to $\ln(k^2)$ of the moment $g(n,k^2)$ of the gluon distribution in a hadron. The equation (3.1) corresponds to the sum of ladder-like diagrams with reggeised gluon exchange. The inhomogeneous term $f^0(n,k^2)$ vanishes at $k^2=0$ due to colour singlet nature of a hadron. Taking into account asymptotic freedom corrections amounts to replacing in the integral equation (3.1) the fixed coupling α by the running one $\alpha(t)$ where $t=\ln(k^2/\Lambda^2)$ (also some lower momentum cut-off should be introduced for consistency). In the leading $\log(Q^2)$ approximation the equation (3.1) after introducing asymptotic freedom corrections reduces to the evolution equation (2.12) in the double logarithmic approximation. The latter corresponds to the approximation when only the singular term $\gamma_{gg}^{sing}(n)=6/(n-1)$ is retained in the eqs. (2.12). If the (differential) evolution equations (2.12) are rewritten in an equivalent integral form one obtains the Volterra type integral equations reflecting the ordering of transverse momenta in the corresponding ladder diagrams generating those equations. On the contrary there is no ordering in transverse momenta in the eq. (3.1) and the corresponding equation is of the Fredholm type (albeit with the unbounded kernel at least for the fixed coupling case). The generalisation of the eq.(3.1) in a way which treats both large logarithms $\log(1/x)$ and $\log(Q^2)$ on equal footing is also possible [2,26].

The equation (3.1) generates the singularity in the n plane located to the right of n=1 and the details concerning its intercept are as follows:

(i) for the fixed coupling case the analytic solution of the eq.(3.1) is possible and the intercept α_P^B of the corresponding (branch point) singularity is [12]:

$$\alpha_P^B = 1 + \frac{12\alpha}{\pi}\ln 2 \tag{3.2}$$

(ii) for the running coupling case there should be infinite number of poles at accumulating at n=1. An estimate based on variational principle gives the following bound for the leading singularity [27]:

$$1 + \frac{3.6\alpha(t_o)}{\pi} \le \alpha_P^B \le 1 + \frac{12\alpha(t_o)}{\pi} \ln 2 \qquad (3.3)$$

where $\alpha(t_o)$ is the QCD coupling at some large scale $k_o^2=\Lambda^2\exp(t_o)$. Assuming $\alpha(t_o)\cong 0.25$ ane gets from (3.3) the following estimate:

$$1.35 \le \alpha_P^B \le 1.7 \qquad (3.4)$$

For other posilible estimates of α_P^B based on an approximate solution of the eq.(3.1) see the ref. [13] and for the numerical solution of the eq.(3.1) and of its generalisation see the ref.[26].

This bare Pomeron intercept should control in particular the small x behaviour of gluon distributions leading to a power law behaviour independently of the scale Q^2:

$$g(x,Q^2) \sim x^{-\alpha_P^B} \qquad (3.5)$$

which is still more singular than the behaviour (2.19).

It should be noticed that even the lower bound in (3.4) is a relatively big number for a magnitude of the bare Pomeron intercept as compared with the typical value <1.1 of the "bare soft Pomeron" intercept obtained phenomenologically from fits to the energy dependence of total hadronic cross sections [7]. As it has already been mentioned the very singular behaviour like (2.19) or (3.5) with $\alpha_P^B>1$ cannot hold for arbitrary small values of x and the next Section is just devoted to a discussion of this problem.

4. Parton overcrowding at small x and the shadowing corrections for parton distributions.

The very steep behaviour of the gluon and sea quark distributions implied by perturbative QCD cannot hold for arbitrary small values of x and/or Q^2 since it would inevitably lead to violation of unitarity giving cross sections exceeding the Froissart bound [29]. We shall now discuss this problem using simple geometrical arguments [2,15] which will also lead to the shadowing corrections as a natural way to restore unitarity. To this aim let us consider the scattering of the highly virtual probe (i.e. gluon of virtuality Q^2) on a hadron. The QCD improved parton model which is physically equivalent to impulse approximation gives in this case the following expression for the corresponding total cross section $\sigma_{g(Q^2)h}$:

$$\sigma_{g(Q^2)h} = \sigma_{g(Q^2)g}\, xg(x,Q^2) \tag{4.1}$$

where

$$\sigma_{g(Q^2)g} \sim \frac{\alpha(t)}{Q^2} \tag{4.2}$$

is the gluon gluon total cross-section (we limit ourselves to the gluon contribution which dominates at small x). $\alpha(t)$ is the running QCD coupling [see the formula (2.13)] and $t=\ln(Q^2/\Lambda^2)$. Assuming that the total cross section $\sigma_{g(Q^2)g}$ is equal to the transverse area of the individual gluon one obtais the simple geometrical interpretation for the rhs. of the formula (4.1): it just represents the transverse area occupied by the soft gluons i.e. by gluons having their x-s in the region $x, x+\Delta x$ with $\Delta x \cong x$ (let us notice that the quantity $xg(x,Q^2)$ is equal by definition to the number of those gluons). We showed in Section 3 and 4 that the number of soft gluons $xg(x,Q^2)$ if calculated in the leading logarithmic approximation of QCD can grow indefinitely in the limit of small x. In the leading $\log(Q^2)$ approximation this behaviour is given by the formula (2.17) and it can be even more singular (i.e. $\sim x^{1-\alpha_P^B}$ with $\alpha_P^B>1$) if the leading terms containing powers of log(1/x) which define the bare Pomeron in perturbative QCD are consistently taken into account. Due to this indefinite increase of gluon distributions the transverse area occupied by the gluons can exceed, for sufficiently small values of x and/or Q^2, the transverse area of a hadron $S_h = \pi R_h^2$ where R_h is the hadronic radius. This parton overcrowding takes place for $x<x_c(Q^2)$ where $x_c(Q^2)$ satisfies the equation:

$$\frac{\alpha(t)}{Q^2} x_c(Q^2) g[x_c(Q^2), Q^2] = \mathrm{const}\,\pi R_h^2 \tag{4.3}$$

When this happens one cannot longer treat the partons as non-interacting i.e. one goes beyond the region of applicability of the impulse approximation and the shadowing of partons has to be taken into account. Those shadowing corrections modify the evolution equation which, in the small x limit, has now the following form [15,16,31]:

$$\frac{\partial^2 xg(x,Q^2)}{\partial \ln Q^2 \partial \ln(1/x)} = \frac{3\alpha(t)}{\pi} xg(x,Q^2) - \frac{[3\alpha(t)]^2}{4r^2Q^2} [xg(x,Q^2)]^2 \tag{4.4}$$

where r^2 is the parameter characterising the hadronic radius which can be, for instance, related to the energy independent part of the slope of the diffractive peak or to the integral of the square of the hadronic form-factor etc. The linear term in the eq.(4.4) corresponds to the approximation of keeping only the most singular term in the small x limit in the ordinary leading $\log Q^2$ QCD evolution discussed in Sec.2. Generalisation of this equation which takes into account complete leading $\log(Q^2)$ evolution kernels is straightforward [16,31].

In the region when the non-linear term in the eq.(4.4) can be neglected one obtains the distributions behaving in the way described in the preceeding Sections. The non-linear term tames however the indefinite increase of gluon distributions and in the limit $x\to 0$ the solution of this equation approaches the saturation limit $xg^{sat}(x,Q^2)$

$$xg^{sat}(x,Q^2) = \frac{4\pi r^2 Q^2}{3\alpha(t)} \tag{4.5}$$

which corresponds to the vanishing of the right hand side of the eq.(4.4). It should be noticed that although the shadowing corrections eliminate the indefinite increase of the parton distributions in the limit $x\to 0$ they introduce at the same time the linear scaling violation. This can have important phenomenological implications for the $p_\perp$ distribution of jets flattening their $p_\perp$ spectrum in the region where the shadowing is important [30]. In practice it is interesting to have an estimate of those corrections for $x>10^{-4}$ and $Q^2>5GeV^2$ or so which is relevant for the jet production up to the SSC energy range (see the next Section). The first order shadowing corrections $\Delta g(x,Q^2)$ to gluon distributions are given by the following formula [17]:

$$\Delta g(x,Q^2) = -\frac{1}{4r^2x}\int_x^1 \frac{dx'}{x'}\int_{Q_0^2}^{Q^2} dQ'^2 \left[\frac{3\alpha(t')}{Q'^2}x'g(x',Q'^2)\right]^2 D(x/x',Q^2;Q'^2) \tag{4.6}$$

where $g(x,Q^2)$ is the leading order gluon distribution in a hadron while the function $D(z,Q^2;Q'^2)$ is the distribution which satisfies the same evolution equation as the gluon distribution but is normalised to 1 at $Q'^2=Q^2$. The result of the estimate of the shadowing corrections is that they are relatively small and do not exceed 15% or so of the leading order gluon distributions for $x>10^{-4}$ [16,17]. It has to be emphasised again that those estimates were based on the "conventional" distributions constrained to behave as $1/x$ at some fixed reference scale. The situation can however be different if the input distributions are assumed to be more singular. Finally let us point out that the shadowing effects are enhanced for the simple combinatorial reasons in the case of the parton distributions in nuclei. The formula (4.6) with $r^2 \to r_A^2 \sim A^{2/3}$ and with $g(x,Q^2) \to g_A(x,Q^2) = Ag(x,Q^2)$ where $g(x,Q^2)$ is the gluon distribution in a nucleon implies that the first order shadowing corrections $\Delta g_A(x,Q^2)$ of parton distributions in nuclei are proportional to $A^{4/3}$ and so one gets $\Delta g_A(x,Q^2)/g_A(x,Q^2) \sim A^{1/3}$ i.e. the effective enhancment of the shadowing corrections to parton distributions in the nuclear enviroment. It has to be remembered that the formula (4.6) when used in the nuclear case is only applicable for sufficiently small values of x [$x < 1/(2mR_A)$] where m is the nucleon mass and R_A the nuclear radius . For bigger values of x the nuclear form factor introduces important effect reducing the magnitude of shadowing [31-33].

5. Minijet production in hadronic collisions and its influence on elastic scattering.

The large $p_\perp$ jet production in hadronic collisions corresponds in the QCD improved parton model to the hard collision of partons (see. Fig.4). The inclusive two-jet cross section is given by the following formula [28]:

$$\frac{d\sigma}{dy_1 dy_2 dp_\perp^2} = \frac{\pi}{s}\, g(x_1,p_\perp^2) g(x_2,p_\perp^2) \hat{\sigma}(\hat{s},\hat{t},\hat{u}) \tag{5.1}$$

where $g(x_{1,2},p_\perp^2)$ are the gluon distributions in the colliding hadrons (we limit ourselves to the contribution of the gluonic jets), $y_{1,2}$ are the rapidities of the produced jets, s is the total cen-ter of mass energy squared of the colliding hadrons and the cross-section $\hat{\sigma}(\hat{s},\hat{t},\hat{u})$ is the the hard gluon gluon scattering cross-section which depends on the invariants $\hat{s},\hat{t},\hat{u}$ describing this scattering. The kinematics of the two jet production implies that $\hat{s} > 4p_\perp^2$ i.e.:

$$x_1 x_2 s > 4p_\perp^2 \tag{5.2}$$

It follows from (5.2) that a "typical" value of x probed in the jet production is $\langle x\rangle \cong p_{\perp}/s^{1/2}$. Defining a minijet as a jet corresponding to moderately large $p_{\perp}$ (i.e. $p_{\perp}\geq p_{\perp}^{min}\cong 2GeV/c$ - $3GeV/c$ or so) we can find that the typical "small x" values which are probed in the mini-jet production are $\langle x\rangle\cong 10^{-2}$ for the CERN $p\bar{p}$ collider ($s^{1/2}$<1 TeV) ,$\langle x\rangle\geq 10^{-3}$ for the Tevatron and $\langle x\rangle\cong 10^{-4}$ for the SSC energy ranges respectively. Integrating the parton model formula over the available phase-space one gets the integrated inclusive jet yield $\sigma^{incl}(s)$ equal (by definition) to a product of the average number of semihard interactions and the total inelastic cross section. It is evident that the energy dependence of $\sigma^{incl}(s)$ is governed by the small x behaviour of gluon distributions. Estimate of this cross-section gives values being already in the milibarn range (i.e. comparable with the cross-section of soft processes) for the CERN $p\bar{p}$ collider energies ($\sigma^{incl}(s)\geq 10$ mb or so). Strong increase of gluon distributions in the region between $x=10^{-2}$ and $x=10^{-4}$ implies that the inclusive minijet yield can exceed 100-200 milibarns or so in the SSC energy range. Behaviour of the corresponding part of the total cross section which corresponds to jet production can be however different. This is due to the fact that at very high energies processes with several semihard interactions (i.e. multiple minijet production) are expected to be important. If this happens then the inclusive jet cross section is not longer equal to the corresponding part of the total cross section. In order to calculate the latter one proceeds in the following steps [8]:

(1) one defines the bare semihard cross-section $\sigma_{SH}^{bare}(s)$ which corresponds to the contribution of the bare QCD Pomeron to hadron-hadron total cross section. An approximate way to estimate this quantity is provided by the integral over the QCD improved parton model formula (4.1) over the restricted region of phase space in order to avoid double counting[6];

(2) one introduces the absorptive part $Im\Omega_{SH}(b^2,s)$ corresponding to semihard interactions for the fixed impact parameter b arriving at the following factorisable form:

$$Im\Omega_{SH}(b^2,s) = f(b^2)\sigma_{SH}^{bare}(s) \qquad (5.3)$$

where $f(b^2)$ can be calculated (for instance) from the hadronic form-factor [8,35].

(3) One introduces multiple scattering and absorption in the eikonal approximation which automatically enforces unitarity. In this approximation one gets the following representation for the cross-section $\sigma_{SH}(s)$ for (mini) jet production:

$$\sigma_{SH}(s) = \pi \int_0^\infty db^2 \, (1-\exp[-2Im\Omega_{SH}(b^2,s)]) \tag{5.4}$$

(4) One appeals to the old theorem [34] which states that the absorptive and rescattering corrections in the hadron-hadron channel cancel in the inclusive quantities. This means that the product of the average number of semihard interactions and the total inelastic cross section is equal to the integral of the QCD improved parton model formula as before.

As an illustration we plot in Fig.5 the three cross-sections: $\sigma^{incl}(s)$, $\sigma^{bare}_{SH}(s)$ and $\sigma_{SH}(s)$ $p\bar{p}$ collision. One can see that at very high energy $\sigma_{SH}(s) \ll \sigma^{bare}_{SH}(s)$ and this means that the unitarity effects are important. One can also see that since $\sigma^{incl}(s) \gg \sigma_{SH}(s)$ the average multiplicity of minijets is expexted to be large and rising rapidly with increasing energy [8].

Unitarity condition connects multiparticle production with the (imaginary part of) elastic scattering amplitude and so the onset of the minijet production has to influence the properties of the elastic scatt-ring as well. In order to analyse this problem one has to combine the semihard interactions with the soft ones in a way which respects unitarity. To this aim one assumes that the eikonal function $\Omega(b^2,s)$ is a sum of the soft and semihard contributions [6-8,35]:

$$\Omega(b^2,s) = \Omega_{soft}(b^2,s) + \Omega_{SH}(b^2,s) \tag{5.5}$$

The imaginary part of the semihard component is given by (5.3). In order to calculate its real part one can use the fact that in the wide energy range the bare cross-section $\sigma_{SH}^{bare}(s)$ has a simple power law parametrisation [7]:

$$\sigma_{SH}(s)=\sigma_0 s^{\beta} \tag{5.6}$$

with $\beta \cong 0.35$. The real part $Re\Omega_{SH}(b^2,s)$ is then related to $Im\Omega_{SH}(b^2,s)$ from analyticity :

$$Re\Omega_{SH}(b^2,s)=tg(\pi\beta/2)*Im\Omega_{SH}(b^2,s) \tag{5.7}$$

Assuming that the soft interactions obey the geometrical scaling law [36] (i.e. that the eikonal $\Omega_{soft}(b^2,s)$ depends upon a single variable $b^2/R^2(s)$ where R(s) is the energy dependent radius) one can extrapolate the geometrical scaling parametrisation of $\Omega_{soft}(b^2,s)$ from the ISR to the collider and higher energy regions. One can study in this way an interesting problem whether the onset of semihard interactions can account quantitatively for various changes observed in elastic $p\bar{p}$ scattering when moving from ISR to collider energy range which are manifestations of violation of geometrical scaling. It has to be observed at this point that the factorised form (5.4) violates manifestly the geometrical scaling. Results of the analysis of elastic $p\bar{p}$ scattering obtained in the ref.[35] are presented in Figs. 6-8. It is interesting to observe that one can quantitatively correlate the onset of the central semihard interactions manifesting itself in the minijet production with the violation of geometrical scaling in elastic scattering (increase of $\sigma_{el}(s)/\sigma_{tot}(s)$ with increasing energy, strong energy dependence of $d\sigma/dt$ at the diffractive minimum etc.). The value of the quantity ρ=ReA/ImA where A is the forward $p\bar{p}$ scattering amplitude which is obtained within this approach is significantly smaller than the value ρ=0.24 obtained experimentally in the CERN $p\bar{p}$ collider energy range [37] (see however the ref.[38] for the more detailed discussion of this problem). Various effects of the onset of the minijet production for changes in multiparticle production with increasing energy are discussed in refs. [7,39-41].

To summarise we tried in those lectures to give an account of the recent developments in the small x physics. We discussed in particular with some detail small x behaviour of parton distributions in perturbative QCD and its phenomenological implications for mini-jet production. Theoretically those problems are related to the analysis of the effects of infrared singularities (responsible , for instance, for the appearence of powers of logarithms of 1/x etc.) for physical quantities decribing the processes in which space-like partons participate. Finally let us emphasise that there exists the whole class of phenomena not discussed here which is related to the infrared properties of processes like fragmentation of jets produced in e^+e^- annihilation etc. reflecting the evolution of **time-like** partons in perturbative QCD. In this case the various coherence effects were found to play the crucial role [42-44] and for the recent discussion of the coherence effects also for the space-like case see the refs.[45,46].

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Figure captions

Fig 1. Inelastic lepton nucleon scattering.

Fig 2. Parton model for the virtual Compton scattering.

Fig 3. Leading $\log(Q^2)$ QCD evolution of gluon distributions in a nucleon in the small x region for different values of x. (. . . .) $x=10^{-2}$, (- - -) $x=10^{-3}$, (———) $x=10^{-4}$, (-. -. -) $x=10^{-5}$.

Fig 4. Two gluon jet production in hadron-hadron collision.

Fig 5. Energy dependence of the cross sections characterising the minijet production. (-. -. -) $\sigma_{SH}(s)$, (———) $\sigma_{SH}^{bare}(s)$, (- - -) $\sigma^{incl}(s)$.

Fig 6. Cross section for minijet production in $p\bar{p}$ collisions within the model of the ref. [35] compared with data.

Fig 7. Energy dependence of σ_{tot}, σ_{el}/σ_{tot} and ρ for $p\bar{p}$ scattering [35].

Fig 8. Differential cross-section $d\sigma/dt$ of $p\bar{p}$ scattering wiithin the model of the ref. [35] compared with the experimental data from the CERN $p\bar{p}$ collider.

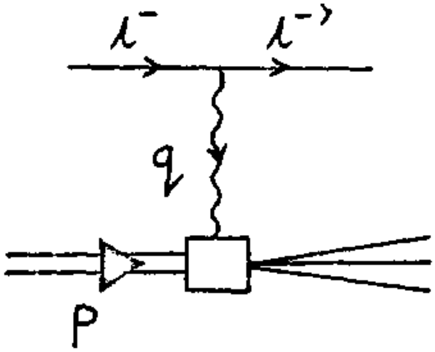
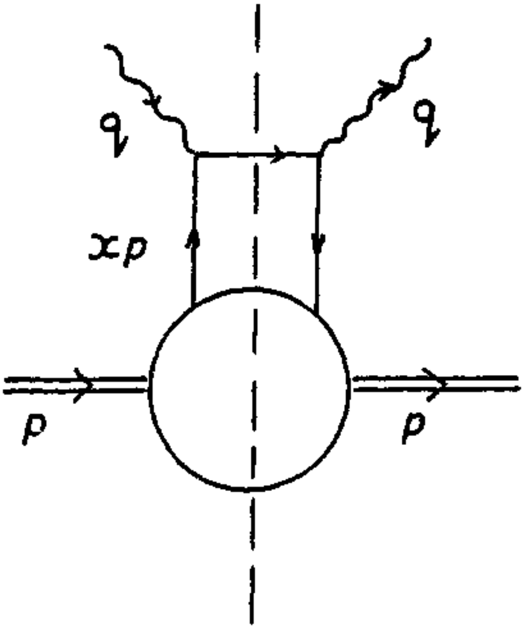


Fig.1

Fig.2



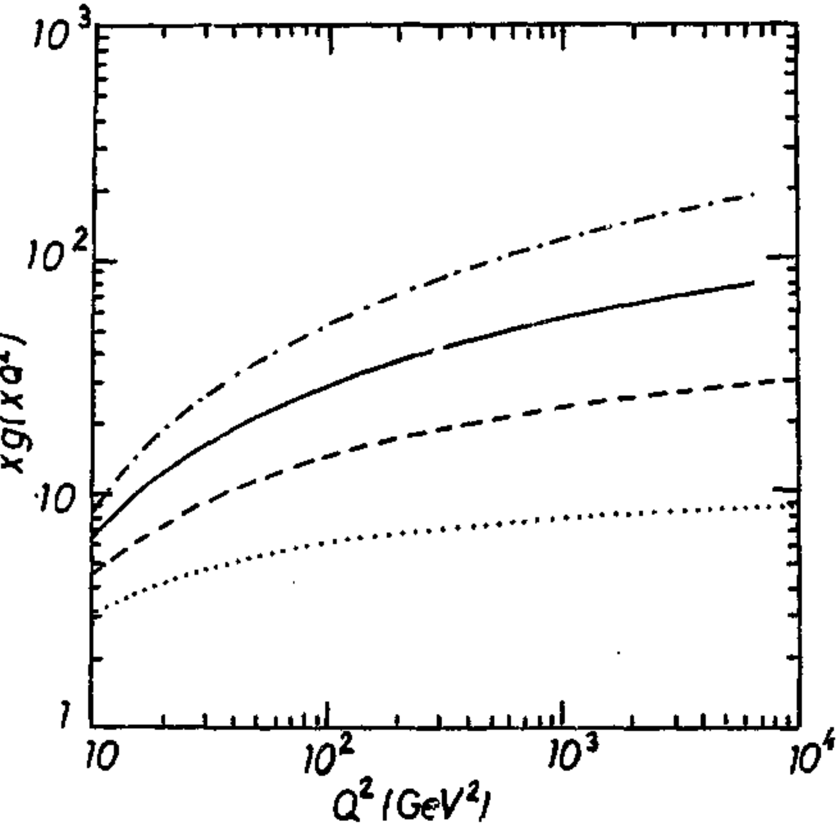
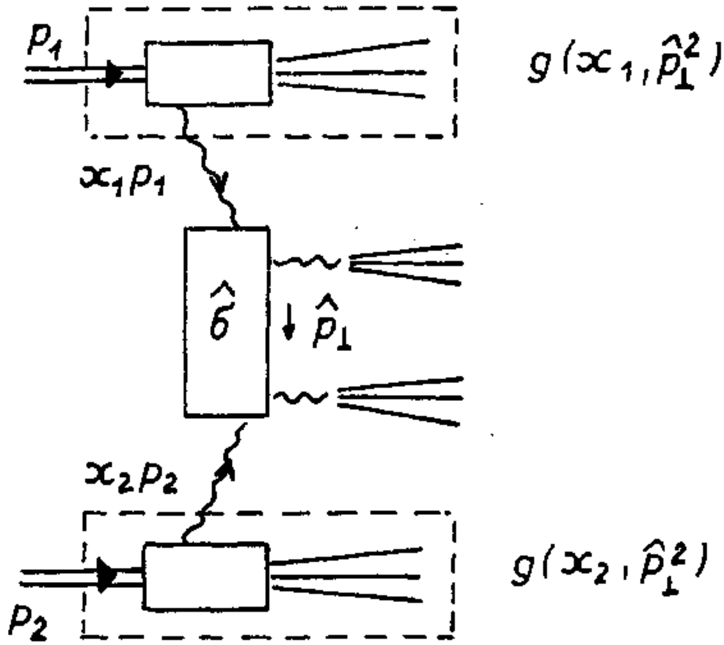


Fig.3



p_1
$g(x_1, \hat{p}_\perp^2)$
$x_1 p_1$
$\hat{\sigma}$
$\hat{p}_\perp$
$x_2 p_2$
$g(x_2, \hat{p}_\perp^2)$
p_2

Fig. 4

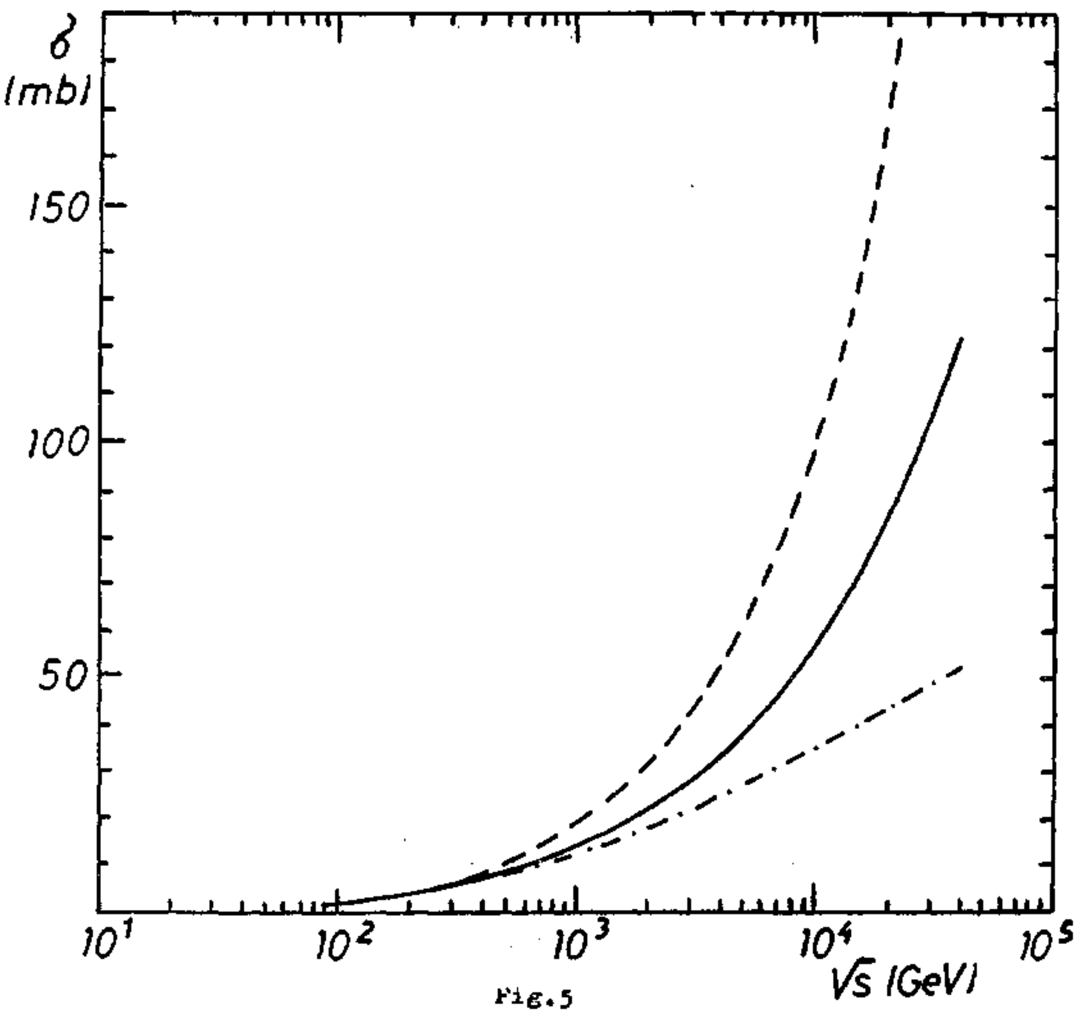
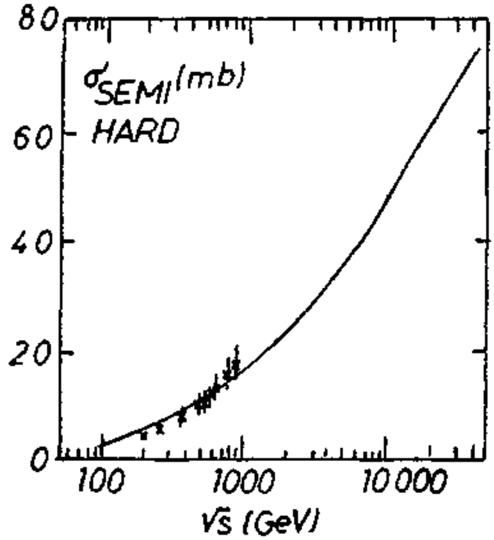
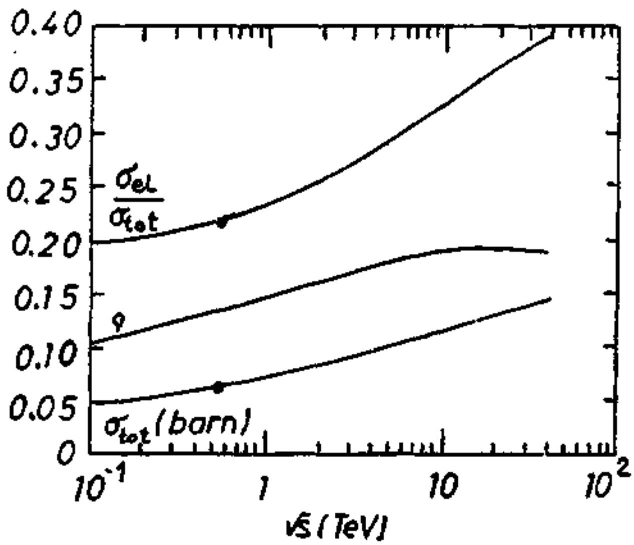


Fig.5



80
60
40
20
0
σ_{SEMI} (mb)
HARD
100
1000
10 000
$\sqrt{s}$ (GeV)

Fig.6



0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0
σ_{el}/σ_{tot}
ρ
σ_{tot} (barn)
10^{-1}
1
10
10^{2}
$\sqrt{s}$ (TeV)

Fig.7

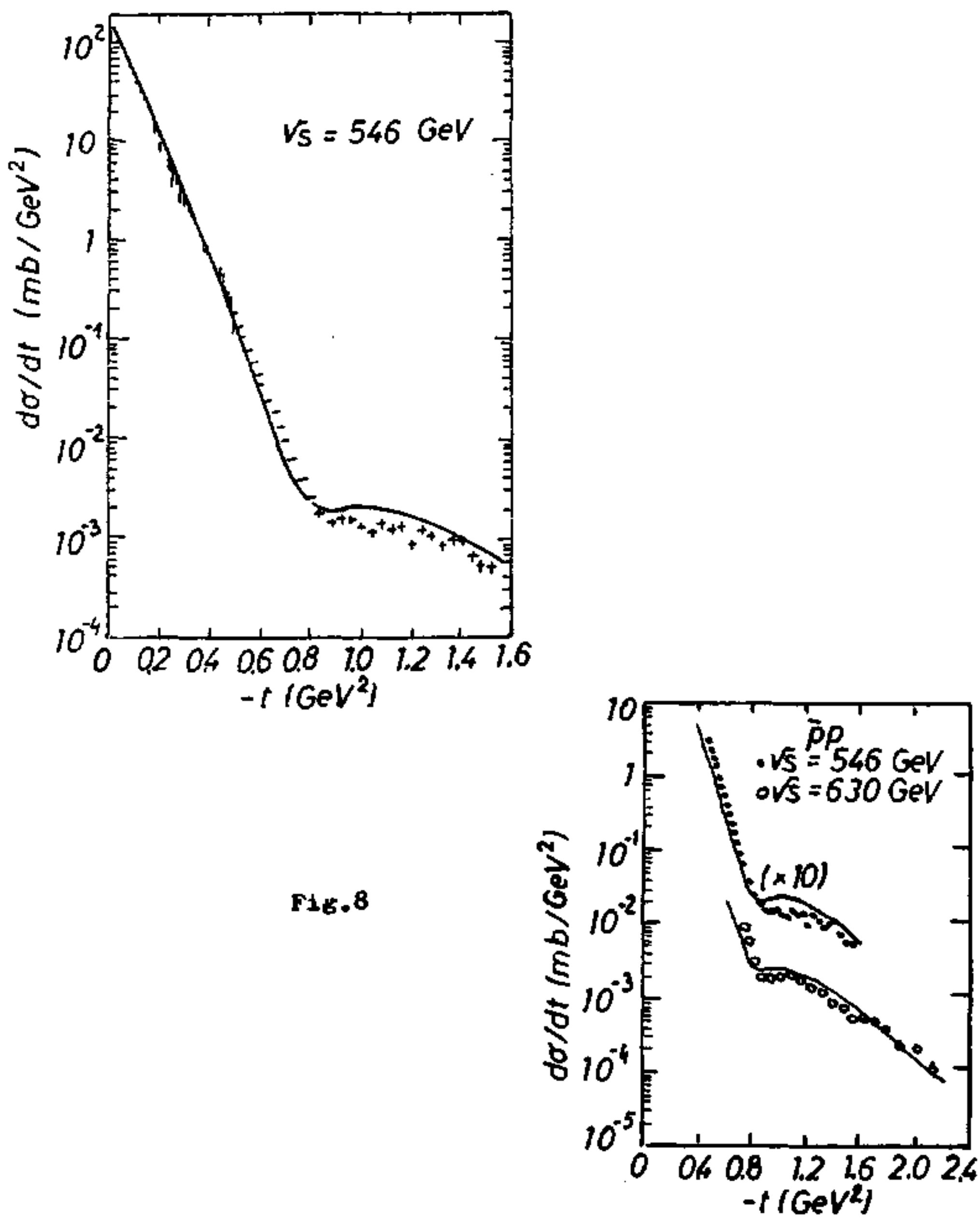


Fig.8

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