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SINGLE-TIME REDUCTION OF BETHE-SALPETER FORMALISM FOR TWO-FERMION SYSTEM

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Abstract

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The single-time reduction method proposed in refs. /10,11/ for the system of two scalar particles is generalized for the case of two-fermion system. A self-consistent procedure of single-time reduction has been constructed both in terms of the Bethe-Salpeter wave function and in terms of the Green's function of two-fermion system. Three-dimensional dynamic equations have been obtained for single-time wave functions and two-time Green's functions of a two-fermion system and the Schrödinger structure of the equations obtained is shown to be a consequence of the causality structure of the local QFT.

Аннотация

Архипов А.А. Одновременная редукция формализма Бете-Солпитера для двухфермионной системы: Препринт ИФВЭ 88-147. -Серпухов, 1988. -44 с., библиогр.:32.

Метод одновременной редукции, предложенный в работах /10,11/ для системы двух скалярных частиц, обобщается на систему двух фермионов. Построена самосогласованная процедура одновременной редукции как в терминах волновых функций Бете-Солпитера, так и в терминах функций Грина двухфермионной системы. Получены трехмерные динамические уравнения для одновременных волновых функций и двухвременных функций Грина системы двух фермионов и показано, что шредингеровская структура полученных уравнений является следствием причинной структуры локальной квантовой теории поля.

INTRODUCTION

At present in the framework of the local quantum field theory there exists a fundamental gauge model of strong interactions, formulated in terms of the well-known QCD Lagrangian (see,e.g., review /1/ and the references therein). The property of the asymptotic freedom discovered in this model /2,3/ allows one to make calculations for the region of small distances in the framework of perturbative theory, using the running coupling constant as a small parameter, and to compare these results with experimentally measurable quantities at large momentum transfers and large transverse momenta. Here one managed to describe a sufficiently large number of experimental data /4/. However QCD faced with considerable difficulties in describing the spectroscopy of hadron states predicted by the theory itself as bound states of quark and gluon fields. On the one hand the experimental observation of the J/ψ and Υ mesons consisting of $c\bar{c}$ and $b\bar{b}$ quarks confirmed our notions about quarks as basic structures of initial fundamental gauge theory, on the other hand it imposed an important problem of describing systems in the framework of QCD. It still awaits its solution.

At the same time the spectroscopy of the J/Ψ and Υ particle families are nicely reproduced in the framework of the potential models with phenomenological potentials /5,6/. Sometimes the system of heavy quarkonium $Q\bar{Q}$, where the $Q\bar{Q}$ interaction is given as a Coulomb-like potential added with a term linearly growing with distance, is declared to be a " hydrogen atom" for strongly interacting systems.

The success of the potential models in describing the spectroscopy of quarkonium systems may be thought to be not accidental and in order to understand why it is so, one should, first of all, clarify what the status of the potential models in the QCD framework is. The solution of this problem would allow one to establish the connection between the fundamental theory and experiment right at the point where at present we have confrontation. Note that the indicated point of the confrontation between theory and experiment is an essentially nonperturbative region.

In a more general case this problem may be formulated as a problem of calculating the interaction potential between quarks proceeding from the first principles. In the case the problem be solved one would manage to predict all the properties of quark systems proceeding from the fundamental QCD Lagrangian. Here great hopes are set on the calculations based on the lattice methods and large efforts are undertaken in this direction /7-9/.

In the present paper we are going to show that there exists a simpler and more consistent way to solve this problem, which is based on the single-time reduction procedure in QFT. The method earlier developed by us for the system of two scalar particles /10,11/ admits a natural generalization and extension onto a two-fermion system, this is what we are intending to do in the given paper. We will show that such generalization can be realized in a self-consistent way both in terms of the Bethe-Salpeter wave functions and in terms of the Grenn's functions of the two-fermion system. Three-dimensional equations of the Scrödinger type will be obtained and a construction which allows one to calculate the effective potential for two-fermion interaction in terms of the QFT basic functions, will be derived.

1. THE BETHE-SALPETER WAVE FUNCTION AND THE GREEN's FUNCTION FOR TWO-FERMION SYSTEM

As an initial object of our consideration we shall treat the Bethe-Salpeter wave function of a two-fermion system for scattering states, which will be determined with the matrix element /12/

$$\Phi(x_1 x_2) = \langle 0 \left| T(\Psi_1(x_1)\Psi_2(x_2)) \right| \Phi; in\rangle, \tag{1}$$

where $\Psi_1(x_1)$ and $\Psi_2(x_2)$ are Heisenberg field operators of spinor particles 1 and 2, $|\Phi;in\rangle$ is a normalized state vector corresponding to the asymptotic configuration of two free particles at $t \to -\infty$, in the in-basis. We shall present the vector $|\Phi;in\rangle$ in the form

$$\left|\Phi; in\right\rangle = \sum_{\sigma_1 \sigma_2} \iint d\mu_1(\vec{p}_1) d\mu_2(\vec{p}_2) \Phi_{in}(\tilde{p}_1\sigma_1, \tilde{p}_2\sigma_2) a^+_{2\,in}(\vec{p}_2\sigma_2) a^+_{1\,in}(\vec{p}_1\sigma_1) \left|0\right\rangle, \tag{2}$$

where $a^+_{1,2}$ are the creation operators of fermions 1 and 2 $\Phi_{in}(\tilde{p}_1\sigma_1, \tilde{p}_2\sigma_2)$ is the wave function of the initial state (wave packet), antisymmetric in its variables and satisfying the normalization condition

$$\sum_{\sigma_1 \sigma_2} \iint d\mu_1(\vec{p}_1) d\mu_2(\vec{p}_2) \left| \Phi_{in}(\tilde{p}_1\sigma_1, \tilde{p}_2\sigma_2) \right|^2 = 1. \tag{3}$$

The invariant measure in the momentum space (an element of single particle phase space) is designated as usually through $d\mu(\tilde{p})$

$$d\mu_1(\vec{p}) = (2\sqrt{\vec{p}^2 + m_1^2})^{-1} d^3\vec{p}.$$

The tilde used throughout the paper implies that the given momentum lies on the mass shell $\tilde{p}_i^2=m_i^2$.

Similar to the scalar case /11/ it is convenient to consider the structure of wave function (1) in the framework of the Bogolubov's axiomatic formulation of quantum field theory. For this purpose let us use the following formula /13/

$$T(\Psi_1(x_1)\Psi_2(x_2)\ldots) = T(\psi_1(x_1)\psi_2(x_2)\ldots S)S^+$$

and rewrite expression (1) in an equivalent form

$$\Phi(x_1x_2) = \langle 0\left|T(\psi_1(x_1)\psi_2(x_2)S)\right|\Phi;out\rangle, \qquad (4)$$

where $\psi_1(x_1)$ and $\psi_2(x_2)$ are asymptotic out-fields of spinor particles and we took into account, that

$$\left|\Phi;in\rangle = S\right|\Phi;out\rangle.$$

Substituting expression (2) into formula (4) and using the commutation relations of Fock representation in the Bogolubov's form we obtain

$$\Phi(x_1x_2) = \iint dy_1dy_2\langle 0\left|\frac{\delta T(\psi_1(x_1)\psi_2(x_2)S)}{\delta\psi_2(y_2)\delta\psi_1(y_1)}\right|0\rangle\Phi^{(0)}(y_1y_2), \qquad (5)$$

where

$$\Phi^{(0)}(y_1y_2)=\sum_{\sigma_1\sigma_2}\iint d\mu_1(\vec{p}_1)d\mu_2(\vec{p}_2)\Phi_{in}(\tilde{p}_1\sigma_1,\tilde{p}_2\sigma_2)u_1(y_1\left|\vec{p}_1\sigma_1)u_2(y_2\right|\vec{p}_2\sigma_2). \qquad (6)$$

For the definition of the function $u(x|\vec{p}\sigma)$ and their properties refer to Appendix. In the R.H.S. of equality (5) we imply summation over spinor indices, which are not written out here explicitly. After functional differentiation in formula (5) and using a generalized Wick theorem /14/, we find

$$\Phi(x_1x_2) = \Phi^{(0)}(x_1x_2) + (G^{(0)}* R * \Phi^{(0)})(x_1x_2) =$$

$$= \Phi^{(0)}(x_1x_2) + (G^{(0)} * R^{(4)} * \Phi^{(0)})(x_1x_2), \qquad (7)$$

where $G^{(0)} = S_1^c S_2^c$, S_i^c are single particle causality Green's functions of spinor particles, the structure of the function R is as follows

$$R(x_1x_2;y_2y_1) = S_1^{c^{-1}}(x_1-y_1)R_2^{(2)}(x_2;y_2) +$$

$$+ R_1^{(2)}(x_1;y_1)S_2^{c^{-1}}(x_2-y_2) + R^{(4)}(x_1x_2;y_2y_1), \qquad (8)$$

where $R_i^{(2)}$ $(i=1,2)$ are the vacuum expectations (VEV) of the second-order radiation operators

$$R_i^{(2)}(x;y) = \frac{1}{i}\langle 0\left|\frac{\delta^2 S}{\delta\bar{\psi}_i(x)\delta\psi_i(y)}S^+\right|0\rangle. \qquad (9)$$

$R^{(4)}$ is the VEV of the fourth-order radiation operator[1]

$$R^{(4)}(x_1x_2;y_2y_1) = \frac{1}{i^2}\langle 0\left|\frac{\delta^4 S}{\delta\bar{\psi}_1(x_1)\delta\bar{\psi}_2(x_2)\delta\psi_2(y_2)\delta\psi_1(x_1)}S^+\right|0\rangle. \qquad (10)$$

The operation $*$ in formula (7) denotes the convolution of the functions in the configuration space. Besides, in the R.H.S. of equality (7) in the term containing convolutions, summation over spinor indices is implied. When going over to the second equation in relation (7) we used the stability of single-particle states

$$S_i^c * R_i^{(2)} * \Phi^{(0)} = 0. \qquad (11)$$

[1]When working with spinor variables we agree for conveniency, that the derivatives $\delta/\delta\psi_i$ are always considered to be right handed and the derivatives over the conjugated field $\delta/\delta\bar{\psi}_i$ are left handed. In this case the derivatives $\delta/\delta\psi_i$ commute with the derivatives $\delta/\delta\bar{\psi}_j$, and the derivatives $\delta/\delta\psi_i$ anticommute with $\delta/\delta\psi_j$ in the same way as the derivatives $\delta/\delta\bar{\psi}_i$ with $\delta/\delta\bar{\psi}_j$.

The two-particle Grenn's function, determined with the equality

$$G(x_1x_2;y_2y_1)=i^2\langle 0\left|T(\Psi_1(x_1)\Psi_2(x_2)\bar{\Psi}_2(y_2)\bar{\Psi}_1(y_1))\right|0\rangle =$$

$$= i^2\langle 0\left|T(\psi_1(x_1)\psi_2(x_2)\bar{\psi}_2(y_2)\bar{\psi}_1(y_1)S)S^+\right|0\rangle \qquad (12)$$

can be presented after partial transformation of the chronological product, as

$$G(x_1x_2;y_2y_1)=\iint dz_1dz_2\langle 0\left|\frac{\delta T(\psi_1(x_1)\psi_2(x_2)S)}{\delta\psi_2(z_2)\delta\psi_1(z_1)}S^+\right|0\rangle \times$$

$$\times S_1^c(z_1-y_1)S_2^c(z_2-y_2). \qquad (13)$$

Using expression (13) for the Green's function one can easily see that linear relation (5) connecting the Bethe-Salpeter wave function with the initial wave function of two spinor particles may be rewritten in the form

$$\Phi(x_1x_2) = [(G * S_1^{c^{-1}} S_2^{c^{-1}}) * \Phi^{(0)}](x_1x_2). \qquad (14)$$

Completely transforming the chronological product in the two-particle Green's function with the generalized Wick theorem, we obtain

$$G = G^{(0)} + G^{(0)} * R * G^{(0)}, \qquad (15)$$

where the structure of the function R is given by equation (8) and $G^0=S_1^cS_2^c$ free Green's function of two spinor particles. With an account of the stability of single-particle states (11) one can present linear relation (14) in the form

$$\Phi(x_1x_2) = [(\bar{G} * G^{(0)^{-1}}) * \Phi^{(0)}](x_1x_2), \qquad (16)$$

where

$$\bar{G} = G - G^{(0)} * (S_1^{c^{-1}} R_2^{(2)} + R_1^{(2)} S_2^{c^{-1}}) * G^{(0)} =$$

$$= G^{(0)} + G^{(0)} * R^{(4)} * G^{(0)}. \qquad (17)$$

As can easily be seen relation (16) is equivalent to the second equality in relation (7), which we shall rewrite in an expanded form indicating all the spinor indices

$$\Phi_{\alpha_1\alpha_2}(x_1x_2) = \Phi^{(0)}_{\alpha_1\alpha_2}(x_1x_2) + \iint dy_1 dy_2 [S_1^c(x_1 - y_1)]^{\beta_1}_{\alpha_1} \times$$

$$\times [S_2^c(x_2 - y_2)]^{\beta_2}_{\alpha_2} \iint dz_1 dz_2 [R^{(4)}(y_1y_2; z_2z_1)]^{\gamma_2\gamma_1}_{\beta_1\beta_2} \Phi^{(0)}_{\gamma_1\gamma_2}(z_1z_2). \quad (18)$$

Summation is implied over the repeated indices in relation (18).

The function $R^{(4)}$ in the R.H.S. of relation (18) is directly connected with the elastic scattering amplitude of two spinor particles. Indeed, using the Bogolubov's reduction formulae we shall obtain the following expression for the matrix element of S-matrix, corresponding to the elastic scattering process of two spinor particles

$$\langle\Phi'; out | S-1 | \Phi; out\rangle = i^2 \iint dx_1 dx_2 \bar{\Phi}^{(0)\prime \alpha_1\alpha_2}(x_1x_2) \times$$

$$\times \iint dy_1 dy_2 \; [R^{(4)}(x_1x_2; y_2y_1)]^{\beta_2\beta_1}_{\alpha_1\alpha_2} \Phi^{(0)}_{\beta_1\beta_2}(y_1y_2),$$

where

$$\bar{\Phi}^{(0)\prime}(x_1x_2) = \sum_{\lambda_1\lambda_2} \iint d\mu_1(\vec{k}_1) d\mu_2(\vec{k}_2) \Phi^{*\prime}_{out}(\tilde{k}_1\lambda_1, \tilde{k}_2\lambda_2) \times$$

$$\times \bar{u}_1(x_1 | \vec{k}_1\lambda_1) \bar{u}_2(x_2 | \vec{k}_2\lambda_2).$$

Relation (18) which will also be called an evolution relation, determines the structure of the Bethe-Salpeter wave function for the system of two spinor particles and is very important. In particular, one can easily pass over from relation (18) to a dynamical Bethe-Salpeter equation if the interaction kernel for two spinor particles is determined with the help of an equation

$$R^{(4)} = K + K * G^{(0)} * R^{(4)}. \tag{19}$$

Then one can easily understand that the Bethe-Salpeter wave function given by equality (18) is the solution of the equation

$$\Phi(x_1x_2) = \Phi^{(0)}(x_1x_2) + (G^{(0)} * K * \Phi)(x_1x_2). \tag{20}$$

The importance of relation (18) is also determined by the fact, that it allows to explicitly take into account causality properties of the local quantum field theory when carrying out single time reduction. In what follows we shall deal with this fact in more detail, however in conclusion to this Section we shall present a formulae which is of interest from the practical point of view, when one of the spinor particles is an antifermion.

Let the Bethe-Salpeter wave function for the fermion-antifermion system be determined with the help of the matrix element

$$\chi(x_1x_2) = \langle 0 \left| T(\Psi_1(x_1)\bar{\Psi}_2(x_2)) \right| \chi; in\rangle =$$

$$= \langle 0 \left| T(\psi_1(x_1)\bar{\psi}_2(x_2)S) \right| \chi; out\rangle, \tag{21}$$

where $|\chi\rangle$ is the vector of the fermion-antifermion system state, which may be presented in the form

$$\left|\chi; out\right\rangle = \sum_{\sigma_1\sigma_2} \iint d\mu_1(\vec{p}_1)\, d\mu_2(\vec{p}_2)\, \chi_{in}(\tilde{p}_1\sigma_1, \tilde{p}_2\sigma_2)\, b^{+}_{2\,out}(\vec{p}_2\sigma_2)\, a^{+}_{1\,out}(\vec{p}_1\sigma_1) \left|0\right\rangle, \tag{22}$$

where a_1^{+} is the creation operator of fermion "1", b_2^{+} is the creation operator of antifermion "2", moreover we consider this case when antifermion "2" is in general not an antiparticle respect to fermion "1" Function χ_{in} is an antisymmetric function of its variables and satisfies the same normalization condition (3) as the function Φ_{in}. Acting in the same way as in the previous case, we find the linear relation of Bethe-Salpeter wave function (21) with the initial wave function of two-particle fermion-antifermion system[1]

$$\chi(x_1x_2)=-\iint dy_1dy_2\langle 0\left|\frac{\delta T(\psi_1(x_1)\bar{\psi}_2(x_2)S)}{\delta\bar{\psi}_2(y_2)\delta\psi_1(y_1)}\right|0\rangle\chi^{(0)}(y_1y_2), \tag{23}$$

where

$$\chi^{(0)}(y_1y_2)=\sum_{\sigma_1\sigma_2}\iint d\mu_1(\vec{p}_1)d\mu_2(\vec{p}_2)\chi_{in}(\tilde{p}_1\sigma_1,\tilde{p}_2\sigma_2)u(y_1|\vec{p}_1\sigma_1)v(y_2|\vec{p}_2\sigma_2). \tag{24}$$

The definition of the functions u and v and their properties are given in Appendix.

Having carried out the functional differentiation in the R.H.S. of relation (23) and using the generalized Wick theorem we shall obtain

$$\chi(x_1x_2)=\chi^{(0)}(x_1x_2) + [(S_1^cS_2^{cT}) * \tilde{R} * \chi^{(0)}](x_1x_2) =$$

$$= \chi^{(0)}(x_1x_2) + [(S_1^cS_2^{cT}) * \tilde{R}^{(4)} * \chi^{(0)}](x_1x_2). \tag{25}$$

Let us present the last equality in relation (25) which follows from the stability of the single-particle states in an expanded form

$$\chi_{\alpha_1}^{\alpha_2}(x_1x_2) = \chi^{(0)}{}_{\alpha_1}^{\alpha_2}(x_1x_2) + \iint dy_1dy_2[S_1^c(x_1-y_1)]_{\alpha_1}^{\beta_1} \times$$

[1]The appearance of the sign "-" in the R.H.S. of eqution (23) as compared with similar relation (5) is connected with our agreement to consider the derivatives $\delta/\delta\bar{\psi}$ left handed and the derivatives $\delta/\delta\psi$ right-handed (see footnote on page 5).

$$\times\ [S_2^{cT}(x_2-y_2)]^{\alpha_2}_{\beta_2}\iint dz_1 dz_2 [\tilde{R}^{(4)}(y_1y_2;z_2z_1)]^{\beta_2;\gamma_1}_{\beta_1;\gamma_2}\Phi^{(0)\gamma_2}_{\gamma_1}(z_1z_2). \quad (26)$$

The functions $\tilde{R}$ and $\tilde{R}^{(4)}$ in relation (25) are connected with the functions R and $R^{(4)}$ introduced above (see formulae (8),(9),(10)) through the equality

$$\left.\begin{aligned}\tilde{R}(x_1x_2;y_2y_1) &= R(x_1y_2;x_2y_1),\\ \tilde{R}^{(4)}(x_1x_2;y_2y_1) &= R^{(4)}(x_1y_2;x_2y_1)\end{aligned}\right\} \quad (27)$$

or in the momentum space

$$\left.\begin{aligned}\tilde{R}(p_1,p_2;k_2,k_1) &= R(p_1,-k_2;-p_2,k_1),\\ \tilde{R}^{(4)}(p_1,p_2;k_2,k_1) &= R^{(4)}(p_1,-k_2;-p_2,k_1)\end{aligned}\right\}. \quad (28)$$

Matrix S_2^{cT} in relation (26) is the matrix transposed to S_2^c over all the variables, both discrete and continuous

$$[S_2^{cT}(x-y)]^{\alpha;}_{;\beta} = [S_2^c(y-x)]^{;\alpha}_{\beta;}.$$

Determining the interaction kernel $\tilde{K}$ for fermion and antifermion with the help of the relation

$$\tilde{R}^{(4)} = \tilde{K} + \tilde{K} * (S_1^c S_2^{cT}) * \tilde{R}^{(4)}, \quad (29)$$

we arrive at a dynamical equation for the Bethe-Salpeter wave function of the fermion-antifermion system

$$\chi(x_1x_2) = \chi^{(0)}(x_1x_2) + (S_1^c S_2^{cT} * \tilde{K} * \chi)(x_1x_2). \quad (30)$$

Similar to the previous case one can easily show that the function $\tilde{R}^{(4)}$ is directly connected with the elastic fermion-antifermion scattering amplitude. It is worth paying attention to relations (27) which show that the function R provide a uniform description of both fermion-fermion and fermion-antiferimon systems.

2. SINGLE-TIME REDUCTION OF THE BETHE-SALPETER WAVE FUNCTION

Two-fermion system. Let us determine the single-time wave function for a two-fermion system by projecting the Bethe-Salpeter wave function (1) onto some space-like hypersurface

$$\tilde{\psi}(n\tau|x_1x_2)=\frac{1}{i^2}\iint\limits_{n\zeta_1=\tau=n\zeta_2} S_1^{(-)}(x_1+n\tau-\zeta_1)S_2^{(-)}(x_2+n\tau-\zeta_2)d\hat{\sigma}_{\zeta_1}d\hat{\sigma}_{\zeta_2}\Phi(\zeta_1\zeta_2), \quad (31)$$

where $S_i^{(-)}$ are negative frequency parts of the permutation functions for the spinor fields, ψ_i, $d\hat{\sigma}_{\zeta_i}=\gamma_\mu^{(i)}d\sigma^\mu_{\zeta_i}$ $(i=1.2)$ $d\sigma_\zeta$ is an lement of a flat space-like hypersurface given by the equation $n\zeta=\tau$, $n^\mu(n^2=1)$ is a unity time-like normal vector to the given flat hypersurface. One can easily get convinced that

$$\tilde{\psi}^{(0)}(n\tau|x_1x_2) = \Phi^{(0)}(x_1+n\tau,x_2+n\tau),$$

where $\Phi^{(0)}$ is determined above by equality (6).

The momentum representation for single-time wave function (31) can conveniently be introduced with the help of the integral transformation

$$\psi(n\tau|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2)=\iint\bar{u}_1(x_1|\vec{p}_1\sigma_1)\bar{u}_2(x_2|\vec{p}_2\sigma_2)d\hat{\sigma}_{x_1}d\hat{\sigma}_{x_2}\tilde{\psi}(n\tau|x_1x_2), \quad (32)$$

where integration in the R.H.S. of equality (32) is carried out over some arbitrary space-like hypersurfaces, besides one can easily check that the result of such integration does not depend on the choice of these surfaces. Since in what follows we shall often find surface integrals let us agree that we shall point out explicitly the hypersurfaces over which we perform integration only in the case when the result depends on the choice of the indicated surfaces. In the R.H.S. of equality (32) we also imply the convolution over the spinor indices. The formula of an inverse transformation to the configuration space has the following form

$$\tilde{\psi}(n\tau|x_1x_2)=\sum_{\sigma_1\sigma_2}\iint d\mu_1(\vec{p}_1)d\mu_2(\vec{p}_2)\psi(n\tau|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2)\times$$

$$\times u_1(x_1|\vec{p}_1\sigma_1)u_2(x_2|\vec{p}_2\sigma_2). \quad (33)$$

Let us also make a Fourier transformation of the single-time wave function over the variable τ

$$\psi(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2)=\int_{-\infty}^{\infty}d\tau\exp(iM\tau)\psi(n\tau|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2). \quad (34)$$

In particular we find

$$\psi^{(0)}(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2)=2\pi\delta(M-n\tilde{p}_1-n\tilde{p}_2)\Phi_{1n}(\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2). \quad (35)$$

The single-time reduction of the Bethe-Salpeter function for a two-fermion system we carry out following the technique proposed by us earlier for the system of scalar particles /11/. Substituting into the R.H.S. of equality (31) the expression for the Bethe-Salpeter wave function from evolution relation (18) we obtain

$$\tilde{\psi}(n\tau|x_1x_2)=\tilde{\psi}^{(0)}(n\tau|x_1x_2)+\iint dy_1dy_2\theta(\tau-ny_1)\theta(\tau-ny_2)\times$$

$$\times S_1^{(-)}(x_1+n\tau-y_1)S_2^{(-)}(x_2+n\tau-y_2)\iint dz_1dz_2R^{(4)}(y_1y_2;z_2z_1)\times$$

$$\times\frac{1}{i^2}\iint S_1^{(-)}(z_1-n\tau-\zeta_1)S_2^{(-)}(z_2-n\tau-\zeta_2)d\hat{\sigma}_{\zeta_1}d\hat{\sigma}_{\zeta_2}\tilde{\psi}^{(0)}(n\tau|\zeta_1\zeta_2). \quad (36)$$

In deriving relation (36) we took into account the fact that

$$\Phi^{(0)}(z_1z_2)=\frac{1}{i^2}\iint S_1^{(-)}(z_1-n\tau-\zeta_1)S_2^{(-)}(z_2-n\tau-\zeta_2)d\hat{\sigma}_{\zeta_1}d\hat{\sigma}_{\zeta_2}\tilde{\psi}^{(0)}(n\tau|\zeta_1\zeta_2)$$

(37)

and used the equality

$$\frac{1}{i}\int\limits_{n\zeta=\tau} S_i^{(-)}(x_1+n\tau-\zeta)\,d\hat{\sigma}_\zeta\, S_i^{(c)}(\zeta-y)=\theta(\tau-ny)S_i^{(-)}(x+n\tau-y). \tag{38}$$

In relation (36) we shall go over to a momentum representation through integral transformation (32) and carry out Fourier tranformation over the variable τ. As a result we obtain

$$\psi(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2)=\psi^{(0)}(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2)+\sum_{\lambda_1\lambda_2}\iint d\mu_1(\vec{k}_1)\,d\mu_2(\vec{k}_2)\,\times$$

$$\times\Big\{\bar{u}_1(\vec{p}_1\sigma_1)\bar{u}_2(\vec{p}_2\sigma_2)(2\pi)^{-8}\int_{-\infty}^{\infty}\frac{d\alpha_1}{\alpha_1-i\epsilon}\int_{-\infty}^{\infty}\frac{d\alpha_2}{\alpha_2-i\epsilon}R(\tilde{p}_1-\alpha_1 n,\tilde{p}_2-\alpha_2 n;\tilde{k}_2,\tilde{k}_1)\times$$

$$\times\, u_1(\vec{k}_1\lambda_1)u_2(\vec{k}_2\lambda_2)\Big\}\,\psi^{(0)}(nM|\tilde{k}_1\lambda_1\tilde{k}_2\lambda_2), \tag{39}$$

where the function R is a Fourier image of VEV of the fourth-order radiation operator

$$R(p_1,p_2;k_2,k_1) = \int dx_1dx_2dy_1dy_2R^{(4)}(x_1x_2;y_2y_1)\,\times$$

$$\times\,\exp(ip_1x_1+ip_2x_2-ik_1y_1-ik_2y_2).$$

In deriving relation (39) we took into account the translation invariance of the function $R^{(4)}$. This property allows one to make simplifications in the R.H.S. of relation (39). Indeed, using the fact that owing to the translation invariance the function R contains a four-dimensional δ function

$$R(p_1,p_2;k_2,k_1) = (2\pi)^4\delta^4(P-K)R(p;k|K),$$

we can present the integral in the R.H.S. of relation (39) in the form

$$(2\pi)^{-8}\int_{-\infty}^{\infty}\frac{d\alpha_1}{\alpha_1-i\epsilon}\int_{-\infty}^{\infty}\frac{d\alpha_2}{\alpha_2-i\epsilon}R(\tilde{p}_1-\alpha_1 n,\tilde{p}_2-\alpha_2 n;\tilde{k}_2,\tilde{k}_1) =$$

$$= (2\pi)^3 n_0\delta^3(\vec{P}-\vec{K}-(nP-nK)\vec{n})(2\pi)^{-7}\int_{-\infty}^{\infty}d\alpha[(\Delta/2+\alpha-i\epsilon)\times$$

$$\times(\Delta/2-\alpha-i\epsilon)]^{-1}R(p-\alpha n;k|K), \tag{40}$$

where $P = \tilde{p}_1+\tilde{p}_2$, $K = \tilde{k}_1+\tilde{k}_2$,

$p = \frac{1}{2}(\tilde{p}_1-\tilde{p}_2)$, $k = \frac{1}{2}(\tilde{k}_1-\tilde{k}_2)$, $\Delta = nP-nK$.

After substituting equality (40) into the R.H.S. of relation (39), we obtain

$$\psi(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2)=\psi^{(0)}(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2) +$$

$$+\frac{1}{\Delta-i\epsilon}\sum_{\lambda_1\lambda_2}\iint d\mu_1(\vec{k}_1)d\mu_2(\vec{k}_2)(2\pi)^3 n_0\delta^3(\vec{P}-\vec{K}-\Delta\vec{n})\times$$

$$\times\Big\{\bar{u}_1(\vec{p}_1\sigma_1)\bar{u}_2(\vec{p}_2\sigma_2)(2\pi)^{-7}\int_{-\infty}^{\infty}d\alpha\Big(\frac{1}{\Delta/2+\alpha-i\epsilon}+\frac{1}{\Delta/2-\alpha-i\epsilon}\Big)\times$$

$$\times R(p-\alpha n;k|K)u_1(\vec{k}_1\lambda_1)u_2(\vec{k}_2\lambda_2)\Big\}\ \psi^{(0)}(nM|\tilde{k}_1\lambda_1\tilde{k}_2\lambda_2), \tag{41}$$

where $\Delta=nP-M$ and we also took into account the fact that the function $\psi^{(0)}$ $(nM|\tilde{k}_1\lambda_1 k_2\lambda_2)$ contains the δ function $\delta(nK-M)$, which allowed us to take Δ out of the integrand. The presence of this δ function gives us a possibility to rewrite relation (41) in an equivalent, more symmetric form

$$\psi(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2)=\psi^{(0)}(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2)+\frac{1}{nP-M-i\epsilon}\sum_{\lambda_1\lambda_2}\iint d\mu_1(\vec{k}_1)d\mu_2(\vec{k}_2)\times$$

$$\times\ T(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2;\ \tilde{k}_2\lambda_2\tilde{k}_1\lambda_1)\psi^{(0)}(nM|\tilde{k}_1\lambda_1\tilde{k}_2\lambda_2), \qquad (42)$$

where

$$T(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2;\ \tilde{k}_2\lambda_2\tilde{k}_1\lambda_1) = \frac{1}{i}(2\pi)^3 n_0\delta^3(\vec{P}_M-\vec{K}_M)\ \times$$

$$\times\ \bar{u}_1(\vec{p}_1\sigma_1)\bar{u}_2(\vec{p}_2\sigma_2)(2\pi)^{-8}\int_{-\infty}^{\infty} d\alpha\left(\frac{1}{\Delta/2+\alpha-i\epsilon}+\frac{1}{\Delta/2-\alpha-i\epsilon}\right)\ \times$$

$$\times\int_{-\infty}^{\infty} d\beta\left(\frac{1}{\Delta'/2+\beta-i\epsilon}+\frac{1}{\Delta'/2-\beta-i\epsilon}\right)R(p-\alpha n;k-\beta n|K_M)u_1(\vec{k}_1\lambda_1)u_2(\vec{k}_2\lambda_2), \qquad (43)$$

$$P_M = P-\Delta n,\quad K_M = K-\Delta' n,$$

$$\Delta = nP-M,\quad \Delta' = nK-M.$$

The R.H.S. of equality (43) determines the continuation of the function T off energy shell both over the particles in the final and initial states. Such continuation is determined, on the one hand, by the requirement for the conservation of the symmetry of the function $R(p;k|K)$, and on the other hand we shall see below it is unambiguously connected with a two-time Green's function for the system of two spinor particles.

Now one can easily go over from relation (42) to a dynamic equation for the single-time wave function of the two-fermion system. For this purpose let us introduce a function V, using the relation

$$T(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2;\ \tilde{k}_2\lambda_2\tilde{k}_1\lambda_1) = V(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2;\ \tilde{k}_2\lambda_2\tilde{k}_1\lambda_1)\ +$$

$$+\sum_{\sigma_1'\sigma_2'}\iint d\mu_1(\vec{p}_1{}')\ d\mu_2(\vec{p}_2{}')\ \times$$

$$\times \frac{V(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2;\ \tilde{p}'_2\sigma'_2\tilde{p}'_1\sigma'_1)T(nM|\tilde{p}'_1\sigma'_1\tilde{p}'_2\sigma'_2;\ \tilde{k}_2\lambda_2\tilde{k}_1\lambda_1)}{nP' - M - i\epsilon}. \qquad (44)$$

Then we can easily see that single-time wave function (42) is the solution of the dynamic equation

$$\psi(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2) = \psi^{(0)}(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2) + \frac{1}{nP-M-i\epsilon} \times$$

$$\times \sum_{\lambda_1\lambda_2} \iint d\mu_1(\vec{k}_1)d\mu_2(\vec{k}_2)V(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2;\ \tilde{k}_2\lambda_2\tilde{k}_1\lambda_1)\psi(nM|\tilde{k}_1\lambda_1\tilde{k}_2\lambda_2). \qquad (45)$$

In the discrete spectrum, when $M<m_1+m_2$ (m_1 and m_2 are fermion masses) the inhomogeneous term in equation (45) vanishes and we come to a homogeneous equation for the single-time wave function of a bound state of two fermions

$$\psi(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2) = \frac{1}{nP-M} \sum_{\lambda_1\lambda_2} \iint d\mu_1(\vec{k}_1)d\mu_2(\vec{k}_2) \times$$

$$\times V(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2;\ \tilde{k}_2\lambda_2\tilde{k}_1\lambda_1)\psi(nM|\tilde{k}_1\lambda_1\tilde{k}_2\lambda_2). \qquad (46)$$

It is also obvious that the equation of the form

$$(nP-M)\psi(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2) = \sum_{\lambda_1\lambda_2} \iint d\mu_1(\vec{k}_1)d\mu_2(\vec{k}_2) \times$$

$$\times V(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2;\ \tilde{k}_2\lambda_2\tilde{k}_1\lambda_1)\psi(nM|\tilde{k}_1\lambda_1\tilde{k}_2\lambda_2) \qquad (47)$$

is satisfied by the single-time wave function of the two-fermion system of continuous and discrete spectra.

It may be shown that the function T introduced through equality (43), coincides on energy shell with the physical amplitude of two fermion elastic scattering. Really using the Bogolubov's reduction technique we obtain

$$\langle\vec{p}_1\sigma_1\vec{p}_2\sigma_2|S-1|\vec{k}_1\lambda_1\vec{k}_2\lambda_2\rangle = i^2(2\pi)^{-2}\delta^4(P-K)\times$$

$$\times\ \bar{u}_1(\vec{p}_1\sigma_1)\bar{u}_2(\vec{p}_2\sigma_2)R(p;k|K)u_1(\vec{k}_1\lambda_1)u_2(\vec{k}_2\lambda_2). \qquad (48)$$

On the other hand from equality (43) we find

$$T(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2;\ \tilde{k}_2\lambda_2\tilde{k}_1\lambda_1)\Big|_{nP=M=nK} = \frac{i}{(2\pi)^3}n_0\delta^3(\vec{P}-\vec{K})\times$$

$$\times\ \bar{u}_1(\vec{p}_1\sigma_1)\bar{u}_2(\vec{p}_2\sigma_2)R(p;k|K)u_1(\vec{k}_1\lambda_1)u_2(\vec{k}_2\lambda_2). \qquad (49)$$

From equations (48) and (49) it follows that

$$\langle\vec{p}_1\sigma_1\vec{p}_2\sigma_2|S-1|\vec{k}_1\lambda_1\vec{k}_2\lambda_2\rangle =$$

$$= 2\pi i\delta(nP-nK)T(n,nK|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2;\ \tilde{k}_2\lambda_2\tilde{k}_1\lambda_1), \qquad (50)$$

which proves the statement made above. The connection of the function T with the physical amplitude of the elastic scattering of two spinor particles together with representation (42) allows us to find a physical, in its essence, quantum mechanical interpretation of the single-time wave function for the two-fermion system introduced by us.

Fermion-antifermion system. The single-time reduction of the Bethe-Salpeter wave function for the fermion-antifermion system follows the scheme used above for the single-time reduction of the two-fermion system. Therefore we shall omit its detailed description and present here only basic relations. Let us determine the single-time wave function of the fermion-antifermion system through the equality[1]

$$\tilde{\psi}(n\tau|x_1x_2)=\frac{1}{i^2}\iint\limits_{n\zeta_1=\tau=n\zeta_2} S_1^{(-)}(x_1+n\tau-\zeta_1)d\hat{\sigma}_{\zeta_1}\chi(\zeta_1\zeta_2)d\hat{\sigma}_{\zeta_2}S^{(+)}(\zeta_2-n\tau-x_2), \quad (51)$$

where $S_2^{(+)}$ is a positive-frequency part of the permutation function for the spinor field. One should bare in mind that the ordering of the functions in the R.H.S. of equality (51) (as well as in all the other surface integrals in this paper) is significant since the differentials $d\hat{\sigma}_{\zeta_i}$ contain the γ-matrix. One can easily get convinced that the substitution of the free Bethe-Salpeter wave function $\chi^{(0)}$ from relation (24) into the R.H.S. of equality (51) yields

$$\psi^{(0)}(n\tau|x_1x_2)=\chi^{(0)}(x_1+n\tau,\ x_2+n\tau).$$

The transition to the momentum representation for the single-time wave function in (51) is realized through the integral transformation

$$\psi(n\tau|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2)=\iint\bar{u}_1(x_1|\vec{p}_1\sigma_1)d\hat{\sigma}_{x_1}\tilde{\psi}(n\tau|x_1x_2)d\hat{\sigma}_{x_2}v(x_2|\vec{p}_2\sigma_2), \quad (52)$$

where integration is carried out over some space-like surfaces, besides similar to the previous case one can again get convinced that the results of such integration are independent of the choice of these surfaces. The formula for the inverse transition into configuration space has the form

$$\tilde{\psi}(n\tau|x_1x_2)=\sum_{\sigma_1\sigma_2}\iint d\mu_1(\vec{p}_1)d\mu_2(\vec{p}_2)u(x_1|\vec{p}_1\sigma_1)\psi(n\tau|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2)\times$$
$$\times\bar{v}(x_2|\vec{p}_2\sigma_2). \quad (53)$$

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[1]We use here the same letter $\tilde{\psi}$ to denote the single-time wave function for the fermion-antifermion system, assuming that no misunderstanding would occur.

Let us also introduce the Fourier transforms of single-time wave function (52) over the variable τ

$$\psi(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2) = \int_{-\infty}^{\infty} d\tau \exp(iM\tau)\, \psi(n\tau|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2).$$

In particular, we have

$$\psi^{(0)}(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2) = 2\pi\delta(M-n\tilde{p}_1-n\tilde{p}_2)\chi_{in}(\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2). \tag{54}$$

Having substituted the Bethe-Salpeter wave function χ from evolution relation (26) into the R.H.S. of equality (51) we arrive at

$$\tilde{\psi}(n\tau|x_1x_2)=\tilde{\psi}^{(0)}(n\tau|x_1x_2)-\iint dx_1' dx_2' \theta(\tau-nx_1')\theta(\tau-nx_2') \times$$
$$\times S_1^{(-)}(x_1+n\tau-x_1')S_2^{(+)T}(x_2+n\tau-x_2')\iint dy_1 dy_2 \tilde{R}^{(4)}(x_1'x_2';y_2y_1) \times$$
$$\times \frac{1}{i^2}\iint S_1^{(-)}(y_1-n\tau-\zeta_1)d\hat{\sigma}_{\zeta_1}\tilde{\psi}^{(0)}(n\tau|\zeta_1\zeta_2)d\hat{\sigma}_{\zeta_2}S_2^{(+)}(\zeta_2+n\tau-y_2). \tag{55}$$

When deriving relation (55) we expressed the free wave function $\chi^{(0)}$ through $\psi^{(0)}$ with the help of relation

$$\chi^{(0)}(y_1y_2) = \frac{1}{i^2}\iint S_1^{(-)}(y_1-n\tau-\zeta_1)d\hat{\sigma}_{\zeta_1}\tilde{\psi}^{(0)}(n\tau|\zeta_1\zeta_2) \times$$
$$\times d\hat{\sigma}_{\zeta_2}S_2^{(+)}(\zeta_2+n\tau-y_2), \tag{56}$$

used equality (38) and the following analogous equality

$$\frac{1}{i}\int_{n\zeta=\tau} S^{c}(x-\zeta)d\hat{\sigma}_{\zeta}S^{(+)}(\zeta-n\tau-y)=-\theta(\tau-nx)S^{(+)}(x-n\tau-y). \tag{57}$$

In passing over to the momentum representaion in (55) with the help of integral transformation (52) and realizing the Fourier transformation over the variable τ, we obtain

$$\psi(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2)=\psi^{(0)}(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2)+\frac{1}{nP-M-i\epsilon}\sum_{\lambda_1\lambda_2}\iint d\mu_1(\vec{k}_1)d\mu_2(\vec{k}_2)\times$$

$$\times\tilde{T}(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2;\ \tilde{k}_2\lambda_2\tilde{k}_1\lambda_1)\psi^{(0)}(nM|\tilde{k}_1\lambda_1\tilde{k}_2\lambda_2),\qquad(58)$$

where

$$\tilde{T}(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2;\ \tilde{k}_2\lambda_2\tilde{k}_1\lambda_1)=-\frac{1}{i}(2\pi)^3 n_0\delta^3(\vec{P}_M-\vec{K}_M)\times$$

$$\times\bar{u}_1(\vec{p}_1\sigma_1)v(\vec{p}_2\sigma_2)(2\pi)^{-8}\int_{-\infty}^{\infty}d\alpha\left(\frac{1}{\Delta/2+\alpha-i\epsilon}+\frac{1}{\Delta/2-\alpha-i\epsilon}\right)\times$$

$$\times\int_{-\infty}^{\infty}d\beta\left(\frac{1}{\Delta'/2+\beta-i\epsilon}+\frac{1}{\Delta'/2-\beta-i\epsilon}\right)\tilde{R}(p-\alpha n;k-\beta n|K_M)u_1(\vec{k}_1\lambda_1)\bar{v}(\vec{k}_2\lambda_2).\ (59)$$

In the R.H.S. of equality (59) we used the same notations as in equality (43). The function $\tilde{R}$ is a Fourier-image of the function $\tilde{R}^{(4)}$

$$\tilde{R}(p_1p_2;k_2k_1)=\int dx_1dx_2dy_1dy_2\tilde{R}^{(4)}(x_1x_2;y_2y_1)\times$$
$$\times\exp(ip_1x_1+ip_2x_2-ik_1y_1-ik_2y_2).$$

When deriving relation (58) we take into account the translation invariance of the function $\tilde{R}^{(4)}$ as well as the circumstance that the initial wave function $\psi^{(0)}$ contains the δ-function (see formula (54)).

If now one defines the function $\tilde{V}$ with the help of the relation

$$\tilde{T}(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2;\ \tilde{k}_2\lambda_2\tilde{k}_1\lambda_1)=\tilde{V}(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2;\ \tilde{k}_2\lambda_2\tilde{k}_1\lambda_1)+$$

$$+\sum_{\sigma_1'\sigma_2'}\iint d\mu_1(\vec{p}_1')\,d\mu_2(\vec{p}_2')\times$$

$$\times\frac{\tilde{V}(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2;\ \tilde{p}_2'\sigma_2'\tilde{p}_1'\sigma_1')\tilde{T}(nM|\tilde{p}_1'\sigma_1'\tilde{p}_2'\sigma_2';\ \tilde{k}_2\lambda_2\tilde{k}_1\lambda_1)}{nP'-M-i\epsilon}, \tag{60}$$

then it can easily be seen that single-time wave function (58) is a solution of the dynamical equation

$$\psi(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2)=\psi^{(0)}(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2)+\frac{1}{nP-M-i\epsilon}\times$$

$$\times\tilde{\Sigma}_{\lambda_1\lambda_2}\iint d\mu_1(\vec{k}_1)\,d\mu_2(\vec{k}_2)\tilde{V}(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2;\ \tilde{k}_2\lambda_2\tilde{k}_1\lambda_1)\psi(nM|\tilde{k}_1\lambda_1\tilde{k}_2\lambda_2). \tag{61}$$

In the discrete spectrum from equation (61) one should obviously go over to a homogeneous equation. As in the case of the two-fermion system we can show that the function $\tilde{T}$ on the energy shell is directly connected with the matrix element of the S-matrix for elastic fermion-antifermion scattering.

$$\langle\vec{p}_1\sigma_1\vec{p}_2\sigma_2\,|S-1|\,\vec{k}_1\lambda_1\vec{k}_2\lambda_1\rangle=$$

$$=2\pi i\delta(nP-nK)\tilde{T}(n,nK|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2;\ \tilde{k}_2\lambda_2\tilde{k}_1\lambda_1). \tag{62}$$

Equality (62) together with representation (58) provide physical reasonings for introducing the single-time wave function of the fermion-antifermion system in form (51).

Equation (61) is fully analogous in its structure to dynamic equation (45) for the single-time wave function of the two-fermion system. The only difference is that in equations (45) and (61) we have two different functions V and $\tilde{V}$, describing the interactions of fermion with fermion and fermion with antifermion, respectively. However it should be noted that the functions V and $\tilde{V}$ are not completely independent since they are determined through the functions R and $\tilde{R}$ which are connected with each other via crossing relations (27),(28)

3. SINGLE-TIME REDUCTION OF TWO-FERMION SYSTEM IN TERMS OF THE GREEN's FUNCTION

In the previous Section we carried out the single-time reduction of the Bethe-Salpeter wave functions and obtained three dimensional dynamic equations for the wave function of the two-fermion system. Since in the quantum field theory one usually deals with the Green's functions, it will be of undoubt interest to generalize the developed technique for the Green's functions. In this Section we present this generalization and construct the procedure of the single-time reduction right in the terms of the Green's functions /15,16/. We shall start our consideration with the R-function or current Green's functions[1]. As an initial relation we take equation (19) connecting the four-point current Green's function with the Bethe-Salpeter interaction kernel

$$R^{(4)}(x_1x_2;y_2y_1)=K(x_1x_2;y_2y_1)+(K*G^{(0)}*R^{(4)})(x_1x_2;y_2y_1), \qquad (63)$$

where

$$G^{(0)}(x_1x_2;y_2y_1)=S_1^c(x_1-y_1)S_2^c(x_2-y_2),$$

$S_i^c(x_i-y_i),(i=1,2)$ are single-time causality Green's function of spinor particles. Remember that the operation $*$ denotes the convolution of the functions in configuration space and as usually the convolution over the spinor indices is implied.

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[1]VEVs of the radiation operators reduce with an accuracy up to quasilocal terms to VEVs of the chronological product of currents /15/, where the name "current Green's functions" comes from.

For our purposes it is convenient to go over from equation (63) to an equivalent relation

$$R^{(4)}(x_1x_2;y_2y_1)=U(x_1x_2;y_2y_1)+(U*g_0*R^{(4)})(x_1x_2;y_2y_1), \tag{64}$$

where the function U is determined from the equation

$$U(x_1x_2;y_2y_1)=K(x_1x_2;y_2y_1)+(K*(G^{(0)}-g_0)*U)(x_1x_2;y_2y_1). \tag{65}$$

The system of equations (64) and (65) is equivalent to initial equation (63) with an arbitrary function g_0. The problem is to find such function g_0 which would allow one to carry out the single-time reduction in a self-consistent way. The self-consistency condition unambiguously determines the function g_0. It has the following form

$$g_0(x_1x_2;y_2y_1)=-\int_{-\infty}^{\infty}d\tau\int_{-\infty}^{\infty}d\tau'\frac{d}{d\tau}[\theta(nx_1-\tau)\theta(nx_2-\tau)]\times$$

$$\times\frac{1}{i^2}\iint S_1^{(-)}(x_1-n\tau-x_1')S_2^{(-)}(x_2-n\tau-x_2')d\hat{\sigma}_{x_1'}d\hat{\sigma}_{x_2'}\times$$

$$\times\frac{1}{i^2}\iint i\tilde{G}_0(n,\tau-\tau'|x_1'x_2';y_2'y_1')d\hat{\sigma}_{y_1'}d\hat{\sigma}_{y_2'}S_1^{(-)}(y_1'+n\tau'-y_1)\times$$

$$\times S_2^{(-)}(y_2'+n\tau'-y_2)\frac{d}{d\tau'}[\theta(\tau'-ny_1)\theta(\tau'-ny_2)], \tag{66}$$

where the function $\tilde{G}_0$ is determined by projecting the free two-particle Green's function

$$i\tilde{G}_0(n,\tau-\tau'|x_1x_2;y_2y_1) = \frac{1}{i^2}\iint_{nx_1'=\tau=nx_2'}S_1^{(-)}(x_1+n\tau-x_1')S_2^{(-)}(x_2+n\tau-x_2')\times$$

$$\times\, d\hat{6}_{x'_1} d\hat{6}_{x'_2} \frac{1}{i^2} \iint\limits_{ny'_1=\tau'=ny'_2} G^{(0)}(x'_1x'_2;y'_2y'_1)\, d\hat{6}_{y'_1} d\hat{6}_{y'_2} S_1^{(-)}(y'_1-n\tau'-y_1) \times$$

$$\times\, S_2^{(-)}(y'_2-n\tau'-y_2). \tag{67}$$

Having calculated the surface integrals in the R.H.S. of equality (67) we obtain an explicit expression for the function $\tilde{G}_0$

$$i\tilde{G}_0(n,\tau-\tau'|x_1x_2;y_2y_1)=\theta(\tau-\tau')S_1^{(-)}(x_1+n(\tau-\tau')-y_1) \times$$

$$\times\, S_2^{(-)}(x_2+n(\tau-\tau')-y_2). \tag{68}$$

The integration in the R.H.S. of equality (66) is carried out over the arbitrary space-like surfaces, the result of integration being independent of the choice of these surfaces. Obviously all the integrations in the R.H.S. of equality (66) for the function g_0 may be performed but we shall not do it since for our purposes it is more convenient to use the representation for the function g_0 in form (66)

In equation (64) we shall pass over from the function $R^{(4)}$ and U to new functions $\tilde{T}$ and $\tilde{V}$ which will be determined through the relations

$$-\iint dx'_1dx'_2S_1^{(-)}(x_1+n\tau-x'_1)S_2^{(-)}(x_2+n\tau-x'_2)\frac{d}{d\tau}[\theta(\tau-nx'_1)\theta(\tau-nx'_2)] \times$$

$$\times\iint dy'_1dy'_2\begin{Bmatrix}R^{(4)}(x'_1x'_2;y'_2y'_1)\\U(x'_1x'_2;y'_2y'_1)\end{Bmatrix}S_1^{(-)}(y'_1-n\tau'-y_1)S_2^{(-)}(y'_2-n\tau'-y_2)\times$$

$$\times\frac{d}{d\tau'}[\theta(ny'_1-\tau')\theta(ny'_2-\tau')]=\begin{Bmatrix}\frac{1}{i}\tilde{T}(n,\tau-\tau'|x_1x_2;y_2y_1)\\[1ex]\frac{1}{i}\tilde{V}(n,\tau-\tau'|x_1x_2;y_2y_1)\end{Bmatrix}. \tag{69}$$

Then from equation (64) we obtain

$$\tilde{T}(n,\tau-\tau'|x_1x_2;y_2y_1)=\tilde{V}(n,\tau-\tau'|x_1x_2;y_2y_1)\ +$$

$$+\int_{-\infty}^{\infty}d\sigma\int_{-\infty}^{\infty}d\sigma'\iint\tilde{V}(n,\tau-\sigma|x_1x_2;x_2'x_1')d\hat{\sigma}_{x_1'}d\hat{\sigma}_{x_2'}\iint\tilde{G}_0(n,\sigma-\sigma'|x_1'x_2';y_2'y_1')\times$$

$$\times\ d\hat{\sigma}_{y_1'}d\hat{\sigma}_{y_2'}\tilde{T}(n,\sigma'-\tau'|y_1'y_2';y_2y_1). \tag{70}$$

Equation (70) is the required three-dimensional reduced Bethe-Salpeter equation for four-point current Green's function.

In equation (70) one may pass over to the momentum representation. For this let us introduce the images of the functions $\tilde{T}$, $\tilde{V}$, $\tilde{G}_0$ in the momentum space via formulae

$$\int_{-\infty}^{\infty}d\tau\ \exp(iM\tau)\iint\bar{u}_1(x_1|\vec{p}_1\sigma_1)\bar{u}_2(x_2|\vec{p}_2\sigma_2)d\hat{\sigma}_{x_1}d\hat{\sigma}_{x_2}\ \times$$

$$\times\iint\tilde{F}(n,\tau|x_1x_2;y_2y_1)d\hat{\sigma}_{y_1}d\hat{\sigma}_{y_2}u_1(y_1|\vec{k}_1\lambda_1)u_2(y_2|\vec{k}_2\lambda_2)\ =$$

$$=\ F(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2;\tilde{k}_2\lambda_2\tilde{k}_1\lambda_1), \tag{71}$$

where $\tilde{F}$ is any of these three functions $\tilde{G}_0$, $\tilde{V}$, $\tilde{T}$; F is its image in the momentum space. The formula of the inverse transformation has the form

$$\tilde{F}(n\tau|x_1x_2;y_2y_1)=\frac{1}{2\pi}\int_{-\infty}^{\infty}dM\ \exp(-iM\tau)\sum_{\substack{\sigma_1\sigma_2\\ \lambda_1\lambda_2}}\iint d\mu_1(\vec{p}_1)d\mu_2(\vec{p}_2)\ \times$$

$$\times\iint d\mu_1(\vec{k}_1)d\mu_2(\vec{k}_2)u_1(x_1|\vec{p}_1\sigma_1)u_2(x_2|\vec{p}_2\sigma_2)F(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2;\tilde{k}_2\lambda_2\tilde{k}_1\lambda_1)\times$$

$$\times\ \bar{u}_1(y_1|\vec{k}_1\lambda_1)\bar{u}_2(y_2|\vec{k}_2\lambda_2). \tag{72}$$

Using explicit expression (68) for a free two-particle Green's function $\tilde{G}_0$ we shall obtain for its image in the momentum space the following expression

$$\tilde{G}_0(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2;\tilde{k}_2\lambda_2\tilde{k}_1\lambda_1) =$$

$$= \frac{2E_1(\vec{p}_1)\delta^3(\vec{p}_1-\vec{k}_1)2E_2(\vec{p}_2)\delta^3(\vec{p}_2-\vec{k}_2)}{n\tilde{p}_1+n\tilde{p}_2-M-i\epsilon}\delta^{\lambda_1}_{\sigma_1}\delta^{\lambda_2}_{\sigma_2}. \tag{73}$$

As a result equation (70) in the momentum space will be presented in the form

$$T(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2;\tilde{k}_2\lambda_2\tilde{k}_1\lambda_1) = V(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2;\tilde{k}_2\lambda_2\tilde{k}_1\lambda_1) +$$

$$+ \sum_{\sigma_1'\sigma_2'}\iint d\mu_1(\vec{p}_1')d\mu_2(\vec{p}_2') \times$$

$$\times \frac{V(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2;\tilde{p}_2'\sigma_2'\tilde{p}_1'\sigma_1')T(nM|\tilde{p}_1'\sigma_1'\tilde{p}_2'\sigma_2';\tilde{k}_2\lambda_2\tilde{k}_1\lambda_1)}{n\tilde{p}_1'+n\tilde{p}_2'-M-i\epsilon}. \tag{74}$$

From relations (69) and (71) it follows that

$$T(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2;\tilde{k}_2\lambda_2\tilde{k}_1\lambda_1) = \frac{1}{i}(2\pi)^3 n_0\delta^3(\vec{P}_M-\vec{K}_M) \times$$

$$\times \bar{u}_1(\vec{p}_1\sigma_1)\bar{u}_2(\vec{p}_2\sigma_2)(2\pi)^{-8}\int_{-\infty}^{\infty} d\alpha\left(\frac{1}{\Delta/2+\alpha-i\epsilon}+\frac{1}{\Delta/2-\alpha-i\epsilon}\right) \times$$

$$\times\int_{-\infty}^{\infty} d\beta\left(\frac{1}{\Delta'/2+\beta-i\epsilon}+\frac{1}{\Delta'/2-\beta-i\epsilon}\right)R(p-\alpha n;k-\beta n|K_M) \times$$

$$\times u_1(\vec{k}_1\lambda_1)u_2(\vec{k}_2\lambda_2), \tag{75}$$

$$V(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2;\tilde{k}_2\lambda_2\tilde{k}_1\lambda_1)=\frac{1}{i}(2\pi)^3 n_0\delta^3(\vec{P}_M-\vec{K}_M)\times$$

$$\times\bar{u}_1(\vec{p}_1\sigma_1)\bar{u}_2(\vec{p}_2\sigma_2)(2\pi)^{-8}\int_{-\infty}^{\infty}d\alpha\left(\frac{1}{\Delta/2+\alpha-i\epsilon}+\frac{1}{\Delta/2-\alpha-i\epsilon}\right)\times$$

$$\times\int_{-\infty}^{\infty}d\beta\left(\frac{1}{\Delta'/2+\beta-i\epsilon}+\frac{1}{\Delta'/2-\beta-i\epsilon}\right)U(p-\alpha n;k-\beta n|K_M)u_1(\vec{k}_1\lambda_1)u_2(\vec{k}_2\lambda_2),\quad(76)$$

where

$$P=\tilde{p}_1+\tilde{p}_2,\quad K=\tilde{k}_1+\tilde{k}_2,$$

$$p=\frac{1}{2}(\tilde{p}_1-\tilde{p}_2),\quad k=\frac{1}{2}(\tilde{k}_1-\tilde{k}_2),$$

$$P_M=P-\Delta n,\quad K_M=K-\Delta' n,$$

$$\Delta=nP-M,\quad \Delta'=nK-M.$$

$R(p;k|K)$ and $U(p;k|K)$ are Fourier images of the translational invariant parts of the function $R^{(4)}$ and U, respectively. The R.H.S. of formula (75) exactly coincides with that in equality (43). Formula (76) with an account of equation (65), which relates the functions U and K, provides an alternative way for calculating the function V directly in terms of the function K - the kernel of the Bethe-Salpeter interaction.

Let us now proceed to the construction of the single-time reduction procedure for the field Green's function (12), which is determined as a VEV of the chronological product of the field operators. The two-time Green's function for the two-fermion system will be determined by the covariant projection operation introduced with formula (67)

$$i\tilde{G}(n,\tau-\tau'|x_1x_2;y_2y_1) = \frac{1}{i^2}\iint\limits_{nx'_1=\tau=nx'_2} S_1^{(-)}(x_1+n\tau-x'_1)S_2^{(-)}(x_2+n\tau-x'_2) \times$$

$$\times\, d\hat{\sigma}_{x'_1}d\hat{\sigma}_{x'_2}\frac{1}{i^2}\iint\limits_{ny'_1=\tau'=ny'_2}\bar{G}(x'_1x'_2;y'_2y'_1)\,d\hat{\sigma}_{y'_1}d\hat{\sigma}_{y'_2}S_1^{(-)}(y'_1-n\tau'-y_1) \times$$

$$\times\, S_2^{(-)}(y'_2-n\tau'-y_2), \tag{77}$$

where the function $\bar{G}$ is determined above by equality (17):

$$\bar{G}(x_1x_2;y_2y_1)=G^{(0)}(x_1x_2;y_2y_1)+(G^{(0)}*R^{(4)}*G^{(0)})(x_1x_2;y_2y_1). \tag{78}$$

The projection from the first term in the R.H.S. of equality (78) has already been calculated (see formula (68)). To calculate the projection from the second term in (78) we shall use the following relation

$$\frac{1}{i^2}\iint\limits_{nx'_1=\tau=nx'_2} S_1^{(-)}(x_1+n\tau-x'_1)S_2^{(-)}(x_2+n\tau-x'_2)\,d\hat{\sigma}_{x'_1}d\hat{\sigma}_{x'_2}G^{(0)}(x'_1x'_2;y_2y_1) =$$

$$=\int_{-\infty}^{\infty}d\tau'\frac{1}{i^2}\iint i\tilde{G}_0(n,\tau-\tau'|x_1x_2;x'_2x'_1)\,d\hat{\sigma}_{x'_1}d\hat{\sigma}_{x'_2}S_1^{(-)}(x'_1+n\tau'-y_1)\times$$

$$\times\, S_2^{(-)}(x'_2+n\tau'-y_2)\frac{d}{d\tau'}[\theta(\tau'-ny_1)\theta(\tau'-ny_2)], \tag{79}$$

$$\frac{1}{i^2}\iint\limits_{ny'_1=\tau'=ny'_2} G^{(0)}(x_1x_2;y'_2y'_1)\,d\hat{\sigma}_{y'_1}d\hat{\sigma}_{y'_2}S_1^{(-)}(y'_1-n\tau'-y_1)S_2^{(-)}(y'_2-n\tau'-y_2) =$$

$$
=-\int_{-\infty}^{\infty} d\tau \frac{d}{d\tau}[\theta(nx_1-\tau)\theta(nx_2-\tau)]\frac{1}{i^2}\iint S_1^{(-)}(x_1-n\tau-y_1')S_2^{(-)}(x_2-n\tau-y_2') \times
$$

$$
\times\, d\hat{\sigma}_{y_1'} d\hat{\sigma}_{y_2'}\, i\tilde{G}_0(n,\tau-\tau'|y_1'y_2';y_2y_1). \qquad (80)
$$

With the help of relations (79) and (80) and with account of definition (69) for the function $\tilde{T}$, the projection from the second term in the R.H.S. of equality (78) may be presented in the form

$$
\frac{1}{i^2}\iint_{nx_1'=\tau=nx_2'} S_1^{(-)}(x_1+n\tau-x_1')S_2^{(-)}(x_2+n\tau-x_2')\, d\hat{\sigma}_{x_1'} d\hat{\sigma}_{x_2'} \frac{1}{i^2} \times
$$

$$
\times\iint_{ny_1'=\tau'=ny_2'} (G_0 * R^{(4)} * G_0)(x_1'x_2';y_2'y_1')\, d\hat{\sigma}_{y_1'} d\hat{\sigma}_{y_2'} S_1^{(-)}(y_1'-n\tau'-y_1)S_2^{(-)}(y_2'-n\tau'-y_2) =
$$

$$
=\int_{-\infty}^{\infty} d\sigma \int_{-\infty}^{\infty} d\sigma' \iint i\tilde{G}_0(n,\tau-\sigma|x_1x_2;x_2'x_1')\, d\hat{\sigma}_{x_1'} d\hat{\sigma}_{x_2'} \times
$$

$$
\times\iint \frac{1}{i}\tilde{T}(n,\sigma-\sigma'|x_1'x_2';y_2'y_1')\, d\hat{\sigma}_{y_1'} d\hat{\sigma}_{y_2'}\, i\tilde{G}_0(n,\sigma'-\tau'|y_1'y_2';y_2y_1). \qquad (81)
$$

As a result for the two-time Green's function we obtain the following expression

$$
\tilde{G}(n,\tau-\tau'|x_1x_2;y_2y_1)=\tilde{G}_0(n,\tau-\tau'|x_1x_2;y_2y_1) +
$$

$$
+\int_{-\infty}^{\infty} d\sigma \int_{-\infty}^{\infty} d\sigma' \iint \tilde{G}_0(n,\tau-\sigma|x_1x_2;x_2'x_1')\, d\hat{\sigma}_{x_1'} d\hat{\sigma}_{x_2'} \times
$$

$$
\times\iint \tilde{T}(n,\sigma-\sigma'|x_1'x_2';y_2'y_1')\, d\hat{\sigma}_{y_1'} d\hat{\sigma}_{y_2'}\, \tilde{G}_0(n,\sigma'-\tau'|y_1'y_2';y_2y_1). \qquad (82)
$$

The substitution of equation (70) into the R.H.S. of obtained relation (82) yields the following equation for the two-time Green's function

$$\tilde{G}(n,\tau-\tau'|x_1x_2;y_2y_1) = \tilde{G}_0(n,\tau-\tau'|x_1x_2;y_2y_1) +$$

$$+\int_{-\infty}^{\infty} d\sigma\int_{-\infty}^{\infty} d\sigma' \iint \tilde{G}_0(n,\tau-\sigma|x_1x_2;x_2'x_1')\, d\hat{\sigma}_{x_1'}\, d\hat{\sigma}_{x_2'} \times$$

$$\times\iint \tilde{V}(n,\sigma-\sigma'|x_1'x_2';y_2'y_1')\, d\hat{\sigma}_{y_1'}\, d\hat{\sigma}_{y_2'}\ \tilde{G}(n,\sigma'-\tau'|y_1'y_2';y_2y_1). \qquad (83)$$

Using integral transformation (71) one can easily rewrite relations (82) and (83) in the momentum space. For instance, equation (83) in the momentum representation has the form

$$G(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2;\ \tilde{k}_2\lambda_2\tilde{k}_1\lambda_1) = G_0(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2;\tilde{k}_2\lambda_2\tilde{k}_1\lambda_1) +$$

$$+\frac{1}{nP-M-i\epsilon}\sum_{\sigma_1'\sigma_2'}\iint d\mu_1(\vec{p}_1')\, d\mu_2(\vec{p}_2')\, V(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2;\tilde{p}_2'\sigma_2'\tilde{p}_1'\sigma_1') \times$$

$$\times\ G(nM|\tilde{p}_1'\sigma_1'\tilde{p}_2'\sigma_2';\tilde{k}_2\lambda_2\tilde{k}_1\lambda_1). \qquad (84)$$

Hence relations (82),(83) provide the solution of the problem of the single-time reduction of the Bethe-Salpeter formalism in terms of the field Green's function. Now the scattering problem of two relativistic particles may be formulated in the form similar to the nonrelativistic theory of potential scattering. The evolution of a two-particle system from the initial state given by the wave function

$$\Psi^{(0)}(n\tau|x_1x_2) = \sum_{\sigma_1\sigma_2}\iint d\mu_1(\vec{p}_1)\ d\mu_2(\vec{p}_2)\Phi_{in}(\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2) \times$$

$$\times\ exp\left[-i(n\tilde{p}_1+n\tilde{p}_2)\tau\right]\ u_1(x_1|\vec{p}_1\sigma_1)\ u_2(x_2|\vec{p}_2\sigma_2)$$

is determined by the relation

$$\tilde{\psi}(n\tau|x_1x_2) =$$

$$= \frac{1}{i^2}\int_{-\infty}^{\infty} d\tau' \iint \Omega(n,\tau-\tau'|x_1x_2;x_2'x_1')\, d\hat{\sigma}_{x_1'} d\hat{\sigma}_{x_2'} \tilde{\psi}^{(0)}(n\tau'|x_1'x_2'), \qquad (85)$$

where the operator Ω is a relativistic analogue of the Möller operator in the nonrelativistic theory of potential scattering

$$\Omega(n,\tau-\tau'|x_1x_2;x_2'x_1') = (\tilde{G} \,\tilde{*}\, \tilde{G}_0^{-1})(n,\tau-\tau'|x_1x_2;x_2'x_1') =$$

$$= (1 + \tilde{G}_0 \,\tilde{*}\, \tilde{T})(n,\tau-\tau'|x_1x_2;x_2'x_1'). \qquad (86)$$

In formula (86) we introduced the operation $\tilde{*}$, which denotes the integration over some space-like surfaces and the integration over the evolution parameter as it is explicitly presented in the R.H.S. of formula (85). The function $\tilde{G}_0^{-1}$ has the form

$$\frac{1}{i}\tilde{G}_0^{-1}(n,\tau-\tau'|x_1x_2;y_2y_1) =$$

$$= \delta'(\tau-\tau')S_1^{(-)}(x_1+n(\tau-\tau')-y_1)S_2^{(-)}(x_2+n(\tau-\tau')-y_2) \qquad (87)$$

and satisfies the relation

$$(\tilde{G}_0 \,\tilde{*}\, \tilde{G}_0^{-1})(n,\tau-\tau'|x_1x_2;y_2y_1) = (\tilde{G}_0^{-1} \tilde{*}\, \tilde{G}_0)(n,\tau-\tau'|x_1x_2;y_2y_1) =$$

$$= \delta(\tau-\tau')S_1^{(-)}(x_1-y_1)S_2^{(-)}(x_2-y_2). \qquad (88)$$

Evolution relation (85) for the single-time wave function has a three-dimensional form contrary to evolution equation (18) for the Bethe-Salpeter wave function, and in this respect it is analogous to the evolution equation in the nonrelativistic scattering theory. However, and what is very important, evolution equation (85) determines the structure of the relativistic two-particle system, since it has been obtained in the framework of the local quantum field theory and is a consequence of the most general properties of the relativistic quantum theory.

4. TRANSITION TO PHYSICAL EQUIVALENT DESCRIPTION OF THE TWO-FERMION SYSTEM

Proceeding from the causality and spectrality properties in the local quantum field theory it has been shown in ref./11/ that one can go over from the single-time wave function of the two-particle system to a physical equivalent single-time wave function, which is determined in the following way. Let us divide the function T in the R.H.S. in equality (42), determining the single-time wave function, into two terms

$$T = T' + T_1, \tag{89}$$

where

$$\begin{aligned} T'(nM|\tilde p_1 \sigma_1 \tilde p_2 \sigma_2; \tilde k_2 \lambda_2 \tilde k_1 \lambda_1) &= i(2\pi)^3 n_0 \delta^3(\vec P_M - \vec K_M) \times \\ &\times \bar u_1(\vec p_1 \sigma_1) \bar u_2(\vec p_2 \sigma_2) \frac{(2\pi)^{-6}}{4} \Big[R(p-\tfrac{\Delta}{2}n;\ k-\tfrac{\Delta'}{2}n | K_M) + \\ &+ R(p-\tfrac{\Delta}{2}n;\ k+\tfrac{\Delta'}{2}n | K_M) + R(p+\tfrac{\Delta}{2}n;\ k-\tfrac{\Delta'}{2}n | K_M) + \\ &+ R(p+\tfrac{\Delta}{2}n;\ k+\tfrac{\Delta'}{2}n | K_M) \Big] u_1(\vec k_1 \lambda_1) u_2(\vec k_2 \lambda_2), \end{aligned} \tag{90}$$

$$\begin{aligned} T_1(nM|\tilde p_1 \sigma_1 \tilde p_2 \sigma_2; \tilde k_2 \lambda_2 \tilde k_1 \lambda_1) &= \frac{1}{i}(2\pi)^3 n_0 \delta^3(\vec P_M - \vec K_M) \times \\ &\times \bar u_1(\vec p_1 \sigma_1) \bar u_2(\vec p_2 \sigma_2) (2\pi)^{-8} \Big\{ i\pi \int_{-\infty}^{\infty} d\alpha \Big(P\frac{1}{\Delta/2+\alpha} + P\frac{1}{\Delta/2-\alpha}\Big) \Big[R(p-\alpha n; k+\tfrac{\Delta'}{2} n | K_M) + \end{aligned}$$

$$+ R(p-\alpha n;k-\frac{\Delta'}{2}n|K_M)] + i\pi\int_{-\infty}^{\infty}d\beta(P\frac{1}{\Delta'/2+\beta}+P\frac{1}{\Delta'/2-\beta})[R(p-\frac{\Delta}{2}n;k-\beta n|K_M) +$$

$$+ R(p+\frac{\Delta}{2}n;k-\beta n|K_M)] + \int_{-\infty}^{\infty}d\alpha(P\frac{1}{\Delta/2+\epsilon}+P\frac{1}{\Delta/2-\epsilon}) \times$$

$$\times \int_{-\infty}^{\infty}d\beta(P\frac{1}{\Delta'/2+\beta}+P\frac{1}{\Delta'/2-\beta})R(p-\alpha n;k-\beta n|K_M)\Big\}u_1(\hat{K}_1\lambda_1)u_2(\hat{K}_2\lambda_2). \qquad (91)$$

As can easily been seen, division (89) the function T into two terms is realized with the help of the well-known formula

$$\frac{1}{\Delta/2\pm\epsilon-i\epsilon} = P\,\frac{1}{\Delta/2\pm\alpha} + i\pi\delta(\Delta/2\pm\alpha),$$

where P is the symbol of the principal value. From the single-time wave function ψ we go over to the wave function ψ', which will be defined with the equation

$$\psi'(nM|\tilde{p}_1 6_1 \tilde{p}_2 6_2) = \psi(nM|\tilde{p}_1 6_1 \tilde{p}_2 6_2) - \psi_1(nM|\tilde{p}_1 6_1 \tilde{p}_2 6_2), \qquad (92)$$

where

$$\psi_1(nM|\tilde{p}_1 6_1 \tilde{p}_2 6_2) = \frac{1}{nP-M-i\epsilon}\sum_{\lambda_1\lambda_2}\iint d\mu_1(\hat{K}_1)\;d\mu_2(\hat{K}_2) \times$$

$$\times T_1(nM|\tilde{p}_1 6_1 \tilde{p}_2 6_2;\tilde{k}_2\lambda_2\tilde{k}_1\lambda_1)\psi^{(0)}(nM|\tilde{k}_1\lambda_1\tilde{k}_2\lambda_2). \qquad (93)$$

For the wave function ψ' we obviously have the following representation

$$\psi'(nM|\tilde{p}_1 6_1 \tilde{p}_2 6_2) = \psi^{(0)}(nM|\tilde{p}_1 6_1 \tilde{p}_2 6_2) + \frac{1}{nP-M-i\epsilon} \times$$

$$\times \sum_{\lambda_1\lambda_2}\iint d\mu_1(\hat{K}_1)\;d\mu_2(\hat{K}_2)T'(nM|\tilde{p}_1 6_1 \tilde{p}_2 6_2;\tilde{k}_2\lambda_2\tilde{k}_1\lambda_1)\psi^{(0)}(nM|\tilde{k}_1\lambda_1\tilde{k}_2\lambda_2). \qquad (94)$$

If now one determines the function V' with the help of the relation

$$T'(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2;\tilde{k}_2\lambda_2\tilde{k}_1\lambda_1) = V'(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2;\tilde{k}_2\lambda_2\tilde{k}_1\lambda_1) +$$

$$+ \sum_{\sigma_1'\sigma_2'} \iint d\mu_1(\vec{p}_1')\ d\mu_2(\vec{p}_2') \times$$

$$\times \frac{V'(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2;\tilde{p}_2'\sigma_2'\tilde{p}_1'\sigma_1')\,T'(nM|\tilde{p}_1'\sigma_1'\tilde{p}_2'\sigma_2';\tilde{k}_2\lambda_2\tilde{k}_1\lambda_1)}{nP' - M}, \qquad (95)$$

$M \to M + i\epsilon,$

then it can easily be established that the wave function ψ' is a solution of the dynamical equation

$$\psi'(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2) = \psi^{(0)}(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2) + \frac{1}{nP-M-i\epsilon} \times$$

$$\times \sum_{\lambda_1\lambda_2} \iint d\mu_1(\vec{k}_1)\ d\mu_2(\vec{k}_2) V'(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2;\tilde{k}_2\lambda_2\tilde{k}_1\lambda_1)\psi'(nM|\tilde{k}_1\lambda_1\tilde{k}_2\lambda_2). \qquad (96)$$

In the discrete spectrum from equation (96) there obviously follows a homogeneous equation for the single-time wave function for the bound state of two fermions. From explicit expression (91) for the function T_1 we can easily see that on the energy shell this function turns into zero

$$T_1(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2;\tilde{k}_2\lambda_2\tilde{k}_1\lambda_1)\Big|_{\Delta = \Delta' = 0} = 0 ,$$

and from (90) it follows that the function T' coincides on the energy shell with physical relativistic invariant amplitude for elastic scattering of two spinor particles. That is why we may say that the wave functions ψ and ψ' give a physical equivalent description of the two-fermion system. The single-time wave function ψ' is attractive, since it satisfies the dynamical equation, for which we have a constructive way of deriving the function V right in the terms of the basic functions of the quantum field theory - VEVs of the radiation operators. This circumstance may be of particular interest from the point of view of studying the exactly-solvable models in the quantum field theory.

In the language of the Green's function division (89) of the function T into two terms and transition to the description of the two-fermion system with the help of the wave function ψ' means a replacement of two-time Green's function G with the Green's function G' which is defined by the equality

$$G'(M) = G(M) - G_1(M),$$

where

$$G_1(M) = G_0(M)T_1(M)G_0(M).$$

It is clear that

$$G'(M) = G_0(M) + G_0(M)T'(M)G_0(M).$$

Taking into consideration relation (95) for the functions T' and V' we obtain an equation for the Green's function

$$G'(M) = G_0(M) + G_0(M)V'(M)G'(M).$$

In conclusion to the present Section it should be noted that the functions V and V' defined with relations (44) and (95), respectively, are complex functions depending on the spectral parameter M. The imaginary (more precisely antihermitian) part of these functions is connected with inelastic channels in two-particle interactions. It may be shown that the following equality holds /11/:

$$(1 + \overset{+}{T}(M)\overset{+}{G_0}(M))(V(M) - \overset{+}{V}(M))(1 + G_0(M)T(M))\Big|_{\text{on energy-shell}} = H, \qquad (97)$$

$$(1 + \overset{+}{T}'(M)\overset{+}{G}_0(M))(V'(M) - \overset{+}{V}'(M))(1 + G_0(M)T'(M))\Big|_{\text{on energy-shell}} = H, \qquad (98)$$

where the function H determines the contribution of all inelastic channels to the unitarity condition. Relations (97) and (98) determine the consistency condition for the dynamical equations with the requirement for the complete two-particle unitarity.

CONCLUSION

The single-time technique, proposed in refs./10,11/ for the Bethe-Salpeter wave function of the two scalar particle system has been generalized in this work and applied to the two-fermion system[1]. A self-consistent procedure of the single-time reduction has been constructed both in terms of the Bethe-Salpeter wave function and of the Green's function. In other words it means that the single-time reduction of the Bethe-Salpeter formalism has been realized in a complete form. The single-time reduction of the Bethe-Salpeter formalism brings us to a three-dimensional dynamical equation for the single-time wave function and two-time Green's functions, moreover the indicated equations having Schrödinger structure. By this structure we imply a possibility to present the full Hamiltonian of the relativistic two-particle system as a sum of the kinematic term and interaction. It is worth stressing that the Schrödinger structure of the dynamic equations arises for our case as a consequence of the causality structure of the local quantum field theory.

When deriving the three-dimensional dynamic equations we did not bare in mind any concrete model of the quantum field theory,

[1]The problem of deriving three-dimensional dynamic equations for the two-particle system in QFT has almost thirty year history, which may be traced e.g., in refs,/15-26/. There are also reviews /27-31/ on the subject.

but used its most general properties, for example, such as the property of frequency dividing the single-particle causality Green's functions, orthogonality property and completeness of the single-particle wave functions. The realization of the single-time reduction technique in terms of the Green's functions show that this technique can also be used in the quantum field theory models, where the single-particle causality Green's functions may differ from the free ones. Namely this situation occurs in the investigation of the infrared singularities in QCD /1/. Another attractive feature of the formalism considered is its universality, which manifests itself in the fact, that it can be applied to the system of particles with an arbitrary tensor structure. In the given formalism scalar, spinor, vector,etc., particles are treated on equal grounds.

It is of undoubted interest to calculate the potential for the interaction of two fermions in traditional quantum field theory models - QED and QCD. A separate article will be devoted to the investigation of the structure of the thus obtained potential. However here we would like to stress once more that the proposed technique for the single-time reduction provides strickt and consistent reasoning for the potential models in elementary particle physics.

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Appendix

In Appendix we present the list of basic formulae and relations.

The function $u(x|p\sigma)$ and $v(x|p\sigma)$, used in the article, are determined in the following way

$$u(x|\vec{p}\sigma)=f_{\tilde{p}}(x)u(\vec{p}\sigma),\quad \bar{u}(x|\vec{p}\sigma)=\overset{*}{f}_{\tilde{p}}(x)\bar{u}(\vec{p}\sigma), \tag{A.1}$$

$$v(x|\vec{p}\sigma)=\overset{*}{f}_{\tilde{p}}(x)v(\vec{p}\sigma),\quad \bar{v}(x|\vec{p}\sigma)=f_{\tilde{p}}(x)\bar{v}(\vec{p}\sigma),$$

where

$$f_{\tilde{p}}(x) = (2\pi)^{-3/2}\exp(-i\tilde{p}x),\quad \tilde{p}^2 = m^2, \tag{A.2}$$

$u(\vec{p}\sigma)$, $v(\vec{p}\sigma)$ - are the Dirac bispinors satisfying normalization and orthogonality conditions

$$\left.\begin{aligned} u^{\dagger}(\vec{p}\sigma)u(\vec{p}\sigma') &= 2E(\vec{p},m)\delta_{\sigma\sigma'}, \\ v^{\dagger}(\vec{p}\sigma)v(\vec{p}\sigma') &= 2E(\vec{p},m)\delta_{\sigma\sigma'}, \end{aligned}\right\} \tag{A.3}$$

$$E(\vec{p},m) = \sqrt{\vec{p}^2+m^2},$$

$$\left.\begin{aligned} \bar{u}(\vec{p}\sigma)u(\vec{p}\sigma') &= 2m\,\delta_{\sigma\sigma'}, \\ \bar{v}(\vec{p}\sigma)v(\vec{p}\sigma') &= -2m\,\delta_{\sigma\sigma'}, \end{aligned}\right\} \tag{A.4}$$

$$\left.\begin{aligned} u^{\dagger}(\vec{p}\sigma)v(-\vec{p}\sigma') &= 0, \\ v^{\dagger}(\vec{p}\sigma)u(-\vec{p}\sigma') &= 0, \end{aligned}\right\} \tag{A.5}$$

$\bar{u}$, $\bar{v}$ are bispinors the Dirac conjugates to u and v. The orthogonality and normalization relations for the functions $u(x|\vec{p}\sigma)$ and $v(x|\vec{p}\sigma)$ have the form

$$\left.\begin{aligned} &\int d^3x\ \bar{u}(x|\vec{p}\sigma)\gamma^0 u(x|\vec{k}\sigma') = 2E(\vec{p},m)\delta^3(\vec{p}-\vec{k})\delta_{\sigma\sigma'}, \\ &\int d^3x\ \bar{v}(x|\vec{p}\sigma)\gamma^0 v(x|\vec{k}\sigma') = 2E(\vec{p},m)\delta^3(\vec{p}-\vec{k})\delta_{\sigma\sigma'}, \end{aligned}\right\} \qquad (A.6)$$

$$\left.\begin{aligned} &\int d^3x\ \bar{u}(x|\vec{p}\sigma)\gamma^0 v(x|\vec{k}\sigma') = 0, \\ &\int d^3x\ \bar{v}(x|\vec{p}\sigma)\gamma^0 u(x|\vec{k}\sigma') = 0, \end{aligned}\right\} \qquad (A.7)$$

or in a covariant form

$$\int \bar{u}(x|\vec{p}\sigma)\, d\hat{\sigma}_x\ u(x|\vec{k}\sigma') = 2E(\vec{p},m)\delta^3(\vec{p}-\vec{k})\delta_{\sigma\sigma'} =$$

$$= \int \bar{v}(x|\vec{p}\sigma)\, d\hat{\sigma}_x\ v(x|\vec{k}\sigma'), \qquad (A.8)$$

$$\int \bar{u}(x|\vec{p}\sigma)\, d\hat{\sigma}_x\ v(x|\vec{k}\sigma') = \int \bar{v}(x|\vec{p}\sigma)\, d\hat{\sigma}_x\ u(x|\vec{k}\sigma') = 0, \qquad (A.9)$$

$$d\hat{\sigma}_x = \gamma_\mu d\sigma_x^\mu .$$

The condition for the completeness of the functions u and v is presented in the form

$$\left.\begin{aligned} &\sum_\sigma \int d\mu(\vec{p})\, u(x|\vec{p}\sigma)\bar{u}(y|\vec{p}\sigma) = \frac{1}{i} S^{(-)}(x-y), \\ &\sum_\sigma \int d\mu(\vec{p})\, v(x|\vec{p}\sigma)\bar{v}(y|\vec{p}\sigma) = \frac{1}{i} S^{(+)}(x-y). \end{aligned}\right\} \qquad (A.10)$$

It can easily be seen that the following relations are fulfilled

$$\left.\begin{aligned}\int S^{(-)}(x-y)\,d\hat{\sigma}_y u(y|\vec{p}\sigma) &= iu(x|\vec{p}\sigma)\ ,\\ \int \bar{u}\ (y|\vec{p}\sigma)\,d\hat{\sigma}_y S^{(-)}(y-x) &= i\bar{u}(x|\vec{p}\sigma),\end{aligned}\right\}\qquad \text{(A.11)}$$

$$\left.\begin{aligned}\int S^{(+)}(x-y)\,d\hat{\sigma}_y v(y|\vec{p}\sigma) &= iv(x|\vec{p}\sigma)\ ,\\ \int \bar{v}\ (y|\vec{p}\sigma)\,d\hat{\sigma}_y S^{(+)}(y-x) &= i\bar{v}(x|\vec{p}\sigma),\end{aligned}\right\}\qquad \text{(A.12)}$$

$$\left.\begin{aligned}\int S^{(-)}(x-y)\,d\hat{\sigma}_y v(y|\vec{p}\sigma) &= \int \bar{v}\ (y|\vec{p}\sigma)\,d\hat{\sigma}_y S^{(-)}(y-x)=0,\\ \int S^{(+)}(x-y)\,d\hat{\sigma}_y u(y|\vec{p}\sigma) &= \int \bar{u}\ (y|\vec{p}\sigma)\,d\hat{\sigma}_y S^{(+)}(y-x)=0,\end{aligned}\right\}\qquad \text{(A.13)}$$

$$\left.\begin{aligned}\int S^{(-)}(x-y)\,d\hat{\sigma}_y S^{(-)}(y-z) &= iS^{(-)}(x-z),\\ \int S^{(+)}(x-y)\,d\hat{\sigma}_y S^{(+)}(y-z) &= iS^{(+)}(x-z),\\ \int S^{(\pm)}(x-y)\,d\hat{\sigma}_y S^{(\mp)}(y-z) &= 0.\end{aligned}\right\}\qquad \text{(A.14)}$$

For the single-particle causality Green's function of the spinor field we use the expression

$$S^{c}(x-y) = \theta(x^0-y^0)S^{(-)}(x-y)-\theta(y^0-x^0)S^{(+)}(x-y). \qquad \text{(A.15)}$$

From (A.14) and (A.15) there follows that

$$\left.\begin{aligned}\int_{n\zeta=\tau} S^{(-)}(x-\zeta)\,d\hat{\sigma}_\zeta S^{c}(\zeta-y) &= i\theta(\tau-ny)S^{(-)}(x-y),\\ \int_{n\zeta=\tau} S^{(+)}(x-\zeta)\,d\hat{\sigma}_\zeta S^{c}(\zeta-y) &= -i\theta(ny-\tau)S^{(+)}(x-y),\\ \int_{n\zeta=\tau} S^{c}(x-\zeta)\,d\hat{\sigma}_\zeta S^{(-)}(\zeta-y) &= i\theta(nx-\tau)S^{(-)}(x-y),\end{aligned}\right\}\qquad \text{(A.16)}$$

$$\int_{n\zeta=\tau} S^{c}(x-\zeta)\, d\hat{\sigma}_{\zeta} S^{(+)}(\zeta-y) = -i\theta(\tau-nx) S^{(+)}(x-y)$$

We shall also present the transformation properties of the functions $u(x|\vec{p}\sigma)$ and $v(x|\vec{p}\sigma)$ respect to the proper orthochronous Lorentz group

$$u'(x'|\vec{p}'\sigma)=S(A)u(x|\vec{p}\sigma)=u(x'|\vec{p}'\sigma')D^{1/2}_{\sigma'\sigma}(\tilde{A}(A,p)),$$
$$\bar{u}'(x'|\vec{p}'\sigma)=\bar{u}(x|\vec{p}\sigma)S^{-1}(A)=D^{+1/2}_{\sigma\sigma'}(\tilde{A}(A,p))\bar{u}(x'|\vec{p}'\sigma') \qquad (A.17)$$

where $A\in SL(2.c)$ is an element of the universal covering group of the proper orthochronous Lorentz group $L^{\uparrow}_{+}$, $\tilde{A}$ is Wigner rotation

$$\tilde{A}(A,p) = [p']^{-1}A[p], \qquad (A.18)$$

where $[p]$ denotes the element from the $SL(2.C)$ group and it is such that $[p]\hat{\overset{0}{p}}[p]^{+} = \hat{p}$, $\hat{p} \equiv p^{\mu}\sigma_{\mu}$, $\hat{\overset{0}{p}} = \overset{0}{p}{}^{\mu}\sigma_{\mu}$, $\overset{0}{p}{}^{\mu} = (m,\vec{o})$, σ_{μ} is the set of the Pauli matrix with the unity one /32/. $\tilde{A}$ is an element of the $SU(2)$ subgroup of the $SL(2.C)$ group. $D^{1/2}$ is the matrix of the finite-dimensional representation of the rotation group ($SU(2)$ group) with a half integer spin. $S(A)$ is the matrix of the spinor representation of the $SL(2.C)$ group. In the representation of the Dirac matrices where the γ_5 matrix has a diagonal form $\gamma_5 = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$, the matrix $S(A)$ has a block-diagonal form

$$S(A) = \begin{pmatrix} A & 0 \\ 0 & A^{+-1}\end{pmatrix}.$$

Finally, $p'=\Lambda(A)p$, $x'=\Lambda(A)x$, where $\Lambda(A)$ is the Lorentz transformation matrix.

It may be shown that the single-time wave functions, introduced in this work, possess the following transformation properties two-fermion system:

$$\left.\begin{aligned}\tilde{\psi}'(n'\tau|x_1'x_2') &= S_1(A)S_2(A)\tilde{\psi}(n\tau|x_1x_2),\\ \psi'(n'M|\tilde{p}_1'\sigma_1'\tilde{p}_2'\sigma_2') &= D^{1/2}_{\sigma_1'\sigma_1}(\tilde{A}_1)D^{1/2}_{\sigma_2'\sigma_2}(\tilde{A}_2)\psi(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2).\end{aligned}\right\} \qquad (A.19)$$

$$n' = \Lambda(A)n,\quad x_1' = \Lambda(A)x_1,\quad x_2' = \Lambda(A)x_2,$$

$$\tilde{p}_1' = \Lambda(A)\tilde{p}_1,\quad \tilde{p}_2' = \Lambda(A)\tilde{p}_2,$$

$$\tilde{A}_1 = [\tilde{p}_1']^{-1}A[\tilde{p}_1],\quad \tilde{A}_2 = [\tilde{p}_2']^{-1}A[\tilde{p}_2],$$

fermion-antifermion system:

$$\left.\begin{aligned}\tilde{\psi}'(n'\tau|x_1'x_2') &= S_1(A)\tilde{\psi}(n\tau|x_1x_2)\,S_2^{-1}(A),\\ \psi'(n'M|\tilde{p}_1'\sigma_1'\tilde{p}_2'\sigma_2') &= D^{1/2}_{\sigma_1'\sigma_1}(\tilde{A}_1)\,\psi(nM|\tilde{p}_1\sigma_1\tilde{p}_2\sigma_2)D^{+1/2}_{\sigma_2\sigma_2'}(\tilde{A}_2).\end{aligned}\right\} \qquad (A.20)$$

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