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L. LAURENT, J.M. RAX

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L. Laurent, J.M. Rax
Association EURATOM-CEA sur la Fusion Contrôlée
Centre d'Etudes Nucleaires Cadarache
13108 Saint-Paul-lez-Durance Cedex
FRANCE

Because of the finite number of coils the toroidal magnetic field of tokamak configurations is sligthly modulated. This distortion (the "ripple") has generally a negligible effect on particle trajectories. However this static perturbation may play an important role when it is resonant. A well known example [1] is the resonance of trapped alpha particles when there exist two integers N_b, N_d such that: $N_b \omega_b - N_d \omega_d = 0$ where ω_b, ω_d are the bounce and precession frequency. This resonance leads to the loss of a significant part of the α particles if the ripple is large enough.

We discuss here another resonance which occurs with fast electrons. In tokamaks discharges a fraction of the electrons (the runaway electrons) can be accelerated up to energies of the order of several tenth of MeV until synchrotron losses equilibrate the loop voltage acceleration. In large tokamaks such electrons are created mainly at the beginning of the discharge when the collisionality is low enough or during disruptions when large loop voltage are generated. For such electrons it is possible to find to integers N_t, N_c such that: $N_t \omega_t - N_c \omega_c = 0$ where ω_t, ω_c are the transit and cyclotron frequency. When this condition is fulfilled the pitch angle of electrons varies drastically. The resonant interaction may affect drastically the energy spectrum of runaway electrons when combined with the cyclotron radiation and even push them in the trapped domain.

This report contains two papers presenting two approaches to study these resonances:

- The first one has been presented at the Technical Committee Meeting on Electron Cyclotron Emission and Electron Cyclotron Resonance Heating (HEFEI China 1989). The resonance effect is considered as an intense cyclotron heating of the electrons by the ripple field in the guiding center frame of the fast particles. This method provides a good understanding of the runaway ripple interaction. Although the decorrelation time of the cyclotron phase is not handled rigorously, quasilinear diffusion coefficients have been obtained. A simple criterion to take into account the radiative cooling by cyclotron radiation of electrons is used.

- In the second one, submitted to Europhysics letters, an Hamiltonian formalism is used. A criterion for the onset of chaotic behaviour is given and the results of the first study are recovered. The interaction between cyclotron radiation and pitch angle scattering is treated more accurately.

[1] A. Becoulet, D.J. Gambier, P.Grua, J.M. Rax, J-P Roubin 16^{th}European conference on Controlled Fusion and Plasma Physics. Vol1 p 267.

Pitch angle scattering of runaway electrons in tokamaks due to cyclotron resonance in the ripple

I-Introduction

It has been observed recently in the Tore Supra tokamak (TS) that the energy of runaway electrons was below 30 Mev, although their confinement time was of the order of 1s. This implies that the average pitch angle of these electrons is of the order of 8° in such a way that the acceleration due to the loop voltage V_l equilibrates the synchrotron losses[1] :

$$\frac{d\gamma}{dt} = \frac{eV_l}{2\pi R m_e c} - \frac{2}{3} r_e c \frac{\gamma^4}{R_c^2} = 0, \tag{1}$$

e and m_e are the electron charge and mas, γ is the relativistic energy normalized to the rest energy, r_e the classical electron radius, c the velocity of light and R_c is the instantaneous radius of curvature of the electron trajectory which, for a given γ, can be expressed as a function of the tokamak major radius R and particle pitch angle φ (*) :

$$R_c = R\left(\frac{1+\sin^2(\varphi)}{1+\frac{R^2\sin^2(\varphi)}{\lambda^2}}\right)^{1/2} \quad \text{with } \lambda = \frac{m_e c\gamma}{eB} \tag{2}$$

This steady pitch angle of 8° is much larger than expected one taking into account the residual effect of collisions or the influence of the electromagnetic turbulence. Direct observations of synchrotron radiation in Textor confirm this large angle[2] .

The aim of this paper is to propose a mechanism which can drastically enhance the perpendicular velocity of electrons in such a way that $R_c \ll R$. The cyclotron interaction of runaway electrons with the field modulation due discrete number of toroidal coils (ripple) provides such a mechanism.

The effect of this interaction can be conveniently evaluated using a Lorentz transformation: in its guiding center frame the electron experiences a dramatic cyclotron heating (close to what is obtained with the undulator of some free electron lasers). Going back to the laboratory frame it appears that this heating becomes a conversion between perpendicular and parallel momentum. The associated angular diffusion coefficient is then evaluated to acess the efficiency of the proposed mechanism.

II-Resonance condition

Because of the finite number of coils the toroidal field is modulated in the toroidal direction. An electron located close to a magnetic surface of minor radius r experiences a modulation which can be written as a function of the poloidal (θ) and toroidal angle (ϕ) :

$$\frac{\delta B}{B} = F_1(\theta)\, e^{iN\phi} \;...\; + F_n(\theta)\, e^{inN\phi} \;+ ... \qquad (3)$$

where N is the number of coils, and the F_n are functions containing information about the ripple localization on the low field side and details of the runaway trajectory. Typically in Tore Supra at mid-radius $F_1(0) = 8.\ 10^{-3}$, $F_2(0) = 7.\ 10^{-5}$.

Along its trajectory an electron experiences this high frequency modulation as bursts of electromagnetic waves occuring on the low field side because of the form factor F_n. If $V_{||} \approx c$ is the velocity of the guiding center the frequency of the bursts due to the toroidal component n is :

$$\omega = \frac{nNV_{||}}{R}. \qquad (4)$$

The cyclotron frequency of this electron is : $\omega_c = \frac{eB}{m_e\gamma}$ where γ is the relativistic factor . The resonance can be written as a condition on the energy $\mathcal{E} = m_e c^2 \gamma$:

$$\gamma = \frac{eBR}{nNcm_e} \quad , \quad \mathcal{E}\,(\mathrm{MeV}) = 299.8\,\frac{BR}{nN}. \qquad (5)$$

For the Tore Supra parameters (B=1.8T, R=2.36m) this condition leads to a series of resonances, the n=2 one occurs at 35 MeV and is a possible candidate for the observed energy limit. The strength of this resonance is estimated in the following section.

III-Angular diffusion coefficient in slab geometry

To estimate the effect of the cyclotron interaction we consider now a simplified model: the electron guiding center is moving in the z direction along the unperturbed field B with a velocity $V_{||}$. A static perturbation of the magnetic field is added in the y direction: $\delta B \cos\left(\frac{nNz}{R}\right)$ where δB correspond to the F_n function . In the guiding center frame (indexed with the subscript ') this perturbation appears as a (nearly) plane electromagnetic wave whose fields are given by a Lorentz transformation as follow:

$$\mathbf{E}' = -V_{\parallel}\gamma_{\parallel}\,\delta B \cos\left(\frac{nN\gamma_{\parallel}.(z'+V_{\parallel}.t')}{R}\right)\mathbf{e}_x \tag{6}$$

$$\mathbf{B}' = \gamma_{\parallel}\delta B \cos\left(\frac{nN\gamma_{\parallel}(z'+V_{\parallel}.t')}{R}\right)\mathbf{e}_y \tag{6bis}$$

$\gamma_{\parallel} = \dfrac{1}{\sqrt{1-V_{\parallel}^2/c^2}}$ is the relativistic factor associated with $V_{\parallel}$. The energy flux of this wave is $\dfrac{c(\delta B\gamma_{\parallel})^2}{\mu_0}$ and a typical value is of 10^{14} W m^{-2}! The variation of energy and parallel momentum due to the interaction of the wave with the particle are :

$$\frac{d\mathcal{E}'}{dt'} = e\,\mathbf{E}'.\mathbf{V}' = -e\,V'_{\perp}\frac{\gamma_{\parallel}V_{\parallel}\,\delta B \sin(\alpha)}{2} \tag{7}$$

$$\frac{dp'_{\parallel}}{dt'} = e\,\mathbf{V}'\times\mathbf{B}'.\mathbf{e}_z = e\,V'_{\perp}\frac{\gamma_{\parallel}\delta B \sin(\alpha)}{2} \tag{7bis}$$

where α is the phase angle between the cyclotron motion and the wave. Going back to the laboratory with a Lorenz transformation one finds $\frac{d\mathcal{E}}{dt} = 0$ as expected, since a magnetic field cannot give energy to a particle, the dynamics of the laboratory parallel momentum is given by:

$$\frac{dp_{\parallel}}{dt} = \left(\frac{dp'_{\parallel}}{dt'} + \frac{V_{\parallel}d\mathcal{E}'}{c^2\,dt'}\right) = e\,V_{\parallel}V_{\perp}\frac{\delta B \sin(\alpha)}{2c} \tag{8}$$

The energy conservation i.e. conservation of p^2 can be used to calculate the the corresponding perpendicular momentum or pitch angle φ variation:

$$\frac{dp_{\perp}}{dt} = -e\,V_{\parallel}\frac{\delta B \sin(\alpha)}{2}, \qquad \frac{d\varphi}{dt} = -\frac{\delta B\,\omega_c \sin(\alpha)}{2B} \tag{9}$$

Let us consider now the actual tokamak geometry. The electron experiences this interaction when F_n has a large value i.e. typically when $\left|\theta\right| < \frac{\pi}{2}$. The duration of this interaction is $\frac{\pi qR}{c}$ where q is the safety factor. Between two interactions there is a time lag of the order of $\frac{2\pi qR}{c}$. The cyclotron phase is lost because of the combined effect of the rotational transform , and the finite width of F_n. Therefore the exchange between parallel and perpendicular momenta is irreversible and can be described by an angular diffusion coefficient (**) :

$$D_{\varphi\varphi} = \frac{\pi qc}{8R} n^2 N^2 \left(\frac{\delta B}{B}\right)^2 . \tag{10}$$

For the n=2 resonance and typical Tore Supra parameter ($\delta B/B = 7.\ 10^{-5}$, N=18, R=2.4 m, q=2) a very large diffusion coefficient is found ($D \approx 620.\ \mathrm{rad}^2\ \mathrm{s}^{-1}$) .

IV-The Ripple cyclotron barrier for runaway electrons

The particle experiences this interaction by bursts of $\frac{nNq}{2}$ periods corresponding to its excursion on the low field side and this causes a resonance broadening: $\frac{\Delta\gamma}{\gamma} = \frac{2}{nNq}$. During the crossing of the resonance the particle dynamics is governed by three effects:

- The cyclotron interaction with ripple increase the pitch angle .
- The loop voltage V_l tend to increase the energy of the electron .
- The electron looses its energy through cyclotron radiation.

When the pitch angle of the electron is 8° the cyclotron losses equilibrate the loop voltage acceleration. For larger pitch angles the electron cannot go beyond the resonance. The combination of the previous effects acts as an effective slowing down if the following ordering is fullfilled:

$$\tau_{rad} \ll \tau_{acc} \quad \text{and} \quad \tau_{dif} \ll \tau_{acc}.$$

where τ_{rad} is the characteristic time scale of the radiative cooling, τ_{dif} the diffusion time associated with the resonant interaction and τ_{acc} the time needed to cross the resonant region according to the free fall acceleration.

A simple criterion to guess if there is an energy blocking is to state that this will happen if the pitch angle diffuses by more than $\frac{\pi}{4}$ during the crossing of the resonance (neglecting the radiative cooling), in such a case with the usual loop voltage the previous time ordering is fullfilled. This condition can be written :

$$\left(D_{\varphi\varphi}\frac{2\pi R\Delta\gamma m_e c^2}{eV_l c}\right)^{1/2} > \frac{\pi}{4}, \qquad \frac{\delta B}{B} > \left(\frac{eV_l}{8nN\mathcal{E}}\right)^{1/2}. \tag{11}$$

This condition is fulfilled for the n=2 resonance at mid-radius, in the Tore Supra case. Therefore the cyclotron ripple resonance is likely to cause an energy blocking around 35 MeV for the low field discharges. It can be predicted that the energy blocking will disappear at high field discharge since the n= 2 resonance will happen at energy of the order of 50 MeV , i.e. close to the synchrotron limit.

V- Conclusion

A new physical effect explaining the energy blocking of runaway electrons in Tore Supra has been described and investigated. We have presented a clear physical picture of the resonant interaction of the runaway electrons with the ripple field and a detailed Hamiltonian study of this mechanism is left to a future work. These resonances should play also play a role in limiting the runaway energy to a few hundred of MeV in forthcoming large devices, or to even lower values if a special device (ondulator, wiggler...) is used to produce larger values of N.

Acknowledgements: The authors wish to acknowledge D[r] Chatelier, A. Geraud and G. Martin for fruitfull discussions.

References

1 M. Chatelier et al. Proc. of the 16th EPS conference on controlled fusion and plasma physics (Venice, 1989) V1 P19.

2 K.H Finken et al. Proc. of the 16th EPS conference on controlled fusion and plasma physics (Venice, 1989) V1 P147.

3 L. Laurent et J.M Rax to be submitted to Euro. Phys. Letters.

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In the second paper the approximation R>> λ has been used. The cyclotron losses in equation (1) of thist paper are equivalent to the one expressed in formula (14) in next one.

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Using equation (9) it is also possible to find the diffusion coefficient for the J action defined in second paper:

$$\frac{\partial J}{\partial t} = \frac{P_{\perp} V_{\parallel}}{2} \left(\frac{\delta B}{B}\right) \sin(\alpha)$$

Hence using the same technique one finds the quasilinear diffusion coefficient of J with agree with formula (13) of second paper within a small factor.

$$D_{JJ} = \frac{\pi R \omega_c q}{4} \left(\frac{\delta B}{B}\right)^2 J\, P$$

STOCHASTIC INSTABILITY OF RUNAWAY ELECTRONS IN TOKAMAKS

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Abstract: The motion of runaway electrons in tokamak field configuration is found to be unstable with respect to a stochastic instability due to non linear cyclotron resonances with the toroidal magnetic field ripple. We demonstrate that neighbouring poloidal harmonics of these relativistic resonances overlap. The resulting instability provides an efficient mechanism to convert the parallel kinetic energy driven by the loop voltage into perpendicular one. When combined with cyclotron losses this conversion acts as an effective radiative slowing down process in agreement with recent observations. Potential applications of this instability are briefly considered.

1.Introduction

When the velocity of runaway electrons approaches the celerity of light (c) all the usual dissipative coupling (collision, turbulence, bremstrahlung) become inefficient. Synchrotron slowing down remains the only mechanism able to influence the electron dynamics and to balance the free fall acceleration due to the electric field. It has been recently observed in the Tore Supra tokamak, during low field discharges, that the energy of runaway electrons was of the order of 30 MeV, i.e. well below the synchrotron limit ($\approx$ 70 MeV), [1] although their confinement time was of the order of 1. s. These two observations imply the existence of an additional slowing down mechanism. A finite level of cyclotron radiation (stemming from the cyclotron motion) to complement the usual synchrotron one (stemming from the toroidal motion) may account for this observed low energy limit.[1] With a pitch angle of the order of few degrees the radiation limit is easily lowered down to 30. MeV in agreement with the observed limit. Direct observations in Textor with an IR camera confirm this unsuspected pitch angle.[2] However a mechanism has to be found to explain this steady pitch angle which corresponds to $m_e v_\perp^2/2 \approx 10$ KeV.

In this letter we propose and investigate a mechanism, based on a new universal instability of the runaway population, which may account for this energy isotropisation in tokamak configurations. Because of the finite number of coil the toroidal field is modulated in the toroidal direction. This magnetic field ripple (MFR) creates a set of non linear cyclotron resonances which causes a stochastic instability.[3,4] This instability allows an irreversible transfer of energy from the parallel degree of freedom to the perpendicular one, then this energy is dissipated through cyclotron radiation. This transfer can be interpretated as an intense cyclotron heating in the guiding center frame[5] of the fast particle.

In the next section we review the non ideal field structure of tokamak configuration. Then we show that the MFR interact resonantly with runaway electron for somes energies. In section 4 and 5 we investigate the possibility of causing an energy blocking under the combined influence of pitch angle scattering, due to quasi-linear interaction with the MFR, and cyclotron radiation. Finally practical formulas are given and it is shown that in Tore Supra the MFR is large enough to explain the experimental results.

2.Ripple structure of Tokamak configurations

Let us consider a tokamak configuration with N coils, a major radius R and a minor radius a. The transverse part, δB, of the MFR can be expressed as a Fourier series in the toroidal angle (ϕ) and poloidal angle (θ) describing a given magnetic surface of radius r and safety factor q:

$$\delta B(r,\theta,\varphi) = \sum_{m=0,n=1}^{m=\infty,n=\infty} \delta B_{mn}(r)\cos(m\theta)\cos(nN\varphi). \tag{1}$$

The longitudinal part of the MFR will not be considered here because the associated particle field coupling is weaker. The structure of the various harmonics δB_{mn} is usually given by the numerical field map of the considered Tokamak. The runaway trajectory is located on a shifted torus ($r \approx r_0 + \Delta r \cos(\theta)$) and the electrons follow helical trajectories, with a pitch of the order of q, on these shifted drift surfaces. This shift between the drift surface and the magnetic surface enhances the first poloidal harmonic effectively experienced by the fast particule. In the following Hamiltonian study δB_{mn} are understood as these modified Fourier coefficients which will be evaluated in section 6. A more convenient system of coordinates consist in a Cartesian basis whose z axis is directed along the trajectory and with the y axis along the drift surface normal. Such a set of coordinates corresponding to an unfolding of the drift surface followed by a pillup of a set of drift surface replica is illustrated by Fig. 1.

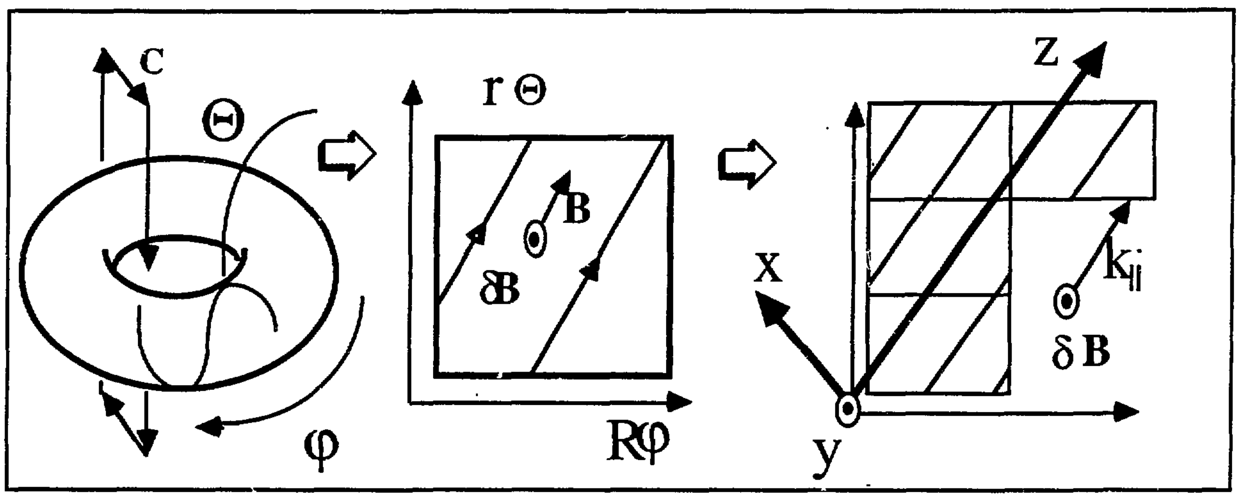

Fig. 1 The set of transformations leading to the Cartesian basis xyz.

With these coordinates the **B** field is approximatively aligned along the z axis and the transverse MFR **δB** stands along the y axis:

$$\mathbf{B}= B.\mathbf{e}_z, \qquad \delta\mathbf{B} = \sum_{m=0,n=1}^{m=\infty,n=\infty} \delta B_{mn} \cos[k_{\|}(m,n).z].\mathbf{e}_y. \tag{2}$$

With the approximation R>>r the wave vector can be expressed as a function of the coresponding poloidal and toroidal numbers: $k_{\|} = [nNq + m]/Rq \approx nN/R$. The transverse variation of the field corresponds to wavenumbers such that $k_{\perp}\rho \approx m\rho/R \ll 1$ (ρ is the Larmor radius of the runaway) and thus can be neglected. With the Coulomb gauge the vectors potential describing the main field and one MFR component are:

$$\mathbf{A}(x) = Bx.\mathbf{e}_y, \qquad \delta\, \mathbf{A}_{mn}(z) = -\, \delta B_{mn} \frac{\sin[k_{\|}(m,n)z]}{k_{\|}(m,n)}.\mathbf{e}_x. \tag{3}$$

The previous unfolding of the trajectory is allowed because we are interested in passing particle, the trapped runaway population (which has already a large pitch angle i.e. large cyclotron losses) is not considered in this study.

3.Non linear relativistic cyclotron resonances

Let us follow the dynamic of a runaway electron characterized by its charge e, mass m_e and relativistic factor γ. Such a suprathermal particle ceases to experience the dissipative influence of the colliding thermal background as soon as it has reached the critical Dreicer [6] kinetic energy $(\gamma_D-1)m_e c^2 < O(1\text{MeV})$. When it reaches few ten of MeV synchrotron radiation losses begin to balance the inductive energy gain, resulting in an upper synchrotron limit $\gamma_S m_e c^2$:

$$\gamma_S m_e c^2 = \left(\frac{3ReV_1}{4\pi r_e}\right)^{1/4} \approx O(70\text{MeV}). \tag{4}$$

r_e the classical electron radius (r_e=2.8 10^{-15} m) and V_1 is the loop voltage of the discharge. In fact between these lower and upper bounds the runaway particle experiences a set of cyclotron resonances with the MFR when $\omega_c = eB/(m_e\,\gamma) = k_{\|}(n,m)c$ i.e:

$$\gamma_n\, m_e c^2 = \frac{BeRc}{nN} \approx O(10,\, 30\text{ MeV}). \tag{5}$$

In the TS low field configuration we obtain the following ordering [Fig. 2]:

$$\gamma_D < ... < \gamma_3 < \gamma_2 < \gamma_S < \gamma_1.$$

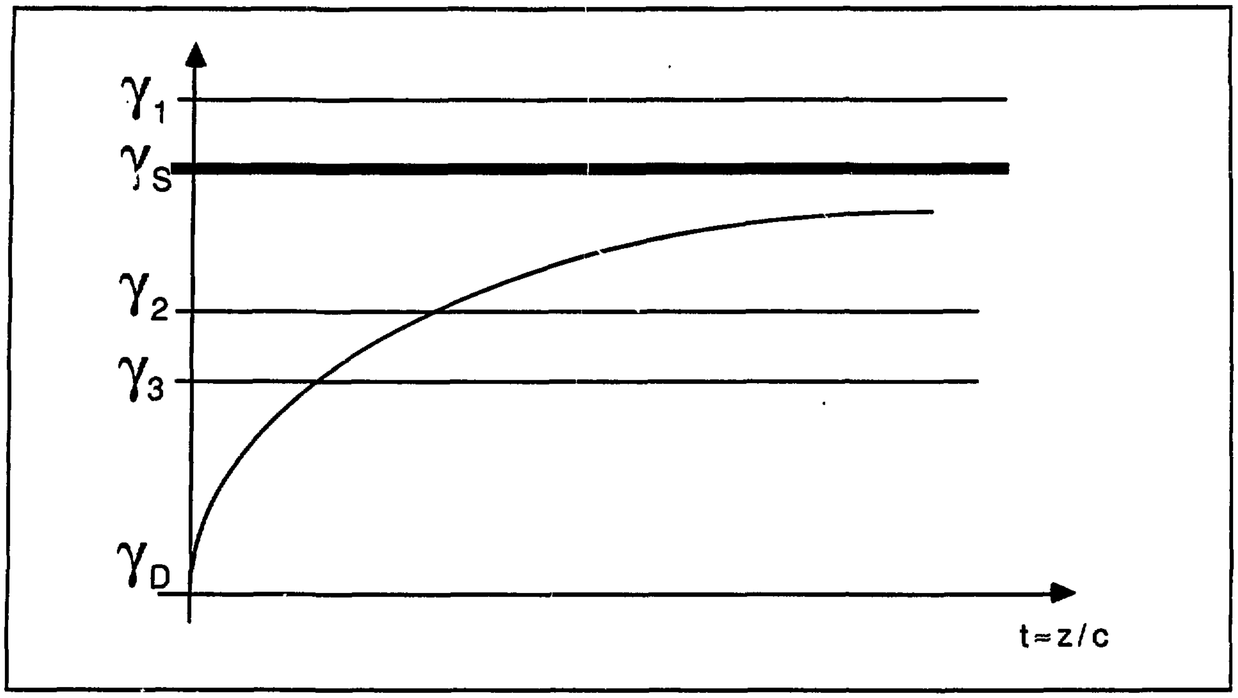

Figure 2: The occurence of the various resonances as a function of time

To analyse the influence of one of these non linear resonances encountered between γ_D and γ_S we introduce the relativistic Hamiltonian for one MFR component:

$$H(x,y,z,\pi_x,\pi_y,\pi_z) = \sqrt{m_e^2c^4 + [\pi + ecA(x) + ec\delta A(z)]^2} + \frac{eV_l\, z}{2\pi R}, \qquad (6)$$

π is the canonical momentum vector conjugated to the position. The electric part of the H function describes the loop voltage acceleration and is associated with the slow times scales of the problem (>O(1s)). Here we are interested by resonant interaction and stochastic diffusion which occur on the fast time scale τ_{dif} thus we can perfom a two time scale analysis and neglect the influence of the loop voltage in the study of the MFR cyclotron resonances. We can define a set of action variables: $P = p_{\parallel}$ and $J = p_{\perp}^2/2\omega_C\gamma m_e$ canonically conjugated to the z drift and to the gyroangle α ($p_{\parallel}$ and $p_{\perp}$ are the usual parallel and perpendicular relativistic momenta). Then we expand the Hamiltonian $H = H_0(A) + \delta H(\delta A)$ around $\delta A = 0$ and express both the integrable part and the perturbative one in term of these angle action variables:

$$H(J,P,\alpha,z) = \sqrt{m_e^2c^4 + P^2c^2 + 2\gamma m_e c^2\omega_c J} + \sum_{m,n,\varepsilon=\pm 1} \frac{e\delta B_{mn}}{2k_{\parallel}} \sqrt{\frac{2\omega_c J}{m_e\gamma}} \sin[k_{\parallel}(m,n).z - \varepsilon.\alpha)].$$

(7)

The perturbation is resonant if $k_{||}(m,n)P \pm m_e\gamma\omega_c = 0$. With the upper sign we recover equation (5). This defines a set P = P(m,n) of parallel resonant curves in the action space J,P (Fig. 1). We will restrict our investigations to the largest components of the series i. e m = 0,1.

4.Stochasticity threshold, random phase approximation

For a given number n there are close resonances coresponding to the different m numbers:

$$P(m,n) = \frac{\gamma\omega_c m_e Rq}{nNq+m} \approx \frac{\gamma\omega_c m_e R}{nN}, \quad P(0,n)\text{-}P(1,n) = \delta P(n) \approx \frac{P(n)}{nNq}. \tag{8}$$

These neighbouring resonances (dashed lines on figure3) may overlap and cause a stochastic instability and a subsequent quasi linear diffusion in the velocity space at constant energy (i.e. a pitch angle scatttering). To evaluate the threshold of this overlapping we calculate the extend of the trapped domain (i.e the island width):

$$\Delta P(m,n) = 4\sqrt{\frac{\delta B_{mn}}{B}\frac{p_{||}p_{\perp}}{2}}\ . \tag{9}$$

Then the Chirikov parameter S_n, which measure the strength of the non linear resonances coupling, enables to find the threshold condition [3,4] for the occurence of a stochastic instability :

$$S_n(r,p_{\perp}) = \frac{\Delta P(0,n)+\Delta P(1,n)}{2\delta P(n)} = 4nNq\sqrt{\frac{\delta B_{0n}(r)}{B}\frac{p_{\perp}}{2p_{||}}} > 0.7. \tag{10}$$

Note that even for Chirikov parameters below 0.7 these resonances play a role, in presence of another pitch angle scattering cause (like residual collisions) the electron can jump from a trapped domain to another. This results in a dramatic enhancement of the diffusion coefficient.

This parameter is to be evaluated with a parallel momentum equal to the resonant ones defined previously and with a perpendicular residual velocity of the order of the thermal energy (we have used the fact that the m=0 and m=1 harmonics have nearly the same amplitude, this will be justified in the following). Assuming $v_{\perp} \approx \frac{c}{20}$, $v_{||} \approx$ c we obtain an approximate criterion on the MFR strength of the n^{th} toroidal harmonic:

$$\frac{\delta B_{0n}}{B} > (nNq)^{-2} \tag{11}$$

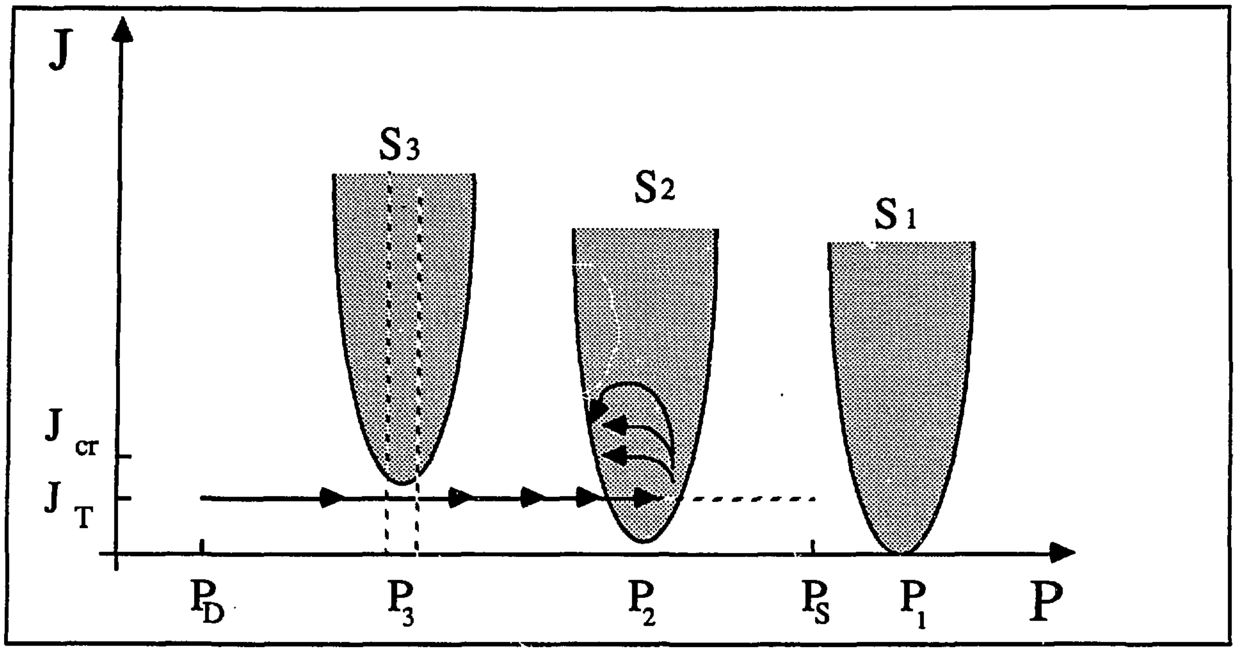

Fig. 3 The action space, the resonant momentum P_n and the instability zones S_n.

When the condition equation (10) is fullfilled (shaded areas of Fig.3) the dynamic becomes stochastic and is described by a quasilinear diffusion of the actions. To evaluate the associated diffusion coefficient we consider the motion of the perpendicular degree of freedom which is governed by the Hamilton equation $\frac{dJ}{dt} = - \frac{\partial(\delta H)}{\partial \alpha}$ and we calculate the perpendicular action increment δJ during a small time step δt as follow:

$$\delta J(\delta t, \Phi) = \frac{\mathrm{Re}\delta B_{0n}}{2nN} \sqrt{\frac{2\omega_c J}{m_e \gamma}} \frac{\sin[(\frac{nNP}{m_e \gamma R} - \omega_c)\delta t + \Phi] - \sin(\Phi)}{\frac{nNP}{m_e \gamma R} - \omega_c}. \tag{12}$$

We have introduced an initial phase Φ which, according to the previous study, can be considered as a random variable above the stochasticity threshold. This random phase approximation (RPA) assumption, legitimated by the stochastic instability, allows to average the square of the equation (12) over the various realizations of this initial random phase.

5.Quasi linear diffusion, cyclotron radiation

The action diffusion coefficient D_{JJ}, obtained with the RPA assumption, is given by equation (13), where we have used the identity $v\sin^2(x/v)/\pi x^2 = \delta(x)$ if $v = 0$ and then averaged over the resonances m = 0 and m = 1:

$$D_{JJ}(J) = \frac{<\delta J \delta J>_\Phi}{\delta t} = \frac{\pi R \omega_c q}{2}\left(\frac{\delta B_{0n}}{B}\right)^2 J\,P \tag{13}$$

The condition (10) insures the occurence of a stochastic instability but is not sufficient to guarantee that the pitch angle scattering will cause an energy blocking. For such a blocking to take place the perpendicular action has to reach a critical value J_{cr} such that the cyclotron energy losses,

$$\frac{d\gamma}{dt} = -\frac{4 r_e \gamma^3 \omega_c^3}{3 m_e c^3} J, \tag{14}$$

overcome the loop voltage energy gain. This critical action will enable us to introduce the dissipative time scale of the problem and is given by:

$$J_{cr} = \frac{3c^2 e V_l}{8\pi R r_e \gamma^3 \omega_c^3}$$

The time needed to reach this angle through the MFR quasi linear diffusion must be smaller than the time needed to cross the resonant zone because of the inductive acceleration. We define these two fast time scales τ_{res} and τ_{dif} as follows:

$$\tau_{dif} = \frac{J_{cr}^2}{D_{JJ}(J_{cr})} = \left(\frac{\delta B_{0n}}{B}\right)^{-2} \frac{3ecV_l}{4\pi^2 R^2 r_e \gamma^4 \omega_c^4 m_e q}, \qquad \tau_{res} = \frac{2\pi R \delta P(n)}{eV_l} = \frac{2\pi R^2 B}{(Nn)^2 q V_l} \tag{15}$$

The condition for an efficient energy blocking writes:

$$\tau_{dif} << \tau_{res}. \tag{16}$$

This time ordering enables to find a MFR lower bound:

$$\left(\frac{\delta B_{0n}}{B}\right)^2 > \frac{3ecV_l^2 n^2 N^2}{8\pi^3 R^4 r_e \gamma^4 \omega_c^4 m_e B},$$

it can be checked that for usual tokamaks and not too large nN values (< 100) this condition is weaker than equation (11), i.e. as soon as the Chirikov parameter is of the order of 0.7 the associated diffusion is large enough for (16) to be fullfilled.

6.Anomalous energy spectrum

Let us consider the figure 3: a fast electron begin to be accelerated near the Dreicer energy coresponding to the Dreicer parallel action P_D, its perpendicular energy remaining close to the thermal one (J_T) while its parallel momentum ($P \approx E/c$) increases linearly with time. Then because of the synchrotron losses its acceleration decreases, thus it takes few seconds to reach the syncrotron limit P_S and to obtain the normal runaway spectra near P_S. The particule crosses resonances $n = \infty....3,2$ appearing for energies $E_n[\mathrm{MeV}] \approx 300 \frac{RB}{nN}$. If for one of the corresponding action P_n the condition equation (10) (i.e. also (16)) is fullfilled, the particule experiences a strong diffusion which converts its parallel energy into perpendicular and electron kinetic energy decreases since cyclotron radiation overcomes loop voltage acceleration. Then, as soon as the runaway reaches an energy below this n^{th} stochastic region S_n, the inductive acceleration will push it again into the stochastic zone and the diffusion dissipation process will take place again and so on, finally resulting in an accumulation of particule near P_n and thus preventing the fast population to reach P_S. Ultimate[illegible] a small fraction ($J \ll J_T$) of fast particules can reach this synchrotron limit, but most of them will stay around P_n.

To assess the general potentiality of the previous mechanism near a given resonance we need to evaluate the MFR amplitude in order to calculate S_n, τ_{dif} and τ_{res}. Rather than using the numerical field map we use a simple formula to describe the ripple. They are deduced from the following modelization. The coils are infinite straight line conductors regularly spaced along an axis with periodicity $2\pi(R+b)/N$ and $2\pi(R-b)/N$ on the high and low field sides. This give a satisfactorily approximation of the of the various real MFR toroidal harmonic in the equatorial plane. If x is the position (minor radius) in this equatorial plane the MFR toroidal components of this simplified structure writes:

$$\Delta B_n(x) = B \left| e^{-nN\left(\frac{b-x}{R+b}\right)} - e^{-nN\left(\frac{b+x}{R-b}\right)} \right| \approx B e^{-nN\left(\frac{b-x}{R+b}\right)}$$

where b is the toroidal coil radius. A good approximation to modelize tokamak ripple is to consider only the first poloidal harmonic as follows:

$$\delta B_n(r,\theta) \approx B\, e^{-nN\left(\frac{b-r}{R+b}\right)} \frac{1+\cos(\theta)}{2}$$

The drift surface of a relativistic runaway is shifted away from the magnetic surface in the horizontal direction by an amount :

$$\Delta r = \frac{\gamma m_e q c}{eB} = \frac{qR}{nN} . \qquad (17)$$

Runaway tend to explore zones on the low field side (i.e. where the ripple is the larger), thus the effective poloidal harmonic experienced by the fast particle can be approximated by:

$$\delta B_{0n} \approx \frac{\Delta B_n(r+\Delta r)}{2} = \frac{1}{2}\left(e^{-N\left[\frac{b-r}{b+R}\right]}\right)^n\left(1+\frac{qR}{(R+b)}\right) \qquad (18)$$

Note that the expression in the first bracket is the ripple of the field for n=1, θ =0 , i.e. what is commonly called " ripple value" in tokamaks.

The following table displays the results for Tore Supra low field configuration with N= 18, B=1.8T, q=2.0, T= 1.5KeV, V_l=1.0V, R=2.40 m, a=0.75m, b=1.30m, r=0.45

n	E_n[MeV]	$\delta B_{0n}/B$	S_n	τ_{res}	τ_{dif}
1	74.7	1.8 10^{-2}	3.0	0.1	3.6 10^{-11}
2	37.3	2.9 10^{-4}	0.77	0.025	1.4 10^{-7}
3	24.9	4.7 10^{-6}	0.15	***	***

The n = 1 resonance is very strong but plays no role under usual conditions, since γ_1 is larger than the synchrotron limit. In fact the quasi linear approximation is not valid because τ_{dif} is shorter than a cyclotron period. However during disruptions the loop voltage can reach very large values and the synchrotron limit can be well above γ_1. In this case this resonance will replace the synchrotron limit preventing the creation of too large energy runaway.

In non disruptive discharges the n=2 MFR cylotron resonance in Tore Supra, behaves as an energy upper bound in agreement with the observed anomalous limit.

The ratio τ_{res}/τ_{dif} is very large hence the scattering angle are very large and the cyclotron energy blocking efficient even when the loop voltage is above 500 V .

The n=3 resonance plays a role only at the edge of the plasma .

7.Conclusions and potential applications

A new universal instability of the runaway population in tokamak configuration has been found. When combined with cyclotron losses one of its major consequence is to act as an effective slowing down mechanism preventing the free fall acceleration toward the synchrotron limit explaining some experimental results of Tore Supra and Textor. The existence of this instability has some other consequences:

-The angular diffusion coefficient is so large (especially at the plasma edge) that runaway electrons can be trapped in local mirrors and may damage the wall.

- It prevents the formation of too large energy electrons during disruptions. However this energy limit scales as RB and is a few hundred of MeV in large tokamaks.

- If these runaway electrons become a serious problem, it is possible to build a "wiggler" creating an artificial field ripple at the plasma edge (about 10^{-4}). It is not dangerous for the bulk confinment since it involves large toroidal numbers i.e. non resonnant, typically N= 120.

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