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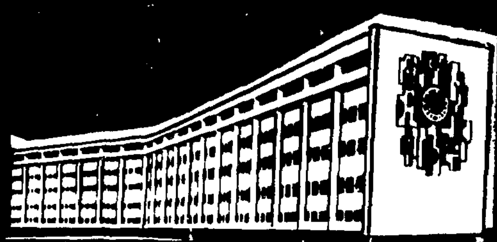

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Взаимно-когерентные свойства изображений квазара, микролинзированного звездной массой

Исследовался вопрос о взаимной когерентности излучения пары изображений квазара, формируемых гравитационной линзой одиночной точечной массы порядка солнечной. Получено аналитическое выражение для степени когерентности путем асимптотического разложения по частоте. В отдельных случаях модуль степени достигает величины 0.0I - 0.02 в радиодиапазоне.

O.P.Verkhoglyadova, A.V.Mandzhos

Mutual Coherent Properties of the Images of the Quasar Microlensed by Star Mass

The paper investigates the problem of mutual coherence of the radiation from two quasar images formed by a single point-mass gravitational lens with the mass of the order of solar mass. The expression for coherence degree is derived by asymptotic expansion in frequency. The coherence degree magnitude attains , in some cases, the values of 0.01 - 0.02 in the radio-frequency range.

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INTRODUCTION

For many years gravitational lens effect remained only an exotic prediction of the theory of relativity and the subject of an abstract studies. However, nearly for ten years the above effect has been a real astrophysical fact due to the discovery of the double quasar Q 0957+561 A,B [1] which represents lens system with the gravitator, i.e. a compact cluster of galaxies as well as other lense systems. At the present time another theoretically predicted idea of quasar microlensing is nearly in the same state as the earlier idea of the lens effect by itself. The essence of the microlensing quasar effect consists in intermediate galaxy star projection on the quasar and considerable increase of its brightness due to lens effect. The present paper discusses microlensing of the central part of the quasar, i.e. its nucleus kern. Microlensing is primarily of interest because of offering possibilities to define the quasar kern structure and transverse velocities of the intermediate galaxy and the quasar. The present paper studies some specific features of the predicted phenomenon.

One of the distinguishing features of the effect is the formation of two or more object images. The image multiplicity was predicted by Einstein [2] and confirmed by the discovery of the first gravitational lens. Being formed by the same radiating surface, the multiple images must be mutually coherent to some degree. Refsdal was the first to call attention to the above fact [3] and, after discovering the first gravitational lens, the same idea was supported by Mitskevich [4]. The question resides in the value of mutual coherence degree of the images. The above value is determined, on the one hand, by lens parameters and, on the other hand, by the sizes and structure of a lensed object. For the cases much like Q 0957+561 A,B lens the problem was generally solved in [5] (more exactly, the gravitator with arbitrary mass distribution and the circle-like object homogeneous in brightness were considered). As shown in [5] , the mutual coherence degree was too small to detect the effect in the case of gravitator mass of the order of galaxy mass and larger. It was concluded from the above that the only promising case was, apparently, quasar microlensing by stars. This problem was studied in [6]. Mutual coherence degree was calculated for some special cases. Compared to "common" lenses, the obtained values proved to be closer to the effect detection limit, but they were not sufficiently large to be detected.

It should be emphasized that the angular dimensions and relative positions of microlensed quasar kern images are such that the above images cannot be detected separately by modern instruments. Only a general flux is accessible to observation, which, with regard to mutual coherence, takes the form:

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\,|\gamma_{12}|\cos\varphi_\gamma$$

where I_1 and I_2 are the fluxes from every image, $|\gamma_{12}|$ is the magnitude and φ_γ is the phase of mutual coherence degree. From the above facts it transpires that the flux will oscillate in the case of a monotonic phase variation. The relative amplitude of oscillations is determined by the value $2\sqrt{I_1 I_2}(I_1+I_2)|\gamma_{12}|$ and the period is determined by the velocity of change in phase φ_γ. Because of this, the estimation of the possible mutual coherence degree magnitudes are of interest for different microlens parametres, namely, single and multiple star-gravitators, different star masses, different mutual distances between the system components, i.e. quasar-star-observer. On the other hand, the study should be made of the factors which can cause regular phase changes, namely, intervening gravitational fields, optical density gradients of the exstragalactic space, gravitator configuration variation in the case of a double star and so on.

In the present paper the mutual coherence degree of images for the case of single star-gravitator is calculated. These calculations are true for arbitrary configuration of radiating surface and arbitrary brightness distribution on it. Owing to the fact that the results were derived in an analytic form, it is possible to obtain those values of lens parametres for which the mutual coherence effect reaches a maximum value.

I. PHOTOMETRIC EFFECTS AND TIME DELAYS

To do the above study one should possess the data on the image structure and the formulae for propagation times from the object to the observer. The present section studies these problems.

The orders of magnitude used are briefly outlined as follows. The effective dimensions of radiative quasar kern surface are believed to be in the range from 10^{15} cm to 10^{17} cm.The distances between the object and the gravitator (the star) can vary from 50 kpc (the star of the galaxy containing the quasar) to 1000Mpc (cosmological distances).The distances between the gravitator-star and the observer are in the same range of distances as the above ones;namely,the distance from a Galaxy star to a cosmologic distant star.The gravitator - star mass is adopted to be in the range from 0,01 $M_{\odot}$ to 1 $M_{\odot}$ .As for refraction angles of rays, these are not over $5\cdot 10^{-4}$ second of arc.at the same parameters.

The following calculations are carried out in terms of linearized Einstein gravitational theory on the approximation of geometrical optics and small refraction angles,and under the assumption of quasimonochromatic radiation.Inasmuch the gravitator is a point - like one,the plane gravitator formalism is used universally where it is convenient.

Besides,it should be emphasized.that although the distances between the objects are thought of as being cosmological,all the calculations should be carried out in the plane statical space,as the introduction of cosmological models does not change sufficiently neither the essence of the problem nor the results. It presents no special problems,when required,to carry out the cosmological generalization.

Further we'll use the following designations: "a" is the distance between the object plane and the gravitator plane, "b" is the distance between the gravitator plane and the observer plane.All three planes are normal to a conventionally introduced axis (a system axis) passing through the gravitator. Similarly oriented two - dimensional systems of Cartesian coordinates with the origin on the axis are introduced.Assume that a particular arbitrary object element has the (ξ , η) coordinates.The observer's coordinates are (u, v) and the radiation passing from the element to the observer crosses the gravitator plane at the point (x , y).The above values,on condition that $\sqrt{\xi^2+\eta^2}, \sqrt{u^2+v^2}, \sqrt{x^2+y^2} \ll a , b$; are associated with the so - called lense equations:

$$\left(1+\frac{b}{a}\right)x - \frac{b}{a}\xi + b\,d_x(x,y) = u, \qquad (1)$$

$$\left(1+\frac{b}{a}\right)y - \frac{b}{a}\eta + b\,d_y(x,y) = v$$

where $\{\alpha_x(x,y), \alpha_y(x,y)\}$ is the two - dimensional deflection angle.

In the case of the point gravitator:

$$\alpha_x = -\frac{2r_g x}{x^2+y^2}; \quad \alpha_y = -\frac{2r_g y}{x^2+y^2} \qquad (2)$$

Further only the point gravitator case will be considered. The general solution of system takes the form:

$$x_{\pm} = \frac{u + \frac{b}{a}\xi \pm \sqrt{(u+\frac{b}{a}\xi)^2 + 8r_g b(1+\frac{b}{a})/(1+A^2)}}{2(1+\frac{b}{a})} \qquad (3)$$

$$y_{\pm} = A x_{\pm}; \qquad A = \frac{v+\frac{b}{a}\eta}{u+\frac{b}{a}\xi}$$

where r_g is the gravitational radius of the gravitator. Two signs before the square root denote that the point gravitator forms two images: the upper sign conforms to the direct image and the lower sign conforms to the reversed one. Without disturbing the generality we consider that $u = v = 0$. It is convenient to introduce the polar coordinates:

$$\xi = r\cos\varphi \qquad x = R\cos\varphi'$$
$$\eta = r\sin\varphi \qquad y = R\sin\varphi' \qquad (4)$$

Then solution (3) takes the form:

$$R_{\pm} = \frac{\sqrt{\frac{b^2}{a^2}r^2 + 8r_g b(1+\frac{b}{a})} \pm \frac{b}{a}r}{2(1+\frac{b}{a})}; \quad \varphi'_{\pm} = \varphi + \frac{\pi}{2}(1 \mp 1) \qquad (5)$$

As in the above case, the upper sign conforms to the direct image and the lower sign conforms to the reversed one. The lens varies the magnitude of radiation flux detected by the observer. In order to derive the flux amplification factor one can use the position common to the gravitational lenses, namely, <u>the radiation intensity along the ray does not vary in the gravitational lens system</u>. It follows from the above facts that for a homogeneous source the flux amplification factor is defined by the ratio of the areas of the radiating surface and its image in the gravitator plane. Indeed, if and are the radiational fluxes detected by the observer, in the cases of the absence and the presence of the lens, respectively, thus,

$$I_0 = J_0 \frac{\Delta S_{\xi\eta}}{(a+b)^2}; \quad I = J_0 \frac{\Delta S_{xy}}{b^2} \qquad (6)$$

where J_o is the source strength (its brightness) , $\Delta S_{\xi\eta}$ is the source area, ΔS_{xy} is the area of the image in the gravitator plane. Hence:

$$I/I_o = \frac{(a+b)^2}{b^2} \cdot \frac{\Delta S_{xy}}{\Delta S_{\xi\eta}} \qquad (7)$$

Further the points on the object surface, producing caustic singularities in the observer locus,play an important role.These points can be called "protocaustical"points" or simply "protocaustics".In the case of the point gravitator there is only one "protocaustic",i.e. the intersection point of the "observer - gravitator" straight line and the object plane.

If the quasar kern is considered as a circle homogeneous in brightness with the radius R_q ,then $\Delta S_{\xi\eta} = \pi R_Q^2$ .As for the area of ΔS_{xy} ,one should distinguish three cases:

/ *i* / protocaustics coincides with the quasar disk centre;

/ *ii* / protocaustics is within the boundaries of the disk but it does not coincide with the disk centre;

/ *iii* / protocaustic is out of the disk.

Let ξ_c be a distance between the protocaustic and disk centre.

/ *i* / The $\xi_c = 0$ case.

This is the simplest case of high symmetry.

$$\Delta S^{\pm}_{xy} = \pi (R^2_{\pm} - R^2_o) \qquad (8)$$

where

$$R_{\pm} = \frac{\sqrt{\frac{b^2}{a^2} R_Q^2 + 8 r_g b(1+\frac{b}{a})} \pm \frac{b}{a} R_Q}{2(1+\frac{b}{a})} \qquad (9)$$

$$R_o = \sqrt{2 r_g b (1+\frac{b}{a})} \qquad (10)$$

The signs in the expression for $R_{\pm}$ are in the same conformity for images.Formula (10) defines so - called Einstein radius, i.e.,the radius of the circle on the gravitator plane,which is the image of the protocaustics on the gravitator plane.Fig.1 presents the direct (broken lines) and the reversed (dotted lines) images of the quasar disk.The Einstein circle is the boundary dividing these images.The numerical values for the

typical case are as follows:
For: $R_Q=10^{16}$ cm; $a = b = 0,5 \times 10^{28}$ cm; $M_S = M_\odot$
we obtain: $R_0 = 3,87 \times 10^{16}$ cm; $R_+ = 4,13 \times 10^{16}$ cm;
$R_- = 3,63 \times 10^{16}$ cm;

$I_+/I_o = 8,26$; $I_-/I_o = 7,26$; $(I_+ + I_-)/I_o = 15,52$

These estimations show that a microlensed quasar metamorphoses into a thin ring (actually,two nested rings,i.e. the direct and reversed images).The brightness of the quasar increases more than by a factor of 15.So,the quasar microlensing in terms of photometric properties represents a rather strong effect.

The formulae for two other cases take the form:

/ *ii* / The $\xi_c > R_Q$ case.

$$\Delta S^{\pm}_{xy} = \frac{\pi b^2 R_Q^2}{2(a+b)^2} + \frac{ab}{2(a+b)^2}\left(\pm J_2 \mp J_3\right) \qquad (11)$$

where the signs are in the same conformity.

$$J_{2,3} = \int_0^{\varphi_m}\left(\xi_c\cos\varphi \pm \sqrt{R_Q^2-\xi_c^2\sin^2\varphi}\right)\sqrt{\frac{b^2}{a^2}\left(\xi_c\cos\varphi \pm \sqrt{R_Q^2-\xi_c^2\sin^2\varphi}\right)^2 + 8 r_g b\left(1+\frac{b}{a}\right)}\,d\varphi \qquad (12)$$

where $\qquad \varphi_m = \arcsin\,(R_Q/\xi_c)$

In the formula (12) the upper sign conforms to J_2 , the lower sign conforms to J_3 .

/ *iii* / The $\xi_c < R_Q$ case.

$$\Delta S^{\pm}_{xy} = \frac{b^2 \pi R_Q^2}{2(a+b)^2} \pm \frac{ab}{2(a+b)^2} J_2 \qquad (13)$$

where J_2 is defined by the above formula with $\varphi_m = \pi$.

If the quasar,gravitator and observer have relative transverse velocities,the protocaustic moves in the object plane.In this case the lens amplification coefficient varies with time.The example of such dependence $I(t)/I_o$ is shown in Fig.2.The lens parametres are the same,as in the above numerical example.Transverse gravitator velocity is considered to be 500 km/s.

The relative time delay of signals, δt ,propagating from the same point on the quasar along the trajectories to the direct and reversed images, is a very important parameter in the problem of the mutual coherence of the images. The propagation time from the quasar to the observer for the arbitrary - type lens is equal to:

$$t=\frac{1}{c}\left\{a+b+\frac{(x-\xi)^2}{2a}+\frac{(y-\eta)^2}{2a}+\frac{x^2}{2b}+\frac{y^2}{2b}\right\}+$$

$$+\frac{r_g}{c}\ln 4ab-\frac{2G}{c^3}\iint\rho(X,Y)\ln\left\{(X-x)^2+(Y-y)^2\right\}dX\,dY \qquad (14)$$

where ρ (X,Y) is the surface mass density of the gravitator.The integration is performed with respect to the entire gravitator plane. In the case of point mass: $\rho(x,y) = M\delta(0,0)$. For the relative time delay we obtain the expression:

$$\delta t=t_+-t_-=\frac{1}{c}\left\{\frac{a+b}{2ab}(R_+^2-R_-^2)-\frac{\xi}{a}(R_++R_-)\right\}-\frac{2r_g}{c}\ln\frac{R_+}{R_-} \qquad (15)$$

or with respect to (9):

$$\delta t=\frac{\xi\sqrt{\frac{b^2}{a^2}\xi^2+8r_g b\left(1+\frac{b}{a}\right)}}{2c(a+b)}-\frac{2r_g}{c}\ln\frac{\frac{b}{a}\xi+\sqrt{\frac{b^2}{a^2}\xi^2+8r_g b\left(1+\frac{b}{a}\right)}}{-\frac{b}{a}\xi+\sqrt{\frac{b^2}{a^2}\xi^2+8r_g b\left(1+\frac{b}{a}\right)}} \qquad (16)$$

Table 1 gives numerical values of the time delays for different lens parametres.

11. THE DEGREE OF MUTUAL COHERENCE

First it is advantageous to investigate a simple case of a single star - gravitator.The relative time delays δt for the different quasar points are different.The above delay is equal to zero if a point is situated on the "observer - star" axis (for the protocaustics).Maximum time delay for the quasar disk points, depending on the distances a,b and the star masses,varies in the range from 5×10^{-6} to 10^{-8} seconds(Table 1).According to physical optics,mutual coherence of the images under the same conditions reaches the maximum if relative time delay for all radiating quasar points does not exceed so - called radiation coherence time (see,for example, [7]).The order of magnitude of coherence time $\Delta\tau$ is defined according to the formula $\Delta\tau\cdot\Delta\nu\sim 1$,where $\Delta\nu$ is the brandwidth of the radiation detector.The brandwidth used for the observations of radiosources is in order of 2 MHz for the VLBI.In the cases of near-by gravitators,small masses or small quasar sizes the value is in accordance with data of Table1.

It is considered that the optimal choice of the brandwidth in the radio - frequency range makes it possible to satisfy the condition that the relative time delay does not exceed the coherence time for all quasar points.

Now it is possible to embark on calculating of the mutual coherence degree. By definition, the above value is equal to:

$$\gamma_{12} = \frac{\langle V_+ V_- \rangle}{\sqrt{I_+ I_-}} \qquad (17)$$

where the brackets $\langle\ \rangle$ designate the averaging over time, V_+ and V_- are the radiation field strengths for two images in the observer position, I_+ and I_- are the corresponding flux densities. The radiation is thought to be unpolarized and quasi - monochromatic with the mean circular frequency ω. According to Born and Wolf [7] one can write:

$$V_\pm(t) = \sum_m A_m (t - t_m^\pm) e^{-i\omega(t - t_m^\pm)} \frac{e^{-i\delta\psi_\pm}}{R^\pm_{ph\,m}} \qquad (18)$$

where the summation is made with respect to all radiating quasar elements, A is the radiation amplitude from m-th element, $t_m^\pm$ are the propagation times from the m-th element to the observer, $R_{ph\,m}$ are the photometric distances from the element to the observer. The above distances are defined, among other things, by radiation - crossed gravitational field of a star gravitator. $\delta\psi$ is the phase jump if the radiation passes through a caustic. The above jump is equal to zero for the direct image and is equal to $\delta\psi_- = \frac{3}{2}\pi$ for the reversed image in the case of a point gravitator. The substitution of (18) into (17) and the standard transition from the sum to the integral on the assumption of statistic radiation independence of each element gives:

$$\gamma_{12} = \frac{e^{i\delta\psi}}{\sqrt{I_+ I_-}} \iint \frac{J(\xi,\eta)\, e^{i\omega\delta t(\xi,\eta)}}{R^+_{ph}(\xi,\eta) R^-_{ph}(\xi,\eta)} d\xi\, d\eta \qquad (19)$$

where integration is performed with respect to the quasar disk,

(ξ, η) are the Cartesian coordinates of disk points, $\mathfrak{I}(\xi, \eta)$ is the surface quasar brightness, $\delta t(\xi, \eta)$ is the relative time delay. Photometric distances are defined from the formula:

$$I = I_0 \frac{(a+b)^2}{R_{ph}^2} \qquad (20)$$

where I is the radiation flux detected by the observer from the point source, I_0 is the same radiation flux under the condition of the gravitator absence.

Before embarking on the calculations of the mutual coherence degree according to formula (19) it is useful, for a special case of a point mass, to analyze generally this formula for the lens with arbitrary mass distribution in the gravitator. The presence of the high - frequency phase factor $e^{i\omega\delta t}$ in the integrand (ω is considered to be large enough) makes it possible to perform the asymptotic expansion of this integral by the stationary phase method. It is well-known (see, for example, [8], [9]) that a main contribution to the double integral of the type:

$$P = \iint_{\mathcal{D}} g(\xi, \eta)\, e^{ik\Phi(\xi,\eta)}\, d\xi\, d\eta \qquad (21)$$

is made by the vicinities of the so-called critical points in the range of the integration $\mathcal{D}$. These critical points are, firstly, the stationary critical points, i.e. the points, where two first derivatives of the phase function $\Phi(\xi, \eta)$ are equal to zero, and, secondly, the irregular critical points, where the continuity of the functions $g(\xi, \eta)$ and $\Phi(\xi, \eta)$ or their derivatives is disturbed.

Thus, in the case of integral (19) the stationary critical point must obey the equations:

$$\frac{\partial \delta t(\xi,\eta)}{\partial \xi} = 0; \quad \frac{\partial \delta t(\xi,\eta)}{\partial \eta} = 0 \qquad (22)$$

Using general expression (14) for the time delay δt and the lens equations (1), too, one can show that (22) reduced to the system:

$$x_2 - x_1 = 0; \quad y_2 - y_1 = 0 \qquad (23)$$

where (x_1, y_1) and (x_2, y_2) are the point coordinates on the gravitator plane, reflecting the object point (ξ, η) in the first and second images, respectively. Inasmuch as the images for a wide class of lenses have no common points (except for the points reflecting the protocaustic), so, according to (23), in this class of lenses the stationary points do not exist. As for the protocaustics, these are the points at which the function $[R_{ph_1}(\xi,\eta) \times R_{ph_2}(\xi,\eta)]^{-1}$ goes to infinity and the continuity of the first-order derivatives of the phase function $\delta t(\xi, \eta)$ is disturbed. In other words, the protocaustics are irregular critical points of the integral (19). Proceeding from the above a conclusion can be made that a main contribution to the double integral expressing the mutual coherence degree of two images in the lens is made by the vicinities of protocaustics, i.e. the points causing the caustic in the observation point.

Considering the problem of the paper itself, i.e. the case of the single point mass gravitator, it is necessary to emphasize that the single protocaustic, as it has already been noted, is the intersection point of the "observer - gravitator" line and the object plane. At this point the maximum value of δt (δt is equal to zero this point and is negative at other points) is attained and the continuity of the first-order derivatives $\partial \delta t / \partial \xi$ and $\partial \delta t / \delta \eta$ is disturbed. In other words, the modular surface of the function $\delta t(\xi, \eta)$ is characterized by the fact that there is a peak in the protocaustic. As will be shown later, the main contribution to the integral (19) is actually made by a small vicinity near the peak.

One can turn to the direct calculation of integral (19). Based on (20) we can obtain for the point gravitator mass:

$$R_{ph}^2 = (a+b)^2 \left| \left[1 - \frac{4a^2 b^2 r_g^2}{(a+b)^2 R^4} \right] \right| \qquad (24)$$

where $R^2 = x^2 + y^2$, and x, y are the coordinates of the intersection point of the gravitator plane by the ray. The fact that the time delay δt is a function only of , namely, the distance between the radiating element and the protocaustic, is very important for further calculations. It might be well to point out the following formula:

$$\lim_{r\to 0}\left(\frac{r}{R^+_{ph}R^-_{ph}}\right)=\frac{1}{\sqrt{2}}r_g^2 a^{-1}b^{-\frac{1}{2}}\left(1+\frac{b}{a}\right)^{-\frac{3}{2}} \tag{25}$$

A circular disk with constant brightness at every point is considered as the simplest case. Here two cases are qualitatively distinguished:

a) The protocaustic coincides with the disk centre.

According to (5) R (and, subsequently, R_{ph}) is the function of r. Because of this after integrating with respect to angular coordinate we obtain:

$$\gamma_{12}=\frac{2\pi J_0 e^{i\delta\Psi_-}}{\sqrt{I_+I_-}}Q;\quad Q=\int_0^{R_Q}\frac{r}{R^+_{ph}R^-_{ph}}e^{i\hat{\omega}\delta\hat{t}(r)}dr \tag{26}$$

where the dimensionless values: $\hat{\omega}=\frac{R_0}{c}\omega$; $\delta\hat{t}=\frac{c}{R_0}\delta t$ are introduced. The integral Q in the expression for γ_{12} should be seeked as the asymptotic expansion, considering the great values of $\hat{\omega}$. It is convenient here to use the method of the stationary phase for the one - dimensional integrals [10]. As a result we obtain:

$$Q=B_1+A_1+o\left(\frac{1}{\hat{\omega}}\right);\quad A_1=\frac{1}{4}\frac{R_0 e^{i\frac{\pi}{2}}}{b\left(1+\frac{b}{a}\right)}\frac{1}{\hat{\omega}} \tag{27}$$

$$B_1=-\frac{R_Q}{R^+_{ph}(R_Q)R^-_{ph}(R_Q)}\cdot\frac{(a+b)R_0 e^{-i\frac{\pi}{2}}}{\sqrt{\frac{b^2}{a^2}R_Q^2+8r_g b\left(1+\frac{b}{a}\right)}}e^{i\omega\delta t(R_Q)}\frac{1}{\hat{\omega}}$$

Note that the term A_1 represents the protocaustic contribution and the term B_1 represents the contribution of the quasar disk boundary. The presence of the boundary contribution to integral (19) is explained by the high symmetry of the above case.

b) Protocaustic is located on the disk, but it does not coincide with the disk centre.

Let the protocaustic be at a distance $\xi_c < R_Q$ from the disk centre. The radiating disk is divided in two regions: δ_0 is the circle of radius $r_0 = R_Q - \xi_c$ with the centre in the protocaustic; σ_1 is the remaining part of the disk.

Finally after integrating with respect to the angular coordinate we obtain:

$$\gamma_{12} = \frac{2 J_o e^{i\delta\psi_-}}{\sqrt{I_+ I_-}} (\pi Q_o + Q_1) \tag{28}$$

$$Q_o = \int_0^{r_o} \frac{r\, e^{i\hat{\omega}\delta \hat{t}(r)}}{R^+_{ph}(r)\, R^-_{ph}(r)} dr \tag{29}$$

$$Q_1 = \int_{r_o}^{R_Q+\xi_c} \arccos\left\{ \frac{r^2 + \xi_c^2 - R_Q^2}{2r\xi_c} \right\} \frac{r\, e^{i\hat{\omega}\delta\hat{t}(r)}}{R^+_{ph}(r)\, R^-_{ph}(r)} dr \tag{30}$$

The integral like Q_o has already been calculated:

$$Q_o = \overset{(o)}{B}_1 - \overset{(o)}{A}_1 + o\left(\frac{1}{\hat{\omega}}\right); \quad \overset{(o)}{A}_1 = \frac{1}{4} \frac{R_o e^{i\frac{\pi}{2}}}{b(1+\frac{b}{a})} \frac{1}{\hat{\omega}}$$

$$\overset{(o)}{B}_1 = -\frac{r_o}{R^+_{ph}(r_o)\, R^-_{ph}(r_o)} \cdot \frac{(a+b) R_o\, e^{-i\frac{\pi}{2}}}{\sqrt{\frac{b^2}{a^2} r_c^2 + 8 r_g b (1+\frac{b}{a})}}\, e^{i\omega\delta t(r_o)} \frac{1}{\hat{\omega}} \tag{31}$$

As for the integral Q_1, the continuity of the first-order derivative of the cofactor
in the integrand is disturbed on the upper limit of the integration $r = R_Q + \xi_c$. The above fact, therefore, should be discussed in detail. It is required that the singular part of the integrand should be isolated under the asymptotic expansion of the integral Q_1 based on the Erdelyi theorem [10] (see the Appendix):

$$\arccos\left\{(r^2 + \xi_c^2 - R_Q^2)/2r\xi_c\right\} = (R_Q + \xi_c - r)^{-\frac{1}{2}} \hat{g}/r \tag{32}$$

$$\hat{g}(r) = (R_Q - \xi_c - r)^{1/2} \arccos\left\{(r^2 + \xi_c^2 - R_Q^2)/2r\xi_c\right\}$$

•

The function $\hat{g}(r)$ with its first-order derivative throughout the closed interval is continuous. The following designations correspond to those of the Erdelyi theorem:

$$g(t) \Rightarrow \frac{r\hat{g}(r)}{R^+_{ph}(r)\, R^-_{ph}(r)}; \quad h_1(t) \Rightarrow -\frac{\sqrt{\frac{b^2}{a^2} r^2 + 8 r_g b (1+\frac{b}{a})}}{(a+b) R_o} \tag{33}$$

$$x \Rightarrow \hat{\omega}; \; t \Rightarrow r; \; \alpha \Rightarrow r_o; \; \beta \Rightarrow R_Q + \xi_c; \; \rho = \sigma = \lambda = 1; \mu = \frac{1}{2}$$

Thus, the Erdelyi theorem leads to the following expression for the integral Q_1:

$$Q_1 = \overset{(1)}{B_1} - \overset{(1)}{A_1} + o\left(\frac{1}{\omega}\right); \quad \overset{(1)}{B_1} = 0 \tag{34}$$

$$\overset{(1)}{A_1} = \frac{\pi \tau_0}{R^+_{ph}(\tau_0)\, R^-_{ph}(\tau_0)} \cdot \frac{(a+b) R_0\, e^{i\frac{\pi}{2}}}{\sqrt{\frac{b^2}{a^2}\tau_0^2 + 8\tau_0 b\left(1+\frac{b}{a}\right)}}\, e^{i\omega\delta t(\tau_0)} \frac{1}{i\hat{\omega}}$$

Inasmuch as $A_1^{(1)} = \pi B_1^{(0)}$, these terms (the contribution of the boundary between the regions σ_0 and σ_1) in a general expression for γ_{12} destroy each other. Finally, we obtain the expression for γ_{12}:

$$\gamma_{12} = -\frac{\pi J_0}{2\sqrt{I_+ I_-}} \frac{c}{b\left(1+\frac{b}{a}\right)} \frac{1}{\omega} \tag{35}$$

With regard to (6) one can write:

$$\gamma_{12} = -\frac{\pi b^2}{2\sqrt{\Delta S^+_{xy}\, \Delta S^-_{xy}}} \frac{c}{b\left(1+\frac{b}{a}\right)} \frac{1}{\omega} \tag{36}$$

To this point the circle-like radiating surface, homogeneous in brightness, has been discussed. We can show that formula (35) is true for an arbitrary radiating surface shape with an arbitrary brightness distribution on it. In this case J_0 designates the brightness in the protocaustic. Note that in this case a contribution to the expression of coherence degree γ_{12}, as was supposed, is made only by the protocaustic and not by the radiating region boundary.

In the case of the high symmetry, when the radiating surface, constant in brightness, is point-like and the protocaustics coincides with the circle centre, the mutual coherence degree is defined by the formulae (26) and (27). However, this case cannot be realized. It should be concluded that <u>mutual coherence degree of two microlensed quasar images is defined by universal formula (35).</u>

The estimations of $|\gamma_{12}|$ according to formula (36) for $a = 0{,}5 \times 10^{28}$ cm/$\sim$1600 Mpc / are summarized in Table 2.

Firstly it should be emphasised that, applying the above results to real quasars, the size R_Q from the Table 2 one can consider as an average area-effective dimension of the radiating surface / $R_Q \sim \sqrt{\Delta S_{\xi\eta}}$ / that is essential if we observe, for example, the disk-like radiating region from the edge.

As it follows from the data of Table 2, beginning from the distances to the gravitator of the order of 160 Mpc and less, the mutual coherence effect of kern images of the quasar microlensed by a single star becomes sufficient and, in some cases, can be detected in the radio - frequency range. Moreover, it is expected that the mutual coherence degree is sufficiently greater for the double and multiple star-gravitators. It should be emphasised that, as the cases of the protocaustic locating on the radiating surface has been discussed, the application of the wave optics might lead to the sufficient corrections. It is required to carry out an analysis of this problem in future. Moreover, two aspects of this problem require a detailed analysis. Firstly, what is the possibility of the microlensing effect itself of the quasar kern. Here different situations in the space should be considered: passage of quasar radiation through the inner and peripheric regions, intersection by a ray of one and the same quasar of a great number of (up to a score) galaxies, probable microlensing of the quasars by the hypothetic small masses in the galaxy coronas and the intergalaxy space and so on.

As for the second aspect, it refers to the expected velocity of change in the mutual coherence degree phase φ_γ. This velocity, as has been already stressed, defines the period of the microlensed quasar flux oscillations. It depends on the specific possible factors causing the above changes: the gravitational fields of the intermediate bodies with due regard for their transverse motions, local gradients of "optical" density on the line of sight and so on.

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APPENDIX

The Erdelyi theorem

(The theorem is presented ad hoc taking into account only the first terms of an asymptotic expansion).

Let $0 < \lambda, \mu \leq 1$, g (t) be the function with the continuous first-order derivative over the interval $\alpha \leq t \leq \beta$, h (t) be the differentiable function and

$$h'(t) = (t-\alpha)^{\rho-1}(\beta-t)^{\sigma-1} h_1(t)$$

where $\rho, \sigma \geqslant 1$ and h_1 (t) is the positive function over the interval $\alpha \leq t \leq \beta$ having the continuous first-order derivative. Then

$$\int_\alpha^\beta g(t)\, e^{ixh(t)} (t-\alpha)^{\lambda-1}(\beta-t)^{\mu-1} dt = B_1(x) - A_1(x)$$

$$A_1 = -\rho^{\frac{\lambda}{\rho}-1} g(\alpha)\left[h_1(\alpha)\right]^{-\frac{\lambda}{\rho}} (\beta-\alpha)^{\mu-1+(1-\sigma)\frac{\lambda}{\rho}}\, \Gamma\left(\frac{\lambda}{\rho}\right) \times$$

$$\times \exp\left(\frac{\pi i \lambda}{2\rho}\right) e^{ixh(\alpha)} x^{-\frac{\lambda}{\rho}} + o\left(x^{-\frac{\lambda}{\rho}}\right) \quad \text{for } x \to +\infty$$

$$B_1 = \sigma^{\frac{\mu}{\sigma}-1} g(\beta)\left[h_1(\beta)\right]^{-\frac{\mu}{\sigma}} (\beta-\alpha)^{\lambda-1+(1-\rho)\frac{\mu}{\sigma}}\, \Gamma\left(\frac{\mu}{\sigma}\right) \times$$

$$\times \exp\left(-\frac{\pi i \mu}{2\sigma}\right) e^{ixh(\beta)} x^{-\frac{\mu}{\sigma}} + o\left(x^{-\frac{\mu}{\sigma}}\right) \quad \text{for } x \to +\infty$$

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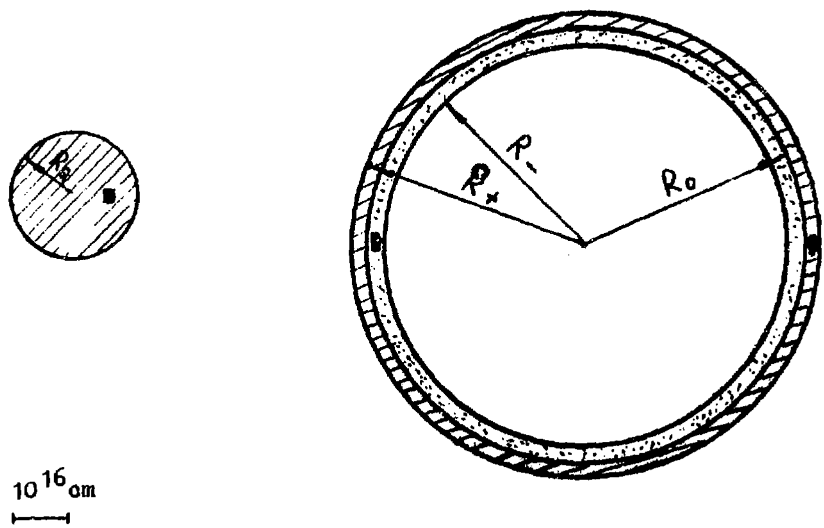

Fig. 1. Object disk and its images in the case of microlensing by star mass.
$a = b = 0,5 \times 10^{28}$ cm, $R_Q = 10^{16}$ cm, $M_s = M_\odot$

the reversed image

the direct image

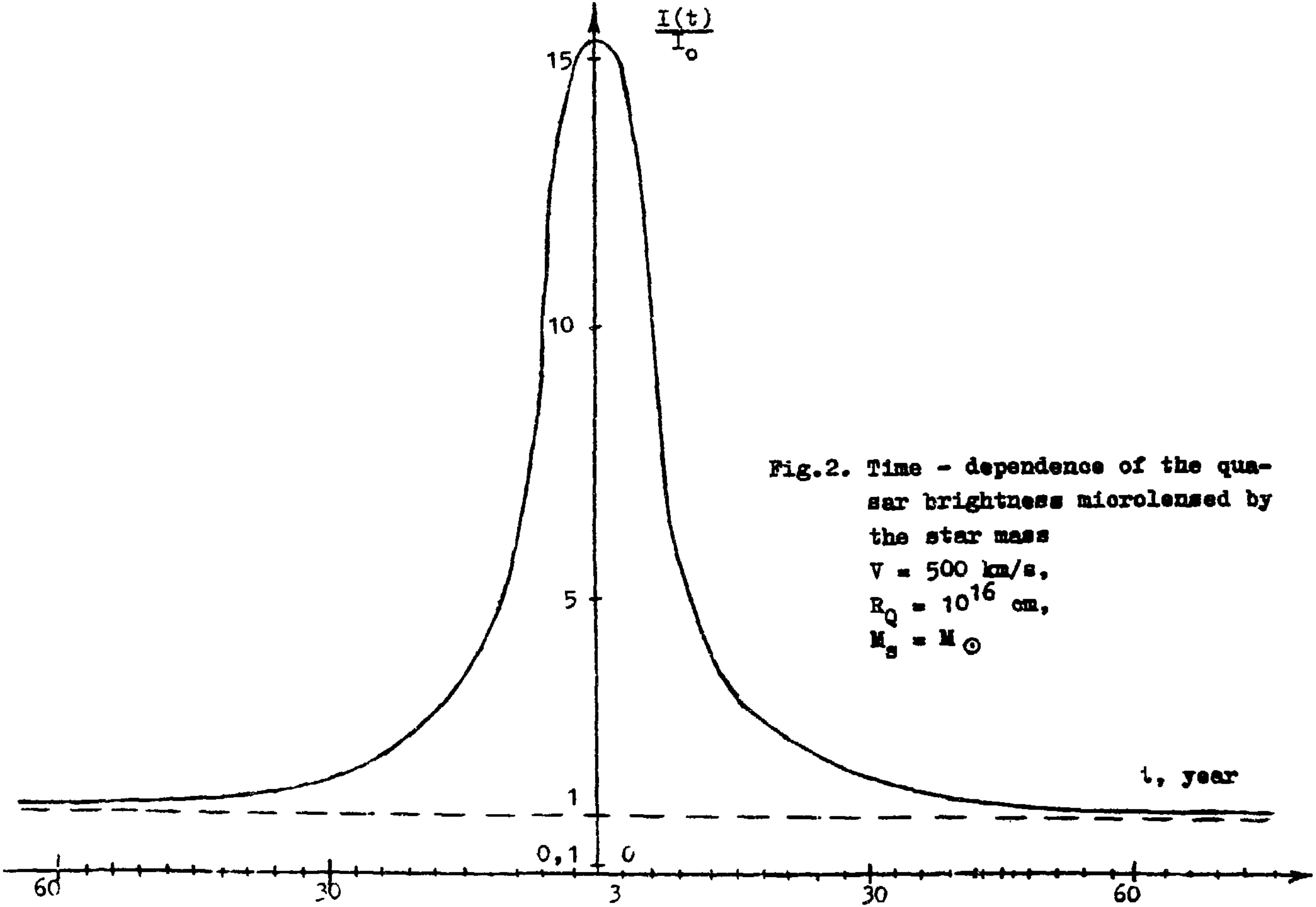

Fig.2. Time - dependence of the quasar brightness microlensed by the star mass
V = 500 km/s,
R_Q = 10^{16} cm,
M_s = $M_\odot$

Table 1.

The values of time delay δt and for optimal brandwidth $\Delta\nu$ for different lens parameters

a, cm	b cm	$M_s/M_\odot$	R_Q cm	δt s	$\Delta\nu$ kHz
		10^{-3}	10^{14}	$1{,}611 \times 10^{-9}$	$6{,}207 \times 10^{5}$
			10^{16}	$2{,}425 \times 10^{-7}$	$4{,}124 \times 10^{3}$
			10^{18}	$1{,}667 \times 10^{-3}$	$5{,}999 \times 10^{-1}$
		10^{-1}	10^{14}	$1{,}611 \times 10^{-8}$	$6{,}207 \times 10^{4}$
			10^{16}	$1{,}622 \times 10^{-6}$	$6{,}165 \times 10^{2}$
			10^{18}	$1{,}683 \times 10^{-3}$	$5{,}942 \times 10^{-1}$
5×10^{27}	5×10^{27}	1	10^{14}	$5{,}095 \times 10^{-8}$	$1{,}963 \times 10^{4}$
			10^{16}	$5{,}098 \times 10^{-6}$	$1{,}962 \times 10^{2}$
			10^{18}	$1{,}786 \times 10^{-3}$	$5{,}599 \times 10^{-1}$
		10	10^{14}	$1{,}611 \times 10^{-7}$	$6{,}207 \times 10^{3}$
			10^{16}	$1{,}611 \times 10^{-5}$	$6{,}207 \times 10^{1}$
			10^{18}	$2{,}425 \times 10^{-3}$	$4{,}124 \times 10^{-1}$
		10^{2}	10^{14}	$5{,}097 \times 10^{-7}$	$1{,}962 \times 10^{3}$
			10^{16}	$5{,}095 \times 10^{-5}$	$1{,}963 \times 10^{1}$
			10^{18}	$5{,}438 \times 10^{-3}$	$1{,}839 \times 10^{-1}$

Table 1.

(the end)

a cm	b cm	$M_s/M_\odot$	R_Q cm	δt s	$\Delta\nu$ kHz
			10^{14}	$6{,}241 \times 10^{-12}$	$1{,}602 \times 10^{8}$
		10^{-3}	10^{16}	$6{,}240 \times 10^{-10}$	$1{,}603 \times 10^{6}$
			10^{18}	$6{,}854 \times 10^{-8}$	$1{,}459 \times 10^{4}$
			10^{14}	$6{,}253 \times 10^{-11}$	$1{,}599 \times 10^{7}$
		10^{-1}	10^{16}	$6{,}240 \times 10^{-9}$	$1{,}603 \times 10^{5}$
			10^{18}	$6{,}246 \times 10^{-7}$	$1{,}601 \times 10^{3}$
10^{28}	$1{,}5 \times 10^{23}$	1	10^{14}	$1{,}984 \times 10^{-10}$	$5{,}040 \times 10^{6}$
			10^{16}	$1{,}973 \times 10^{-8}$	$5{,}068 \times 10^{4}$
			10^{18}	$1{,}973 \times 10^{-6}$	$5{,}068 \times 10^{2}$
			10^{14}	$6{,}369 \times 10^{-10}$	$1{,}570 \times 10^{6}$
		10	10^{16}	$6{,}241 \times 10^{-8}$	$1{,}602 \times 10^{4}$
			10^{18}	$6{,}240 \times 10^{-6}$	$1{,}603 \times 10^{2}$
			10^{14}	$2{,}147 \times 10^{-9}$	$4{,}658 \times 10^{5}$
		10^{2}	10^{16}	$1{,}975 \times 10^{-7}$	$5{,}063 \times 10^{3}$
			10^{18}	$1{,}973 \times 10^{-5}$	$5{,}068 \times 10^{1}$

Table 2.

The values for mutual coherence degree of microlensed quasar images for different distances to gravitator, denoted by b and the detecting radiation wavelength is λ. The value $2\sqrt{I_+ I_-}/(I_+ + I_-)$ is close to unity for all cases.

b cm	$M_s / M_\odot$	R_Q cm	λ cm	$\lvert\gamma_{12}\rvert$	Distance scale to the gravitators
	1	10^{16}	18	$1,850 \times 10^{-5}$	
	1	10^{15}	18	$1,850 \times 10^{-4}$	b $\gtrsim$ 1600 Mpc
5×10^{27}	0,01	10^{15}	18	$1,850 \times 10^{-3}$	cosmological
	0,01	10^{15}	90	$9,248 \times 10^{-3}$	distances
	1	10^{16}	18	$4,338 \times 10^{-5}$	
	1	10^{15}	18	$4,338 \times 10^{-4}$	b $\gtrsim$ 160 Mpc
5×10^{26}	0,01	10^{15}	18	$4,338 \times 10^{-3}$	mean
	0,01	10^{15}	90	$2,169 \times 10^{-2}$	distances
	1	10^{16}	18	$1,693 \times 10^{-4}$	
	1	10^{15}	18	$1,693 \times 10^{-3}$	b $\gtrsim$ 10 Mpc
3×10^{25}	0,01	10^{15}	18	$1,693 \times 10^{-2}$	in the boundaries
	0,01	10^{15}	90	$8,464 \times 10^{-2}$	of the Local
					supercluster
	1	10^{16}	18	$5,340 \times 10^{-4}$	
	1	10^{15}	18	$5,340 \times 10^{-3}$	b $\gtrsim$ 1 Mpc
3×10^{24}	0,01	10^{15}	18	$5,340 \times 10^{-2}$	near galaxies
	0,01	10^{15}	90	$2,670 \times 10^{-1}$	
	1	10^{16}	18	$4,359 \times 10^{-3}$	
	1	10^{15}	18	$4,359 \times 10^{-2}$	b $\gtrsim$ 15 kpc
$4,5 \times 10^{22}$	0,01	10^{15}	18	$4,359 \times 10^{-1}$	Galaxy
	0,01	10^{15}	90	1,0	

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