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NUMERICAL EVALUATION OF CRACKED PIPES UNDER DYNAMIC LOADING

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ABSTRACT

In order to apply the leak-before-break concept to piping systems, the behavior of cracked pipes under dynamic, and especially seismic, loadings must be studied. A simple finite element model of a cracked pipe has been developed and implemented in the general purpose computer code CASTEM 2000.

The model is a generalization of the approach proposed by Paris and Tada (1). Considered loads are bending moment and axial force (representing thermal expansion and internal pressure).

The elastic characteristics of the model are determined using the Zahoor formulae for the geometry-dependent factors. Owing to the material behavior plasticity must be taken into account.

To represent the crack growth, the material is defined by two characteristic values : J_{1c} which is the level of energy corresponding to crack initiation and the tearing modulus, T, which governs the length of propagation of the crack. For dynamic loads, unilateral conditions are imposed to represent crack closure.

The model has been used for the design of dynamic tests to be conducted on shaking tables. Test principle is briefly described and numerical results are presented.

Finally, evaluation of margin, due to plasticity, in comparison with the standard design procedure is made.

NOTATIONS

ϕ : rotation due to the crack
ϕ^e : elastic part of ϕ
ϕ^p : plastic part of ϕ
e : elongation due to the crack
e^e : elastic part of e
e^p : plastic part of e
M : bending moment
P : axial effort
R : mean radius of the pipe
t : wall thickness
2θ : present angle of the crack
$2\theta_0$: initial angle of the crack
K_1 : stress intensity factor (mode I)
J : Rice's integral
J_{1c} : value of J at initiation
J_e : elastic part of J
J_p : plastic part of J
T : tearing modulus
σ_0 : flow stress
[M] : mass matrix
[C] : viscous damping matrix
[K] : stiffness matrix
$\ddot{X}$: relative acceleration
$\dot{X}$: relative velocity
X : relative displacement
g : gravity acceleration

INTRODUCTION

In the numerical evaluation of cracked pipes, two main possibilities exist. The first one is to conduct an analysis with a one dimensional mesh, except in the surroundings of the crack where the analysis would be three dimensional. As this is felt quite uneasy and impossible in current design procedures, an alternative way is to develop a special element (hinge element) which integrates the behavior of a cracked section in a one dimensional model (Figure 1).
This paper describes the formulation of such an element and shows application to dynamic calculation.

FORMULATION OF THE ELEMENT

The cracked pipe element model is a global model which takes into account the flexibility due to a circumferential through-wall crack. It is composed of two nodes, initially at the same location, and it simulates the behavior of the cracked section. Considered loads are bending moment (mode I) and axial force. The associated kinematic variables are rotation due to the crack, ϕ, and elongation due to the crack, e. For the other degrees of freedom, continuity is ensured using penalization technique.
Figure 2 shows the geometry of a cracked section.

Elastic characteristics

The elastic characteristics of the cracked element are determined using the Zahoor formulae (2) for the geometry dependent factors F_m and F_f. The expression for the stress intensity factor in fracture mode I can be written as:

$$K_I = K_f M + K_e P \tag{1}$$

$$\text{with } K_f = \frac{\sqrt{\pi R \theta}}{\pi R^2 t} F_f \quad \text{and} \quad K_e = \frac{\sqrt{\pi R \theta}}{2\pi R t} F_m ; \tag{2}$$

$$F_f \left(\frac{\theta}{\pi}\right) = 1 + A \left(4.60 \left(\frac{\theta}{\pi}\right)^{\frac{3}{2}} + 2.64 \left(\frac{\theta}{\pi}\right)^{4.24}\right)$$

$$F_m \left(\frac{\theta}{\pi}\right) = 1 + A \left(5.33 \left(\frac{\theta}{\pi}\right)^{\frac{3}{2}} + 18.77 \left(\frac{\theta}{\pi}\right)^{4.24}\right) \tag{3}$$

$$\text{with} \begin{cases} A = \left(0.125 \left(\frac{R}{t}\right) - 0.25\right)^{\frac{1}{4}} & \text{for } 5 < \frac{R}{t} < 10 \\ A = \left(0.4 \left(\frac{R}{t}\right) - 3.\right)^{\frac{1}{4}} & \text{for } 10 < \frac{R}{t} < 20 \end{cases} \tag{4}$$

F_f and F_m are functions of the crack angle, θ, and of the ratio R/t. The validity of Zahoor formulae is restricted to $0 < \theta < 100°$ and $5 < R/t < 20$.

Using these expressions, one can easily compute the Rice's integral J :

$$J = \frac{K_I^2}{E} \tag{5}$$

Now, let's denote U the strain energy due to the presence of the crack. We have $U = \int_A J \, dA$, where A is the crack area. Using Castigliano's theorem, the flexibility matrix can be derived and we can write:

$$\begin{pmatrix} \phi \\ e \end{pmatrix} = \begin{bmatrix} C_m & C_{mp} \\ C_{mp} & C_p \end{bmatrix} \begin{pmatrix} M \\ P \end{pmatrix} \tag{6}$$

$$\text{with} \quad C_m = \left(\frac{1}{M}\frac{\partial U}{\partial M}\right)_{P=0} \quad C_p = \frac{1}{P}\left(\frac{\partial U}{\partial P}\right)_{M=0} \quad C_{mp} = \frac{1}{M}\left(\frac{\partial U}{\partial P}\right)_{M=0} \tag{7}$$

Finally, the stiffness matrix K is obtained by inverting the flexibility matrix.

Crack growth

As far as crack growth is concerned, the material is supposed to be characterized by two intrinsic values: J_{Ic} which is the level of energy corresponding to crack initiation and the tearing modulus, T, which governs the crack length when propagation occurs (Figure 3). The cracked pipe element is determined by its initial crack angle $2\theta_0$ and its present crack angle 2θ.

The criterion for crack growth is written as :

$$J < J_{Ic} + TR(\theta - \theta_0) \tag{8}$$

For a given loading point (M,P), the new angle is obtained using an iterative procedure to solve the equation:

$$J(\theta) - J_{Ic} - TR(\theta - \theta_0) = 0 \tag{9}$$

Figures 5 and 6 show moment-crack length curves for different values of the tearing modulus and of the initial crack angle.

At the present time, only elastic propagation has been implemented.

Plastic characteristics

Due to the material behavior in piping systems, plasticity must be taken into account in the formulation of the cracked pipe element. Two basic assumptions are made :

- the material has an elastic perfectly plastic behavior determined by the values of Young's modulus E and the limit stress σ_0 (flow stress).
- in the cracked section, the stress field is governed by a limit stress diagram (Figures 3 and 4).

With these hypothesis, integrating the stress field along the cracked section gives a relation between M and P which is the load function F. Its expression can be written as:

$$F(M,P,\theta) = \frac{M}{M_0} - \cos \frac{1}{2}\left(\theta + \pi \frac{P}{P_0}\right) + \frac{1}{2} \sin \theta \tag{10}$$

$$\text{with } M_0 = 4\,\sigma_0 R^2 t \quad \text{and} \quad P_0 = 2\,\sigma_0 \pi R t \tag{11}$$

Figure 7 shows the typical shape of the load function for different length of crack.

In case of elastic-plastic behavior, total displacements are the sum of elastic and plastic displacements:

$$\begin{cases} e = e^e + e^p \\ \phi = \phi^e + \phi^p \end{cases} \tag{12}$$

Using Hill's principle, we can write:

$$\begin{cases} F(M,P,\theta) = 0 \\ d\phi^p = \frac{\partial F}{\partial M} d\lambda \\ de^p = \frac{\partial F}{\partial P} d\lambda \end{cases} \tag{13}$$

This system is solved using an iterative algorithm.

As for displacements, J is separated into two parts, elastic and plastic:

$$J = J_e + J_p \tag{14}$$

J_p is evaluated by integrating the following expression:

$$dJ^p = - \frac{\partial(dW)}{\partial A} \tag{15}$$

where dW is the work of external forces under imposed displacements.
J_p is related to plastic displacements by :

$$dJ^P = - \frac{1}{2Rt} \left(\frac{\partial M}{\partial \theta} d\phi^P + \frac{\partial P}{\partial \theta} de^P\right) \quad (16)$$

Crack closure

As the cracked pipe element is to be used in dynamic calculations, the problem of crack closure has to be considered. As a matter of fact, one can see that the behavior of the cracked section is different when the crack is open and when the crack is closed. In the formulation, these two behaviors are represented using the general technique of Lagrange's multipliers. As the elongation e is the relative displacement, and the rotation ϕ the relative rotation of the two nodes of the element, the following unilateral conditions are imposed:

$$\begin{cases} e > 0 \\ \phi > 0 \end{cases} \quad (17)$$

We can remark that the Lagrange's parameters associated with these relations are equal to zero when the crack is open and respectively to the axial force and bending moment in the cracked section when the crack is closed.

Figure 8 is an illustration of the discrepancy in the computed dynamic response of a cracked pipe to a snap-back with or without crack closure.

APPLICATION: DESIGN OF DYNAMIC TESTS

The above described model has been used to make preliminary calculations in order to design dynamic tests on cracked pipes to be conducted on shaking tables.

As static four-point bending tests have already been performed, the same pipe is chosen for dynamic test. Geometrical characteristics of the test specimen are shown in Figure 9, and the experimental setup for static tests is presented in Figure 10.

Principle of dynamic test

The aim of the test is to have a cracked pipe subjected to an inertial load such as a dynamic bending moment. This goal is reached by adding inertial masses at each end of the straight pipe. The test setup is given a vertical acceleration by a shaking table. A schematic of the test setup is shown in Figure 11.

Adjustment of the model

First of all, the finite element model has been used for calculation of the static four-point bending test. The parameter σ_0 of the model (flow stress) is chosen to obtain the same maximum moment as in the test. Results of a test and a computation for moment-rotation curve are shown in Figure 12. We can notice that the elastic stiffness is well represented by the model.

Dynamic calculation

The input displacement of the shaking table is an increasing sinusoid of the form:

$$\begin{cases} d(t) = d_0 \left(\frac{t}{t_c}\right)^2 \sin \omega_e t & \text{for } t < t_c \\ d(t) = d_0 \sin \omega_e t & \text{for } t > t_c \end{cases} \quad (18)$$

with $\omega_e = 2\pi f_e$

This ensures that displacement, velocity and acceleration of the shaking table are equal to zero for $t = 0$.

For this particular calculation, we chose $t_c = 0.75$ s and f_e equal to the first eigenfrequency of the cracked pipe (e.g., including stiffness due to the crack).

d_0 is adjusted so that the maximum acceleration is equal to the maximum capability of the shaking table (2g). Displacement, velocity and acceleration are shown on figures 13 to 15.

A large amount of damping is assumed, because this computation is devoted to design of the test. The damping matrix is of the form $[C] = a[K]$ and a damping ratio of 10 % is assumed in the first mode.

The standard formalism for computing seismic response of structures is used. Then, the equations to be solved are:

$$[M]\ddot{X} + [C]\dot{X} + [K]X = - [M] (a(t) + g) U \quad (19)$$

in which U is the unit vector of the vertical direction.

These equations are solved using a Newmark algorithm which takes into account the nonlinear effects of plasticity.

Results are presented in Figure 16 to 19. For this calculation, the length of crack is constant.

Analysis of results

Numerical results have to be analyzed in order to know:

- whether instability will be reached or not
- whether initiation will occur or not.

For the first point, the criterion is based on rotation due to the crack. It is assumed that when rotation ϕ reaches a critical value ϕ_c, instability appears. From the results of static tests, we determined that $\phi_c = 6°$. Figure 18 shows that instability will be reached. The required value of maximum acceleration corresponding to the time of instability thus defined is $a_{max} = 12$ ms^{-2}.

For the second point, we have to look at the computed values of J. With a value of $J_{1c} = 1$ MPa m, we can assess that initiation will occur during the dynamic test (Figure 19).

MARGIN EVALUATION

In order to apply the leak-before-break concept, margins resulting from the standard linear design procedure must be evaluated. The developed finite element model gives an opportunity to compare linear and nonlinear calculations and to determine the margin. This exercise is made on the particular example of the dynamic test described above.

To be consistent with the criterion of instability used with the results of nonlinear calculation, the equivalent criterion for the linear case would be $M = M_c$ at the crack location, in which M_c is the bending moment corresponding to the rotation due to the crack, ϕ_c.

Let's denote $\tilde{a}(t)$ the value of a(t) for t ≤ 0.62 s which is the time of instability in the nonlinear calculation.

The margin is the value of m that gives a maximum moment equal to M_c for the acceleration a(t)/m.

Figure 20 shows the moment versus time curve of the linear system (no crack nor plasticity) for the acceleration a(t). If we call M_1 the maximum moment for t ≤ 0.62 s and M_p the moment due to the masses weight, we can write:

$$m = \frac{M_1 - M_p}{M_c - M_p} \quad (20)$$

For the example studied here, we have M_c = 11.8 kNm, M_p = 8.83 kNm and M_1 = 38.5 kNm. The margin si then m ≈ 10.

CONCLUSION

The formulation of a special cracked pipe element has been proposed. This model has ben implemented in the CASTEM 2000 computer code.

Its application to static and dynamic calculations gave very encouraging results and showed that it is a simple and helpful tool for leak-before-break applications.

Further steps of development will include taking into account hardening of the material and coupling between plasticity and crack propagation.

The influence of the frequency of excitation on the dynamic response and on margins has also to be investigated.

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Paris, P.C. and Tada, H., "The Application of Fracture Proof Design Methods Using Tearing Instability Theory to Nuclear Piping Postulating Circumferential Through Wall Cracks" NUREG/CR-3464 (1983)

Proceedings of the CSNI/NRC Workshop on Ductile Fracture Mechanics, Southwest Research Institute, San Antonio, Texas (1984) Compiled by M.F. Kanninen

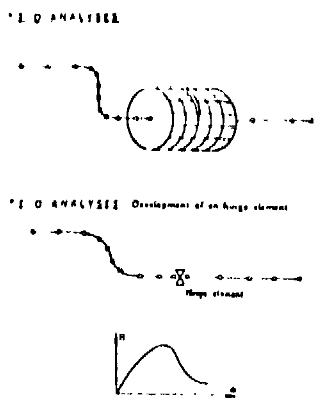


FIGURE 1 Different analyses for cracked pipe lines

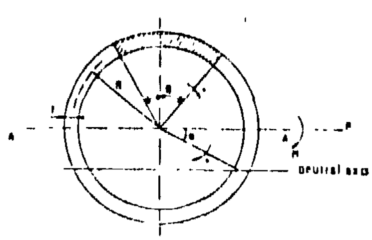


FIGURE 2 Geometry of a cracked pipe section

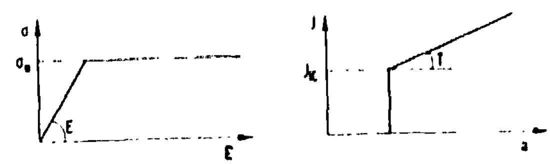


FIGURE 3 Mechanical characteristics of the material

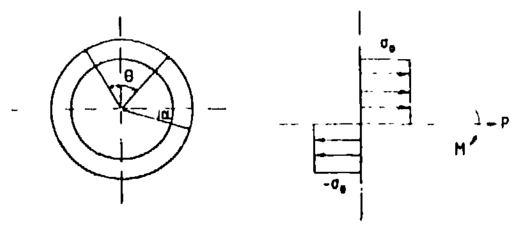
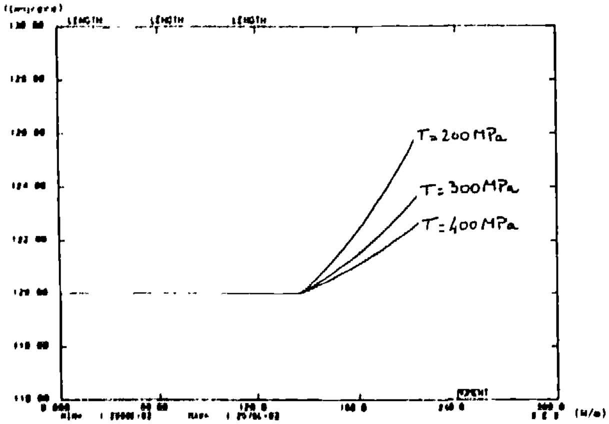
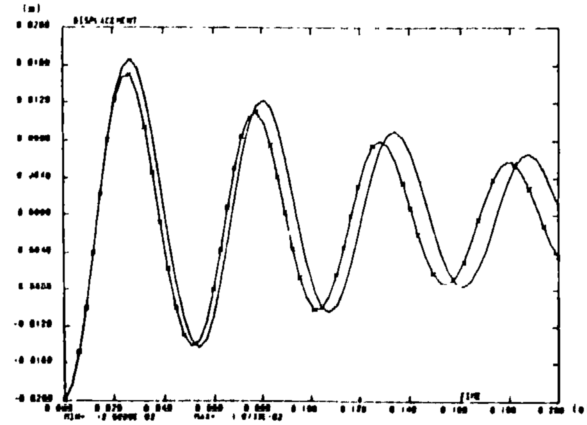


FIGURE 4 Plastic characteristics limit stress diagram

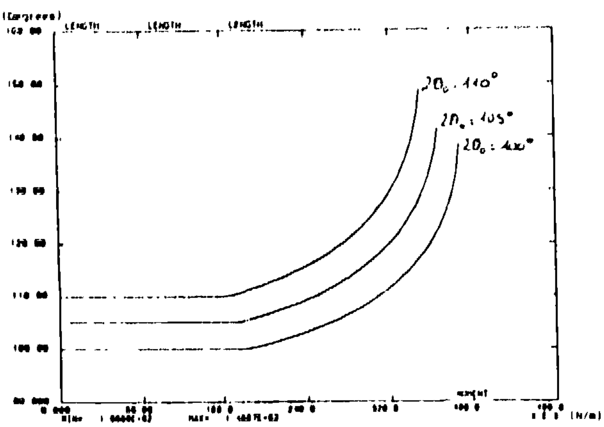


CRACK LENGTH VERSUS BENDING MOMENT Figure 5 CASTEM2000



COMPARISON OF RESPONSE WITH AND WITHOUT CRACK CLOSURE CASTEM2000

Figure 8



CRACK LENGTH VERSUS BENDING MOMENT Figure 6 CASTEM2000

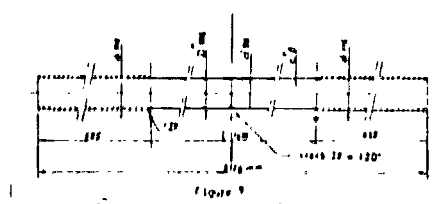


Figure 9

Geometrical characteristics of the test pipe

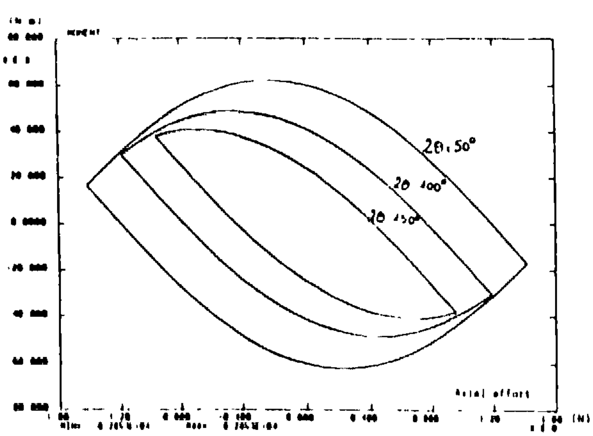


Figure 7

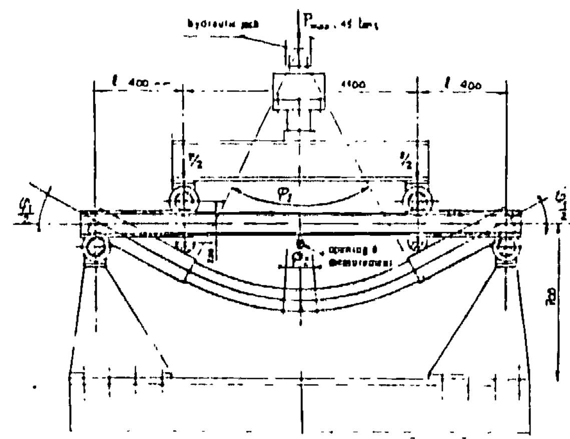


Figure 10

Experimental set up for static test

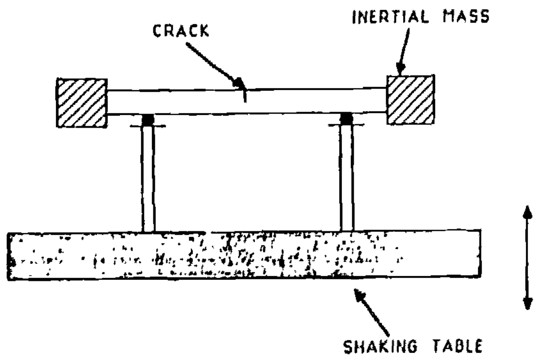
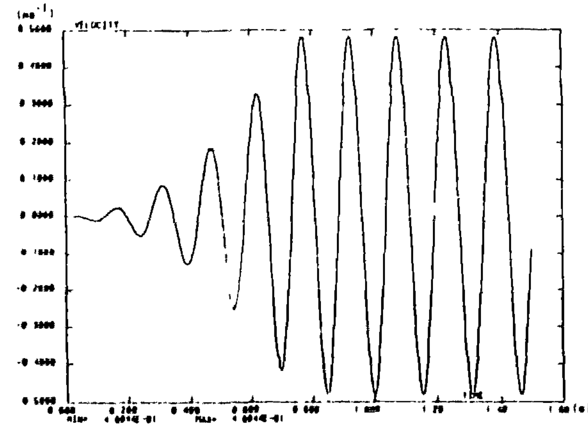


Figure 11
Experimental set up for dynamic test



VELOCITY OF SHAKING TABLE VERSUS TIME Figure 14 CASTEM2000

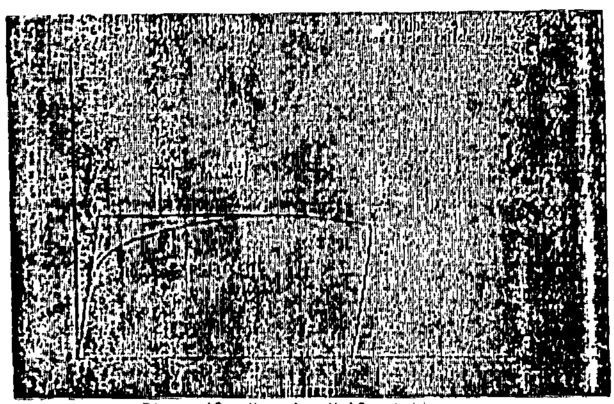
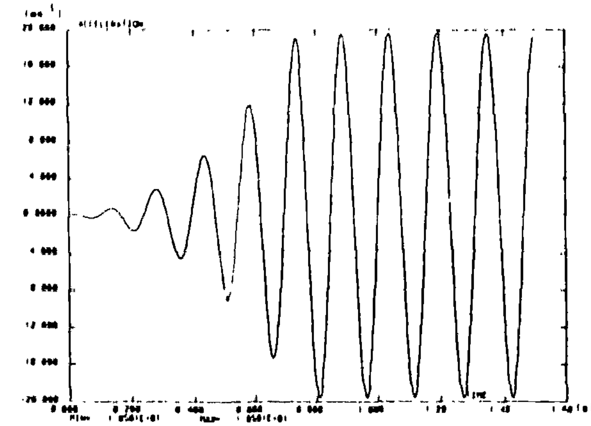
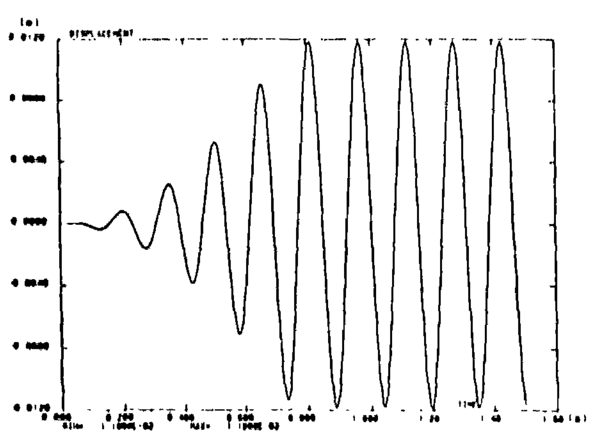


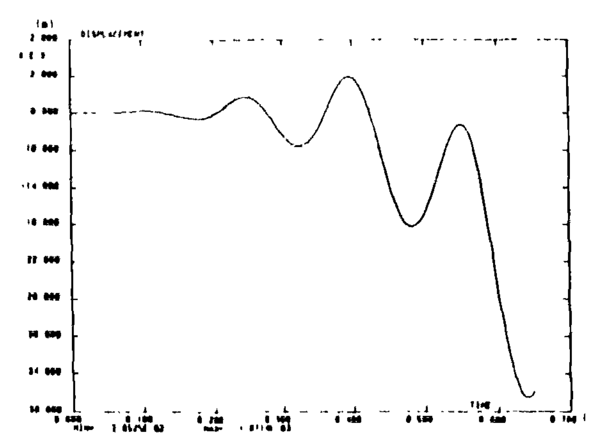
Figure 12 - Moment - Half rotation curve



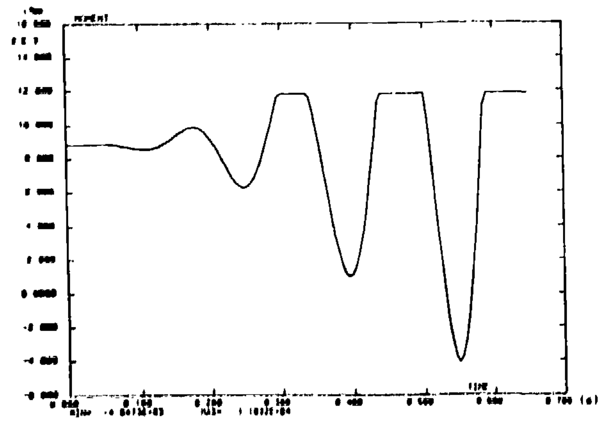
ACCELERATION OF SHAKING TABLE VERSUS TIME Figure 15 CASTEM2000



DISPLACEMENT OF SHAKING TABLE VERSUS TIME Figure 13 CASTEM2000



END MASSES RELATIVE DISPLACEMENT VERSUS TIME Figure 16 CASTEM2000



MOMENT AT CRACK LOCATION VERSUS TIME

Figure 17

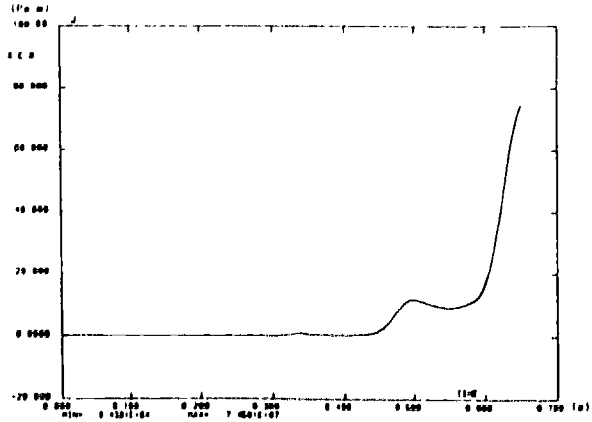


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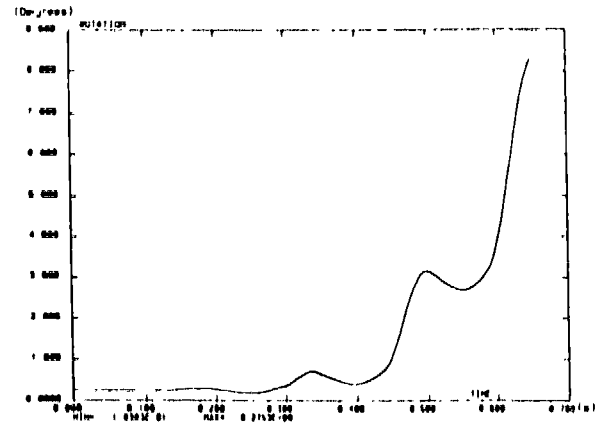
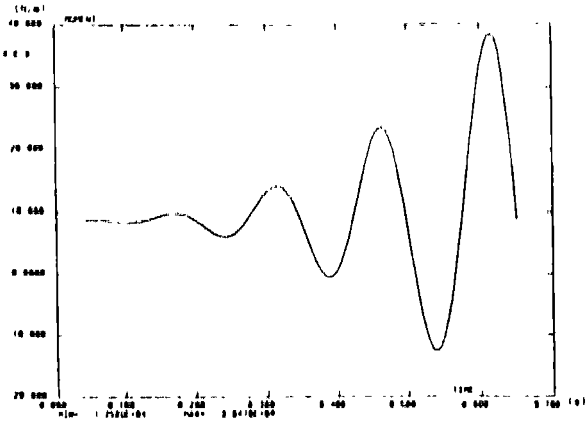
Figure 19



ROTATION DUE TO CRACK VERSUS TIME

Figure 18



MOMENT VERSUS TIME IN LINEAR CALCULATION

Figure 20