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INTRINSIC SPACE CHARGE RESONANCES AND THE SPACE CHARGE LIMIT*

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Abstract

A study has been done of the dependence of the space charge limit on the choice of ν-values using a simulation program. This study finds a strong dependence of the space charge limit on the location of the ν-values relative to the intrinsic space charge resonances, which are driven by the space charge forces due to the beam itself. Four accelerators were studied. For some of these accelerators the study suggests that the space charge limit can be increased by about a factor of 2 by proper choice of the ν-values. The lower order 1/2 and 1/4 intrinsic resonances appear to be the important resonances. There is some evidence for effects due to the 1/6 and 1/8 intrinsic resonances, particularly for larger synchrotrons.

1. Introduction

This study investigates the effect of the choice of the ν-values, ν_x, ν_y on the space charge limit. The study finds a strong dependence of the space charge limit on the location of ν_x, ν_y relative to certain intrinsic resonances, particularly the 1/4 and 1/2 resonances. The intrinsic resonances are the resonances driven by the space charge forces due to the beam. These are not the resonances driven by magnet field errors. The study finds that the space charge limit drops sharply at ν_x, ν_y which are just above the intrinsic 1/4 resonances, that are driven by the space charge forces. Peaks in the space charge limit are usually found at ν_x, ν_y which are just below these 1/4 resonances. The ν_x, ν_y corresponding to these peaks may be the desirable choice of ν_x, ν_y to give the highest space charge limit. For some accelerators, the study suggests that the space charge limit can be increased by a factor of 2 by proper choice of the ν-values.

Four accelerators were studied. These include the Fermilab booster, the AGS booster, the AGS and the PS booster. All these accelerators showed the above described correlation with the location of the 1/4 intrinsic resonances. Three of the accelerators are operating accelerators. The simulation program used in this study has been used to compute the space charge limit for these accelerators,[1] and the results agree with measurements within about a factor of two.

The intrinsic 1/4 resonances considered here are driven by the space charge forces due to the beam itself. The 1/4 resonances are the lowest order non-linear resonance generated this way, as only even order resonances are generated. Every other 1/4 resonance is also a 1/2 resonance which also can be generated by the space charge forces in the beam, and this 1/2 resonance may also be contributing to the drop in the space charge limit. Some 1/2 resonances are also strongly driven by the magnetic fields of the accelerator magnets. This effect will also show up in the study and is a well known effect. The primary emphasis of this study is on the effect of the resonances generated by the space charge forces due to the beam itself. 1/2 resonances driven by the magnetic field of the magnets can be recognized by the increase in the β functions of the lattice in the absence of space charge forces.

2. The Space Charge Model

The following study of the dependence of the space charge limit on the choice of ν-values makes use of a model developed in some previous work[1] on the space charge limit. In this model, the intrinsic space charge limit plays an important role. The intrinsic space charge limit is the space charge limit in the absence of magnetic field errors, and is due to the forces generated by the beam itself.

In studies[1] of three operating accelerators, which include the AGS, the PS Booster, and the Fermilab booster, it was found that the computed intrinsic space charge limit was fairly close to the experimentally observed space charge limit. This result plus studies of the effects of resonances due to magnetic field errors suggest that the intrinsic space charge limit provides an upper bound for the space charge limit which is not far from what is actually achieved by operating accelerators.

The resonances present due to magnetic field errors, if strong enough, can prevent the accelerator from achieving the intrinsic space charge limit. However, the effects of these resonances were found to be appreciable only when the beam intensity gets close to the intrinsic space charge limit. Well below the intrinsic space charge limit, there is little beam growth due to magnetic field error driven resonances, and the space charge forces tend to stabilize these resonances. These results depend on keeping the magnetic field errors below a certain limit.[1] This limit appears to be achievable in present day conventional magnet accelerators.

In this study, only the intrinsic space charge limit is computed. It is assumed that the space charge limit is primarily determined by the intrinsic space charge limit, and that the effects of resonances driven by magnetic field errors can be kept under control.

In the following sections the dependence of the intrinsic space charge limit on the choice of the tune, ν_x and ν_y, is studied for four different accelerators. Three of these accelerators, the Fermilab booster, the AGS and the PS booster are operating accelerators. The AGS booster is being constructed at present. For each accelerator the tune, ν_x and ν_y, was varied by changing the field gradients in the accelerator, and the intrinsic space charge limit was computed for each choice of ν_x, ν_y. This requires a considerable amount of computing, as the intrinsic space charge limit is found by increasing N_b, the number of protons per bunch, and observing the subsequent growth until an N_b is reached where some part of the beam reaches the aperture limit. For each accelerator the actual lattice of the accelerator is used in the tracking. The ν_x, ν_y are usually varied close to the diagonal $\nu_x = \nu_y$. For some accelerators, tunes off the diagonal were studied.

The simulation program used to compute the intrinsic space charge limit is described in reference 1.

3. Fermilab Booster

This accelerator has a particularly simple resonance pattern. It has 24 cells and no superperiod structure. The intrinsic 1/4 resonances are at $\nu = 6, 12, 18, \ldots$ The operating ν-values are $\nu_x = 6.7$, $\nu_y = 6.8$ which is just above the $\nu = 6$ resonance.

Figure 1 shows the computed intrinsic space charge limit $N_{b,L}$ as a function of ν_x. $N_{b,L}$ is the number of protons/bunch at which the beam will grow to the aperture limit. For each

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ν_x, $\nu_y = \nu_x + 0.1$. The curve drops to a low value just above the 1/4-resonance, $\nu_x = 6$ at $\nu_x = 6.2$. In this figure, and in the following figures, the 1/4 resonances are indicated by vertical dashed lines. The operating ν-values is indicated by the large dot, and has $N_{b,L} = 0.9 \times 10^{11}$/bunch. The curve then rises to a peak at $\nu_x = 8.2$, where $N_{b,L} = 1.9 \times 10^{11}$/bunch which is about a factor 2 higher than the $N_{b,L}$ at $\nu_x = 6.7$. The $\nu = 12$ resonance is also a 1/2-resonance which is strongly driven by the accelerator magnets. The β-functions become large near $\nu_x = 12$. They also become large near $\nu_x = 0$.

The dip in $N_{b,L}$ near $\nu_x = 8.4$ may be associated with the 1/6 resonances, $\nu = 8$. It will be seen that small dips like this one, which are not correlated with the 1/4 resonances, are found for several of the accelerators studied. These may be attributed to higher order intrinsic resonances like the 1/6 or 1/8. The presence of effects due to 1/6 or 1/8 resonances appears to be more clear in larger accelerators with higher periodicity such as the proposed Low Energy Booster for the SSC. The beam is assumed to be injected with emittance $\epsilon_x = 10.9$, $\epsilon_y = 9.4$ mm-mrad. The aperture limits are assumed to $\pm$ 31 mm horizontally and $\pm$ 24 mm vertically.

Comparison with measured results[1] indicate that the computed intrinsic space charge limit shown in Fig. 1 is about a factor 2 higher than the measured results. All the space charge limits shown in the figures in this study should be reduced by this factor of 2.

4. AGS Synchrotron

The resonance pattern for this accelerator is made more complicated by the existence of a superperiod, $N_s = 12$. There are 60 cells in the lattice, which produce strong harmonics near n=60.

The $N_s = 12$ periodicity has 1/4 resonances at $\nu = 3, 6, 9, 12, \ldots$ The 1/4 resonances generated by higher harmonic near n=60 are outside the ν-range of interest. The operating ν-values for the AGS are $\nu_x = 8.6$, $\nu_y = 8.8$.

Figure 2 shows the computed intrinsic space charge limit, $N_{b,L}$, as a function of ν_x for the AGS. For each ν_x, $\nu_y = \nu_x + 0.2$. The curve drops to a low value just above the 1/4 resonance $\nu_x = 6$. At the operating ν-value, $\nu_x = 8.6, N_{b,L} = 0.37 \times 10^{13}$/bunch and it is close to the peak $N_{b,L}$ between the two 1/4 resonances $\nu_x = 6$ and $\nu_x = 9$. The curve dips again just above the next 1/4 resonance at $\nu_x = 9$. The curve then dips again just above the 1/4 resonance at $\nu_x = 12$. The peak at about $\nu_x = 11$ is about $N_{b,L} = .8 \times 10^{13}$ which is about a factor of 2 higher than the $N_{b,L} = 0.37 \times 10^{13}$ computed at the operating $\nu_x = 6.7$.

The beam is assumed to be injected with emittance $\epsilon_x = 20$, $\epsilon_y = 5.1$ mm-mrad. The aperture limits are assumed to be at $\pm$ 35 mm horizontally and $\pm$ 32 mm vertically. One should remember that the results for $N_{b,L}$ in all the figures should be divided by a factor of 2 in order to get better agreement with measured results for the Fermilab booster, the AGS and the PS booster.

5. AGS Booster

This accelerator has a superperiod, $N_s = 6$, which excites 1/4 resonances at $\nu = 1.5, 3, 4.5, 6, \ldots$ There are 24 cells, and the quadrupoles have the period $N = 24$, and the β-functions, and thus the beam shape, have a strong 24 harmonic. The 1/4 resonances at $\nu = 6, 12, 18$ are then particularly strong. The period 6 in the space charge forces would appear to come from the $1/\rho^2$ focussing term in the dipoles and from the dispersion function since the dipoles have the period of 6.

Figure 3 shows the computed intrinsic space charge limits, $N_{b,L}$ as a function of ν_x for the AGS Booster. For each ν_x, $\nu_y = \nu_x + 0.1$. The proposed operating ν-values for the Booster is $\nu_x = 4.82$, $\nu_y = 4.83$. The curve has dips just above the 1/4 resonance $\nu = 3, 4.5, 6, 7.5, 9$.

The $\nu = 12$ is a 1/2 resonance that is strongly driven by the magnets in the accelerator.

At the operating ν-values $\nu_x = 4.82$, $\nu_y = 4.83$, the space charge limit found is $N_{b,L} = 1.5 \times 10^{13}$ protons/bunch. By moving to the peak at $\nu_x = 5.82$, $\nu_y = 5.83$, $N_{b,L}$ can be increased to $N_{b,L} = 2 \times 10^{13}$. In section 6, results are found for ν-values further away from $\nu_x = \nu_y$ diagonal, and $N_{b,L}$ was increased to $N_{b,L} = 2.5 \times 10^{13}$ protons/bunch. Altogether, this study suggests that the space charge limit of the AGS booster might be increased by about 70% by proper choice of the ν-values. The choice of ν-values is further discussed in section 7.

The beam is assumed to be injected with emittances $\epsilon_x = 33$, $\epsilon_y = 5.1$ mm-mrad. The aperture limits are assumed to be $\pm$ 48 mm horizontally and $\pm$ 32 mm vertically.

6. The PS Booster

This accelerator has a period of 16 with 16 cells and no superperiod. The 1/4 resonances are at $\nu = 4, 8, 12, \ldots$ The operating ν-values are $\nu_x = 4.25$, $\nu_y = 5.60$ and are appreciably off the diagonal $\nu_x = \nu_y$.

Figure 4 shows the computed intrinsic space charge limit, $N_{b,L}$, as a function of ν_x for the PS booster. For each $\nu_x, \nu_y = \nu_x + 1.35$. There is a strong 1/2 resonance at $\nu = 8$ which is driven by the field of the accelerator magnets. As the ν-values are being varied along the line $\nu_y = \nu_x + 1.35$, the 1/2 resonance at $\nu = 8$ is crossed at two places; the $\nu_y = 8$ resonance is crossed at $\nu_x = 6.5$, and the $\nu_x = 8$ resonance at $\nu_x = 8$. The effects of the 1/2 resonance are felt at ν_x as low as $\nu_x = 5.5$. The 1/4 resonance near $\nu = 4$ is crossed at 3 values of ν_x at $\nu_x \simeq 2.6, 3.2$ and 4.0 corresponding to the resonances $4\nu_y = 16, 2\nu_x + 2\nu_y = 16$ and $4\nu_x = 16$ respectively. There appears to be some correlation of $N_{b,L}$ with these 1/4 resonances, although the correlation is not so clear here because of the strong 1/2 resonance at $\nu = 8$ and the off diagonal ν-values.

At the operating ν-values $\nu_x = 4.25$, $\nu_y = 5.60, N_{b,L} = 0.5 \times 10^{13}$ protons/bunch. This is fairly close to the peak $N_{b,L} = 0.65 \times 10^{13}$. It was also observed that moving ν_x, ν_y closer to the diagonal $\nu_x = \nu_y$ considerably reduced the intrinsic space charge limit.

7. Off Diagonal ν-Values

Most of the studies reported in the previous sections were done with ν-values that moved along a line parallel to the $\nu_y = \nu_x$ resonance line and usually fairly close to this line. The one exception was the PS Booster where ν was varied along the line $\nu_y = \nu_x + 1.35$. For the PS Booster, the intrinsic space charge limit was also studied along the line $\nu_y = \nu_x + 0.1$ which is closer to the $\nu_x = \nu_y$ line. Much smaller values of $N_{b,L}$ were found along the $\nu_y = \nu_x + 0.1$. This is shown in Fig. 5.

Because of the results with the PS Booster, some studies were done with off-diagonal values with the other accelerators studied. It was found that the peak values of $N_{b,L}$ generally get somewhat larger for ν-values off the diagonal. Sometimes $N_{b,L}$ was considerably increased; sometimes $N_{b,L}$ changed only slightly. No consistent quantitative pattern was noticed.

For the AGS Booster, $N_{b,L}$ could be increased from the peak value of $N_{b,L} = 2 \times 10^{13}$ to $N_{b,L} = 2.5 \times 10^{13}$ by moving ν_x, ν_y off the diagonal to any of the following ν-values: 1) $\nu_{x,y} = 4.82, 5.83$; 2) $\nu_{x,y} = 5.64, 5.01$; and 3) $\nu_{x,y} = 5.92, 5.73$.

The first choice, $\nu_x = 4.82$, $\nu_y = 5.83$ was found by starting at the proposed operating point $\nu_x = 4.82$, $\nu_y = 4.83$, and moving away from the diagonal by increasing ν_y until a peak space charge limit was reached at $\nu_y = 5.83$. Choices 2 and 3 were found by starting at the ν-values which give the highest space charge limit in Fig. 3, which are $\nu_x = 5.82$, $\nu_y = 5.83$, and then moving perpendicular to the diagonal until peaks were found on either side of the diagonal.

This study suggests a strong dependence of the space charge limit on the location of the ν-values relative to the intrinsic space resonances. Because of the approximations made in the simulation program, some experimental confirmation of these results would be very desirable.

Reference

1. G. Parzen, Nucl. Inst. and Methods, A281 (1989), p. 413-425.

Fig. 1 Intrinsic space charge limit versus ν_x for the Fermilab booster. $\nu_y = \nu_x + 0.1$.

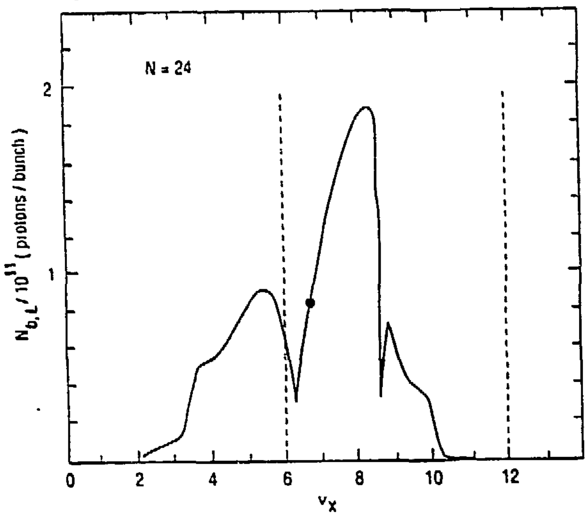

Fig. 2 Intrinsic space charge limit versus ν_x for the AGS synchrotron. $\nu_y = \nu_x + 0.2$.

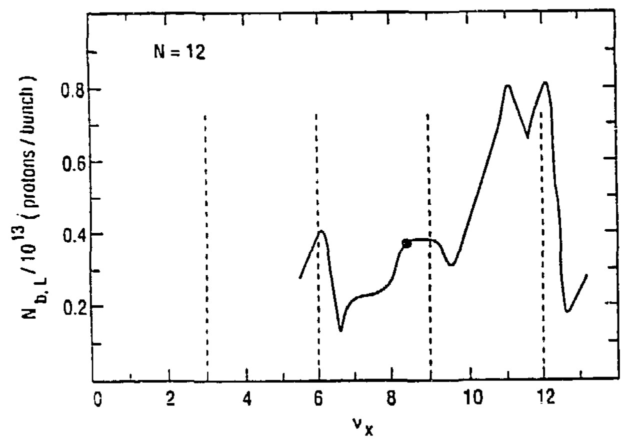

Fig. 3 Intrinsic space charge limit versus ν_x for the AGS booster. $\nu_y = \nu_x + 0.01$.

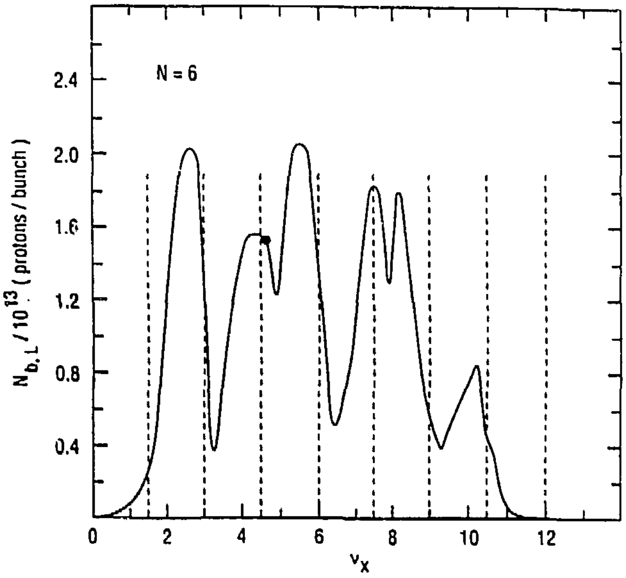

Fig. 4 Intrinsic space charge limit versus ν_x for the PS booster. $\nu_y = \nu_x + 1.35$.

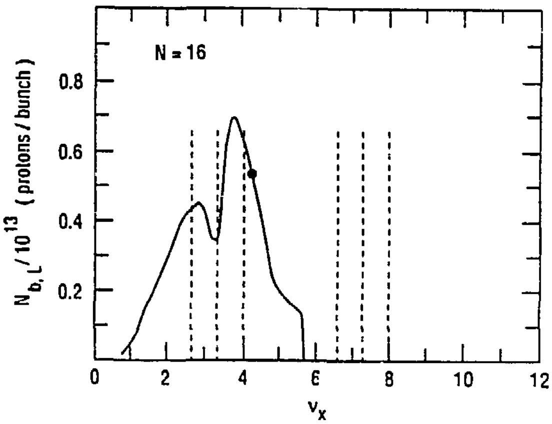

Fig. 5 Intrinsic space charge limit versus ν_x for the PS booster at different distances from the diagonal $\nu_x = \nu_y$.