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ABSTRACT

The cluster model of Buck, Dover and Vary has been used to calculate partial widths for alpha decay from the ground states of all nuclei for which experimental measurements exist. The cluster-core potential is represented by a simple three-parameter form having fixed diffuseness, a radius which scales as $A^{1/3}$ and a depth which is adjusted to fit the Q-value of the particular decay. The calculations yield excellent agreement with the vast majority of the available data, and some typical examples are presented.

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The calculation of alpha decay widths presents one of the oldest challenges in nuclear physics. It was one of the first problems to which quantum mechanics was applied, and although Gamow [1] and Condon and Guerney [2] immediately succeeded in obtaining a qualitative explanation in terms of barrier penetration, the quest for quantitative predictions of the absolute values of alpha decay rates has remained largely unfulfilled ever since [3, 4].

The traditional approach has been through R-matrix theory, where unfortunately the final results are extremely sensitive to the choice of an arbitrary channel radius [5]. Another major difficulty has been the attempt to describe the initial state in terms of oscillator wave functions, which do not produce a sufficiently highly correlated "alpha" particle like structure, because of the need to truncate the shell model space so as to keep within numerically manageable bounds. This may be overcome to a large extent by using explicit cluster wave functions [6]. Even when this is done however, there is still a high sensitivity to the choice of the alpha-core nuclear potential which, through its influence on the barrier, can easily cause variations of several orders of magnitude in the predicted decay rates.

In this Letter we show that the Buck Dover Vary cluster model [7] can give an excellent description of the vast majority of alpha decays from the ground states of atomic nuclei across the periodic table. This model was introduced to describe the properties of alpha particle cluster states in ^{16}O and ^{20}Ne. In these nuclei it was outstandingly successful in describing the partial widths, Γ_α, of the states above the alpha emission threshold, (see also [8] and [9]). It has also been applied to $\alpha-\alpha$ scattering [10] and the exotic decay modes of heavy nuclei [11], where it gives an equally good account of the appropriate cluster emission rates.

In view of this impressive pedigree we were encouraged to apply it to alpha decay in heavy nuclei. We employ a consistent set of alpha-core potential parameters, including a fixed diffuseness, a radius varying like $A^{1/3}$ and a depth chosen to reproduce the Q-value of the particular decay, and we thereby counter the objections of arbitrariness levelled at previous approaches. We now proceed to outline the basis of the model and present some typical results.

The model uses a local potential, V(r), to describe the effective interaction between a cluster and core. This potential can be obtained from a double folding integral,

$$V(r) = \int\int \rho_1(\mathbf{r}_1)\rho_2(\mathbf{r}_2)U(|\,\mathbf{r}+\mathbf{r}_2-\mathbf{r}_1\,|)d^3r_1d^3r_2, \tag{1}$$

where $\rho_1(\mathbf{r}_1)$ and $\rho_2(\mathbf{r}_2)$ are the cluster and core densities and $U(|\,\mathbf{r}_1-\mathbf{r}_2\,|)$ is an effective nucleon-nucleon interaction. It may also be conveniently and accurately approximated by the simple three parameter form,

$$V(r) = \frac{-V_0\left(1+\cosh\left(\frac{R}{a}\right)\right)}{\cosh\left(\frac{r}{a}\right)+\cosh\left(\frac{R}{a}\right)}. \tag{2}$$

where V_0 is the depth of the potential, R is its radius parameter and a is its diffuseness. Bound and resonant state wave functions describing the relative motion of the cluster about the core are then obtained by solving the single particle Schrödinger equation.

The main requirements of the Pauli exclusion principle are satisfied by choosing the quantum numbers of relative motion n (the number of interior nodes in the radial wave function) and L (the orbital angular momentum) to obey a Wildermuth condition, $2n+L \geq N$. Here N is a constant integer chosen large enough to correspond to the microscopic situation in which the cluster nucleons all occupy orbitals above those already occupied by the core nucleons. In all cases studied below we take $N = 22$, which is typical for heavy nuclei, but find that our results do not depend critically on the precise value of N within fairly wide limits. Any remaining effects of antisymmetrisation may be absorbed into the effective potential.

We initially looked at the $^{212}Po \rightarrow {}^{208}Pb + \alpha$ decay, because charge densities for ^{208}Pb and ^{4}He are available from electron scattering experiments [12] and provide a necessary ingredient of the folding integral of Eq.(1). The simplest possible potential, V(r), was then obtained by using a delta-function for the effective nucleon-nucleon interaction and converting the charge densities to the required matter densities as suggested by Cook [13]. The shape of the resulting folded potential could be almost perfectly reproduced by assigning the values R=6.3 fm and a=0.75 fm in Eq.(2). We preferred to continue using the "cosh" potential to investigate other heavy systems, since the relevant charge densities are, in general, unavailable. To do this we maintained the diffuseness as a=0.75 fm in all cases and scaled the radius in proportion to $A^{1/3}$ (where A is the mass number of the core).

Once the nuclear part of the cluster-core potential had been specified, we calculated the width for alpha emission from the ground state using the two potential approach to the decay of a quasi-stationary state developed recently by Gurvitz and Kalbermann [14] The two potentials involved are the total cluster-core potential, $V_T(r)$, (containing nuclear, Coulomb and centrifugal terms) and a modified potential, $V_M(r)$, obtained by rounding off $V_T(r)$ at its maximum value (occurring at $r=r_B$). To be more explicit, $V_M(r) = V_T(r)$ for $r \leq r_B$ and $V_M(r) = V_T(r_B)$ for $r \geq r_B$. Then one finds a bound state energy, E_0, by solving the Schrödinger equation with the potential $V_M(r)$, and uses it in conjunction with the other potential $V_T(r)$ to define three turning points r_0, r_1 and r_2 (in order of increasing distance from the origin). An expression for the decay width of a resonant state is then developed in terms of these quantities.

In the semi-classical limit this essentially leads to the Gamow formula with a well determined pre-exponential factor. If we assume that our heavy nucleus emits an alpha particle from its ground state (spin I_i) to produce a core also in its ground state (spin I_f) with a unique cluster-core relative orbital angular momentum L, then this gives a width Γ where

$$\Gamma = \frac{(2I_i+1)}{(2I_f+1)(2L+1)} \frac{F\hbar^2}{4\mu} \exp\left[-2\int_{r_1}^{r_2} k(r)dr\right]. \quad (3)$$

where μ is the reduced mass of the cluster-core system, $k(r)$ is the semi-classical wave number

$$k(r) = \left[\frac{2\mu}{\hbar^2}(E_0 - V_T(r))\right]^{\frac{1}{2}}. \quad (4)$$

and F is the semi-classical bound state normalization factor

$$F \int_{r_0}^{r_1} \frac{1}{k(r)} \cos^2\left[\int_{r_0}^{r} k(r')dr' - \frac{\pi}{4}\right] dr = 1. \quad (5)$$

The Langer modification is made so that even when L=0 three turning points are always obtained and all our integrals are well defined. We therefore write

$$V_T(r) = V(r) + V_C(r) + \frac{\hbar^2}{2\mu}(L+1/2)^2. \quad (6)$$

Here V(r) is the nuclear part of the cluster-core potential, parametrised as in Eq.(2). The Coulomb potential, $V_C(r)$, is that appropriate to a point-like alpha particle interacting with a uniformly charged spherical core of radius R_C. The value of R_C = 5.5 fm was used for ^{208}Pb [12], and scaled in proportion to $A^{1/3}$ for all other cores. The precise value of R_C is not crucial, within reasonable limits. The final term in Eq.(6) is of course the centrifugal potential, including the Langer modification.

The widths calculated with these formulae are extremely sensitive to the Q-values of the decays. We have taken these quantities from the recently updated mass defect tables of Wapstra and Audi [15]. However, we note that the masses of nuclei far from stability must inevitably be systematically extrapolated from the better known values of their more stable neighbours, and that this is potentially a source of serious error in our calculations.

The other main source of uncertainty in the practical application of these formulae is sometimes our lack of knowledge of the spins I_i and I_f. Very often we are fortunate and both I_i and I_f are known and angular momentum conservation leads to a unique value of L. An obvious example of this is when an even-even nucleus decays, since then all angular momenta are zero. A slightly less favourable situation arises when I_i and I_f are known, but several values of L are possible. Then we always take the smallest L-value consistent with triangulation. In the worst cases however, I_i or I_f (or both) are unknown, and we then assume that $I_i = I_f$ and set L=0. Overall, this is the safest strategy, since removing two protons and two neutrons from a high j-orbital will often leave the total spin unchanged. In particularly unfavourable cases however, this procedure can lead to errors as large as a factor of 100 in our values for Γ.

Figure 1 shows a comparison between our predicted alpha widths and the corresponding experimental values for the Po isotopes. All those isotopes are presented for which a lifetime and alpha decay branching ratio have been experimentally determined [16]. The agreement between our theoretical calculations and the measured values is generally very impressive, and the striking change in Γ_α, by 16 orders of magnitude, as the neutron number passes through the magic value of 126 is well reproduced. One other notable feature is highlighted by the ^{197}Po $\rightarrow$ ^{193}Pb + α decay. We have performed two calculations here. One, using the tentatively suggested spins of $I_i = (3/2^-)$, $I_f = (13/2^+)$ and L=5 and the other with $I_i = I_f$ and L=0. In view of the far better agreement of this latter calculation with experiment, and the natural way that it fits in with the trend established by the other isotopes, we suggest that the proposed spin-parity assignments for the ground states of ^{197}Po and ^{193}Pb could be profitably reexamined.

Using the same set of cluster-core potential parameters, we have gone on to investigate all reported alpha decays from nuclear ground states for which a partial alpha width has been experimentally determined. These range in mass from ^{109}Te $\rightarrow$ ^{105}Sn + α (where we calculate Γ_α = 9.9x10^{-24} MeV compared with the experimental 5.7x10^{-24} MeV) up to ^{266}109 $\rightarrow$ ^{262}107 + α (where we calculate 7.9x10^{-18} MeV compared with the experimental 1.3x10^{-19} MeV, which is quite acceptable given our ignorance of the ground state spins). Our agreement with the experimental values of Γ_α across this entire mass range is generally excellent (usually within a factor of 10). There are one or two poor spots, notably in a few rare earth nuclei which are known to be highly deformed, and in a few of the transuranic nuclei, where the ground state masses are not very reliably known. Nevertheless, the overall picture, with so few free parameters, is highly satisfactory.

Figure 2 shows an example of our results in a lighter mass region. We plot our calculated values for the partial widths, Γ_α, for alpha decay of 17 Pt isotopes, and again find excellent agreement with experimental measurements. Two decays appear to be badly reproduced. Firstly, the ^{183}Pt $\rightarrow$ ^{179}Os + α width is overestimated by our calculation, but would be reduced if our assumption that $I_i = I_f$ and L=0 (giving a statistical factor of unity) were incorrect. Secondly, the experimental value of Γ_α for ^{188}Pt $\rightarrow$ ^{184}Os + α is actually an upper limit, since it is only known that the branching ratio for alpha decay of ^{188}Pt is less than 0.01%. Finally, we calculate Γ_α for the decay ^{190}Pt $\rightarrow$ ^{186}Os + α to be 2.3x10^{-41} MeV compared with the measured value of 3.5x10^{-41} MeV. It would have been inconvenient to plot this result on the same scale as the others. We thus have a good theoretical description of the alpha decays of all 18 Pt isotopes for which experimental data are available.

In Figure 3 we go to the heavier mass region and compare the calculated and experimental widths for alpha decay of 19 Th isotopes. The agreement is again very good, with the most notable discrepancy being the ^{217}Th $\rightarrow$ ^{213}Ra + α value. This is another case where our predicted value would be reduced, closer to the experimental one, if our assumption that $I_i = I_f$ and L=0 were to prove incorrect, thus giving a statistical factor less than unity. The effects of the neutron shell closure at 126 are again clearly discernible, although they are not as marked as for the Po isotopes.

We have applied the Buck Dover Vary cluster model to alpha decay from the ground states of all nuclei for which corresponding experimental data are available. The parent nuclei involved have masses ranging from 109 to 266, and the widths range from 10^{-14} MeV to 10^{-45} MeV. We have succeeded in obtaining generally excellent agreement over these entire ranges. This must indicate that we are reproducing the barrier extremely well with a simple three-parameter cluster-core potential having fixed diffuseness a=0.75 fm, a radius which scales like $A^{1/3}$ and a depth which is varied to reproduce the measured Q-value of each decay (and turns out to lie between 200 MeV and 250 MeV). It must also indicate that our cluster wave functions are a very close approximation to physical reality, and hence provides prima facie evidence for strong alpha clustering across the entire periodic table.

After we completed this work we became aware of a similar study of reduced alpha widths for even-even nuclei with neutron number ≥ 84 and proton number ≤ 84 [17]. The authors of this study have concentrated on the long lived stable nuclei and the proton rich nuclei in their selected mass region. Our calculations for those particular nuclei are in essentially perfect agreement with theirs, and in addition we fit all other known ground state $\rightarrow$ ground state alpha decays.

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FIGURE CAPTIONS

FIGURE 1 - A comparison of the calculated (circles) and measured (crosses) partial alpha decay widths, Γ_α, for the Po isotopes as a function of the daughter nucleus mass number, A. Two calculated values are presented for $^{197}Po \rightarrow {}^{193}Pb + \alpha$ corresponding to $I_i = 3/2^-$, $I_f = 13/2^+$ with L=5 (lower circle) and $I_i = I_f$ with L=0 (upper circle)

FIGURE 2 - A comparison of the calculated (circles) and measured (crosses) partial alpha decay widths, Γ_α, for the Pt isotopes as a function of the daughter nucleus mass number, A. The experimental value for $^{188}Pt \rightarrow {}^{184}Os + \alpha$ is an upper limit.

FIGURE 3 - A comparison of the calculated (circles) and measured (crosses) partial alpha decay widths, Γ_α, for the Th isotopes as a function of the daughter nucleus mass number, A.

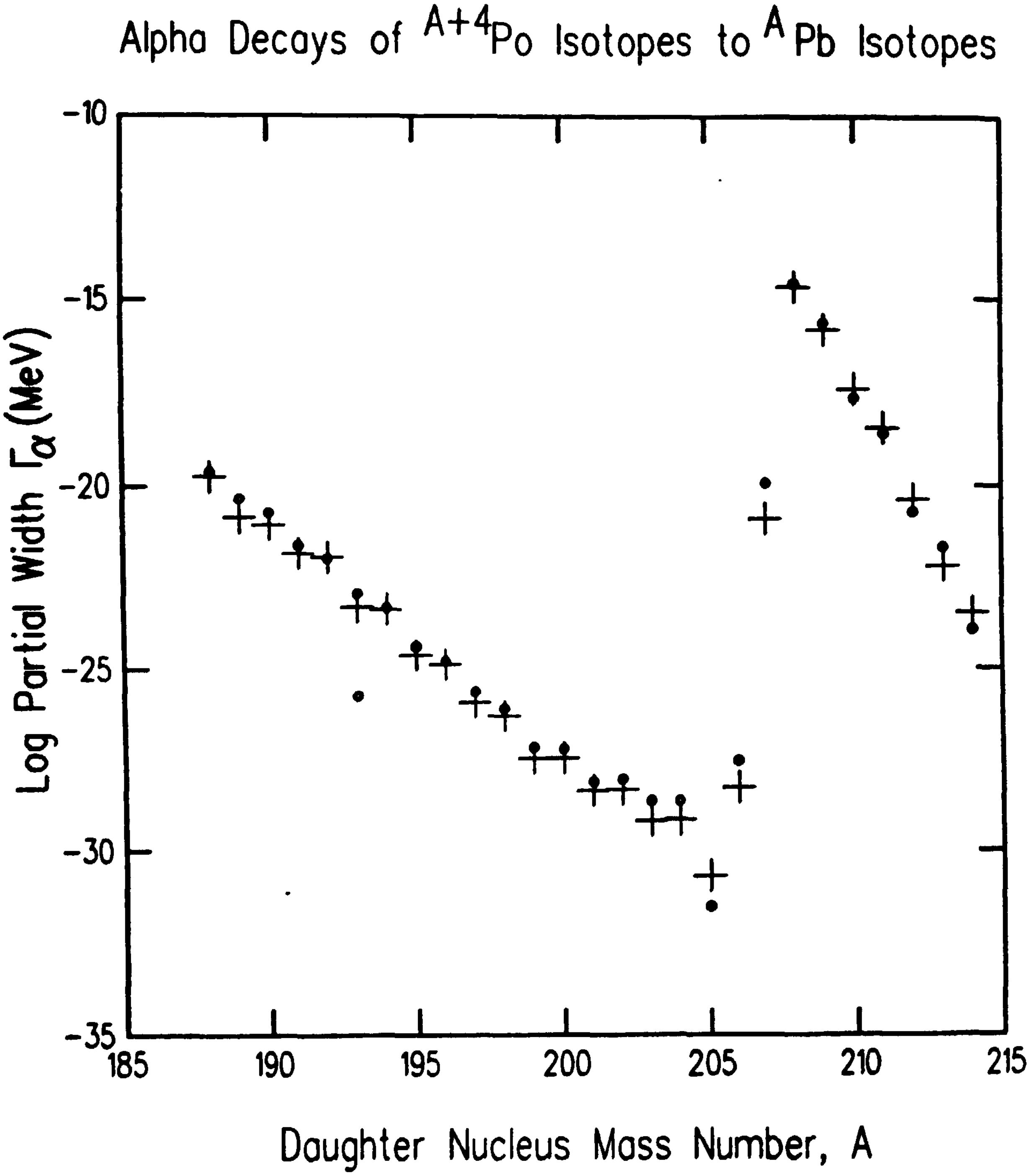
Alpha Decays of A+4Po Isotopes to APb Isotopes
-10
-15
-20
-25
-30
-35
185
190
195
200
205
210
215
Log Partial Width Γα (MeV)
Daughter Nucleus Mass Number, A

Fig. 1

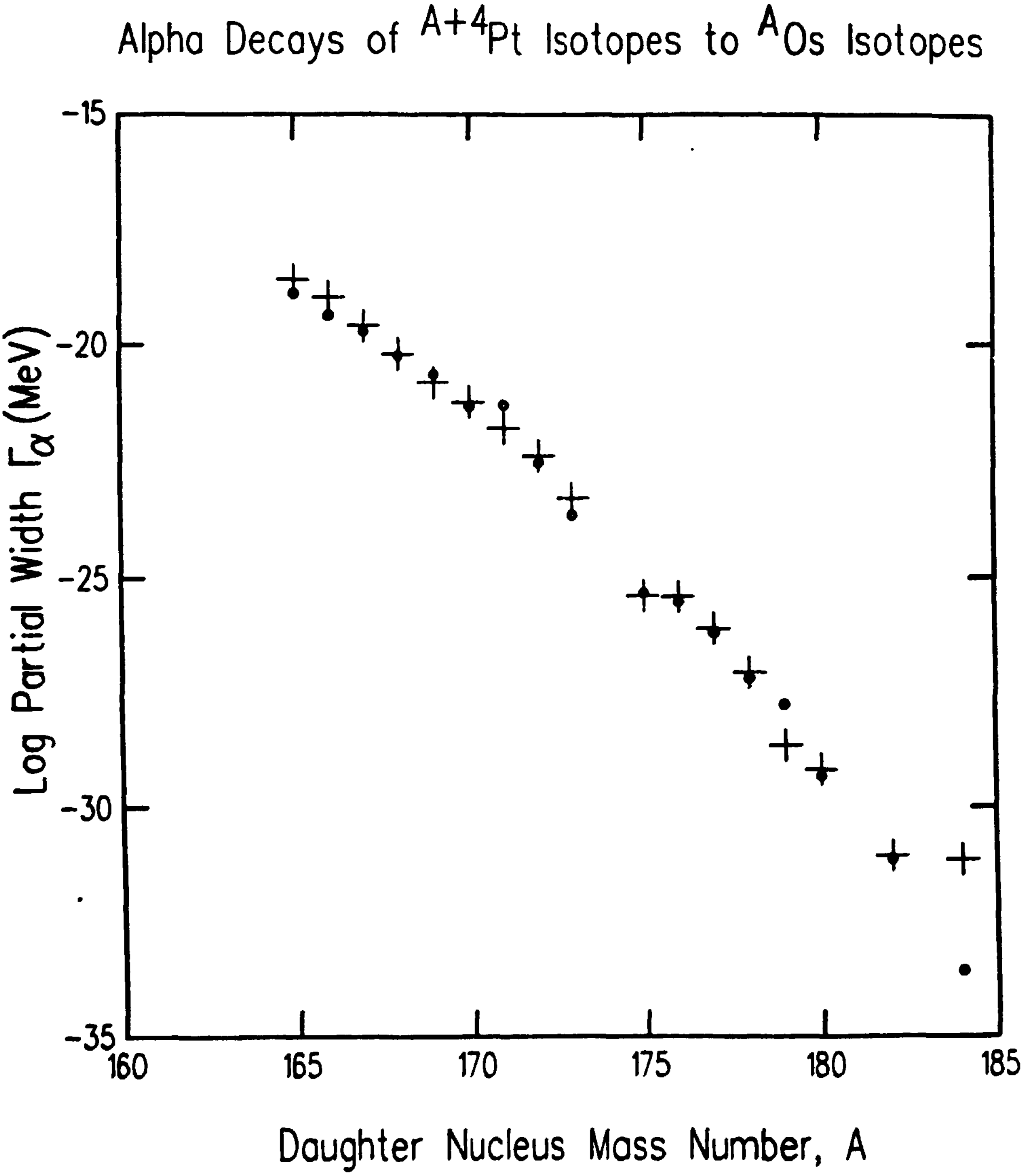
Alpha Decays of A+4Pt Isotopes to AOs Isotopes
Log Partial Width Γα (MeV)
-15
-20
-25
-30
-35
160
165
170
175
180
185
Daughter Nucleus Mass Number, A

Fig. 2

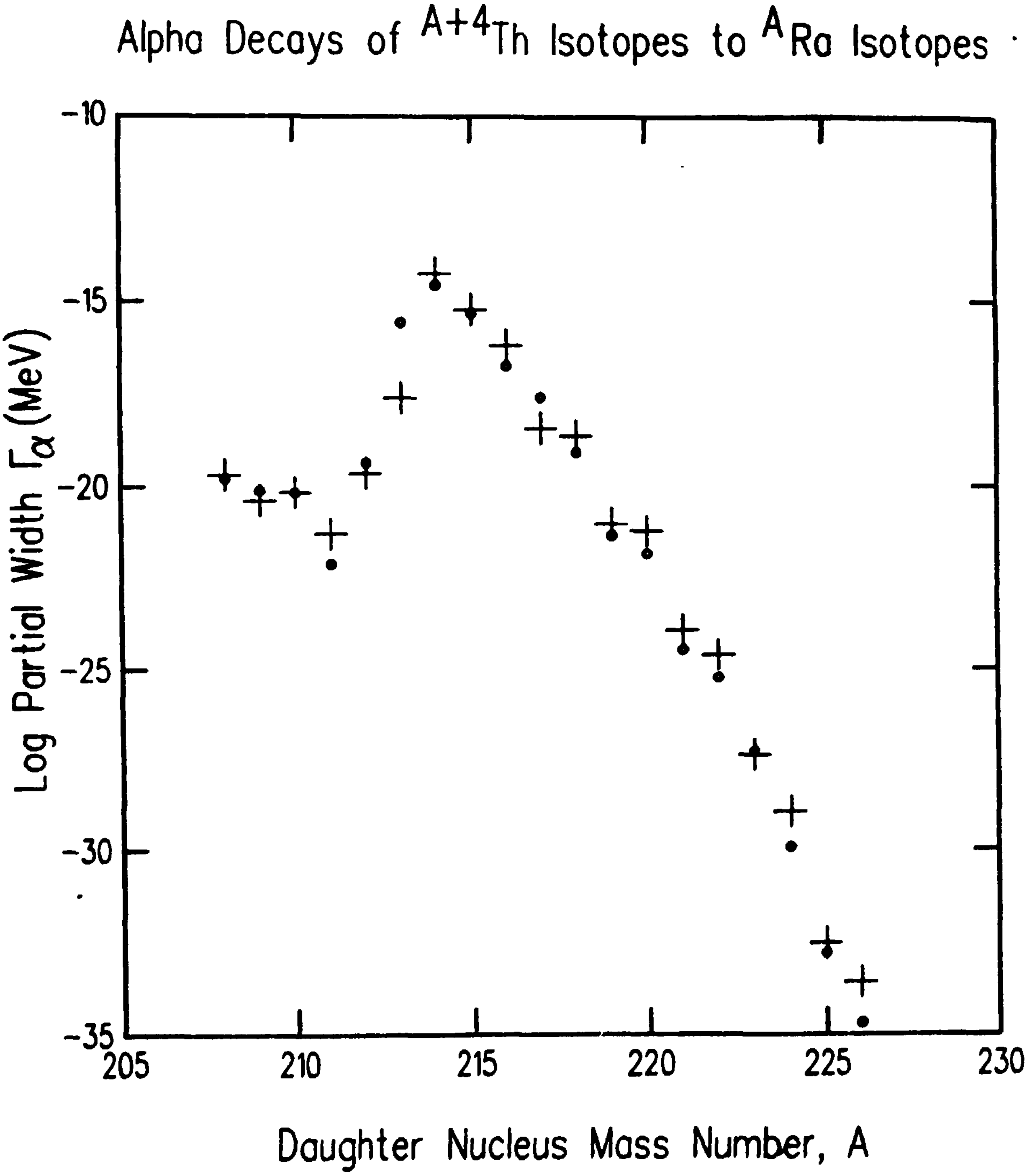
Alpha Decays of $^{A+4}$Th Isotopes to ^{A}Ra Isotopes
Log Partial Width Γ_α (MeV)
-10
-15
-20
-25
-30
-35
205
210
215
220
225
230
Daughter Nucleus Mass Number, A

Fig. 3