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ABSTRACT

We compute the one–loop free energy of the open neutral string gas in a constant electromagnetic background. Starting from this result we show that the Hagedorn temperature of this hot string gas depends on the background electric field. The larger the electric field, the lower the Hagedorn temperature is.

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1. INTRODUCTION

There are many questions related to the string theories for which up to now a procedure that permits to investigate and understand them in a completely satisfactory way does not exist. To this kind of problems, belong the choice of a unique vacuum, the symmetries of the string theory, and others.

In order to gain insight within current formulation of string theories in these and many other analogous problems, it is common to try to probe strings under extreme conditions as at high temperatures or at high energies, as well as to see the possible effects on the theory of non–trivial background fields.

The asymptotically high–energy region of the string theory has been explored in different works under the assumption that one cannot only neglect the masses of higher spin modes but also one can make the calculations around a trivial vacuum. In these cases it was shown that the interactions among the higher spin ($s \geq 2$) modes asymptotically disappear. Such result seems rather natural when one realizes that under the above assumption one has, in the asymptotical high–energy region, a theory of higher spin gauge massless particles in a flat space, and according to the no–go theorem, such theory is a free one. However, as it has been shown in [1], the theory of high spin interacting gauge fields exists in an anti–de Sitter (AdS) space. This means that in a phase where the vacuum of the string becomes non–trivial (with a gravity anti–de Sitter background where the no–go theorem does not work) the asymptotical interactions do not disappear and it would be possible to obtain another asymptotical regime. In fact, in a finite theory a non–trivial background might in principle give non negligible effects even in the high–energy region. This remark will be corroborated particularly, as we will show below, in the results of the present letter.

During the last years the problems of string theories in non–trivial backgrounds have motivated many investigations [2–10] and as a consequence many interesting effects have been found. For example in [3, 5], by considering the open string theory in electromagnetic background, an instability was found associated to the electric field (this instability was obtained in [3] due to the Born–Infeld type effective action, while in [5] it appeared at the classical level); on the other hand, in [10] it was pointed out that the existence of a cosmological background may give rise to the emergence of Jeans–like instabilities.

Moreover, the thermodynamics of string should also be important for the early universe. It can provide us remarkable deviations from standard cosmology, which might be able to explain many puzzles of the present cosmological picture.

In the present letter, we investigate the interrelation between the interesting properties of the string theories at finite temperature and the non less exciting features of the strings in non–trival backgrounds. We do this by computing the one–loop free energy of a hot gas of open neutral strings in a constant electromagnetic background. As we will show below, the Hagedorn temperature of such a gas differs from the value that corresponds to the free string case and, in fact, it becomes dependent on the electric field. This dependence is such that the Hagedorn temperature vanishes at the critical value of the electric field.

2. FREE ENERGY OF THE NEUTRAL STRING GAS IN A CONSTANT ELECTROMAGNETIC BACKGROUND

Let us compute now the one–loop free energy of an open neutral gas in the presence of a constant electromagnetic background. In order to do this we shall use two independent well–known methods [11].

The first one takes into account the fact that the one–loop free energy of the string gas in a constant electromagnetic background can be obtained, similarly to the free string case, by summing the free energies of the single string modes, since the strings in such a constant background do not interact with each other. To use this procedure we need to know the single string spectrum and the degeneracy of the single string states.

In the second method one calculates the string–loop path integral on a target space $S^1 \times R^{25}$ with β (the inverse temperature) the circumference of the compactified time dimension. As we will see below, in the presence of $F_{\mu\nu}$ winding modes appear not only for the zero mode of the time coordinates, but also for the zero mode of the first space coordinate, because these two modes are linked by the boundary conditions.

2.1 Sum of free energies

Let $F_{\mu\nu}$ be the constant electromagnetic field strength. By a convenient rotation in the D–dimensional space–time one can always put this antisymmetric tensor in a standard block diagonal form. In Euclidean coordinates, the rescaled field strength has the form

$$f_{\mu\nu} = \frac{q\, F_{\mu\nu}}{T} = \begin{bmatrix} 0 & f_1 & & & \\ -f_1 & 0 & & & \\ & & \ddots & & \\ & & & 0 & f_{D/2} \\ & & & -f_{D/2} & 0 \end{bmatrix} \tag{1}$$

where T is the string tension and q is the magnitude of the charge in each end of the string. For a neutral string (the case of interest here) the charges q_1, q_2 in the ends are equal and opposite, $q_1 = -q_2 = q$.

The electric field E and the magnetic field components H_α in the Minkowski space can be written after analytical prolongation in terms of the Euclidean invariants $f_i (\, i = 1, \ldots, D/2)$ as follows

$$\begin{aligned} e &= \frac{q\,E}{T} = i\, f_1 \\ h_\alpha &= \frac{q\,H_\alpha}{T} = -f_{\alpha+1}, \quad \alpha = 1, \ldots, \frac{D}{2} - 1\,. \end{aligned} \tag{2}$$

The light–cone gauge square mass operator of the open neutral string in electromagnetic background is given by the following expression [6]:

$$M^2 = -\sum_{\alpha=1}^{D/2-1} \frac{e^2 + h_\alpha^2}{1 + h_\alpha^2} \left[(P^{2\alpha})^2 + (P^{2\alpha+1})^2\right] +$$

$$2\pi T(1 - e^2) \left[\sum_{n=0}^{\infty} \sum_{\alpha=1}^{D/2-1} n \left(\bar{a}_n^{2\alpha} a_n^{2\alpha} + \bar{a}_n^{2\alpha+1} a_n^{2\alpha+1}\right) - 1\right] . \tag{3}$$

The transversal components of the total momentum operator of the string $P^{2\alpha}_{2\alpha+1}$ and the transversal oscillators $\bar{a}_n{}^{2\alpha}_{2\alpha+1}$, $a_n{}^{2\alpha}_{2\alpha+1}$, satisfy the same commutation relations as the corresponding operators of the free string case:

$$[P^i, P^j] = 0 \quad [P^i, a_n^j] = [P^i, \bar{a}_n^j] = 0$$

$$[a_n^i, \bar{a}_m^j] = \delta^{ij} \delta_{nm} . \tag{4}$$

It is worth pointing out two essential differences between the square mass operator (3) and the same operator in the free string theory. First, in Eq.(3) M^2 depends explicitly on the total momentum. Second, for the non zero electric field an "effective" tension $T' = T(1 - e^2)$ in the number operator part of M^2 appears. At the critical electric field $e_c = 1$ this "effective" tension vanishes. As we will see below, this modification in the tension results to be decisive for the thermodynamical properties of the theory.

As we have already argued the one–loop string free energy can be found by summing the free energies (weighted by the correspondent degeneracy of the state) of the single string modes whose mass m_n is given by the eigenvalues of the mass operator (3),

$$m_n^2 = -\sum_{\alpha=1}^{D/2-1} \frac{e^2 + h_\alpha^2}{1 + h_\alpha^2} \left[(p^{2\alpha})^2 + (p^{2\alpha+1})^2\right] + 2\pi T(1 - e^2)(n - 1) \tag{5}$$

with $n = 0, 1, \ldots$

The one–loop free energy of one single mode of mass m_n is given by the well–known formula

$$F_n = \frac{1}{\beta} \int \frac{d^{D-1}p}{(2\pi)^{D-1}} \ell n \left[1 - e^{-\beta\sqrt{p^2 + m_n^2}}\right] . \tag{6}$$

Taking into account Eq.(5), the expression under the square root in the exponent of Eq.(6) can be put in the form

$$p^2 + m_n^2 = (p^1)^2 + \sum_{\alpha=1}^{D/2-1} \frac{1 - e^2}{1 + h_\alpha^2} \left[(p^{2\alpha})^2 + (p^{2\alpha+1})^2\right] + \bar{m}_n^2 \tag{7}$$

where

$$\bar{m}_n^2 = 2\pi T(1 - e^2)(n - 1) . \tag{8}$$

Changing variables in (6) as

$$
\begin{aligned}
p^1 &= k^1 , \\
p^{2\alpha} &= \sqrt{\frac{1+h_\alpha^2}{1-e^2}}\, k^{2\alpha} , \\
p^{2\alpha+1} &= \sqrt{\frac{1+h_\alpha^2}{1-e^2}}\, k^{2\alpha+1} ,
\end{aligned} \tag{9}
$$

passing then from k^i to spherical coordinates and after this, expanding the logarithm, we arrive at

$$
F_n = \frac{1}{\beta}(1-e^2)^{-\frac{D}{2}-1} \left[\prod_{\alpha=1}^{D/2-1}(1+h_\alpha^2)\right] \frac{1}{2^{D-2}\pi^{\frac{D-1}{2}}\Gamma\left(\frac{D-1}{2}\right)} \cdot \sum_{r=1}^{\infty}\int_0^\infty dk\, k^{D-2}\, \frac{e^{-r\beta\sqrt{k^2+\bar{m}_n^2}}}{r} , \tag{10}
$$

$$
k = \sqrt{\sum_{i=1}^{D-1}(k^i)^2} . \tag{11}
$$

After an appropriate change of variables we can use the formulas

$$
K_\nu(z) = \frac{\left(\frac{z}{2}\right)^\nu \Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\nu+\frac{1}{2}\right)} \int_1^\infty dt\, e^{-zt}(t^2-1)^{\nu-\frac{1}{2}} \tag{12}
$$

$$
\frac{d}{dz}\left(z^{-\frac{D-2}{2}} K_{\frac{D-2}{2}}(z)\right) = z^{-\frac{D-2}{2}} K_{\frac{D}{2}}(z) \tag{13}
$$

for the modified Bessel function $K_\nu(z)$ in order to put Eq.(10) in the form

$$
F_n = -(1-e^2)^{-\frac{D-2}{2}} \left[\prod_{\alpha=1}^{D/2-1}(1+h_\alpha^2)\right] (2\pi)^{-\frac{D}{2}} \cdot \sum_{r=1}^{\infty}\int_0^\infty \frac{dt}{t^{\frac{D}{2}+1}}\, e^{-\bar{m}_n^2 \frac{t}{2}}\, e^{-\frac{\beta^2 r^2}{2t}} . \tag{14}
$$

Denoting now

$$
t = \frac{2\pi\tau}{1-e^2} \tag{15}
$$

and taking into account the definition of the Jacobi function θ_3,

$$
\theta_3(0,\tau) = \sum_{n=-\infty}^{\infty} e^{i\pi\tau n^2} \tag{16}
$$

we obtain that the free energy of a single string mode is

$$
F_n = -(1-e^2)\left[\prod_{\alpha=1}^{12}(1+h_\alpha^2)\right] 2^{-27}\pi^{-26} \int_0^\infty \frac{d\tau}{\tau^{14}}\, e^{-2\pi^2 T(n-1)\tau} \cdot \left[\theta_3\left(0, \frac{i\beta^2(1-e^2)}{4\pi^2\tau}\right) - 1\right] . \tag{17}
$$

Here we have already evaluated the dimension in its critical value for the bosonic strings $D = 26$. The factor $(1-e^2)\prod_{\alpha=1}^{12}(1+h_\alpha^2) = \det(1+f_{\mu\nu})$ is the square of the Born–Infeld action. Note that the electric field enters also in the argument of θ_3.

To obtain the free energy of the hot string gas we must multiply Eq.(17) by the degeneracy of each level n and then sum over n. The degeneracy d_n of the levels coincides with that of the free string theory because the background does not modify the spectrum of the excitation but only the distance between the levels.

Thus the sum in n is standard [12], therefore we finally obtain that the free energy of the hot neutral string gas in constant electromagnetic background is given by

$$F = \sum_{n=0}^{\infty} d_n\, F_n = -\det(1+f_{\mu\nu})\,2^{-27}\pi^{-26}\,.$$
$$\cdot \int_0^\infty \frac{d\tau}{\tau^{14}}\,\eta^{-24}(i\pi T\tau)\left[\theta_3\left(0, \frac{i\beta^2(1-e^2)}{4\pi^2\tau}\right) - 1\right] \qquad (18)$$

where the Dedekind function $\eta(\chi)$ is defined as

$$\eta(\chi) = e^{\frac{i\pi\chi}{12}}\prod_{n=1}^{\infty}(1-e^{2i\pi\chi n})\,. \qquad (19)$$

When the temperature is zero the expression (18) reduces, save a minus sign, to the cosmological constant of the theory. At the zero electric field, it coincides with the result found in [4] for neutral string in pure magnetic fields. On the other hand, at zero electromagnetic field Eq.(18) reproduces the free energy of the free open string theory [13]. Starting from the free energy (18), one can analyze what will be the Hagedorn temperature of our gas. It is performed by studying the ultraviolet behaviour of the integral in (18), i.e., when the Teichmüller parameter τ goes to zero.

From the asymptotical behaviours

$$\eta^{-24}(i\pi T\tau) \xrightarrow[\tau\to 0]{} (\pi T\tau)^{12}\, e^{\frac{2}{T\tau}}$$
$$\theta_3\left(0, \frac{i\beta^2(1-e^2)}{4\pi^2\tau}\right) - 1 \xrightarrow[\tau\to 0]{} e^{-\beta^2(1-e^2)/4\pi\tau} \qquad (20)$$

it follows straightforwardly that the condition for the integral (18) to be ultraviolet finite is

$$\beta^2 \geq \beta_H^2 = \frac{8\pi}{T(1-e^2)} \qquad (21)$$

which defines the Hagedorn temperature

$$T_H = \sqrt{\frac{(1-e^2)T}{8\pi}}\,. \qquad (22)$$

As we anticipated in the introduction, the Hagedorn temperature of the neutral string gas in constant electromagnetic background depends on the electric field. The larger electric field, the lower Hagedorn temperature is. Note that this effect is a direct consequence of the modification in the tension (see Eq.(3)).

2.2 Path integral procedure

For the sake of brevity we shall not enter here in the details of the calculus of the free energy by this alternative procedure, limiting ourselves to remark only the main points in which differences exist with respect to the free string case. The explicit computations will be published elsewhere.

As we have already argued, in this approach we should calculate the one–loop path integral of the string theory over a spacetime with the time direction compactified to a circle of circumference β.

Hence, we must start from the one–loop integral in the target space $S^1 \times R^{25}$.

$$\Gamma = \int [dg][dx] e^S . \tag{23}$$

After taking the orthonormal gauge the integral in the metrics g reduces to a parametric integral in the Teichmüller parameter τ. In the one–loop approximation the world sheet can be taken as a flat annulus with internal radius $a < 1$ and external radius equal to 1. The internal radius a is related to the Teichmüller parameter: $ln a = -1/T\tau$.

The Euclidean action S of the open neutral string in electromagnetic background can be written in polar coordinates as

$$S = -\frac{T}{2} \int \int_\Sigma dr\, d\theta \left[r(\partial_r X^\mu)^2 + \frac{1}{r}(\partial_\theta X^\mu)^2 - 2i\, f^{\mu\nu} \partial_r X_\mu \partial_\theta X_\nu \right] . \tag{24}$$

Varying S we can see that the boundary conditions, when the world sheet is the flat annulus, take the form

$$r\, \partial_r X^\mu - i f^{\mu\nu} \partial_\theta X_\nu = 0 \qquad \begin{matrix} r = 1 \\ r = a \end{matrix} . \tag{25}$$

In order to integrate in Eq.(23) over the string coordinates X^μ, we must split them in its zero mode and nonzero mode parts. As the time direction has been compactified to a circle, the zero mode in this direction acquires winding modes proportional to an integer number n (the winding number). Thus

$$X_0 = Y_0 + \frac{n\beta}{2\pi}\, \theta + \xi_0 \tag{26}$$

where ξ_0 is the nonzero mode part.

On the other hand, the winding modes must satisfy the boundary conditions (25). Therefore, it must appear also winding modes in the zero mode part of the spatial coordinate X_1

$$X_1 = Y_1 - i f_1 \frac{n\beta}{2\pi}\, \ell n\, r + \xi_1 . \tag{27}$$

This is really a non–trivial feature: the first spatial direction is able to feel the compactification in the time direction! However, it does not seem so strange if one remembers that the integrations must always be done in space of functions that satisfy the boundary conditions of the theory (in analogy with the case of QFT at finite temperature where the boundary conditions associated to the temperature imply that one must integrate over the functions that are periodical (for bosonic fields) or antiperiodical (for fermionic fields) in the imaginary time). In the present case the antisymmetric character of $F_{\mu\nu}$ yields to boundary conditions that mix the coordinates x_0 and x_1, and therefore the coordinate x_1 must also have windings.

From (26) and (27) we see that the correct splitting is

$$X_\mu = Y_\mu + \frac{n\beta}{2\pi}\,\theta\,\delta_{\mu 0} - i f_1 \frac{n\beta}{2\pi}\,\ell n\, r\,\delta_{\mu 1} + \xi_\mu\,. \tag{28}$$

After substituting Eq.(28) and the action (24), this can rewritten as

$$S = S_0 + \frac{n^2\beta^2}{2\pi}\,T(1 + f_1^2)\,\ell n\, a \tag{29}$$

where $S_0 = S(\xi)$. Note that all the dependence on the temperature is concentrated in the second term in (29). This term is not affected by the integral in ξ but only by the Teichmüller parametric integral.

Using (29) in Eq.(23), integrating over Y, ξ and summing in n as usual, we obtain a result from which the free energy of the string gas may be subtracted straighforwardly. The expression obtained in such a way coincides with the free energy (18) calculated by the other method. This corroborates our previous conclusion concerning the Hagedorn temperature.

3. CONCLUDING REMARKS

In this article we have shown that a non–trival background is able to change the behaviour of the physical characteristics of the string theory even in the asymptotical region (high temperatures, energies, etc.). Particularly, we found that the critical temperature (Hagedorn temperature) essentially depends on the electromagnetic background, decreasing as the electric field tends to its critical value $qE = T = (2\pi\alpha')^{-1}$, (this critical electric field is associated for the charged open string to the Born–Infeld type action, obtained as an effective action of the open string theory in paper [3]).

Such dependence on the background in the asymptotical region is a non–trivial property of the finite theories as is the case of the string theories, quite differently to the case of asymptotically free theories where such a property does not take place.

We would like to bring here some arguments (from the physical and mathematical point of views) related to the possible existence of a phase transition in the considered electromagnetic background around the critical point for the electric field. As it can be seen from the expression for the effective mass of the excitation modes (see Eq.(7)), when the electric field tends to its critical value, the masses of the higher spin modes tend to zero. Hence, it is quite natural to hope the spontaneous appearance in this critical point, of a massless higher spin fields symmetry (similar to the class of symmetries of massless higher spin fields presented in papers [1] by Vasiliev and one of the authors (E.S.F.) of the present paper). A very important feature shown in [1] was that this new symmetry of higher spin fields can be realized only in an AdS–space, where the interactions among themselves and with gravitation of the higher spin fields do not break the initial higher spin symmetry. This means that in a trivial vacuum only a symmetry of noninteracting massless high spin fields can take place. On the contrary, the interactions may exist only in an AdS–background.

Then, the scenario for the existence of a phase transition might be understood in the following way. As the electric field approaches to its critical value, due to the creation of graviton pairs (associated to the loop radiative processes of the string field theory) the structure of the vacuum of the theory changes, appearing a condensate of gravitons with an AdS–background, in which the higher spin fields are able to remain massless, taking place in this form the realization of a gauge higher spin symmetry.

It is worth remarking that during the process of the phase transition the mathematical singularities of the theory are conserved. The non–analyticity (in α'^{-1}) of the initial theory in the critical value of the electric field, that is reflected in the mass of the higher spin modes, takes place in the AdS–phase in the form of a non–analyticity in the cosmological constant (AdS–radius). It is precisely such non–analyticity which guarantees (see [1]) the conservation of the symmetry of the gauge higher spin fields, in spite of the interactions of these with the gravity and among themselves. As the above mentioned interaction does not disappear in the asymptotical regime, it gives rise to the dependence of the physical magnitudes on the background even in the asymptotical region.

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