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THE TENSION AS PERTURBATIVE PARAMETER IN STRING THEORY

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Abstract

We propose an approach to string theory where the zero order theory is the null string. We find an explicit form of the propagator for the null string in the momentum space. We show that considering the tension as perturbative parameter, the perturbative series is completely summable and we find the propagator of the bosonic open string with tension T.

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1.- Introduction

Recently it has been discussed in the literature that string theories are particularly simple in the small tension limit [1,2]. The scattering amplitudes have a compact form and, in principle, they can be used to extract important physical informations.

On the other hand, it is well known that the string tension is proportional to square of the Planck mass and, in consequence, the string theory with $T << 1$ is, of course, a high energy theory.

Nevertheless, as was emphatized by Schild ten years ago [3], a formulation for the string with zero tension does exist (null string). In this object, all points move with the velocity of light, the hamiltonian constraints have a simple structure and this object can be quantized along similar lines to the conventional Nambu string.

In two recents articles [4,5] we have studied extensively this class of strings and we have proposed a normal ordering for the dynamical variables of the theory. We have showed that the critical dimension of the bosonic (fermionic) string is 26 (10). The physical spectrum is massless and the theory is a genuine pure gauge theory.

The problem then is, how does the symmetry breaking occurs ? or in other words, how to generate the string tension ?.

A similar problem was studied by Isham [6] and independently by Teitelboim [7] some years ago for the gravitational field case.

In the context of quantum gravity the idea used by Isham - Teitelboim was to truncate the conventional generator $\mathcal{H}_\perp$ defined by

$$\mathcal{H}_\perp = G^{ijkl}\pi_{ij}\pi_{kl} + \sigma g^{(3)\frac{1}{2}} R^{(3)}, \qquad i,j,k,l,= 1,2,3, \tag{1}$$

eliminating the " potential " term

$$\sigma g^{(3)\frac{1}{2}} R^{(3)}. \tag{2}$$

Then, they argued that this "potential" can be introduced as a perturbation considering the theory with $\mathcal{H}_\perp^{(0)} = G^{ijkl}\pi_{ij}\pi_{kl}$ as a "free" one. As was shown by Pilati [8] and Teitelboim [7] this truncated gravity is precisely the strong coupling limit of quantum gravity and the perturbation theory developed with (2), is just a perturbative procedure to recuperate conventional quantum gravity. *

This procedure is, nevertheless, technically very complicated and in the context of quantum gravity (in the strong coupling limit), only the free propagator is known [9], while the following terms of the pertubative expansion are not known.

On the other hand, from a geometrical point of view, the truncation process of $\mathcal{H}_\perp$ is equivalent to a change of the conventional spacetime structure by a new one where the

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* In fact, it is possible to demostrate a closed analogy between this approach and lattice gauge theory. In the last theories in the strong coupling limit the magnetic field does not exist, and therefore these theories are truncated.

relativistic group is replaced by the Caroll group, i.e. by a group where each point of the manifold is causally desconected with the neighbour point (i.e. $c \longrightarrow 0$) [10,11].

In this manifold the metric tensor is degenerate and the physics, as it is usually known, is only recuperated if we can sum all the perturbative series.

The null string is precisely an example of a truncated theory with respect to the conventional Nambu string, where the normal deformation generator is :

$$\mathcal{H}_\perp = \frac{1}{2}(P^2 + T^2 X'^2). \tag{3}$$

with T the tension of the string.

In the zero tension limit the two dimensional metric is degenerate and the spacetime geometry is Carollian.

In this article we will use the ideas developed in ref. [4,5] in order to propose an aproach to null string theory that allows, perturbatively, to recuperate to **all orders** the tension of the Nambu string. Our result is probably the first example of a generally covariant theory (in the context of Isham - Teitelboim approach) in that the perturbative series is completely summable.

The order of this paper is the following : In section 2, using the Faddeev - Popov method, we derive an expression for the null string propagator in the proper time gauge . In section 3 we develope our approach and show explicitly the summability of the perturbative series and we calculate the open string propagator with tension T. Finally, in section 4, we give the conclusions .

2.- Propagators for Null String Theory

The null string action is :

$$S = \int d^2\sigma (P^\mu \dot{X}_\mu - N_\perp \mathcal{H}_\perp - N_1 \mathcal{H}_1), \tag{4}$$

where $N_\perp$ and N_1 are Lagrange multipliers and $\mathcal{H}_\perp$, $\mathcal{H}_1$ are :

$$\begin{aligned} \mathcal{H}_\perp &= \frac{1}{2}P^2, \\ \mathcal{H}_1 &= P \cdot X', \end{aligned} \tag{5}$$

($X' = \frac{\partial X}{\partial \sigma}$).

The constraints algebra of (5) is :

$$\begin{aligned} [\mathcal{H}_\perp(\sigma), \mathcal{H}_\perp(\sigma\prime)] &= 0, \\ [\mathcal{H}_\perp(\sigma), \mathcal{H}_1(\sigma\prime)] &= (\mathcal{H}_\perp(\sigma) + \mathcal{H}_\perp(\sigma\prime))\ \delta'(\sigma - \sigma\prime), \\ [\mathcal{H}_1(\sigma), \mathcal{H}_1(\sigma\prime)] &= (\mathcal{H}_1(\sigma) + \mathcal{H}_1(\sigma\prime))\ \delta'(\sigma - \sigma\prime), \end{aligned} \tag{6}$$

It is easy to see that the action (4) is invariant under the following transformations,

$$\delta X^\mu = \epsilon_\perp P^\mu + \epsilon_1{}' X^\mu, \tag{7a}$$

$$\delta P^\mu = (\epsilon_1 P^\mu)', \tag{7b}$$

$$\delta N_\perp = \dot\epsilon_\perp + N_\perp{}'\epsilon_1 - N_\perp\epsilon_1{}' + N_1{}'\epsilon_\perp - N_1\epsilon_\perp{}', \tag{7c}$$

$$\delta N_1 = \dot\epsilon_1 + N_1{}'\epsilon_1 - N_1\epsilon_1{}', \tag{7d}$$

if the $\epsilon_\perp$ parameters satisfy at the end points :

$$\epsilon_\perp(\tau_1, 0) = 0 = \epsilon_\perp(\tau_2, 0), \tag{8a}$$

$$\epsilon_\perp(\tau, 0) = 0 = \epsilon_\perp(\tau, \pi). \tag{8b}$$

The equation (8a) is a necessary and sufficient condition to cancel the term $\frac{1}{2}\epsilon_\perp(\sigma,\tau)P^2$ that appear in the variation of the action (4) [12,13].

The condition (8b) is an ambiguity that appears in the null string case, and that in the Nambu string is related to the spatial boundary conditions (open and closed string). As it is discussed in the literature [14,15], the null string string has not spatial boundary conditions because the X'^2 term is absent in (5a).

In order to quantize this theory it is necessary to fix the gauge. Observing (8a), we conclude that a possible gauge is :

$$\dot N_\perp = 0, \tag{9a}$$

$$N_1 = 0, \tag{9b}$$

(proper time gauge) [7].

But, as was emphatized in [7], the condition (9) is not completely satisfactory because the σ dependence is not fixed.

In the spirit of the gauge fixing procedure, $N_\perp(\sigma) + C$ (C constant) is arbitrary and the unique requisite that we impose on (9) is that $\dot N_\perp = 0$ fixed the ϵ's at the end points. Without loss of generality, we suppose that $N_\perp(\sigma)$ is zero everywhere and the unique contribution is the constant zero mode $C = N_\perp(0)$.

According with these arguments the path integral amplitude is :

$$G[X_1(\sigma), X_2(\sigma)] = \int \mathcal{D}N_\perp \mathcal{D}N_1 \mathcal{D}P^\mu \mathcal{D}X_\mu \delta[\dot N_\perp]\delta[N_1] det(M).$$

$$\exp[\, i \int d^2\sigma (P^\mu \dot X_\mu - N_perp\mathcal{H}_\perp - N_1\mathcal{H}_1)], \tag{10}$$

where M is a matrix which is determined from (7c) - (7d) and by direct calculation is :

$$M=\begin{pmatrix}\partial_\tau{}^2+\dot N_1'+N_1{}'\partial_\tau-\dot N_1\partial_\tau-N_1\partial_\tau\partial_\sigma & \dot N_\perp+N_\perp'\partial_\tau-\dot N_\perp\partial_\tau-N_\perp\partial_\tau\partial_\sigma\\ 0 & \partial_\tau+N_1'-N_1\partial_\sigma\end{pmatrix}. \quad (11)$$

Replacing (11) in (10) and integrating in N_1 ,

$$G[X_2(\sigma),X_1(\sigma)]=\int\mathcal{D}N_\perp\mathcal{D}P^\mu\mathcal{D}X_\mu.$$

$$.\delta[\dot N_\perp]det(\partial_\tau{}^2)det(\partial_\tau)\exp[i\int d^2\sigma(P^\mu\dot X_\mu-N_\perp\mathcal{H}_\perp]. \quad (12)$$

As we have argumented, only the zero mode of $N_\perp(\tau,\sigma)$ constributes and the path integral in $N_\perp$ can be replaced by an ordinary integral in $N_\perp(0)$. The computation of $det(\partial_\tau{}^2)$ is perfomed using the boundary conditions $X(\tau_1,\sigma)=X_1$ and $X(\tau_2,\sigma)=X_2$. * The determinant of ∂_τ is indeterminate and it can be taken out of the path integral. Following the arguments of the ref. [7], the integration limit of $N_\perp(0)$ must be $(0,\infty)$ by causality requirements. Thus, (12) is :

$$G[X_2(\sigma),X_1(\sigma)]=$$

$$=\mathcal{N}\int_0^\infty dT\int\mathcal{D}P^\mu\mathcal{D}X_\mu\exp[i\int d^2\sigma(P^\mu\dot X_\mu-\frac{N_\perp}{2}P^2)], \quad (13)$$

$(T=N_\perp(0)\Delta\tau)$.

If we make the trivial substitution $P^\mu\dot X_\mu=-\dot P^\mu X_\mu+\partial_\tau(P^\mu X_\mu)$ and we integrate in X_μ, we find that the factor $\delta[\dot P]$ appears, i.e. $P=P(\sigma)$ and eq. (13) is :

$$G[X_2(\sigma),X_1(\sigma)]=\mathcal{N}\int_0^\infty dT\int\mathcal{D}P^\mu(\sigma)\exp[\,i\int_0^\pi d\sigma P^\mu\Delta X_\mu+iT\int_0^\pi d\sigma\;P^2],$$

$$=\int\mathcal{D}P^\mu(\sigma)\frac{\exp[\,i\int_0^\pi d\sigma P^\mu(\sigma).\Delta X_\mu]}{\int_0^\pi d\sigma P^2(\sigma)-i\epsilon}, \quad (14)$$

$$=\langle X_1\|\frac{1}{L_0^N-i\epsilon}\|X_1\rangle. \quad (15)$$

This expression is an explicit representation for the propagator in the momentum space. In the notation (15), L_0^N is the zero mode of the " Virasoro operator " defined by :

* The choosing these boundary conditions is equivalent, at quantum level, to choose a Weyl ordering for operators ; thus in this approach have not critical dimensions according with the results of ref. [14] and [15]. I would like to thank M. Ruiz-Altaba for a discussions on this point

$$L_0^N = \int_0^\pi d\sigma P^2(\sigma) = \sum_n P_n^2.$$

Basically formulae (14) and (15) stablish that the null string is an infinite set of non interacting massless relativistic particles [16].

This last argument is also the reason that shows that the world sheet geometry is Carollian. Finally, we note that (14) and (15) are explicit solutions of the functional difussion equation of the string [17,18,19].

3.- Perturbation Theory of Null Strings

In this section we will elaborate an approach in which we consider the null string as the zero order theory and all terms that can be to added at the normal deformation generator ($\mathcal{H}_\perp$) are considered as perturbations. Nevertheless, the terms that can be added to $\mathcal{H}_\perp$ must be consistent with the closure of the algebra. Thus, it is very easy to see that the unique perturbation term is $\frac{T^2}{2}X'^2$.

Following the Isham - Teitelboim aproach [7] the starting point is the Feynman path integral [20].

The transition amplitude in the proper time gauge is :

$$K[X_2(\sigma), X_1(\sigma)] =$$
$$\int \mathcal{D}N_\perp \mathcal{D}N_1 \mathcal{D}P_\mu \mathcal{D}X^\mu \delta[\dot{N}_\perp]\delta[N_1] det(M).$$
$$\exp[i\int d^2\sigma(P^\mu \dot{X}_\mu - N_\perp \mathcal{H}_\perp - N_1\mathcal{H}_1 - \frac{T^2}{2}X'^2)] \tag{16}$$

From of (16) it is easy to see that the effective transition amplitude is :

$$K[X_2(\sigma), X_1(\sigma)] =$$
$$= \int_0^\infty dT \int \mathcal{D}P_\mu \mathcal{D}X^\mu \exp[i\int d^2\sigma(P^\mu \dot{X}_\mu - \frac{N_\perp(0)}{2}P^2) - i\int d^2\sigma \frac{N_\perp(0)}{2}T^2X'^2], \tag{17}$$

$(T = N_\perp(0)\Delta\tau)$.

Expanding the perturbation term :

$$\exp[-i\int d^2\sigma \frac{N_\perp(0)}{2}T^2X'^2] = 1 + (-i)\int d^2\sigma \frac{T^2}{2}X'^2 +$$
$$+\frac{(-i)^2}{2!}\int d^2\sigma d^2\sigma\prime \frac{T^2}{2}X'(\sigma)^2\frac{T^2}{2}X'(\sigma\prime)^2 + \ldots +$$
$$+\frac{(-i)^n}{n!}\int d^2\sigma d^2\sigma\prime \ldots d^2\sigma^{(n)} \frac{T^2}{2}X'(\sigma)^2 \ldots \frac{T^2}{2}X'(\sigma^{(n)})^2 + \ldots \tag{18}$$

Replacing (18) in (17),

$$K[X_2(\sigma),X_1(\sigma)] = \langle X_2 \| \frac{1}{L_N^0 - i\epsilon} \| X_1\rangle +$$

$$+(-i)\int_0^\infty dT \int \mathcal{D}P_\mu \mathcal{D}X^\mu \exp[i\int d^2\sigma (P^\mu \dot{X}_\mu - \frac{N_\perp(0)}{2}P^2)] \int d^2\sigma \frac{T^2}{2}X'^2 +$$

$$\frac{(-i)^2}{2!}\int_0^\infty dT \int \mathcal{D}P_\mu \mathcal{D}X^\mu \exp[i\int d^2\sigma (P^\mu \dot{X}_\mu - \frac{N_\perp(0)}{2}P^2)] \, .$$

$$\int d^2\sigma d^2\sigma\prime \frac{T^2}{2}X'(\sigma)^2 \frac{T^2}{2}X'(\sigma\prime)^2 + ... \qquad (19)$$

The first term of the expansion (19) is, of course, the null string propagator . The second term can be written in the following form :

$$(-i)\int_0^\infty dT \int \mathcal{D}P_\mu \mathcal{D}X^\mu \exp[\, i\int d^2\sigma (P^\mu \dot{X}_\mu - \frac{N_\perp(0)}{2}P^2)] \int d^2\sigma \frac{T^2}{2}X'^2 =$$

$$\frac{d}{da_1^2}\left[\int_0^\infty dT \int \mathcal{D}P_\mu \mathcal{D}X^\mu \exp[i\int d^2\sigma (P^\mu \dot{X}_\mu - \frac{N_\perp(0)}{2}(P^2 + T^2 a_1^2 X'^2))]\right]_{a_1^2=0} , \qquad (20)$$

i.e. as the derivative of the propagation amplitude (evaluated in $a_1^2 = 0$) of the string with effective tension $a_1 T$, a_1 being a real parameter.

The third term admits a similar representation to (20), but in this case it is necessary to introduce an additional parameter and thus succesively the higher terms.

Using this trick, (19) can be written of the following form :

$$K[X_2(\sigma),X_1(\sigma)] = \langle X_2 \| \frac{1}{L_0^N - i\epsilon} \| X_1\rangle +$$

$$+\sum_{n=1}^{\infty}\frac{1}{n!}\frac{\partial^n}{\partial a_1{}^2 \partial a_2{}^2 \ldots\partial a_n{}^2}\left[\int_0^\infty dT \int \mathcal{D}P_\mu \mathcal{D}X^\mu\right.$$

$$\left.\exp[i\int d^2\sigma (P^\mu \dot{X}_\mu - \frac{N_\perp(0)}{2}(P^2 + T^2(a_1^2 + a_2^2 + \ldots\ldots + a_n^2)X'^2)]\right]_{a_1^2=0,a_2^2=0,\ldots,a_n^2=0} . \qquad (21)$$

Of course, the following step is to evaluate term by term the infinite series that appears in (22). *

In order to calculate the series, it is more conveniente to work in the Fourier representation and we consider the open string case. Here $[0,\pi]$ can be extended to $[-\pi,\pi]$ and we define :

* We note that (21) is the analogous, for the strings theories , of the Lippman - Schwinger equations.

$$X^\mu(\sigma) = X^\mu(-\sigma) \quad , P^\mu(\sigma) = P^\mu(-\sigma) \tag{22}$$

The $X^\mu(\sigma)$ and $P^\mu(\sigma)$ expansions are :

$$X^\mu(\sigma) = \sum_n X_n^\mu e^{-in\sigma} \quad , P^\mu(\sigma) = \sum_n P_n^\mu e^{in\sigma} \tag{23}$$

and of course, we have $X_n^\mu = X_{-n}^\mu$, $P_n^\mu = P_{-n}^\mu$.

The evaluation of the first term of the sum in (21) gives :

$$-\langle X_2 \| \frac{\sum_n n^2 T^2 X_n^2}{(\sum_n P_n^2)^2} \| X_1 \rangle.$$

The second term is :

$$\langle X_2 \| \frac{(\sum_n n^2 T^2 X_n^2)^2}{(\sum_n P_n^2)^3} \| X_1 \rangle.$$

And the m-th term is :

$$\langle X_2 \| \frac{(-1)^m (\sum_n n^2 T^2 X_n^2)^m}{(\sum_n P_n^2)^{m+1}} \| X_1 \rangle.$$

Replacing in (21) :

$$K[X_2(\sigma), X_1(\sigma)] = \langle \| \frac{1}{\sum_n P_n^2} (1 - \frac{\sum_n n^2 T^2 X_n^2}{\sum_n P_n^2} +$$

$$+ \frac{(\sum_n n^2 T^2 X_n^2)^2}{(\sum_n P_n^2)^2} - \ldots\ldots + \frac{(-1)^m (\sum_n n^2 T^2 X_n^2)^m}{(\sum_n P_n^2)^m} + \ldots) \| X_1 \rangle =$$

$$= \langle X_2 \| \frac{1}{\sum_n P_n^2} \cdot \frac{1}{1 + \frac{\sum_n n^2 T^2 X_n^2}{\sum_n P_n^2}} \| X_1 \rangle =$$

$$= \langle X_2 \| \frac{1}{L_0 - i\epsilon} \| X_1 \rangle \tag{24}$$

where L_0 is the zero mode of the Virasoro operator,

$$L_0 = \sum_{n>0} (P_n^2 + T^2 n^2 X_n^2)$$

(24) is the propagator of the open bosonic string with tension T [21].

4.- Conclusions

In this paper we have calculated the open string propagator with tension T considering the null string as a 0-th order theory.

Of course, the main problem remains, i.e. how to introduce interactions at the tree level or loops ?. In this approach the perturbation $\frac{T^2}{2}X'^2$ introduces only 2-point function corrections negleting the loops corrections.

To relate this paper with the works of Gross and Mende [1] and Amati, Ciafaloni and Veneziano [2] it is necessary to calculate higher order corrections, but the problem of how to incorparate interactions in the null strings theories is open.

On the other hand, using the Isham - Teitelboim approach, the perturbation implemented by an ultralocal theory [22], in general, has a clear physical sense only if it is possible to sum all the perturbative series. The partial summation is non local and the physical interpretation is not clear. In this sense, the string is a system highly priveleged with respect to gravity or p-branes where the perturbative series is not summable.

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