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THEORY OF THE QUANTUM HALL EFFECTS IN LATTICE SYSTEMS *

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ABSTRACT

The Fractional Quantum Hall Effect is identified as an Integral Quantum Hall Effect of electrons on a lattice with an even number of statistical flux quanta. A variational wavefunction in terms of the Hofstadter lattice eigenstates is proposed.

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Although similar in appearance, the physics behind the Fractional Quantum Hall Effect (FQHE) seems different from the physics of the Integral Quantum Hall Effect (IQHE). The IQHE is essentially a single-particle phenomenon due to the quantization of the electron orbits in a strong magnetic field and to the electron localization due to disorder. The FQHE is a subtle many-particle effect which arises from a condensation of a two-dimensional electron gas into a new state of matter as a result of the electron-electron interactions. The main features of the IQHE are well understood [1,2] and remaining open problems are those related to the precise width and shape of Hall steps which need a better understanding of the localization concept in the presence of a magnetic field [3,4]. Also, there is a standard picture of the Fractional Quantum matter based on Laughlin's theory [5] and the hierarchy models [6,7]. A scaling theory for both the effects has been presented as well [8,9].

Recent experiments have been performed across a boundary between FQHE and IQHE and show a transmission from the $\nu = 1$ to $\nu = 2/3$ fractional effect [10,11]. These experiments suggest that the two phenomena could be considered in the same theoretical framework. The present paper makes an attempt towards a unified treatment of the IQHE and FQHE using a gauge-invariant tight-binding model on a lattice [12] and the concept of anyons [13,14]. A variational electronic wavefunction of the FQHE is constructed through the many-particle wavefunction describing the IQHE of anyons with even statistical parameter in our lattice.

First consider a system of fermions which hop among sites of a two-dimensional lattice with a uniform magnetic field perpendicular to the plane so that there is a flux ϕ per plaquette in units of flux quantum $\phi_o \equiv hc/e$. The one-particle Hamiltonian has the form

$$H = -\sum_{\langle ij\rangle,\sigma} (t_{ij} c^{+}_{i\sigma} c_{j\sigma} + \text{c.c.}) \qquad (1)$$

where the sum is over nearest-neighbor bonds $\langle ij\rangle$, $c^{+}_{j\sigma}$ creates an electron spin σ at site j and

$$t_{ij} = t \exp[2\pi i \int_i^j \vec{A} \cdot d\vec{l}\,] \qquad (2)$$

represents the hopping matrix-elements in the presence of a vector potential considered in the Landau gauge $\vec{A} = (0, Bx)$.

Let us now consider a magnetic field corresponding to $\phi = p/q$ (p,q incommensurate integers) flux quanta per plaquette and integer filling factor ν so that there is an average flux per electron equal to ν^{-1} (in units ϕ_o). Then the complete energy spectrum of (1) was obtained by Hofstadter [15]. The corresponding eigenproblem can be solved in the first magnetic Brillouin zone $(-\pi/q \le k_x < \pi/q\ ,\ -\pi \le k_y < \pi)$ and the eigenstates are given by

$$| u_{\vec{k}\sigma} \rangle = \sum_{m=0}^{q-1} a_m c^{+}_{\sigma}(k_x + 2\pi\phi m\ ,\ k_y)\, | 0 \rangle \qquad (3)$$

where $c^{+}_{\sigma}(k_x, k_y)$ is the inverse Fourier transform of the fermionic

creation operator $c^{+}_{j\sigma}$ and $| 0 \rangle$ being the vaccum state. The coefficients a_m satisfy the "Harper's" difference equation [15]. It is well-known [16,17] that the above system shows IQHE, i.e. $\sigma_H = \nu e^2/h$. The corresponding many-fermion wave function is given by a single Slater-determinant of the "Bloch-Landau" orbitals $u_{\vec{k}\sigma}(z_j) \equiv \langle z_j | u_{\vec{k}\sigma} \rangle$ which satisfy the position and crystal-momentum periodicity conditions. The complex number $z_j = l_j + i m_j$ locates the j-particle on the lattice (unit bond length is assumed).

In the following, we attach to each electron an infinitely thin flux tube with flux a_s so that we have an anyon gas on a lattice [18,19]. The concept of anyons was first introduced and studied by Wilczek in 1982 [13]. Exchange of two anyons multiply the anyon-wavefunction by a phase factor $(-1)^{1+a_s}$, a_s being the statistics parameter. If $a_s = 2n$, the anyons obey Fermi statistics and the particle-particle correlations are the same as for $a_s = 0$. Hence, we expect our system to exhibit QHE corresponding to a filling factor ν_c. It should be noted that the addition of $a_s = 2n$ flux quanta per electron must be accompanied by change in the size of the system keeping with that way the total flux (statistical and background flux) and therefore the applied magnetic field constant. We can determine the ν_c of the above QHE-state using the interplay of statistics (attached flux) and magnetic field [18] : A system of particles each carrying flux a_s is equivalent in a mean field sense to a system of particles in a magnetic field such that there is an average flux a_s per particle. In our case, the resulting mean-field many-particle state is characterised by $(2n + \nu^{-1})$ flux quanta available to each particle and therefore shows QHE at $\nu_c = \frac{\nu}{2n\nu + 1}$ which gives a hierarchy of fractional states equivalent to that of ref. [9].

We now may write down a variational wave function for the ν_c-state as a resulting state of an IQHE of anyons-2n in our lattice:

$$\Psi_{\nu_c}(\{z_j\}) = \prod_{i<j}^{N} f(z_i - z_j)\, \Psi_\nu(\{z_j\}) \tag{4}$$

where $\{z_j\}$ labels the electron positions on the lattice and f is an even-function of its argument and can be determined by analogy of the many-particle state in the presence of a singular gauge first discussed by Wilczek [20,21]. Using the appropriate Landau-gauge version of the arguments we finally obtain

$$\Psi_{\nu_c}(\{z_j\}) = \prod_{i<j}^{N} (e^{2\pi z_j} - e^{2\pi z_i})^{2n} \det\{u_{\vec{k}>_0}(z_j)\} \tag{5}$$

Considering that odd-denominator fractions are associated with spin-polarized ground states, we have to construct the det in (5) occupying energy levels with the same spin orientation. The above many-particle wave function is expected to be a good variational state in the presence of repulsive interactions and the two-particle density matrix to vanish like r^{2n} as the inter-particle separation r tends to zero. Counting the zeros of the written state we can interpret the resulting fractional filling as the binding of ν-electrons to $(2n\nu + 1)$-

vortices in the thermodynamic limit. The binding of nodes to fermions in a FQHE-state was pointed out some years ago by Halperin [6].

In conclusion, we have presented a gauge-lattice model in which Fractional Hall states come from Integral Hall states attaching to each electron magnetic flux tubes carrying an even number of flux quanta provided that the "actual" magnetic field is kept constant. We have proposed lattice-variational wave functions at the corresponding filling factors.

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