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FORMULATION OF HAMILTONIAN MECHANICS WITH EVEN AND ODD POISSON BRACKETS

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ՊՈՒԱՍՈՆԻ ԶՈՒՅԳ ԵՎ ԿԵՆՏ ՓԱԿԱԳԾԵՐՈՎ ՀԱՄԻԼՏՈՆՅԱՆ ՄԵԽԱՆԻԿԱՅԻ ՁԵՎԱԿԵՐՊՈՒՄԸ

Տվյալ հոդվածում քննարկվում է Պուասոնի կենտ փակագծի և կենտ համիլտոնյանի կառուցումը, ելնելով Փազային գերտարածությունում տրված դինամիկայից, որը որոշվում է Պուասոնի զույգ փակագծով, և զույգ համիլտոնյանով: Նոր փակագիծը և համիլտոնյանը չեն փոխում շարժման հավասարումները:

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А.П.НЕРСЕСЯН,О.М.ХУДАВЕРДЯН

ФОРМУЛИРОВКА ГАМИЛЬТОНОВОЙ МЕХАНИКИ С ЧЕТНОЙ И НЕЧЕТНОЙ СКОБКАМИ ПУАССОНА

В работе исследуется возможность построения нечетной скобки Пуассона и нечетного гамильтониана по заданной динамике в фазовом суперпространстве-четной скобке Пуассона и четному гамильтониану, так что переход к новой структуре не меняет уравнений движения.

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O.M. KHUDAVERDYAN, A.P. NERSESSYAN

FORMULATION OF HAMILTONIAN MECHANICS WITH EVEN AND ODD POISSON BRACKETS

A possibility is studied as to construct the odd Poisson bracket and odd Hamiltonian by the given dynamics in phase superspace - the even Poisson bracket and even Hamiltonian, so the transition to the new structure does not change the equation of motion.

Yerevan Physics Institute

Yerevan 1987

Introduction

In the recent years, in the field theory studies, certain interest has been attracted to the odd Poisson bracket - the Butten bracket [1,2,3] . In particular, D.V.Volkov et al. have been working out a concept, according to which the odd Poisson bracket is an initial object for superhamiltonian mechanics [4,3]. In Ref.[4] these authors have noted that in the formulation of classical supersymmetric Witten mechanics the Witten hamiltonian

$$H = \frac{p^2 + W^2(q)}{2} + \theta_1 \theta_2 W'(q), \tag{1}$$

$W(q)$ is superpotential

may be replaced by a more elementary one:

$$\tilde{H} = Q_1, \qquad (Q_1 = -p\theta_2 + W(q)\theta_1, \qquad Q_2 = p\theta_1 + W(q)\theta_2). \tag{2}$$

In this case, from the even Poisson bracket giving the dynamics of the system (1)

$$\{p,q\}_0=1\ ,\ \ \{\theta_\alpha,\theta_\beta\}_0=\delta_{\alpha\beta}\ ,\ \ \{p,\theta_2\}_0=\{q,\theta_2\}_0=0\ ,\quad \alpha,\beta=1,2$$

one should turn to a specially chosen odd bracket $\{\ ,\ \}_1'$ constructed in [4] such that equations of motion would not change*:

$$\dot f=\{H,f\}_0\equiv\{\bar H,f\}_1'\ .$$

In the case $W(q)=q$ in (1) (harmonic superoscillator) the bracket $\{\ ,\ \}_1'$ takes a canonical form: $\{\ ,\ \}_1'=\{\ ,\ \}_1$

$$\{p,\theta_1\}_1=\{q,\theta_2\}_1=1\ ,\quad \{p,\theta_2\}_1=\{q,\theta_1\}_1=\{\theta_\alpha,\theta_\beta\}_1=0\ . \tag{3}$$

The superalgebra of the motion integrals

$$\{Q_\alpha,\theta_\beta\}_0=2\delta_{\alpha\beta}H\ ,\qquad \{Q_\alpha,F\}_0=\epsilon_{\alpha\beta}Q_\beta$$
$$F=\frac{Q_1Q_2}{2H}=\theta_1\theta_2 \tag{4}$$

will preserve its form:

$$\{\bar Q_\alpha,\bar Q_\beta\}_1'=2\delta_{\alpha\beta}\bar H\ ,\qquad \{\bar Q_\alpha,F\}_1'=\epsilon_{\alpha\beta}\bar Q_\beta \tag{5}$$

where

$$\bar F=-\frac{i}{2}Q_2\ ,\qquad \bar Q_1=H\ ,\qquad \bar Q_2=i\,(2F-H)\ .$$

In this work we'll try to generalize the statement of [4] for the case of arbitrary supersymmetric mechanics.

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* The use of the odd bracket for the description of dynamics see also in [5] .

Let H be the supersymmetric Hamiltonian in $(2n, m)$-dimensional phase superspace $E^{2n,m}$ with coordinates $(p_1, \dots, p_n, q_1, \dots, q_n, \theta_1, \dots, \theta_m)$ in which the system dynamics is given by canonical even Poisson bracket:

$$\{f, g\}_0 = \sum_{i=1}^{n} \left(\frac{\partial f}{\partial p^i} \frac{\partial g}{\partial q^i} - \frac{\partial f}{\partial q^i} \frac{\partial g}{\partial p^i} \right) - \sum_{\alpha=1}^{m} (-1)^{p(f)} \frac{\partial f}{\partial \theta^\alpha} \frac{\partial g}{\partial \theta^\alpha} . \tag{6}$$

Can we reformulate the system dynamics in terms of odd Poisson bracket, i.e. to construct a new (odd) Hamiltonian $\bar{H}$ and new odd bracket $\{\ ,\ \}_1'$ (not necessarily canonical) such that the motion equations would be unchanged, i.e.

$$\{H, f\}_0 \equiv \{\bar{H}, f\}_1' \tag{7}$$

for any function f on the phase space?

In Section 1 we'll remind the definitions of the Poisson bracket in superspace (for details see [6,7]).

In Section 2 we'll arrive at results of Ref.[4] by a more geometric manner.

In Section 3 we'll suggest an explicit construction of the $\{\ ,\ \}_1'$ bracket satisfying (7) for an arbitrary supersymmetric Hamiltonian H. Here the role of the new Hamiltonian will be played by the supercharge of Hamiltonian H : $\bar{H} = Q$ where $\{Q, Q\}_0 = 2H$.

In Section 4 we'll construct solutions of Eq.(7) for one class of integrable superhamiltonian systems.

1. Recall some definitions.

$\{\ ,\ \}$ is called an even Poisson bracket on the phase superspace $E^{2n,m}$ if for functions of definite parity on $E^{2n,m}$

$$p(\{f,g\}) = p(f) + p(g),$$

$$\{f,g\} = -(-1)^{p(f)p(g)}\{g,f\} \tag{8a}$$

(superanticommutativity condition)

$$\{h, fg\} = \{h,f\}g + (-1)^{p(f)p(g)}\{h,g\}f \tag{8b}$$

(the Leibnitz differentiation rule)

$$(-1)^{p(f)p(h)}\{f,\{g,h\}\} + (-1)^{p(g)p(f)}\{g,\{h,f\}\} + (-1)^{p(h)p(g)}\{h,\{f,g\}\} = 0 \tag{8c}$$

(the Jacobi identity)

In (8) $p(f)$ is the f function parity. The action of on inhomogeneous elements (functions of uncertain parity) continues by linearity:

$$\{f, g\lambda + h\mu\} = \{f,g\}\lambda + \{f,h\}\mu .$$

To the nondegenerate* even Poisson bracket there corresponds the nondegenerate differential closed even 2-form which, for example, for the canonical bracket (6) has the form:

$$\omega = \sum_{i=1}^{n} dp_i \wedge dq_i - \sum_{\alpha=1}^{m} (d\theta_\alpha)^2 . \tag{9}$$

We arrive at the definition of the odd Poisson bracket (the Butten bracket) if in the definitions (8a,c) we make

---

* The bracket $\{\ ,\ \}$ is nondegenerate if for any nonconstant function + there exists such function g that $\{f,g\} \neq 0$

everywhere replacement $p(f) \to p(f)+1$. In the ($2n, 2n$)-dimensional phase space to the nondegenerate Butten bracket there corresponds the nondegenerate differential closed odd 2-form. Locally, such a form (the analog of Darboux theorem) always may be reduced to the canonical form [8] (similarly as the even form to that of (9)):

$$\omega = \sum_{i=1}^{n} (dp_i d\theta_i + dq_i d\theta_{n+i}) . \tag{10}$$

To the canonical form (10) there corresponds the canonical odd bracket:

$$\{f,g\}_1 = \sum_{i=1}^{n} \left(\frac{\partial f}{\partial p^i} \frac{\partial g}{\partial \theta^i} + \frac{\partial f}{\partial q^i} \frac{\partial g}{\partial \theta^{n+i}} + (-1)^{p(f)} \left(\frac{\partial f}{\partial \theta^i} \frac{\partial g}{\partial p^i} + \frac{\partial f}{\partial \theta^{n+i}} \frac{\partial g}{\partial q^i} \right) \right).$$

In the superspace with the even (odd) Poisson bracket the system's dynamics is given by the even (odd) Hamiltonian H . Equations of motion have the form:

$$\dot{f} = \{H, f\} .$$

2. Return now to our problem. The construction of the odd bracket $\{\ ,\ \}_1'$ and the new odd Hamiltonian $\bar{H}$ satisfying (7) is especially simple if the supersymmetric Hamiltonian H has a sufficient amount of integrals of motion (see for details the end of Section 4). Precisely this situation is the case for the Hamiltonian (1) in [4] - in the (2,2)-dimensional phase space there is one even motion integral H and two odd Q_1 , Q_2 , and $\{Q_\alpha, Q_\beta\}_0 = 2\delta_{\alpha\beta} H$. We'll obtain $\{\ ,\ \}_1'$ (constructed in [4] by direct solving the differential

equation (7)) by defining it on the basic functions:

$$\{Q_1, \tau\}_1' = \left\{\frac{Q_2}{2H} + g,\ H - f\right\}_1' = 1,$$

$$\{\tau, H-f\}_1' = \left\{\tau, \frac{Q_2}{2H} + g\right\}_1' = \{Q_1, H-f\}_1' = \left\{Q_1, \frac{Q_2}{2H} + g\right\}_1' = 0, \tag{11}$$

where f and g are arbitrary functions of the motion integrals,(f is even and g is odd) τ is the "time" - function such that

$$\{H, \tau\}_0 = 1. \tag{12}$$

To the bracket $\{\ ,\ \}_1'$ there corresponds two-form

$$\omega = dQ_1 d\tau + d\left(\frac{Q_2}{2H} + g\right) d(H-f)\ . \tag{13}$$

If we put $\bar{H} = Q_1$ just like in [4] , then the relation (7) will be fulfilled, because it is evidently fulfilled for the basic functions

$$\tau,\ H-f\ ,\quad Q_1\ ,\quad \frac{Q_2}{2H} + g\ .$$

The arbitrariness in the definition of the $\{\ ,\ \}_1'$ bracket in Ref.[4] arising in the direct solving of differential equations (7) corresponds to the arbitrariness in the choice of f and g functions in (11) and τ in (12) (the τ may be added to by an arbitrary even function of the motion integrals H, Q_1, Q_2). We'll satisfy the relations (5) too, if we also put

$$\bar{Q}_1 = (H-f) + Q_1\left(\frac{Q_2}{2H} + g\right) = H + Q_1 g + (F-f)$$

$$\bar{Q}_2 = -i\Big((H-f) - Q_1\big(\frac{Q_2}{2H}+g\big)\Big) = i(2f-H) + i(F-f) + iQ_1 g$$
$$\bar{F} = i(H-f)\Big(\frac{Q_2}{2H}+g\Big) = \frac{i}{2}Q_2 + ig(H-f) - \frac{ifQ_2}{2H} \; . \tag{14}$$

3. Turn now to the general case.

Let H be a supersymmetric Hamiltonian. The dynamics is determined by the canonical Poisson bracket (6). Let Q be the odd integral of motion:

$$\{H, Q\}_0 = 0 \;, \qquad p(Q) = 1 \, .$$

Without violating generality we may assume that Q is a supercharge:

$$\{Q, Q\}_0 = 2H \tag{15}$$

Indeed, $\{Q,Q\}_0 \neq 0$, since the bracket (6) is sign-defined in the odd sector and hence $\tau(\{Q,Q\}_0) = \sum_{\alpha=1}^{m} \tau\Big(\frac{\partial Q}{\partial\theta^\alpha}\Big)\,\tau\Big(\frac{\partial Q}{\partial\theta^\alpha}\Big) > 0$, where $\tau(h)$ is the number part of superquantity h . Therefore, the replacement of Q by $\frac{Q}{\{Q,Q\}_0}\sqrt{2H}$ will bring to (15). (To the motion integral whose "square" is nonzero ($\{\hat{Q}, Q\} \neq 0$) there corresponds the supertransformation of the action which adds to the lagrangian a full derivative).

The main observation is that if $\{\;,\;\}$ is the even Poisson bracket and a_i is the set of even and α_i is the set of odd functions, then

$$\{f, g\}_1' = \sum_i \Big(\{a_i, f\}\{\alpha_i, g\} + (-1)^{p(f)}\{\alpha_i, f\}\{a_i, g\}\Big) \tag{16}$$

satisfies the relations (8a), (8b) for the odd bracket $\left(p(f) \to p(f)+1\right.$ in (8a,c)), and under certain conditions imposed on a_i and α_i (in particular, if all commutators $\{a_i, a_j\}$, $\{a_i, \alpha_j\}$, $\{\alpha_i, \alpha_j\}$ are constant) also the condition (8c); i.e. $\{\ ,\ \}'_1$ in (16) is the odd Poisson bracket. We'll search for the solution of Eq.(7) using the ansatz (16).

For example, for the supersymmetric Hamiltonian H with supercharge Q we take (16) putting $a = \sqrt{H}$, $\alpha = -\frac{Q}{\sqrt{H}}$,

$$\{a, a\}_0 = \{a, \alpha\}_0 = 0, \qquad \{\alpha, \alpha\}_0 = 2.$$

$$\{f, g\}'_1 = -\left(\{\sqrt{H}, f\}_0 \{\tfrac{Q}{\sqrt{H}}, g\}_0 + (-1)^{p(f)} \{\tfrac{Q}{\sqrt{H}}, f\}_0 \{\sqrt{H}, g\}_0\right) \tag{17}$$

The bracket (17) may be identically rewritten in the form:

$$\{f, g\}'_1 = -\frac{1}{2H}\left(\{H, f\}_0 \{Q, g\}_0 + (-1)^{p(f)} \{Q, f\}_0 \{H, g\}_0\right) + \tag{17a}$$
$$+ (-1)^{p(f)} \frac{Q}{2H^2} \{H, f\}_0 \{H, g\}_0$$

From (17) it is obvious that

$$\{Q, f\}'_1 = \{\frac{Q}{\sqrt{H}}, Q\}_0 \{\sqrt{H}, f\}_0 = \{H, f\}_0$$

That is the transition to the odd bracket (17) together with the transition to odd Hamiltonian $\tilde{H} = Q$ does not change the motion equations.

Of course, the bracket (17) has one serious drawback - it is degenerate. However, note, that in constructing our odd bracket we always have certain arbitrariness (see, e.g. Sections 2 and 4), since we practically keep to motion equations alone. In a class of all brackets satisfying (7) at a fixed choice $\bar{H} = Q$ of the new Hamiltonian the bracket (17) is extracted by the condition of maximally possible degeneracy.

It is interesting to notice that in the usual case (if there are no fermion variables and both $\{\ \}$ and $\{\ \}'$ connected by ansatz (16) are usual Poisson brackets) the straightforward calculations give us that concommutant of these two brackets is equal to zero, i.e.

$$\{f,\{g,h\}'\}+\{f,\{g,h\}\}' \quad + \text{cyclic permutation} = 0,$$

where

$$\{f,g\}' = \sum_i \left(\{a_i,f\}\{\alpha_i,g\} - \{\alpha_i,f\}\{a_i,g\}\right).$$

4. Certainly, the substitution (16) can be used to construct an odd bracket, less degenerate than (17), and a new odd Hamiltonian such that the system's dynamics would not change.

Consider, e.g., the following class of dynamical systems: dynamics on the $(2n,m)$-dimensional phase superspace is determined by the Hamiltonian H and canonical Poisson bracket (6); here there are n even integrals of motion $J_1, J_2, \dots, J_n$ and m odd $q_1, \dots, q_m$, and

$$\{J_i,\ J_k\}_0 = \{J_i,q_\alpha\}_0 = 0, \qquad \{q_\alpha, q_\beta\}_0 = \delta_{\alpha\beta}.$$

We'll call such systems integrable. It can be readily shown by elementary calculations like those carried out in the standard symplectic geometry (see, e.g. [9]) that the phase space is divided into ($n,0$)-dimensional tori $T(J_{i0}, q_{\alpha 0})$ determined by the conditions $J_i = J_{i0}$, $q_\alpha = q_{\alpha 0}$. In this case into the vicinities of any torus T one can introduce the coordinates ($\varphi_1,\dots,\varphi_n$) so that $\{J_i, \varphi_K\}_0 = \delta_{iK}$, $\{\varphi_i, q_\alpha\}_0 = \{\varphi_i, \varphi_K\}_0 = 0$. Clearly, $H = H(J_1, \dots, J_n)$. Indeed, n vector fields $IdJ_K\,(IdJ_K : \omega(IdJ_K, \eta) = dJ_K(\eta))$ are tangential to ($n,0$)-dimensional tori T and $dH(IdJ_K) = \{H, J_K\}_0 = 0$, hence $H = H(J_i, q_\alpha)$. On the other hand, $\frac{\partial H}{\partial q^\alpha} = \{H, q_\alpha\}_0 = 0$, hence $H = H(J_1,\dots,J_n)$.

In the case $m \geqslant n$, for integrable Hamiltonians we can construct a less degenerate than in Sect. 3 odd bracket $\{\ ,\ \}_1'$ and odd Hamiltonian $\bar{H}$, both satisfying Eq. (7) as follows. Consider the odd bracket (16), putting

$$a_i = J_i \ , \quad \alpha_i = -q_i \ , \quad i = 1, \dots, n$$

and introduce the new odd Hamiltonian

$$\bar{H} = \sum_{i=1}^{n} \omega_i(J) q_i \ , \tag{18}$$

where $\omega_i(J) = \dfrac{\partial H(J)}{\partial J_i}$.

Then

$$\{f, g\}_1' = -\sum_{i=1}^{n} \left(\{J_i, f\}_0 \{q_i, g\}_0 + (-1)^{p(f)} \{q_i, f\}_0 \{J_i, g\}_0 \right) . \tag{19}$$

Obviously,

$$\{\bar{H}, f\}_1' = \sum_{i=1}^{n} \{q_i, \bar{H}\}_0 \{J_i, f\}_0 = \sum \frac{\partial H}{\partial J_i} \{J_i, f\}_0 = \{H, f\}_0$$

i.e. Eqs. (7) hold.

Note, that for integrable Hamiltonians in ($2n, 2n$)-dimensional phase space we can directly construct the nondegenerate odd bracket so that the odd Hamiltonian-supercharge of the former Hamiltonian would describe the system dynamics. We'll generalize the construction formulated in Section 2.

Let τ_i be such functions that

$$\{H, \tau_i\}_0 = 1 , \quad i = 1, \dots, n \tag{20}$$

For example, $\tau_i = \frac{\varphi_i}{\omega_i} \left(\omega_i = \frac{\partial H(J)}{\partial J_i}\right)$

Determine $\{\ ,\ \}_1'$ by relations on basic functions $\tau_1, \dots, \tau_n$, $J_1, \dots, J_n$, $Q_1, \dots, Q_{2n}$ ($Q_\alpha = \sqrt{2H} q_\alpha, \alpha = 1, \dots, 2n$ are supercharges, $\{Q_\alpha, Q_\beta\}_0 = 2\delta_{\alpha\beta} H$)

$$\{Q_i, \tau_j\}_1' = \left\{\frac{Q_{n+i}}{2H}, J_j\right\}_1' = \delta_{ij} , \quad i, j = 1, \dots, n$$

$$\{Q_i, Q_j\}_1' = \left\{Q_i, \frac{Q_{n+j}}{2H}\right\}_1' = \{Q_i, J_j\}_1' = \{\tau_i, \tau_j\}_1' = \tag{21}$$

$$= \left\{\tau_i, \frac{Q_{n+j}}{2H}\right\}_1' = \{\tau_i, J_j\}_1' = \left\{\frac{Q_{n+i}}{2H}, \frac{Q_{n+j}}{2H}\right\}_1' = \{J_i, J_j\}_1' = 0$$

$$\omega = \sum_{i=1} \left(dQ_i d\tau_i + d\frac{Q_{n+i}}{2H} dJ_i\right)$$

and introduce the new Hamiltonian

$$\bar{H} = Q_1 + Q_2 + \dots + Q_n \tag{22}$$

which is a supercharge of the former one $\{\bar{H}, \bar{H}\}_0 = 2nH$.

Clearly, Eqs.(7) at determination (21) of the odd bracket $\{\ ,\ \}_1'$ and at determination (22) of the odd Hamiltonian $\bar{H}$ will be satisfied automatically, since they will be obviously satisfied on basic functions $\tau_i, J_i, Q_i, Q_{i+n}$ $(i = 1, \dots, n)$

At such determination of the new bracket the algebra of integrals of motion will not change its form:

$$\{Q_\alpha, Q_\beta\}_0 = 2\delta_{\alpha\beta} H \longrightarrow \{\bar{Q}_\alpha, \bar{Q}_\beta\}'_1 = 2\delta_{\alpha\beta}\bar{H}$$

if we put immediately after $\bar{H} = Q_1 + \ldots + Q_n$

$$\bar{Q}_i = \mathcal{J}_i + \bar{H}\frac{Q_{n+i}}{2H}$$

$$\bar{Q}_{n+i} = -i\left(\mathcal{J} - \bar{H}\frac{Q_{n+i}}{2H}\right).$$

At $n = 1$ this construction transforms into one formulated in Section 2.

Of course, the determination of the bracket (21) as well as of the bracket (11) in Section 2 contains arbitrariness in the choice of $2n$ even and $2n-1$ odd functions of integrals of motion. When replacing in the determination of the bracket (21)

$$\tau_i \to \tau_i + \ell_i, \quad i = 1, \ldots, n$$

$$\mathcal{J}_i \to \mathcal{J}_i + f_i,$$

$$Q_\alpha \to Q_\alpha + g_\alpha, \quad \alpha = 1, \ldots, 2n$$

provided that $\sum\limits_{\alpha=1}^{n} g_\alpha = 0$, where ℓ_i, f_i are even, g_α are odd functions of integrals of motion, Eqs.(7) will not be violated.

This work suggests two ways of constructing an odd bracket and odd Hamiltonian which just like the former even bracket and even Hamiltonian describe the system dynamics. The first way consists in constructing nondegenerate bracket by giving it on canonical basic functions in case if the system possesses a sufficient amount of integrals of motion. The second way consists in direct expressing the odd bracket through the even one by means of the ansatz (16).

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ФОРМУЛИРОВКА ГАМИЛЬТОНОВОЙ МЕХАНИКИ С ЧЕТНОЙ И НЕЧЕТНОЙ СКОБКАМИ ПУАССОНА

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