IC/90/268

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

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INTERNATIONAL ATOMIC ENERGY AGENCY

UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

1990 MIRAMARE-TRIESTE

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and
United Nations Educational Scientific and Cultural Organization
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ABSTRACT

Within the context of Barut's self–field approach, we write the exact expression of the spontaneous atomic decay rate [*Phys. Rev.* **A37**, 2284 (1988)], in the long wavelength approximation, in terms of electric– and magnetic–like multipole contributions which are related to the matrix elements of the transition charge and current distributions of the relativistic electron. A numer of features of these expressions are discussed and their generalization to interacting composite systems is also pointed out.

MIRAMARE – TRIESTE
August 1990

* To be submitted for publication.

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1 INTRODUCTION

In Self-Energy Quantum Electrodynamics (SEQED) advanced by Barut *et. al.*[1-3] spontaneous emission from one electron atoms is treated as a self-field attribute in a fashion close in spirit to the classical idea of radiation reaction. So far in this theory only a first iteration of the action functional of the matter plus radiation field has been considered in direct correspondence with first order perturbation theory. To this order of iteration, account has been made[2-4] of the electron's anomalous magnetic moment $(g-2)$, the Unruh and Casimir effects, the Lamb shift and others besides atomic spontaneous emission[5] and absorption.

The subject of this paper is again spontaneous emission. We have been encouraged by the success of our formulation in producing precise atomic decay rates for some of the low-lying Hydrogenic excited states reported elsewhere[5]. Moreover, we have recently employed[6] our general relativistic formula in calculating the decay rates of the metastable $2S$ states in the Hydrogenlike atoms and ions with values of the atomic number Z ranging from 1 to 92. Agreement between our results and those of other formulations as well as with experiment is good especially for high Z values where a relativistic treatment is essential.

In this paper, we bring our exact formula [5] for the atomic transition rates one step closer to the familiar language and terminology of the standard theory. This goal is fulfilled by making the long wavelength approximation which is less severe than the dipole limit. We show that, in this regime, our expression, which was arrived at fully relativistically, in fact contains contributions from all the electric and magnetic multipoles of the radiating system, namely the electron, with slightly modified definitions for the multipole moments that automatically exclude contribution from the electric 2^0-pole (monopole).

2 THE THEORY

Within the context of SEQED, we have arrived[5] at the following general expression for the Einstein A-coefficient of spontaneous emission, or the transition probability per unit time, for the decay of an atomic state n into the state s

$$\begin{aligned} W_{n\to s} &= -2\, Im(\Delta E_n^{SE}) \\ &= -\frac{\pi}{2}\int d^3k T^{\mu}_{ns}(\mathbf{k})T_{sn}(-\mathbf{k})_{\mu}\delta(E_s - E_n + |\mathbf{k}|) \\ &= -\frac{\pi\omega}{2}\int d\Omega_k T^{\mu}_{ns}(\mathbf{k})T_{sn}(-\mathbf{k})_{\mu}, \end{aligned} \tag{1}$$

where n and s stand for the totality of the respective state's quantum numbers n, ℓ, J and M. In the last step in equation (1) the radial integration over $|\mathbf{k}|$ has been carried out resulting in the understanding that $|\mathbf{k}|$ is to be replaced everywhere by $\omega \equiv E_n - E_s$, by virtue of the delta function. Moreover, $d\Omega_k = \sin\theta_k d\theta_k d\phi_k$ and the quantities

$$T^{\mu}_{ns}(\mathbf{k}) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3r\, j^{\mu}_{ns}(\mathbf{r}) e^{i\mathbf{k}\cdot\mathbf{r}}, \tag{2}$$

are Fourier transforms, or transition form factors, of the electron current

$$j^{\mu}_{ns}(\mathbf{r}) = -e\bar{\psi}_n(\mathbf{r})\gamma^{\mu}\psi_s(\mathbf{r}). \tag{3}$$

The wavefunctions are everywhere the well-known exact solutions of the Dirac equation for a single electron in the Coulomb field of the atomic nucleus, as is explained in reference [5]. Equation (1) is thus exact and has been the basis of our decay rate calculations referred to above[5,6].

3 THE LONG WAVELENGTH APPROXIMATION

Equation (1) has been shown[5] to reduce to its well-known nonrelativistic counterpart when the dipole limit ($e^{i\mathbf{k}\cdot\mathbf{r}} \approx 1$) is made. Although retaining the dipole term is believed to be sufficient for most practical purposes in atomic physics calculations, this may be too severe for radiation from atoms with high values of the atomic number Z, where the relativistic corrections become important. We show below that more terms can yet be retained when a less severe approximation is made. In fact we shall demonstrate that the decay rate is a sum of contributions from all multipoles of the system, establishing in this way some degree of resemblance between our fully relativistic semiclassical formulation and the familiar theory of nonrelativistic multipole radiation from atoms.

In expanded form, equation (1) can be written as

$$W_{n\to s} = -\frac{\pi\omega}{2} \int d\Omega_k \{|T^0_{ns}(\mathbf{k})|^2 - |\mathbf{T}_{ns}(\mathbf{k})|^2\}, \tag{4}$$

where it is straightforward to show that $T^0_{sn}(-\mathbf{k}) = T^0_{ns}(\mathbf{k})^{\dagger}$ and that $\mathbf{T}_{sn}(-\mathbf{k}) = \mathbf{T}_{ns}(\mathbf{k})^{\dagger}$.

Next we look at

$$T^0_{ns}(\mathbf{k}) \;\; = \;\; -e\int \frac{d^3r}{(2\pi)^{\frac{3}{2}}} \psi^{\dagger}_n(\mathbf{r})\psi_s(\mathbf{r}) e^{i\mathbf{k}\cdot\mathbf{r}}$$

$$= -\frac{4\pi e}{(2\pi)^{\frac{3}{2}}}\sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell} i^\ell Y_{\ell m}(\hat{k})\int d^3r g_\ell(\omega r)Y^*_{\ell m}(\hat{r})\rho_{ns}(\mathbf{r}), \tag{5}$$

where use has been made of the expansion

$$e^{i\mathbf{k}\cdot\mathbf{r}} = 4\pi\sum_{\ell m} i^\ell g_\ell(\omega r)Y^*_{\ell m}(\hat{r})Y_{\ell m}(\hat{k}), \tag{6}$$

in which g_ℓ is a spherical Bessel function and where

$$\rho_{ns} = \psi_n^\dagger(\mathbf{r})\psi_s(\mathbf{r}). \tag{7}$$

The long wavelength approximation amounts to retaining the first term in the power series expansion of the spherical Bessel function

$$g_\ell(\omega r) \approx \frac{(\omega r)^\ell}{(2\ell+1)!!}. \tag{8}$$

Thus equation (5) becomes

$$T^0_{ns}(\mathbf{k}) \approx \frac{1}{\sqrt{2\pi^2}}\sum_{\ell m}(-1)^{m+1}\frac{\sqrt{2\ell+1}}{(2\ell+1)!!}(i\omega)^\ell Y_{\ell m}(\hat{k})\ e\ \left(Q^{(e)}_{\ell,-m}\right)_{ns}, \tag{9}$$

where

$$\left(Q^{(e)}_{\ell m}\right)_{ns} = \sqrt{\frac{4\pi}{2\ell+1}}\int \rho_{ns}(\mathbf{r})r^\ell Y_{\ell m}(\hat{r})d^3r, \tag{10}$$

are matrix elements of the classical electric multipole moments (the (e) here stands for *electric*)

$$Q^{(e)}_{\ell m} = \sqrt{\frac{4\pi}{2\ell+1}}r^\ell Y_{\ell m}(\hat{r}). \tag{11}$$

Hence the first term in equation (4), after carrying out the remaining angular integration, becomes

$$-\frac{\pi\omega}{2}\int d\Omega_k|T^0_{ns}(\mathbf{k})|^2 = \sum_{\ell m} w^{(e)}_{\ell m}, \tag{12}$$

where $w^{(e)}_{\ell m}$ is the 2^ℓ-pole electric transition rate per unit time and is given by

$$w_{\ell m}^{(e)} = -\frac{1}{4\pi}\frac{2\ell+1}{[(2\ell+1)!!]^2}\omega^{2\ell+1}\ e^2 \mid \left(Q_{\ell,-m}^{(e)}\right)_{ns} \mid^2. \tag{13}$$

Equation (13) is precisely what one gets for the transition probability per unit time from the electric part of a multipole expansion[7] of the radiation field, apart from a factor of $-\frac{2(\ell+1)}{\ell}$ which we shall discuss below. The factor $\frac{1}{4\pi}$ is a result of the system of units we are adopting here whereby $\hbar = c = 1$ and the fine structure constant $\alpha = e^2/4\pi$. The minus sign in (13) implies that $w_{\ell m}^{(e)}$ is a *growth rate* for the population of state n and a *depletion rate* for that of s. In other words, $w_{\ell m}^{(e)}$ corresponds to absorption of radiation rather than emission.

Similarly, the second term in the expression for the transition probability per unit time, equation (4), yields

$$\frac{\omega\pi}{2}\int d\Omega_k |\mathbf{T}_{ns}(\mathbf{k})|^2 = \sum_{\ell m} w_{\ell m}^{(m)}, \tag{14}$$

where the (m) stands for *magnetic* and

$$w_{\ell m}^{(m)} = \frac{1}{4\pi}\frac{2\ell+1}{(2\ell+1)!!}\omega^{2\ell+1}\ e^2 \mid \left(\mathbf{Q}_{\ell,-m}^{(m)}\right)_{ns} \mid^2. \tag{15}$$

In (15) we have used the notation

$$\left(\mathbf{Q}_{\ell,m}^{(m)}\right)_{ns} = \sqrt{\frac{4\pi}{2\ell+1}}\int r^\ell Y_{\ell m}(\hat{r})\vec{j}_{ns}(\mathbf{r})d^3r, \tag{16}$$

with

$$\vec{j}_{ns}(\mathbf{r}) = \psi_n^\dagger(\mathbf{r})\vec{\alpha}\psi_s(\mathbf{r}). \tag{17}$$

In perfect analogy with the electric multipole contributions to the transition probability per unit time, equation (13), one is tempted to call $w_{\ell m}^{(m)}$ the 2^ℓ-pole magnetic transition probability per unit time, except here the departure from the standard usage of the term is quite apparent. In relativistic Quantum Mechanics, the velocity operator describing the jittering motion (*Zitterbewegung*) of a Dirac particle is given by the $\vec{\alpha}$ matrices. The fast irregular motion of the electron, termed *Zitterbewegung*, is associated with its spin motion. Hence, one is led to intuitively interpret $e\vec{j}_{ns}$ as the matrix element of the physical current associated with this motion, which results in the magnetic effects just like in the classical picture. It is in this sense that we call $w_{\ell m}^{(m)}$ the 2^ℓ-pole magnetic transition probability per unit time and it is in that sense we call

$$\mathrm{Q}_{\ell,m}^{(m)} = \sqrt{\frac{4\pi}{2\ell+1}} r^{\ell} Y_{\ell m}(\hat{r})\vec{\alpha} \tag{18}$$

the 2^{ℓ}-pole magnetic moments of the electron. Putting (13) and (15) together, we get the following expression for the total transition probability per unit time in the long wavelength approximation

$$\begin{aligned} W_{n\to s} &= \sum_{\ell m} \left(w_{\ell m}^{(e)} + w_{\ell m}^{(m)} \right) \\ &= -\frac{1}{4\pi} \sum_{\ell m} \frac{2\ell+1}{[2\ell+1!!]^2}\, \omega^{2\ell+1}\, e^2\, \left\{ \left| \left(Q_{\ell,-m}^{(e)} \right)_{ns} \right|^2 - \left| \left(\mathrm{Q}_{\ell,-m}^{(m)} \right)_{ns} \right|^2 \right\}. \end{aligned} \tag{19}$$

4 DISCUSSION and CONCLUSIONS

By making the long wavelength approximation, we have managed to write the transition probability per unit time in one electron atoms in terms of the matrix elements of suitably defined electric and magnetic multipole moments of the radiating system. The intuitive picture we draw for spontaneous emission (and absorption) of radiation from such atoms is thus close in spirit to the intuitively clear one drawn from the ideas of the classical radiation from oscillating charges. In our formulation, the radiating system is an electron undergoing *Zitterbewegung.*

We maintain that the full decay rate of the transition $n \to s$ should be calculated using the exact analytic expression of reference [5] or equation (19) above as an approximation. A decomposition of the transition probability into electric and magnetic contributions, effected artificially in the conventional theory, is rendered meaningless in our approach due to the presence of the negative sign in equation (13) for $w_{\ell m}^{(e)}$.

Our main results in this paper are equations (13) and (15). Equation (13) is to be compared, if need be, with[7]

$$w_{\ell m}^{(e)} = \frac{2(\ell+1)}{\ell} \frac{2\ell+1}{[(2\ell+1)!!]^2} \omega^{2\ell+1}\, e^2\, \left| \left(Q_{\ell,-m}^{(e)} \right)_{ns} \right|^2. \tag{20}$$

of the standard theory. Immediately one recognizes the following differences between (13) and (20):

1. The minus sign in (13): We have good reason to believe that the existence of this sign in our version of the theory is crucial, provided that (13) and

(15) are taken together and equation (19) is used in practical calculations. In calculating the $2S \rightarrow 1S$ transition rate, for example, using the exact formula developed from equation (1) above, we discovered that contribution from both terms is the same up to a good number of decimal places resulting in a null final result if one is not careful enough. In a double precision calculation, however, we found that the finite result of the standard magnetic dipole calculation can be reproduced[5,6]. This transition is, of course, forbidden in the nonrelativistic limit.

2. The $\frac{1}{4\pi}$ factor in (13) comes from our system of units.

3. The factor of 2 in equation (20) is a result of summing, in the standard approach, over the two photon polarization states. In our formulation of the problem, polarization of the emitted radiation is an attribute of the spin of its source which is, in turn, automatically accounted for through the use, from the start, of spin-dependent Dirac-Coulomb wavefunctions. See also references [5] and [6] for a discussion of this factor.

4. The factor $\frac{\ell+1}{\ell}$ in (20) comes from a choice[7] of the arbitrary constant in the electric photon wavefunction. In the standard theory, radiation from the 2^0-pole (monopole) corresponding to $\ell = 0$ is absolutely ruled out ($J_n = 0 \not\rightarrow J_s = 0$) by the transversality of the radiation field, yet if one naively takes the limit as $\ell \rightarrow 0$, one gets

$$w_{00}^{(e)} \sim \frac{\int \psi_n^\dagger \psi_s d^3r}{\ell} = \frac{0}{0}, \tag{21}$$

an indeterminate quantity, where orthogonality of the wavefunctions has been invoked. In contradistinction, our result gives automatically

$$w_{00}^{(e)} \sim \int \psi_n^\dagger \psi_s d^3r = 0. \tag{22}$$

This result is a byproduct of our original elimination of the radiation field[1–3] from the picture in favor of its source which is, in turn, described by orthonormal Dirac - Coulomb wavefunctions involving no arbitrary constants. Moreover, this arbitrary factor tends to unity for large ℓ and is therefore important only for the lowest order multipoles.

What we have labeled as a magnetic contribution to the transition probability per unit time, equation (15), is based upon vector magnetic multipole moments defined for our purposes in perfect analogy with their electric counterparts (in terms of a transition current distribution as opposed to a transition charge distribution). The magnetic multipole moments of the standard theory are scalar quantities which in the $\ell = 1$ case are related to the spin and orbital magnetic dipole moments of the electron[8].

In conclusion, we wish to stress here that the real test of our theory of spontaneous emission should be to compare the results of its exact version directly with those of the experiments. Equation (20) above is the multipole-like approximation to our exact spontaneous decay rate formula given in detail elsewhere[5].

The advantage of making the long wavelength approximation, provided that it is good for a certain practical calculation is twofold. On the one hand, one can now break the problem of calculating a decay rate into small pieces corresponding to multipole contributions of increasing order. On the other hand, writing the expression for the decay rate in terms of multipole contributions may be directly generalized to the case of multielectron atoms and to composite systems such as positronium using the many-body form of the action functional.

ACKNOWLEDGMENTS

The author would like to thank Professor A. O. Barut for continuous encouragement and many fruitful discussions. This work is an outgrowth of a collaboration with him. I also thank Professor Abdus Salam, the Atomic Energy Agency and UNESCO for hospitality at the International Center for Theoretical Physics, Trieste where part of this work has been done.

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