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EXACT DIAGONZALIATION OF THE INTERACTING PROPAGATOR FOR THE 2D-ELECTRON GAS IN A MAGNETIC FIELD

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ABSTRACT

The spatial dependence of the exact one electron propagator for an interacting 2D-electron gas in a magnetic field is shown to be the same as for a non-interacting gas. This happens whenever the translational symmetry is unbroken in the ground state. The result may be extended to a more general class of systems The translational symmetry also implies that the density of states has the same kind of discrete character as in the non-interacting case. This is shown explicitly in the Hartree-Fock approximation by solving the Dyson equation.

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1. INTRODUCTION

The Quantum Hall Effect (QHE) has been one of the most exciting subjects of research in condensed matter physics in this decade [1),2)]. A great deal of investigations have been devoted to clarify the theoretical understanding of this phenomenon [3)]. Nowadays, it seems that the so-called integral effect is already well understood. The situation with the fractional QHE is however, different For this one, in spite of many results and ideas which support its existence, a generally accepted explanatory picture is still missing.

In this work we intend to present come technical results which may have interesting consequences for the analysis of the FQHE. These conclusions arise from a discussion of the symmetry properties of the interacting 2D electron gas placed in a homogeneous magnetic field. The relevant symmetries of the system are the translational (cocycle) and rotational ones [4),5)]. It is shown that the exact quasiparticles of the interacting plasma have the same spatial form as the solutions of the one-particle Schrödinger equation in magnetic fields. The quantitiative explanation of this result is that the cocyle symmetry is the representative of the translational invariance of the system which implies the existence of an infinite number of conservation laws. This conclusion is valid also in a usual 3D interacting plasma and moreover should arise in several other situations for systems placed in a homogeneous magnetic field.

A conclusion derived from the above result is that the degeneracy of any quasiparticle state should be infinite. This follows because the degenerate wave functions give the representation space for all the transformations of the cocycle group. Such a property is checked here in the Hartree-Fock approximation.

The resulting spectra differ from that of the free case only by a definite shift of the Landau level energies. The spectra also resemble the ones previously calculated in the one-loop approximation [6)].

In Sec.2 the important symmetries of the system are reviewed and main relations among the symmetry generators are presented. Sec.3 is devoted to show the commutativity of the free-particle Schrödinger equation and the electron mass operator of the interacting problem. Finally in Sec.4 the Dyson equation is solved explicitly in the Hartree-Fock approximation.

2. COCYLE AND ROTATIONAL SYMMETRY PROPERTIES

In this section, as was already mentioned in the introduction the relevant symmetry properties in question are presented and some general relations among their corresponding generators are derived.

The 2D electron gas confined to a plane $(x_1,x_2,0)$ and placed in a homogeneous magnetic field is characterized by the second quantized statistical Hamiltonian [7]

$$H = \int \psi^+_\alpha(x)\left[(\vec{p}-\tfrac{e}{c}\vec{A})^2/(2m^*)-\mu\right]\psi_\alpha(x)\,d\vec{x}$$

$$+\frac{1}{2}\int \psi^+_\alpha(x)\psi^+_\beta(x')U(|\vec{x}-\vec{x}'|)\psi_\beta(x')\psi_\alpha(x)\,d\vec{x}\,d\vec{x}', \qquad (1)$$

where ψ_α and ψ^+_α are the Pauli spinor electron fields and according to the 2-dimensionality of the problem

$$x \equiv (x_1,x_2,t)\ ,\quad \vec{x} = (x_1,x_2),$$

$$\vec{p} \equiv -i\hbar\left(\frac{\partial}{\partial x_1},\frac{\partial}{\partial x_2}\right).$$

In (1) the time arguments t and t' are coincident and the usual convention for the sums in the spatial and spinor indices is used, μ is the chemical potential and the interaction U is taken as a central and spin-independent one. For a constant magnetic field B, along the third axis, the vector potential is

$$\vec{A}(\vec{x}) = \frac{B}{2}(-x_2\vec{i}+x_1\vec{j}). \qquad (2)$$

The Hamiltonian (1) is invariant under a cocycle transformation which is defined through its action over the field operators as follows:

$$U_{\vec{b}}\psi_\alpha(\vec{x},t)U^{-1}_{\vec{b}} = \exp\left[\frac{ie}{\hbar c}\vec{A}(\vec{b}).\vec{x}\right]\psi_\alpha(\vec{x}-\vec{b},t) \equiv C_{\vec{b}}\psi_\alpha(x), \qquad (3)$$

where $U_{\vec{b}}$ is the unitary operator representing the transformation in the second quantized Fock space [7]. Thus, whenever the symmetry is unbroken in the ground state, the 2D-electron system will also possess such an invariance. The ground state is characterized by its density matrix ρ . Therefore, the symmetry is unbroken when ρ is invariant, i.e. if

$$\rho = U_{\vec{b}}\,\rho\, U_{\vec{b}}\ . \qquad (4)$$

Further on it will be useful to work with the generators of the transformation. Their form in the co-ordinate representation may be readily obtained if one takes $\vec{b}$ as an infinitesimal vector. In this case, the operator $c_{\vec{b}}$ takes the form

$$C_{\vec{b}}\psi_\alpha(x) = \psi_\alpha(x) + \frac{\vec{b}}{i\hbar}\cdot(\vec{p}+\frac{e}{c}\vec{A})\psi_\alpha(x)$$

$$= \psi_\alpha(x) + \frac{b_j}{i\hbar}G^c_j(x)\psi_\alpha(x)\ . \qquad (5)$$

From (5) it can been seen that the generators of the cocycle symmetry G^c_i are closely connected to the covariant derivative

$$G_i(x) = p_i - \frac{e}{c}A_i(x)\ . \qquad (6)$$

One can easily show the following set of relations between the four operators G_i and G^c_i :

$$[G^c_i,G^c_j] = -i\frac{e\hbar}{c}F_{ij}, \qquad (7)$$

$$[G_i,G_j] = i\frac{e\hbar}{c}F_{ij}, \qquad (8)$$

$$G^*_i = -G^c_i, \qquad (9)$$

$$[G^c_i,G_j] = 0, \qquad (10)$$

where the 2D electromagnetic field tensor F_{ij} is given by

$$F_{ij} = B\epsilon^{ij3}, \qquad (11)$$

in terms of the Levi-Civita tensor.

Let us now define the quadratic operators

$$G^2 = G_iG_i\ , \qquad (12)$$

$$G^2_c = G^c_iG^c_i\ . \qquad (13)$$

The substitution of (6) for G_i into (12) leads to the following relation:

$$G^2 = G_c^2 - \frac{2eB}{c} L_3, \tag{14}$$

in which the third component of the angular momentum operator is defined by

$$L_3 = \epsilon^{3ij} x_i p_j. \tag{15}$$

The use of (7)-(10) allows us to show that the operators G^2, G_c^2 and L_3 are mutually commuting. That is

$$[G_c^2, L_3] = 0, \tag{16}$$

$$[G^2, L_3] = 0, \tag{17}$$

$$[G^2, G_c^2] = 0. \tag{18}$$

The operator L_3 is the generator of the rotations of the system around the magnetic field axis. Then the Hamiltonian (1) is invariant under the transformation

$$\Psi_\alpha(x) \to R_\theta \Psi_\alpha(x) R_\theta^{-1} = \Psi_\alpha(x) + \theta L_3 \Psi_\alpha(x), \tag{19}$$

where θ is an infinitesimal parameter. In addition, assume that the ground state retains the rotational symmetry which means

$$[\rho, R_\theta] = 0. \tag{20}$$

Before ending this section, let us present the commutation relations between the cocycle generators and the angular momentum L_3 and other expressions derived from them. These relations take the form

$$[G_i^c, L_3] = -i\hbar \epsilon^{3ij} G_j^c. \tag{21}$$

Now we shall define the new operators

$$G_\sigma^c = \xi_j^{(\sigma)} G_j^c \quad , \sigma = \pm 1. \tag{22}$$

in which the two vectors appearing are given as

$$\xi_j^{(\sigma)} \equiv \frac{1}{\sqrt{2}}(1, \sigma i). \tag{23}$$

The G_σ^c play the role of operators for raising ($\sigma = +1$) and lowering ($\sigma = -1$) of the angular momentum quantum number. This property arises from their commutation relations

$$[G_\sigma^c, L_3] = -\sigma\hbar G_\sigma^c \quad , \quad \sigma = \pm 1. \tag{24}$$

The commutativity of all the G_i^c and G^2 implies the same property for the G_σ^c . The relation (24) leads to

$$L_3 G_\sigma^c |m\rangle = (m+\sigma)\hbar G_\sigma^c |m\rangle. \tag{25}$$

where $|m\rangle$ denotes the eigenfunctions of L_3 .

In the next section the above relations will be used to show the commutativity of the free particle Schrödinger Hamiltonian with the exact mass operator for the electrons.

3. DYSON EQUATIONS AND WARD IDENTITIES

The section starts by deriving the Ward identities which are satisfied by the one-electron Green function and its inverse kernel. These relations together with the results of the previous section will be used to show the commutativity of the mass operator with the free particle Schrödinger Hamiltonian.

The Dyson equation for the electron Green function in the system under study may be stated as follows [7]:

$$\left(i\hbar\frac{\partial}{\partial t} - G^2/(2m^*) + \mu\right) G_{\alpha\beta}(x,x') - \int \Sigma_{\alpha\beta'}(x,x'') G_{\beta'\beta}(x'',x') dx'' = \hbar\, \delta_{\alpha\beta}\, \delta(x-x'). \tag{26}$$

In (26) the kinetic energy term has been expressed in terms of G^2; $G_{\alpha\beta}$ and $\Sigma_{\alpha\beta}$ are the electron Green function and the mass operator, respectively. The relation (26) may be restated in the form

$$\int G^{-1}_{\alpha\beta'}(x,x'')G_{\beta'\beta}(x'',x')dx''=\hbar\delta_{\alpha\beta}\delta(x-x'), \tag{27}$$

where the inverse Green function $G^{-1}_{\alpha\beta}$ is defined as

$$G^{-1}_{\alpha\beta}(x,x')=(i\hbar\frac{\partial}{\partial t}-G^2/(2m^*)+\mu)\delta(x-x')\delta_{\alpha\beta}-\Sigma_{\alpha\beta}(x,x'). \tag{28}$$

In (28) the first term on the right-hand side is the inverse kernel of the free particle propagator.

Let us obtain the Ward identities for $G_{\alpha\beta}$ and $G^{-1}_{\alpha\beta}$ which are induced by the invariances. This is performed by introducing the infinitesimal similarity transformation (3), (5) in the definition of the Green function as follows:

$$\begin{aligned}
G_{\alpha\beta}(x,x')&=-i\,Tr[\rho T\{\Psi_\alpha(x)\Psi^+_\beta(x')\}]\\
&=-i\,Tr[U_B\rho T\{\Psi_\alpha(x)\Psi^+_\beta(x')\}U_B^{-1}]\\
&=-i\,Tr[\rho T\{U_B\Psi_\alpha(x)U_B U_B^{-1}\Psi^+(x')U_B^{-1}\}]\\
&=-i\,Tr[\rho T\{C_B(x)\Psi_\alpha(x)C^*_B\Psi^+(x')\}]\\
&=G_{\alpha\beta}(x,x')+\frac{b_j}{2\hbar}\Big[(-i\hbar\frac{\partial}{\partial x_j}+\frac{e}{c}A_j(\vec{x}))G_{\alpha\beta}(x,x')\\
&\quad-G_{\alpha\beta}(x,x')(i\hbar\frac{\overleftarrow{\partial}}{\partial x'_j}+\frac{e}{c}A_j(\vec{x}'))\Big].
\end{aligned} \tag{29}$$

From expression (29) the following identity arises;

$$(-i\hbar\frac{\partial}{\partial x_i}+\frac{e}{c}A_i(\vec{x}))G_{\alpha\beta}(x,x')=G_{\alpha\beta}(x,x')(i\hbar\frac{\overleftarrow{\partial}}{\partial x'_i}+\frac{e}{c}A_i(\vec{x}')), \tag{30}$$

which in terms of G_i and G^c_i takes the form

$$G^c_i(x)G_{\alpha\beta}(x,x')=G_{\alpha\beta}(x,x')[\overleftarrow{G}^c_i(x')]^*. \tag{31}$$

In (29)-(31) and further on the inverse arrow $\leftarrow$ over a differential operator means that the derivatives are acting on the left-hand side expressions. The corresponding relation for the inverse Green function following from (31) is

$$G^c_i(x)G^{-1}_{\alpha\beta}(x,x')=G^{-1}_{\alpha\beta}(x,x')[\overleftarrow{G}^c_i(x')]^*. \tag{32}$$

The identity expressing the rotational symmetry may be derived in a similar way

$$L_3(x)G^{-1}_{\alpha\beta}(x,x')=G^{-1}_{\alpha\beta}(x,x')[\overleftarrow{L}_3(x')]^*. \tag{33}$$

The relations (31), (32) and (33) together with the results in Sec.2, are all which is needed to obtain the desired result. After multiplying (32) by G^c_i and summing over i the following expression is obtained:

$$G^2_c(x)G^{-1}_{\alpha\beta}(x,x')=G^{-1}_{\alpha\beta}(x,x')[\overleftarrow{G}^2_c(x')]^*. \tag{34}$$

Then from (34), (33) and (14) it follows that

$$G^2(x)G^{-1}_{\alpha\beta}(x,x')=G^{-1}_{\alpha\beta}(x,x')[G^2(x')]^*, \tag{35}$$

which after taken G^{-1} as given in (28) leads to

$$G^2(x)\Sigma_{\alpha\beta}(x,x')=\Sigma_{\alpha\beta}(x,x')[\overleftarrow{G}^2(x')]^*. \tag{36}$$

Thus the relation (36) shows that the mass operator kernel commutes with the operator G^2 which is proportional to the free-particle Schrödinger Hamiltonian in a magnetic field. The commutativity of G^c_i and L_3 with $\Sigma_{\alpha\beta}$ follows in a similar manner.

Finally, the following symbolic commutation relations among the kernels of G^2, L_3 and Σ arise

$$[\Sigma,G^2]=0\ , \tag{37}$$

$$[\Sigma,L_3]=0\ , \tag{38}$$

$$[G^2,L_3]=0\ . \tag{39}$$

Hence these three operators can have a common set of orthogonal eigenfunctions. The common set for G^2 and L_3 is completely determined. This is due to the fact that G^2 is proportional to the Hamiltonian of a 2D electron in the magnetic field and L_3 is the projection of the angular momentum along the field. The eigenfunctions of these operators with the vector potential (2) are determined unambiguously. Thus one can conclude that this basis is the common one for these three operators. The explicit form of the eigenfunctions is the following:

$$\varphi_\alpha^{(q)}(\vec{x}) = \varphi_n^m(\vec{x})\,\rho_\alpha^{(\beta)}, \quad \rho_\alpha^{(\beta)} = \delta_{\alpha\beta}, \tag{40}$$

where

$$\varphi_n^m(\vec{x}) = \frac{(\frac{1}{2})^{|m|/2}\sqrt{n^*!}\; e^{im\theta}}{r_0\sqrt{2\pi\Gamma(n^*+|m|+1)}}\, L_{n^*}^{|m|}(\tfrac{1}{2}r^2/r_0^2)(r/r_0)^{|m|}\, e^{-\frac{1}{4}r^2/r_0^2}$$

$$n^* = n-|m|\theta(m);\ n=0,1,\ldots\infty;\ m=n,n-1,\ldots,-\infty; \qquad \alpha,\beta=1,2. \tag{41}$$

The super index $q = (n,m,\beta)$ denotes the set of quantum numbers (β is the spin one), $\theta(x)$ is the heaviside function, $r_0 = \sqrt{\frac{\hbar c}{|e|B}}$ is the orbital radius parameter and (r,θ) are the usual cylindrical co-ordinates. The functions φ_n^m satisfy the eigenvalue equations

$$L_3\varphi_n^m = \hbar m\varphi_n^m, \tag{42}$$

$$G^2\varphi_n^m = 2m^*\epsilon_n\varphi_n^m, \tag{43}$$

with

$$\epsilon_n = \frac{|eB\hbar|}{m^*c}(n+\tfrac{1}{2}). \tag{44}$$

It must be stated that the functions given in (40) and (41) correspond to negative values of the charge e and positive values of the magnetic field parameter B.

4. DYSON EQUATION IN THE HARTREE-FOCK APPROXIMATION

The results of the previous sections will be employed below in solving the Dyson equation in the mean field approximation.

The Hartree-Fock approximation for the Dyson equation (26) corresponds to expressing the mass operator as given by [7]

$$\Sigma_{\alpha\beta}(x,x') = i\,U(|\vec{x}-\vec{x}'|)\left[G_{\alpha\beta}(\vec{x},\vec{x}',t-t'-\delta)\right]_{\delta\to 0^+}\delta(t-t'). \tag{45}$$

In (45) $G_{\alpha\beta}$ is the exact one-electron propagator. The usual consideration of the compensating background of positive charges has been employed in order to cancel out the infinite electron self interaction term [7].

Also, we shall consider the Coulomb interaction

$$U(|\vec{x}-\vec{x}'|) = \frac{e^2}{|\vec{x}-\vec{x}'|}. \tag{46}$$

Now let us begin the search for a solution of the self consistent set of Eqs.(26) and (45). Firstly, $G_{\alpha\beta}$ will be expressed in the form dictated by its commutativity with G^2 and L_3

$$G_{\alpha\beta}(x,x') = \frac{\delta_{\alpha\beta}}{2\pi}\int d\omega\, e^{-i\omega(t-t')}\sum_{n,m} G_n^m(\omega)\varphi_n^m(\vec{x})\varphi_n^{m*}(\vec{x}'), \tag{47}$$

where the frequency ω is the Fourier conjugate variable of the time difference $t-t'$, the φ_n^m are given in (41) and the sum is over all the allowed values of m and n defined in (41).

After substituting (47) into (26) the following equation is obtained:

$$[\hbar\omega - G^2/(2m^*)+\mu]\varphi_\alpha^{(q)}(\vec{x}) - \int d\vec{x}'\,\Sigma_{\alpha\beta}(\vec{x},\vec{x}',\omega)\varphi_\beta^{(q)}(\vec{x}') = \frac{\hbar}{G_n^m(\omega)}\varphi_\alpha^{(q)}(\vec{x}). \tag{48}$$

Then, the raising and lowering operators of the angular momentum, G_σ^c, may be applied to both sides of (48) in order to show that $G_n^m(\omega)$ is independent of the index m. This property is a consequence of the commutativity of G_σ^c, with G^2 and $\Sigma_{\alpha\beta}$.

The m-independence of the G_n^m and the relation [6]

$$\sum_{m=n}^{-\infty}\varphi_n^m(\vec{x})\varphi_n^{m*}(\vec{x}') = \varphi_n^0(0)\varphi_n^{0*}(\vec{x}'-\vec{x})\, e^{\frac{ie}{\hbar c}\vec{A}(\vec{x})\cdot(\vec{x}-\vec{x}')}, \tag{49}$$

allow us to write for the Fourier transform of the exact $G_{\alpha\beta}$ the following general expression:

$$G_{\alpha\beta}(\vec{x},\vec{x}',\omega)=\sum_{n=0}^{\infty} G_n(\omega)\varphi_n^0(0)\varphi_n^{0*}(\vec{x}'-\vec{x})\, e^{\frac{ie}{\hbar c}\vec{A}(\vec{x}).(\vec{x}-\vec{x}')}\,\delta_{\alpha\beta}\,. \tag{50}$$

In (50), the m-independence of G_n^m has been used by the notation $G_n^m(\omega) = G_n(\omega)$.

On the other hand, the commutativity of the cocycle symmetry operators with G^2 and $\Sigma_{\alpha\beta}$ permits us to transform (48) into a more useful expression. Operating on both sides of (48) with the transformation $C_{\vec{b}}$ defined in (3), we have

$$(\hbar\omega-\epsilon_n+\mu)\varphi_\alpha^{(q)}(\vec{x}-\vec{b})\, e^{\frac{ie}{\hbar c}\vec{A}(\vec{b}).\vec{x}} - \int d\vec{x}' \sum_{\alpha\beta}(\vec{x},\vec{x}',\omega)\varphi_\beta^{(q)}(\vec{x}'-\vec{b})\, e^{\frac{ie}{\hbar c}\vec{A}(\vec{b}).\vec{x}'} = \frac{\hbar}{G_n(\omega)}\varphi_\alpha^{(q)}(\vec{x}-\vec{b})\, e^{\frac{ie}{\hbar c}\vec{A}(\vec{b}).\vec{x}}\,, \tag{51}$$

where $q = (n,m,\beta)$ is, as before, the set of quantum numbers.

The Fourier transformed mass operator in (51) is then given by

$$\Sigma_{\alpha\beta}(\vec{x},\vec{x}',\omega) = \frac{ie^2\delta_{\alpha\beta}}{|\vec{x}-\vec{x}'|}\sum_{n'}\left\{\int_{-\infty}^{\infty}\frac{G_{n'}(\omega)}{2\pi}e^{i\delta\omega}d\omega\right\}_{\delta\to 0^+} \cdot\varphi_{n'}^0(0)\varphi_{n'}^{0*}(\vec{x}'-\vec{x})\, e^{\frac{ie}{\hbar c}\vec{A}(\vec{x}).(\vec{x}-\vec{x}')}\,. \tag{52}$$

After substituting (52) into (51) and taking the limit $\vec{b}\to\vec{x}$, the following equation for $G_n(\omega)$ is obtained (the $m = 0$ value of the index has been selected):

$$G_n(\omega) = \frac{\hbar}{\hbar\omega - \epsilon_n^{(HF)}+\mu}\,, \tag{53}$$

where the Hartree-Fock corrected energy has the form

$$\epsilon_n^{(HF)} = \epsilon_n - \sum_{n'} e^2\nu_{n'}\int d\vec{x}\,\frac{\varphi_n^0(\vec{x})\varphi_{n'}^{0*}(\vec{x})}{|\vec{x}|}\,. \tag{54}$$

In this expression $\nu_{n'}$ is proportional to the filling factor of the Landau level with index n' and is defined by

$$\nu_{n'} = -i\left[\int_{-\infty}^{\infty}\frac{G_{n'}(\omega)}{2\pi}e^{i\delta\omega}d\omega\right]_{\delta\to 0^+}. \tag{55}$$

It is necessary to mention here that we are assuming that only a few energy levels appearing here are completely filled, i.e. the chemical potential lies in a gap of the energy spectrum. Then the calculation of $\nu_{n'}$, through (55) gives

$$\nu_{n'} = \begin{cases} 1\,, & \mu > \epsilon_{n'}^{(HF)} \\ 0\,, & \mu < \epsilon_{n'}^{(HF)}\,. \end{cases} \tag{56}$$

Above, we have obtained the solution of the mean field equation for the one-electron Green function. It is not difficult to calculate explicitly the integral in (54) for all n and n' . However, we have performed the integration for the case in which only the first Landau level is filled, i.e. $\nu_{n'} = 0$ for $n' > 0$. In such a case the energies take the form

$$\epsilon_n^{(HF)} = (n+\tfrac{1}{2})\frac{|eB\hbar|}{m^*c} - \frac{1}{2}\frac{\sqrt{2\pi}}{n!}(\tfrac{1}{2})_n\, e^2/r_0\,,\ n=1,2,\ldots\infty,$$
$$\epsilon_0^{(HF)} = \frac{1}{2}\frac{|eB\hbar|}{m^*c} - \sqrt{\frac{\pi}{2}}\, e^2/r_0\,,$$
$$(\tfrac{1}{2})_n = \frac{1}{2}\cdot\frac{3}{2}\cdot\frac{5}{2}\cdots\frac{(2n-1)}{2}\,. \tag{57}$$

For the Green function and the density of states in the general case, the following formulae arise

$$G_{\alpha\beta}(\vec{x},\vec{x}',\omega) = \sum_{n=0}^{\infty}\frac{\hbar\,\delta_{\alpha\beta}\varphi_n^0(0)\varphi_n^{0*}(\vec{x}'-\vec{x})\, e^{\frac{ie}{\hbar c}\vec{A}(\vec{x}).(\vec{x}-\vec{x}')}}{\hbar\omega - \epsilon_n^{(HF)}+\mu+i\delta\,\mathrm{sign}(\epsilon_n^{(HF)}-\mu)}\,, \tag{58}$$

$$\frac{dN}{d\epsilon} = -\frac{1}{\pi}\,\mathrm{sign}(\epsilon)\int \mathrm{Im}\, G_{\alpha\alpha}(\vec{x},\vec{x},\epsilon/\hbar)\, d\vec{x}$$

$$= 2\sum_{n=0}^{\infty} \frac{1}{2\pi r_0^2}\cdot \delta(\epsilon - \epsilon_n^{(HF)} + \mu). \qquad (59)$$

The relation (59) shows that the interaction does not smooth out the density of states. This density preserves its discrete character in the Hartree-Fock approximation.

Finally, we would like to remark that it seems that the extension of the discussions presented here may be useful for the understanding of the FQHE. This will be the subject of further communications. Also, we want to add that after finishing this work we have noticed that the diagonalization property presented here constitutes the extension to the non-relativistic many-body theory and statistical physics of the result of Ritus in the context of quantum electrodynamics [8]. We also want to acknowledge the commentaries and suggestions of G. Baskaran who also informed us about the results in Refs.9 and 10 for which the here presented conclusions become a generalization.

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