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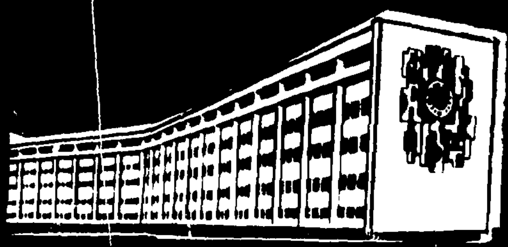

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Электронная нелинейность в поле мощной ленгмюровской волны в плазме

Получена система уравнений, определяющих динамику развития параметрической неустойчивости интенсивных ленгмюровских колебаний плазмы для двух и трехмерных декартовых геометрий.

V.M.Chernousenko, V.M.Kuklin, I.P.Panchenko, I.V.Romensky

Electron Nonlinearity in the Field of Powerful Langmuir Wave in Plasmas

The system of equations that determines the dynamics of the development of parametric instability of intensive Langmuir plasma oscillations is obtained for two- and three-dimensional Cartesian geometries.

One of the generally accepted mechanisms of energy dissipation of high-frequency Langmuir vibrations proves to be the process of creating local regions of reduced plasma density, i.e. cavities with resultant collapse [1]. Most of the papers dealing with Langmuir collapse consider the situation at which the vibrations energy density is lower than that of the internal energy of plasmas ($W \ll n_0 T$). This allows to take into consideration only one form of the nonlinearity - the dependence of ions concentration on the intensity of high-frequency vibrations. However the energy density may exceed the modulation instability threshold ($W \sim n_0 T \left(\frac{rd}{a_0}\right)^2$) . In this case the Langmuir waves become nonlinear. In Ref. [2] the nonlinear system of equations for $W \gtrsim n_0 T$ is obtained. A cumbersome form of these equations makes it impossible to carry out a complete analysis of the effect of proper electron nonlinearities on the dynamics of the collapse. It is easy to show [3] that in the case of one-dimensional vibrations the contribution of electron nonlinearities equals zero. It is exhibited in Refs. [4,5] that under general conditions for the development of modulation instability the electron nonlinearities not only stop the Langmuir waves collpase but also preserve its explosive character. However, in the case of strongly anisotropic cavity the situations, that result in stabilization of collapse are possible [6]. In Ref. [7] the dynamics of 3-dimensional collapse with and without account of electron nonlinearities is analyzed numerically.

When plasma is affected by strongly exact beams of charged particles or external radiation of large intensity there arise the states at which $W/n_0 T \gg 1$. Then the strong parametric instabilities with great increments $\sim (m_e/m_i)^{1/3} \omega_0$ where ω_0 is the pumping wave frequency, are being developed in plasmas. This leads to the excitation of a wide spectrum of short-wave vibrations with the wave length comparable with the value of electrons space oscillation. Since the oscillation velocity of electrons exceeds much their heat velocity one may assume $T_e = 0$. It is quite natural that the role of electron nonlinearities in such a case may become dominant for the plane sheets model. The system of equations describing the interaction between the pump wave and Langmuir spectrum of plasmas for the phane sheets model is obtained and integrated numerically in Ref. [8]. Similarly to the case of modulation instability the electron nonlinearities contribution to the dynamics of the process for one-dimensional case is nonexistent. Therefore it is fairly interesting to extend the statement of the problem to the cases when the dimensionality is larger than one.

In the present paper we obtained the system of equations that determine the dynamics of the development of parametric instability of intensive Langmuir vibrations in plasmas for the two- and three-dimensional Cartesian geometries.

We consider an arbitrary three-dimensional region occupied by plasma (the two-dimensional case results from a trivial convolution of all ratios by one of indices). Let the periodic boundary conditions in all directions be existent.

We consider the system of quasi-hydrodynamical equations for electrons and ions of plasma being in the field of intensive vibrations. As the wave length of the most intensive increasing vibrations is sufficiently small, $\lambda \simeq 2\pi/K_0 = 2\pi U_0/\omega_0 \ll \lambda_0$, where λ_0 is the pump wave length, U is the amplitude of electrons oscillator velocity, ω_0 is the pump wave frequency, the pump may be regarded as spacially homogeneous

$$E = E_0 \sin(\omega_0 t + \tilde{\varphi})$$

where $\tilde{\varphi}$ is slowly varying phase that takes into account the inverse action of exciting vibrations on the pump wave. Assuming that the electric fields are potential, performing the spacial Fourier transformation and going over to the oscillator coordinate system we obtain the equations describing the motion of electrons

$$\begin{aligned}
&\frac{\partial}{\partial t}\nu_p + iK_0 n_0 \nabla_p \vec{\theta}_p = -iK_0 \nabla_p \sum_m \nu_m \vec{\theta}_{p-m} \\
&\frac{\partial}{\partial t}\vec{\theta}_p - \frac{iK_0 e}{m_e}\nabla_p \Psi_p = -iK_0 \sum_m \left(\vec{\theta}_{p-m}\cdot\nabla_m\right)\vec{\theta}_m \qquad (1) \\
&K_0^2 \Delta_p \Psi_p = 4\pi e\left(\delta n_p \exp\left(i a_p \sin(\omega_0 t + \tilde{\varphi})\right) - \nu_p\right)
\end{aligned}$$

Here δn_p is the Fourier transform of a varying ion density

$$\nu_p = n_p \exp\left(-i a_p \cdot \sin\left(\omega_0 t + \tilde{\varphi}\right)\right)$$

$$\vec{\vartheta}_p = \vec{V}_p \exp\left(-i a_p \cdot \sin\left(\omega_0 t + \tilde{\varphi}\right)\right) \qquad (2)$$

$$\mathcal{Y}_p = \varphi_p \exp\left(-i a_p \cdot \sin\left(\omega_0 t + \tilde{\varphi}\right)\right) \qquad a_p = \frac{\kappa_0 e E_0}{m_e \omega_0^2} \cdot P_1$$

n_p , V_p , φ_p are the Fourier transforms of density, electrons velocity and the electric field potential, respectively. P is the multiindex $P = \{P_1 ; P_2 ; P_3\}$

$$\left.\begin{aligned} \nabla_p &= P_1 \vec{j}^1 + P_2 \vec{j}^2 + P_3 \vec{j}^3 = P_i \vec{j}^i \\ \Delta_p &= P_1^2 + P_2^2 + P_3^2 \end{aligned}\right\}$$

are the Fourier transforms of the corresponding differential operators

We seak for the solutions in

$$\nu_p = U_p^0 + U_p^{\pm 1} \cdot e^{\pm i \omega_0 t} + U_p^{\pm 2} \cdot e^{\pm 2 i \omega_0 t}$$

$$\vec{\theta}_p = \vec{V}_p^0 + \vec{V}_p^{\pm 1} \cdot e^{\pm i \omega_0 t} + \vec{V}_p^{\pm 2} \cdot e^{\pm 2 i \omega_0 t} \qquad (3)$$

$$\mathcal{Y}_p = \varphi_p^0 + \varphi_p^{\pm 1} \cdot e^{\pm i \omega_0 t} + \varphi_p^{\pm} \cdot e^{\pm 2 i \omega_0 t}$$

Substituting this representation into the system of equations we get

$$
\left\{
\begin{aligned}
U_p^{0} &= \delta n_p \cdot J_0(a_p) + \frac{\Delta_p \cdot K_0^4}{4\pi m_e \omega_0^2} \sum_m \left(\nabla_m \cdot \nabla_{p-m}\right) \varphi_m^{+1} \cdot \varphi_{p-m}^{-1} \\
V_p^{0} &= + \frac{K_0^3}{4\pi m_e \omega_0} \sum_m \left(\nabla_m \cdot \Delta_{p-m} - \Delta_m \nabla_{p-m}\right) \varphi_m^{+1} \cdot \varphi_{p-m}^{-1} \\
\varphi_p &= - \frac{e K_0^2}{m_e \omega_0^2} \sum_m \left(\nabla_m \cdot \nabla_{p-m}\right) \varphi_m^{+1} \varphi_{p-m}^{-1} \\
U_p^{\pm 2} &= - \frac{1}{3} \delta n_p J_2(a_p) e^{\pm 2i\tilde{\varphi}} + \frac{K_0^4}{3 \cdot 4\pi m_e \omega_0^2} \sum_m (\nabla_p \cdot \nabla_e)(\nabla_m \nabla_e + 2\Delta_m) \varphi_m^{\pm 1} \varphi_{p-m}^{\pm 1} \\
\varphi_p^{\pm 2} &= \frac{4\pi e}{K_0^2 \Delta p} \left[\frac{4}{3} \delta n_p J_2(a_p) e^{\pm 2i\varphi} - \frac{K_0^4}{3 \cdot 4\pi m_e \omega_0^2} \sum_m (\nabla_p \nabla_e)(\nabla_m \nabla_e + 2\Delta_m) \varphi_m^{\pm 1} \varphi_{p-m}^{\pm} \right] \\
\vec{V}_p^{\pm 2} &= \pm \frac{e K_0}{2\omega_0 m_e} \left[\nabla_p \varphi_p^{\pm 2} - \frac{e K_0^2}{m_e \omega_0^2} \sum_m (\nabla_e \nabla_m) \nabla_m \varphi_m^{\pm 1} \cdot \varphi_{p-m}^{\pm 1} \right]
\end{aligned}
\right. \tag{4}
$$

$$
K_0^2 \Delta_p \left[\pm 2i \frac{\partial}{\partial t} \varphi_p^{\pm 1} + \frac{\Omega_e^2 - \omega_0^2}{\omega_0} \varphi_p^{\pm} \right] + 4\pi e \omega_0 \delta n_p \cdot J_1(a_p) =
$$

$$
\begin{aligned}
&= \frac{4\pi e n_0}{\omega_0} K_0 \nabla_p \Big[\sum_m \left(\vec{V}_m^{0} \nabla_{p-m}\right) \vec{V}_{p-m}^{\pm 1} + \left(\vec{V}_{p-m}^{\pm 1} \cdot \nabla_m\right) \vec{V}_m^{0} + \\
&+ \left(\vec{V}_m^{\pm 2} \cdot \nabla_{p-m}\right) \vec{V}_{p-m}^{\mp 1} + \left(\vec{V}_{p-m}^{\mp 1} \nabla_m\right) \vec{V}_m^{\pm 2} \mp \frac{\omega_0}{n_0} \Big(U_m^{0} \vec{V}_{p-m}^{\pm 1} + \\
&+ U_{p-m}^{\pm 1} \cdot \vec{V}_m^{0} + U_m^{\pm 2} \vec{V}_{p-m}^{\mp 1} + U_{p-m}^{\mp 1} \cdot \vec{V}_m^{\pm 2} \Big) \Big]
\end{aligned} \tag{5}
$$

$$
\vec{V}_p^{\pm 1} = \pm \frac{e K_0}{m_e \omega_0} \nabla_p \varphi_p^{\pm 1} \quad ; \qquad U_p^{\pm 1} = - \frac{K_0^2 \Delta_p}{4\pi e} \varphi_p^{\pm 1}
$$

Since ions have large mass they move only slowly. Neglecting in hydrodynamics equations the ion nonlinearities we obtain the following equations

$$\frac{\partial^2 \delta n_p}{\partial t^2} + \Omega_i^2 \delta n_p \left(1 - J_0^2(a_p) + \frac{2}{3} J_2^2(a_p)\right) + \Omega_i^2 J_1(a_p) [\varphi_p^{\pm 1} e^{\mp i\tilde{\varphi}} -$$

$$- \varphi_p^{\mp 1} e^{\pm i\varphi}] \frac{\Delta_p K_0^2}{4\pi e} = \frac{\Omega_i^2 K_0^4}{4\pi m_e \omega_0^2} \Big[J_0(a_p) \Delta_p \sum_m (\nabla_m \nabla_{p-m}) \varphi_m^{\pm 1} \varphi_{p-m}^{\pm 1} + \quad (6)$$

$$+ \frac{1}{3} J_2(a_p) \sum_m (\nabla_p \nabla_{p-m})(\nabla_{p+m} \nabla_m) [\varphi_m^{\pm 1} \varphi_{p-m}^{\pm 1} e^{\mp 2i\tilde{\varphi}} + \varphi_m^{\mp 1} \varphi_{p-m}^{\mp 1} e^{\pm 2i\tilde{\varphi}}]\Big]$$

$$\frac{\partial}{\partial t}(\vec{E}_0 e^{i\tilde{\varphi}}) = 8\pi i e \sum_p \left\{ U_m^0 \vec{V}_{-m}^{+1} + U_m^{+1} \vec{V}_{-m}^{0} + U_m^{+2} \vec{V}_{-m}^{-1} + U_m^{-1} \vec{V}_{-m}^{+2} \right\} \quad (7)$$

When P_2 and P_3 tend to zero the system is transformed to equations obtained in Ref. [8] . When the pump wave amplitude tends to zero ($a_p \to 0$) the equations are transformed to Eqs. described in Ref. [2] (taking into account that $T_e = 0$).

The case when plasma is placed in the strong magnetic field ($\vec{B}_0 \parallel \vec{E}$) proves to be of certain interest. Then the electrons move quasi-one-dimensionally, and the ions can move arbitrarily. Performing analogous calculations we obtain the following system of equations: in the 2-dimensional case

$$\frac{\partial}{\partial t} U_{p,q} - \frac{i}{2\omega_0}\left[\frac{p^2}{p^2+q^2}\Omega_e^2 - \omega_0^2\right] U_{p,q} + \frac{i p^2 \omega_0}{2(p^2+q^2)} \delta n_{p,q} J_1(a_p) = \frac{i\omega_0 p}{2 n_0} \sum_{m,r} \frac{1}{p-m} \cdot$$

$$\cdot \Big\{ \Big[U_{p-m,q-r} J_0(a_m) + \frac{3m^2}{3m^2+4r^2} U_{p-m,q-r}^{*} J_2(a_m) e^{2i\tilde{\varphi}} \Big] \delta n_m + \frac{1}{n_0} \sum_{s,h} \frac{1}{3(m-s)} \cdot \quad (8)$$

$$\Big[\frac{3msr}{3m^2+4r^2} U_{s,h} U_{m-s,m-h} U_{p-m,q-r}^{*} + r^2 U_{s,h} U_{m-s,r-h}^{*} U_{p-m,q-r} \Big] \Big\}.$$

$$\frac{\partial^2 \delta n_{p,q}}{\partial t^2} + \Omega_i^2 \delta n_{p,q} \left(1 - J_0^2(a_p) + \frac{2}{3} \frac{p^2}{3p^2+4q^2} J_2^2(a_p)\right) + \Omega_i^2 J_1(a_p) \cdot \quad (9)$$

$$[U_{p,q} e^{-i\tilde{\varphi}} - U_{m}^{*} e^{+i\tilde{\varphi}}] = \frac{\Omega_i^2}{n_0} \Big\{ \sum_{m,r} J_0(a_p) \frac{p^2+q^2}{m(p+m)} U_{m,r} U_{p-m,q-r}^{*} + \frac{3p(p^2+q^2)}{4q^2+3p^2} J_2(a_p) \cdot$$

$$\frac{1}{p-m} \left[U_{m,r} U_{p-m,q-r} e^{-2i\varphi} + U_{m,r}^{*} U_{p-m,q-r}^{*} e^{+2i\tilde{\varphi}} \right] \Big\}.$$

Here ρ, m, S are the indices of Fourier expansion in the direction of the critical pump field vector, q, r, h are the indices of Fourier expansion in the direction perpendicular to the vector of pump electric field. The electron nonlinearity mechanism will, probably, effect essentially the dynamics of plasma density cavities and lessen the efficiency of ion component heat as compared to the one-dimensional case[9].

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