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COLLECTIVE OSCILLATIONS OF TWIN BOUNDARIES IN HIGH TEMPERATURE SUPERCONDUCTORS AS AN ACOUSTIC ANALOGUE OF TWO-DIMENSIONAL PLASMONS

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ABSTRACT

Low frequency collective oscillations in a superlattice consisting of alternating highly anisotropic layers are considered. Such superstructure may be formed in the ferroelastic near the structural phase transition by alternation of twins. For the surface waves, propagating along the layers, the conditions and the range of existence of those with the dispersion law $\omega \sim k^{1/2}$, characteristic for two-dimensional plasmons, have been analyzed for a solid-state system with consideration for elastic anisotropy and retardation of acoustic waves. In Ref.[1] such excitations ("dyadons") were used in an attempt to explain the anomalies of low temperature thermodynamic and kinetic characteristics of high-T_c superconductors. We have shown that the similarity of the densities of the matching phases and the retardation of elastic waves in the crystal narrow the range of existence of dyadons, but high elastic anisotropy of the solid phases enlarges the range of existence of such excitations in solid-state systems. The example of possible crystalline geometry of the phase matching, for which there arise collective excitations of the type under consideration, is found. For transverse and longitudinal waves propagating across the layers, the existence is proved of low frequency acoustic branches separated by a wide gap from the nearest optical branches.

In Refs. [1] and [2] the explanation of low temperature anomalies of the specific heat of high-T_c superconductors was made with recourse to low frequency collective excitations ("dyadons") with the characteristic dispersion law $\omega \sim k^{1/2}$ that were thought to be associated with coexistence of various high-T_c superconductor phases near the structural phase transition, preceding the superconducting transition. It should be noted that such excitations can exist not only in the model considered in Refs. [1] and [2], but are of a fairly general character and were studied. Thus, the excitations with the dispersion law were obtained in Refs. [3, 4] as a part of a study of collective oscillations of the dislocation wall, and in Ref. [5] when dealing with surface elastic waves in the layer-substrate system. In Refs. [3-5] it was also shown that the retardation of acoustic waves in crystals and the elastic anisotropy of contacting media essentially affect the range of existence of the acoustic analogue of two-dimensional plasmons (AA2DP), i.e. the surface waves with the characteristic dispersion law $\omega \sim k^{1/2}$. In particular, when the densities of the layer and the substrate are similar (as is the case for matching of various phases near the structural phase transition), AA2DP can exist only in a highly anisotropic elastic system [5]. We shall study elastic surface waves in a two-phase solid state system near the structural phase transition and the conditions under which the dispersion law of these waves has the form characteristic for AA2DP. We shall also consider low-frequency elastic oscillations in the superlattice formed by alternating layers of highly anisotropic phases.

Let us consider the two-phase A-B-A system consisting of a layer of the B phase which is disposed in the A phase. Near the structural phase transition the B layer may be formed by the array of twins and the A phase may be the parent phase (see e.g. Refs. [1, 2]). The solid A-B-A system may be realized by the alternating layers of mutually disoriented twin superlattices also. We shall be interested in small amplitude elastic oscillations of such a system, in which the amplitude of the elastic stresses σ_{ik} is smaller than the threshold coercive forces of the motion start of twin boundaries with respect to the lattice [6] and, naturally, the elastic strain amplitude is small as against the spontaneous strain $u_{ik}^{(o)}$ of domains. It means that there is no motion of twin boundaries with respect to the lattice - the number of atoms in each of the matched phases is constant. We shall consider low-frequency collective excitations localized near the B layer and having a wavelength λ much larger than the twin array period ℓ. In the present long-wavelength oscillations $\lambda \gg \ell \gg \xi$ (ξ is the domain wall width), the both phases may be described as homogeneous media with effective elastic moduli and densities [7, 8], i.e. neglecting the domain structure of the media. Because of the smallness of orthorhombic distortions in metal-oxide compounds near the tetragonal-orthorhombic (TO) structural transition [9], the contacting A and B phases may be regarded as disoriented tetragonal crystalline media with similar acoustic parameters (density ρ and elastic moduli C_{ik}). Since near the TO transition the transverse sound velocity $((C_{11} - C_{12})/2\rho)^{1/2}$ is softening, both the phases are highly anisotropic, i.e. the elastic anisotropy parameter is

$$\eta = \frac{C_{11} - C_{12}}{2C_{66}} << 1 .$$

It was shown in Ref. [5] that AA2DP can exist in a binary acoustic system (a layer on a substrate) consisting of two identical highly anisotropic crystals coupled in a certain manner. Now we shall investigate in the ternary A-B-A acoustic system the conditions of existence of long-wave AA2DP of pure shear polarization in the A-B (and B-A) boundary plane.

Let the Z axis be normal to the boundaries of the B layer and the plane $Z = 0$ coincide with its middle plane. When pure shear acoustic waves of horizontal polarization propagate along the X axis, the elastic displacements of waves localized near the boundary of the B layer are:

$$U_y = (B_1 \sin qz + B_2 \cos qz)\, e^{ikx - i\omega t} \tag{1}$$

$$\text{for} \quad -\frac{h_B}{2} < Z < \frac{h_B}{2} ,$$

$$U_y = A e^{\varkappa (z + \frac{h_B}{2}) + ikx - i\omega t} \tag{2}$$

$$\text{for} \quad Z < -\frac{h_B}{2} ,$$

$$U_y = C e^{-\varkappa (z - \frac{h_B}{2}) + ikx - i\omega t} \tag{3}$$

$$\text{for} \quad Z > \frac{h_B}{2} .$$

Here we have introduced the following notations:

$$q = \left[\frac{\omega^2}{S_{B\perp}^2} - K^2 \frac{S_{B\parallel}^2}{S_{B\perp}^2} \right]^{1/2}, \qquad \varkappa = \left[\frac{S_{A\parallel}^2}{S_{A\perp}^2} K^2 - \frac{\omega^2}{S_{A\perp}^2} \right]^{1/2}, \tag{4}$$

ω is the frequency, K is the wave number, h_B is the B layer thickness, $S_{A\parallel}$ and $S_{B\parallel}$ are the velocities of bulk pure transverse elastic waves propagating parallel to the interface and polarized in the plane, and $S_{A\perp}$ and $S_{B\perp}$ are the velocities of transverse waves of the same polarization, propagating in the direction perpendicular to the interface (the subscript A referring to the A media, the subscript B referring to the B medium). The difference in the general case between the transverse wave velocities perpendicular and parallel to the interface is due to elastic anisotropy of the contacting phases. Using the equations of the elasticity theory for both the media and the usual boundary conditions, viz. the continuity of displacements and surface stresses at the interface, we arrive at the following dispersion equation in the present approximation:

$$\rho_A S_{A\perp}^2 \sqrt{\frac{S_{A\parallel}^2}{S_{A\perp}^2} K^2 - \frac{\omega^2}{S_{A\perp}^2}} =$$

$$= \pm \rho_B S_{B\perp}^2 \sqrt{\frac{\omega^2}{S_{B\perp}^2} - K^2 \frac{S_{B\parallel}^2}{S_{B\perp}^2}} \left(tg \frac{h_B}{2} \sqrt{\frac{\omega^2}{S_{B\perp}^2} - K^2 \frac{S_{B\parallel}^2}{S_{B\perp}^2}} \right)^{\pm 1}, \tag{5}$$

where the sign plus indicates symmetric modes in which $B_1 = 0$ and $A = C$ for the displacements (1)-(3), and the sign minus indicates antisymmetric modes in which $B_2 = 0$ and $A = -C$; ρ_A and ρ_B are the densities of the media. Eq. (5) is valid when the sagittal plane and also the interface plane are the planes of mirror symmetry in both matching media. (The sagittal plane is the plane, in which the normal to the boundary and the propagating vector $\vec{K}$ of the wave are disposed). In particular, in the case the transverse waves polarized in the interface plane do not interact with longitudinal waves and therefore are pure shear waves. As can be seen from Eq. (5), in the long-wavelength limit $K h_B \ll 1$ only symmetric modes can be surface waves with $\varkappa > 0$.

It follows from Eq. (5) that the symmetric frequency modes (that with a zero critical frequency) can have a range of wave numbers (or frequencies) in which the dispersion law of surface shear waves in the A-B-A system is similar to that of two-dimensional plasmons. Indeed, if we have

$$S_{B\parallel} \ll S_{A\parallel}, \quad \rho_A S_{A\perp} \ll \rho_B S_{B\perp}, \tag{6}$$

$$\frac{\rho_A}{\rho_B} \frac{S_{A\perp}}{S_{A\parallel}} \ll K h \ll \min\left\{ \frac{\rho_B S_{B\perp}^2}{\rho_A S_{A\perp} S_{A\parallel}}, \frac{\rho_A S_{A\perp} S_{A\parallel}}{\rho_B S_{B\parallel}^2} \right\}, \tag{7}$$

the dispersion law of the wave is

$$\omega^2 = K \frac{2 \rho_A S_{A\perp} S_{A\parallel}}{\rho_B h_B}. \tag{8}$$

Note that the dispersion law (8) of AA2DP does not include the twin structure period ℓ, as in Refs. [1] and [2] in the same limit $K\ell \ll 1$.

Let us analyze the conditions (6) and (7) of the existence of AA2DP in various acoustic systems.

In the simplest case of an elastically isotropic B layer inside an elastically isotropic A medium we have $S_{A\perp} = S_{A\parallel} = S_A$, $S_{B\perp} = S_{B\parallel} = S_B$. Since $\min\{\rho_B S_B^2/\rho_A S_A^2, \rho_A S_A^2/\rho_B S_B^2\} \leq 1$ then, as it is seen from (6) and (7), a surface shear wave with the dispersion law (8) can only exist in this system under the conditions:

$$\frac{\rho_A}{\rho_B} \ll K h \ll 1, \quad \rho_A \ll \rho_B, \quad S_B \ll S_A, \tag{9}$$

i.e. in a sufficiently long-wavelength region and for a very heavy, highly sound-retarding B layer. From the conditions (9) we see that AA2DP do not exist in the system of an elastic isotropic layer in the elastic isotropic medium when the layer and the medium have an equal density, even with considerable sound velocity decrease in the layer (a vanishingly small elastic modulus of the layer), i.e. in this case AA2DP can exist only in highly anisotropic media. Note that if $h_B/2$ is replaced by H the dispersion equation (5), corresponding to the symmetric mode, describes pure shear surface waves propagating in the system comprising a layer of the thickness H on a semi-infinite substrate [5]. If it is an elastic isotropic layer on an isotropic substrate, we arrive at the well-known Love waves [10].

Now let us consider the AA2DP range of existence in a system of a highly anisotropic B layer in a highly anisotropic A medium. In particular, in this case AA2DP can exist in a system consisting of two identical highly anisotropic crystals matched so that

$$S_{A\perp} = S_{B\parallel} \ll S_{A\parallel} = S_{B\perp}, \quad (\rho_A = \rho_B). \tag{10}$$

If the conditions (10) are fulfilled, the dispersion law (8) is realized in the following wavelength range:

$$\frac{S_{A\perp}}{S_{A\parallel}} \ll \kappa h \ll \frac{S_{A\parallel}}{S_{A\perp}} , \qquad (11)$$

which increases with elastic anisotropy of the contacting crystals. The situation of (10) can be realized in tetragonal metal-oxide high-T_c superconductors near the structural phase transition. Since near the TO transition $C_{66}, C_{44} \gg (C_{11}-C_{12})/2$, then the conditions (10) are realized if the boundary (110) of the phase A is connected with the B layer bounded by (001) planes in such a manner that the [001] direction in the A medium coincides with the [110] direction in the B layer and determines the direction of propagation of a surface shear wave. The sagittal plane in the case coincides with the common ($1\bar{1}0$) plane of the A and B phases, and the pure shear wave is polarized in the interface plane along [$1\bar{1}0$], so that $S_{A\perp}=S_{B\parallel}=((C_{11}-C_{12})/2\rho)^{1/2} \ll S_{A\parallel}=S_{B\perp}=(C_{44}/\rho)^{1/2}$. Therefore this geometry of matching of contacting solid phases really gives rise to collective excitations with the dispersion law (8) in the wavelength range (11).

Note that dispersion equation (5) for the symmetric mode in the case of $(\omega^2-\kappa^2 S_{B\parallel}^2)\, h_B/S_{B\perp} \ll 1$ is the following:

$$\rho_A S_{A\perp}^2 \sqrt{\frac{S_{A\parallel}^2}{S_{A\perp}^2}\kappa^2 - \frac{\omega^2}{S_{A\perp}^2}} = \frac{\rho_s}{2}\left(\omega^2 - \kappa^2 S_{B\parallel}^2\right) , \qquad (12)$$

where the value $\rho_s=\rho_B h_B$ corresponds to the density of a layer B per unit surface. The right-hand side of Eq. (12) describes the surface impedance generated by the thin layer B and the dispersion equation (8) for AA2DP may be obtained when this impedance includes only the mass loading $\rho_s (\kappa S_{B\parallel} \ll \omega)$. In the long-wavelength limit $\kappa h_B \ll 1$ the mass loading may be effectively taken into consideration if the layer B is regarded as infinitely thin but having a finite surface density ρ_s , which leads to the following boundary conditions at the plane of the layer :

$$\begin{aligned} &\sigma_{ni}^{(1)} - \sigma_{ni}^{(2)} = -\rho_s \ddot{u}_i^{(s)} , \\ &u_i^{(1)} = u_i^{(2)} = u_i^{(s)} , \qquad (i = x, y, z) \end{aligned} \qquad (13)$$

where $\sigma_{ni}=\sigma_{ki} n_k$ and unit vector $\vec{n}$ is directed along the normal to the interface from the medium 1 to the medium 2. This description of the long-wavelength properties of a thin (with the thickness down to atomic) intermediate layer in the A-B-A system is equivalent to the description of the dynamic properties of a plane crystal defect (a stacking fault type) with consideration for the surface mass ρ_s (see e.g. Refs. [11, 12, 13]). As it was shown in Refs. [12, 14, 15, 16], near such a plane defect there can exist three types of surface waves: an SH wave (a pure transverse wave polarized in the defect plane), an SV wave (a quasi-transverse wave polarized in the vertical plane) and a quasi-longitudinal L wave polarized in the layer plane. The symmetric SH wave which is described by Eq.(12), in the case of a heavy strongly decelerate layer gives rise to an AA2DP with the dispersion law (8).

Let us use this impedance approach to find the dispersion law and the range of existence of AA2DP of SV and L polarizations, because rigorous dispersion equations for the surface waves of such polarizations at the plane crystal defect are rather cumbersome even for isotropic media (see e.g. [14]).

As in the case of AA2DP of pure shear polarization we shall consider such crystalline geometry where the transverse waves polarized in the sagittal plane possess high anisotropy near the TO transition, e.g. when the A-B boundary plane coincides with the (010) plane of the A phase. In the surface waves the elastic displacements have the following form:

$$U_y = [A_1 \exp(\gamma_1 y) + A_2 \exp(\gamma_2 y)] \exp\{i(Kx - \omega t)\} ,$$
$$U_x = i\,[A_1 \Gamma_1 \exp(\gamma_1 y) + A_2 \Gamma_2 \exp(\gamma_2 y)] \exp\{i(Kx - \omega t)\} , \qquad (I4)$$

where the y axis is directed along the normal to the interface from the medium I to the medium 2, the parameters γ_1 and γ_2 are the eigenvalues with positive real part $(Re\,\gamma_{1,2} > 0)$ of the characteristic equation of bulk oscillations polarized in the (OOI) plane:

$$C_{11} C_{66} \gamma^4 - \gamma^2 [C_{11}(C_{11} K^2 - \rho\omega^2) + C_{66}(C_{66} K^2 - \rho\omega^2) -$$
$$-(C_{12} + C_{66})^2] + (C_{11} K^2 - \rho\omega^2)(C_{66} K^2 - \rho\omega^2) = 0 \qquad (I5)$$

and the parameters $\Gamma_1^{(1,2)}$ and $\Gamma_2^{(1,2)}$ are the eigenvectors of the bulk oscillation equation

$$\Gamma_1^{(1)} = -\Gamma_1^{(2)} \equiv \Gamma_1 , \qquad \Gamma_2^{(1)} = -\Gamma_2^{(2)} \equiv \Gamma_2 ,$$
$$\Gamma_i = \frac{\rho\omega^2 + C_{11}\gamma_i^2 - C_{66} K^2}{K\gamma_i (C_{12} + C_{66})} \qquad (i = 1, 2) . \qquad (I6)$$

In the case by using boundary conditions (I3) at the plane $y = 0$ we can present the dispersion equations of the AA2DP in the following form:

$$(\gamma_1 + \gamma_2)(C_{66} K^2 - \rho\omega^2) C_{11} = \frac{\rho_s \omega^2}{2} (C_{66} K^2 + C_{11}\gamma_1\gamma_2 - \rho\omega^2) \qquad (I7)$$

– for the SV waves;

$$(\gamma_1 + \gamma_2)\gamma_1\gamma_2 C_{66} C_{11} = \frac{\rho_s \omega^2}{2} (C_{66} K^2 + C_{11}\gamma_1\gamma_2 - \rho\omega^2) \qquad (I8)$$

– for the L waves,

where the values $\gamma_1\gamma_2$ and $\gamma_1 + \gamma_2 = (\gamma_1^2 + \gamma_2^2 + 2\gamma_1\gamma_2)^{1/2}$ can be obtained with the help of the Vielte theorem from the Eq. (I5).

For the slow surface waves $(\rho\omega^2 \ll C_{44} K^2,\ C_{66} K^2,\ \rho\omega^2 \lesssim (C_{11} - C_{12}) K^2)$ propagating in highly anisotropic tetragonal crystal with $\eta \ll 1$, the Eqs. (I7), (I8) take the same form:

$$\sqrt{C_{11} C_{66}}\, [K^2 (C_{11} - C_{12})(C_{11} + C_{12} + 2C_{66}) - 2\rho\omega^2 (C_{11} + C_{66})]^{1/2} =$$
$$= \frac{\rho_s \omega^2}{2} (C_{11} + C_{66}) . \qquad (I9)$$

As it is seen from Eq. (I9) the AA2DP in the system can exist only if the phase velocity of these waves satisfies the following restriction:

$$\frac{\omega}{K} \ll \left[\frac{(C_{11} - C_{12})(C_{11} + C_{12} + 2C_{66})}{2(C_{11} + C_{66})\rho} \right]^{1/2} \equiv v_0 . \qquad (20)$$

Near the TO transition, when $\eta \ll 1$, the velocity v_0 tends to the value $[(C_{11} - C_{12})/\rho]^{1/2}$, which coincides with the limiting velocity of bulk waves in the system [I7, I8]. From Eq. (I9) in the limit (20), we obtain the same dispersion law for the AA2DP of both SV and L polarizations in the sagittal plane:

$$\rho_s \omega^2 = \frac{2K}{C_{11}+C_{66}} \left[C_{11} C_{66} (C_{11}-C_{12})(C_{11}+C_{12}+2C_{66}) \right]^{1/2}. \qquad (21)$$

If the phase A is an elastic isotropic one $(C_{11}-C_{12}=2C_{66})$, Eq. (21) yields:

$$\rho_s \omega^2 = 4K \frac{C_{11} C_{66}}{C_{11}+C_{66}} \qquad (22)$$

Note that in addition to the restrictions (20) there is another necessary condition of realization of the surface waves dispersion law (21), (22), namely the slowness of the velocity of the transverse or longitudinal waves of the present polarization in the intermediate layer B:

$$v_0 \gg \frac{\omega}{K} \gg v_{t\parallel} \qquad (23)$$

or

$$v_0 \gg \frac{\omega}{K} \gg v_{e\parallel} \quad , \qquad (24)$$

where $v_{t\parallel}$ and $v_{e\parallel}$ are the velocities of the transverse and longitudinal waves, propagating along the layer B. The dispersion law (22) for the AA2DP of both SV and L polarizations in elastic isotropic medium can be obtained from Ref. [16] in the corresponding limit (23) or (24)). Near the TO transition the limiting velocity (20) $v_0 \; ((C_{11}-C_{12})/\rho)^{1/2}$ is of the order of the minimum velocity of acoustic waves in the system and therefore neither of the conditions (23) or (24) of the existence of the AA2DP can be fulfilled. For others orientations

of the sagittal plane and the plane of the intermediate layer B with respect to the crystalline axes of the A phase, the limiting velocity of existence of AA2DP is of the same order of magnitude as (20). In such a way our analysis demonstrates that the conditions of existence of the AA2DP in highly anisotropic systems are more favourable for pure shear elastic waves with horizontal polarization (see Eqs. (6, 7, 8)). Thus, the retardation of elastic waves and the similarity of the densities of the matching phases cause the essential restrictions on the origination and the range of existence of the AA2DP. However, in the general case, high elastic anisotropy enlarges the range of existence of such collective excitations in solid state systems.

Till now it has been assumed that the B layer is in an unbounded medium A. A more realistic case however is the alternation of A and B layers.

Let us consider a superlattice consisting of alternating A and B layers having the thickness h_A and h_B. For a couple of neighbouring layers forming a unit cell of the superlattice, the elastic displacement for a pure shear wave polarized in the boundary plane differs from the form of (1)-(3) and has the form:

$$U_y = (B_1 \sin q_B z + B_2 \cos q_B z) e^{ik_x x - i\omega t} \qquad (25)$$

$$\text{for} \quad -\frac{h_B}{2} \leqslant z \leqslant \frac{h_B}{2},$$

$$U_y = \left[A_1 \,\mathrm{Sh}\, \varkappa_A \left(z - \frac{h_A+h_B}{2}\right) + A_2 \,\mathrm{Ch}\, \varkappa_A \left(z - \frac{h_A+h_B}{2}\right) \right] e^{ik_x x - i\omega t} \qquad (26)$$

$$\text{for} \quad \frac{h_B}{2} \leqslant z \leqslant \frac{h_B}{2} + h_A ,$$

where
$$q_B = \left(\frac{\omega^2}{S_{B\perp}^2} - K_x^2 \frac{S_{B\parallel}^2}{S_{B\perp}^2} \right)^{1/2}, \quad \varkappa_A = \left(\frac{S_{A\parallel}^2}{S_{A\perp}^2} K_x^2 - \frac{\omega^2}{S_{A\perp}^2} \right)^{1/2}, \tag{27}$$

$S_{A\perp}$, $S_{A\parallel}$, $S_{B\perp}$ and $S_{B\parallel}$ are the velocities of transverse plane waves polarized in the interface plane and propagating perpendicular and parallel to the interface in the A and B media (in Eqs. (25) and (26) we assume for definiteness $S_{A\parallel} > S_{B\parallel}$). In this case, the dispersion equations for inhomogeneous pure shear surface waves propagating along the layers are:

$$\rho_A S_{A\perp}^2 \varkappa_A \, th \frac{\varkappa_A h_A}{2} = \rho_B S_{B\perp}^2 q_B \, tg \frac{q_B h_B}{2} \tag{28}$$

- for waves symmetric in both layers A and B, the displacements (25) and (26) in which correspond to $B_1 = 0$ and $A_1 = 0$;

$$\rho_A S_{\perp A}^2 \varkappa_A \, th \frac{\varkappa_A h_A}{2} = -\rho_B S_{B\perp}^2 q_B \, ctg \frac{q_B h_B}{2} \tag{29}$$

- for waves symmetric in the A layers and antisymmetric in B layers: $B_2 = 0$ and $A_1 = 0$;

$$\rho_A S_{A\perp}^2 \varkappa_A \, cth \frac{\varkappa_A h_A}{2} = \rho_B S_{B\perp}^2 q_B \, tg \frac{q_B h_B}{2} \tag{30}$$

- for waves symmetric in B layers and antisymmetric in A layers: $B_1 = 0$ and $A_2 = 0$;

$$\rho_A S_{A\perp}^2 \varkappa_A \, cth \frac{\varkappa_A h_A}{2} = -\rho_B S_{B\perp}^2 q_B \, ctg \frac{q_B h_B}{2} \tag{31}$$

- for waves antisymmetric in both A and B layers: $B_2 = 0$ and $A_2 = 0$.

When $\varkappa_A h_A \to \infty$, formulae (27)-(30) are transformed into the dispersion equation (5) for an inhomogeneous shear waves localized near the B layer in the unbounded A medium.

Let us analyze all the variety of the waves (28)-(31) in the long-wavelength limit $K_x h_A \ll 1$, $K_x h_B \ll 1$. In this limit, from formula (28) for symmetric-symmetric waves we obtain the velocity of the bulk transverse wave propagating along the layers in an effective thin-layered medium:

$$\frac{\omega^2}{K_x^2} = \frac{\rho_A S_{A\parallel}^2 h_A + \rho_B S_{B\parallel}^2 h_B}{\rho_A h_A + \rho_B h_B} , \tag{32}$$

which in the case of isotropic layers $S_\parallel = S_\perp = S$ transforms into the expression obtained by Rytov [7, 8].

For symmetric-antisymmetric waves, Eq. (29) yields in the limit the following dispersion law:

$$\omega^2(K_x) = \frac{4\rho_B S_{B\perp}^2}{h_A h_B \rho_A} + S_{A\parallel}^2 K_x^2 , \tag{33}$$

whence such waves correspond to antiphase oscillation of neighbouring homogeneous A layers. Such oscillations are the case, when $\varkappa_A h_A \ll 1$ and $q_B h_B \ll 1$, i.e.

$$\rho_A h_A \gg \rho_B h_B , \quad \frac{\rho_A S_{A\perp}^2}{h_A} \gg \frac{\rho_B S_{B\perp}^2}{h_B} . \tag{34}$$

If these inequalities are valid, then, as is seen from (33),

$$\omega(0) \ll \frac{S_{A\perp}}{h_A} , \quad \omega(0) \ll \frac{S_{B\perp}}{h_B} ,$$

i.e. the gap $\omega(0)$ in the spectrum $\omega = \omega(K_x)$ of such optical oscillations lies much lower than the characteristic frequencies of transverse phonon quantization both in the layer A and the layer B. Similarly, it follows from Eq. (30), when the conditions are fulfilled

$$\rho_B h_B >> \rho_A h_A , \quad \frac{\rho_B S_{B\perp}^2}{h_B} >> \frac{\rho_A S_{A\perp}^2}{h_A} \quad , \tag{35}$$

which are opposite to the conditions (34), the long-wavelength antisymmetric-symmetric oscillations obey the dispersion law:

$$\omega^2(K_x) = \frac{4\rho_A S_{A\perp}^2}{h_A h_B \rho_B} + S_{B\parallel}^2 K_x^2 \ . \tag{36}$$

It describes antiphase oscillations of homogeneous neighbouring B layers.

The dispersion equations (28)-(3I) were obtained in the consideration of symmetric and antisymmetric transverse waves propagating in a layered medium along the layers. However, the same types of waves can be obtained in the study of a transverse wave propagating at an arbitrary angle to the superlattice axis. The displacement $\tilde{u}_y$ in the wave should be sought, according to the Floquet theorem, in the form:

$$\tilde{u}_y = u_y(z)\, e^{iK_z z + iK_x x - i\omega t} \ , \tag{37}$$

where K_z is the wave number along the normal to the layers and $u_y(z)=u_y(z+h)$, $h = h_A + h_B$ is the superlattice period. In particular, for any two neighbouring layers, the quantity $u_y(z)$ in (37) has the form (25) and (26) in the intervals $-h_B/2 \leq z \leq h_B/2$ and $h_B/2 \leq z \leq h_B/2 + h_A$, respectively. The dispersion equation for the transverse pure shear wave characterized by the wave numbers K_x and K_z is

$$\cos K_z h = \cos q_B h_B \, ch\, \varkappa_A h_A + \frac{1}{2}\left(\frac{\varkappa_A \rho_A S_{A\perp}^2}{q_B \rho_B S_{B\perp}^2} - \frac{q_B \rho_B S_{B\perp}^2}{\varkappa_A \rho_A S_{A\perp}^2} \right) \sin q_B h_B \, sh\, \varkappa_A h_A \, , \tag{38}$$

where the parameters q_B and $\varkappa_A$ are related as in (27). Note that in the case of isotropic layers $(S_\perp = S_\parallel = S)$, Eq. (38) transforms into the equation obtained in Ref. [I7]. From (38), when $\cos K_z h = 1$, we obtain Eqs. (28) and (3I), while when $\cos K_z h = -1$ we obtain Eqs. (29) and (30). Therefore, Eqs. (28)-(3I) specify the boundaries of the ranges of allowed frequencies of transverse waves with a fixed wave number K_x along the superlattice layers. Thus, when $K_x = 0$ the first allowed band is between 0 and $\omega_{max}^{(1)}(0) \equiv \omega^{(1)}(K_z = \pi/h, K_x = 0)$ (the superscript I indicates the number of the superlattice acoustic band). If the conditions (34) are fulfilled, we have $\omega_{max}^{(1)}(0) = 2 S_{B\perp}(\rho_B/\rho_A h_A h_B)^{1/2}$ and if the conditions (35) are fulfilled, $\omega_{max}^{(1)}(0) = 2 S_{A\perp}(\rho_A/\rho_B h_A h_B)^{1/2}$, i.e. in both the cases, all frequencies in the first allowed band lie much lower than the characteristic frequencies of the transverse quantization in both A and B layers. The boundaries of all other bands are determined in fact by the transverse quantization frequencies. Thus, for the lower boundary of the second allowed band we obtain $\omega_{min}^{(2)}(0) \equiv \omega^{(2)}(K_z = \pi/h, K_x = 0) \backsim \min\{\pi S_{A\perp}/h_A \ , \ \pi S_{B\perp}/h_B\}$, i.e. when the conditions (34) or (35) are fulfilled, the frequency $\omega_{min}^{(2)}(0)$ is indeed much higher than $\omega_{max}^{(1)}(0)$.

From Eq. (38) the velocity V of the transverse acoustic wave propagating in the superlattice along its axis in the limit $K_z h << 1$ can be found:

$$V = h\left[\frac{h_B^2}{S_{B\perp}^2} + \frac{h_A^2}{S_{A\perp}^2} + \frac{h_B h_A}{S_{B\perp} S_{A\perp}}\left(\frac{\rho_A S_{A\perp}}{\rho_B S_{B\perp}} + \frac{\rho_B S_{B\perp}}{\rho_A S_{A\perp}}\right)\right]^{-1/2}. \qquad (39)$$

When the conditions (34) are fulfilled, expression (39) transforms into the following:

$$V = S_{B\perp}\, h \sqrt{\frac{\rho_B}{\rho_A\, h_A\, h_B}} \qquad (40)$$

and under the conditions (35) we obtain

$$V = S_{A\perp}\, h \sqrt{\frac{\rho_A}{\rho_B\, h_A\, h_B}} \qquad (41)$$

As may be seen from the comparison of expressions (40) and (41) with (33) and (36) ones, the value of the frequency $\omega_{max}^{(1)}(0) \backsim V\pi/h$, i.e. the whole acoustic branch in the first folded Brillouin zone $(-\pi/h \leqslant K_Z \leqslant \pi/h)$ is defined by the low velocity V and has a low frequency. In this case, as has been mentioned, when $K_x = 0$ the acoustic band is separated from the nearest optical band by a wide forbidden gap $(\omega_{min}^{(2)}(0) \gg \omega_{max}^{(1)}(0))$.

It should be noted that the conditions (34) and (35), providing the existence of the low frequency acoustic branch which is separated from the nearest optical band by a wide forbidden gap, are fulfilled in superlattices consisting of alternating heavy (thick), rigid layers with light (thin), soft intercalations. In such a system, both transverse and longitudinal acoustic waves propagating along the superlattice axis will be slow. In the case of a superlattice formed by alternating thin layers of highly anisotropic parent and product phases having similar densities, the above situation can be realized, if layers of the parent phase A are much wider than those of the product phase B and the layers are matched so that

$$\frac{S_{A\perp}}{S_{B\perp}} = \frac{S_{B\parallel}}{S_{A\parallel}} \gg \sqrt{\frac{h_A}{h_B}} \gg 1, \quad (\rho_A = \rho_B). \qquad (42)$$

The conditions (42) are opposite to the conditions (6-7) of existence of AA2DP near a thin B layer in the present system. Therefore, the geometry of matching of A and B layers satisfying the conditions (42) differs from the above described geometry of the phases matching, which provides the fulfillment of (10) by interchange of A and B layers. Note that, as it is seen from (33) and (42), because of high elastic anisotropy of the layers the branch $\omega_{max}^{(1)}(K_x)$ has low dispersion, since its dependence on K_x is determined by the slow velocity $S_{A\parallel} \ll S_{A\perp} = S_{B\parallel}$.

Thus, we have studied the range of existence of surface waves with the dispersion law $\omega \sim K^{1/2}$ typical for the spectrum of two-dimensional plasmons with consideration for acoustic wave retardation and high elastic anisotropy of the matched phases of the solid-state A-B-A system. Besides, the situation has been analyzed where in a superlattice formed by alternating layers in the case of $K_x = 0$ there arises a low-frequency acoustic branch separated by a wide gap from the nearest optical oscillation band. The existence of such low-frequency slow acoustic branch causes the density of low-frequency oscillatory states to increase and thereby appreciably affects the low-temperature thermodynamics and kinetics of such systems [18]. Similar features of low-frequency oscillations must be the case also for complex layered compounds, whose unit cells contain not only heavy, rigid layers, but also light, soft layers. This can be expected to be characteristic of the most common quasi-two-dimensional high temperature superconductors (e.g., lanthanum, yttrium or bismuth compounds).

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