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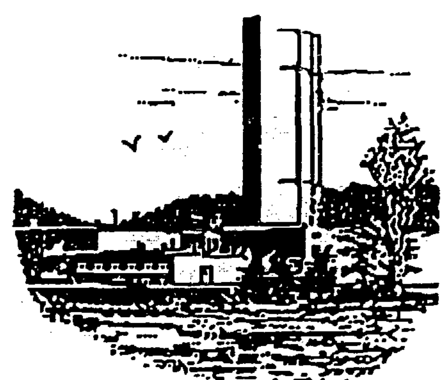

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A Neural-Network Approach to the Problem of Photon-Pair Combinatorics

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Abstract

A recursive neural-network algorithm is applied to the problem of correctly pairing photons from π^0, η, and higher resonance decays in the presence of a large background of photons resulting from many simultaneous decays. The method uses the full information of the multi-photon final state to suppress the selection of false photon pairs which arise from the many combinatorial possibilities. The method is demonstrated for simulated photon events under semirealistic experimental conditions.

I. Introduction — The Combinatorial Problem

Photon measurements provide a means to investigate a broad range of phenomena in relativistic hadron-hadron, hadron-nucleus, and nucleus-nucleus collisions. Although directly radiated photons are highly interesting, by far the dominant source of photons is from the decay of neutral pions, etas, and higher resonances. Therefore, in order to extract the direct photon component, the contribution from decays must be determined. When there is only a single meson in the final state of a reaction, such as a π^0 or η, which decays into two gammas, there can be no ambiguity to reconstruct the mass and momentum of the decaying meson. On the other hand, as soon as there are more than two photons in the final state, it becomes possible that photons from two different decays may accidentally have a calculated invariant mass which makes the false photon pair appear as an actual decay meson, leading to a possible ambiguity as to which are the true photon pairs. It is apparent that while the number of true photon pairs increases only as $n = \frac{N_\gamma}{2}$, the number of combinatorial possibilities increases as $n \cdot (N_\gamma - 1)$, and so it might be expected that this combinatorial background will become overwhelming at high gamma multiplicity N_γ. In this paper we describe a technique which uses the information carried in the multi-photon final state to select the most probable set of photon pairs, suppressing the selection of unwanted combinatorial background pairs. The selection is accomplished using a neural-network type algorithm.[1])

To illustrate the combinatorial problem more clearly we consider a semirealistic example. We imagine an experiment performed at the AGS to measure photons with a complete solid angle coverage for reactions of protons on Au at 12 GeV. We assume the photon detector to have an energy resolution typical of Pb-glass and that the detector has no low energy threshold so that the experiment has complete acceptance for all emitted photons. We have used the LUND model for nucleus-nucleus reactions, FRITIOF,[2]) to calculate the expected photon production for 10,000 minimum bias events for p + Au at 12 GeV. For these minimum bias events the average gamma multiplicity is $< N_\gamma >= 4.7$ with a multiplicity distribution extending up to $N_\gamma = 24$.

In fig. 1a the distribution of invariant masses, $M_{\gamma\gamma}$, is shown calculated using all possible gamma pairs in each event for the 12 GeV p + Au minimum bias events. The mass distribution is observed to be dominated by a smooth continuous background which results from the many photon combinations within each event. The form of this combinatorial background can be obtained by mixing photons from different events so that all resonance decays are removed in the resulting spectrum. Such mixed events have been constructed by mixing the photons from each of N_γ events of multiplicity N_γ to produce N_γ mixed events of multiplicity N_γ. This procedure gives exactly the same gamma multiplicity distribution as in the real events. The resulting combinatorial background spectrum is shown in fig. 1b. The invariant mass spectrum of the true photon pairs as calculated in FRITIOF, which one would like to extract from fig. 1a, is shown in fig. 1c. The π^0 peak at 135 MeV is seen rather clearly above the background in fig. 1a, while there is only a slight indication for the η peak at 549 MeV. In

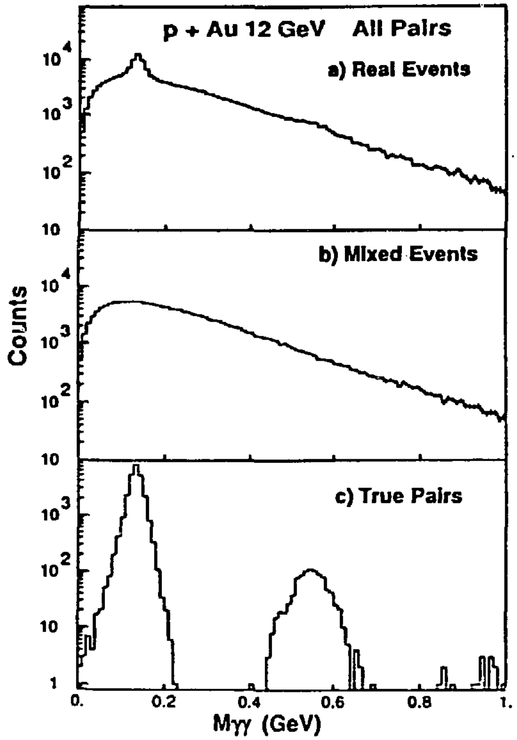

Fig. 1. The gamma-gamma invariant mass distribution for reactions of protons on Au at 12 GeV with complete photon acceptance. The invariant mass is calculated for all gamma-gamma combinations for (a) real events, and (b) mixed events with the same distribution of total gamma multiplicities as the real events. The mass distribution of the true pairs of (a) are shown in (c).

order to extract the invariant mass distribution of the true pairs, one would then subtract the appropriately normalized combinatorial background spectrum of fig. 1b from the spectrum obtained by taking all photon pairs for the real events (fig. 1a). The resulting statistical uncertainty, σ, in extracting the true yield, N, for each of the mesons is given by

$$\frac{\sigma}{N} = \sqrt{\frac{2-Q}{Q \cdot N}}, \tag{1}$$

where we have defined the quality, Q, as the peak-to-total ratio in the mass region of the π^0 or η; i.e., Q =(correct pairs)/(correct + combinatorial pairs) which may vary between 0 and 1. It is clear from this expression that, in order to minimize the uncertainty in extracting the yield, it is desirable to have a large value for the quality. Unfortunately, the quality decreases with increasing gamma multiplicity and the associated increase in the combinatorial background.

II. The Neural-Network Approach

It is clear that the only information that is carried by any single pair of photons which may be used to determine decisively whether or not they form a true pair is their invariant mass. Unfortunately, this information allows one to determine that they form a combinatorial pair only when they have an invariant mass which does not correspond to a physical particle. It, therefore, provides no assistance to identify combinatorial pairs in the mass region of the true pairs where the identification is needed. However, there is additional information in the multi-photon final state which can be used to help identify the combinatorial photon pairs. It may be realized that if photon pairs are selected with the requirement that each photon be used in only one pair and that the overall probability of the chosen pairs be maximized, then the correct pairs will be selected preferentially, even though the probabilities of the individual chosen pairs may not themselves be the largest. This suggests that the combinatorial pairs might be removed or strongly suppressed with only these two requirements: (1) each photon should only be used in one pair, and (2) the probability of the selected pairs should be maximized.

In recent years it has been found that methods modeled after a neural network provide a powerful means for solving such optimization problems and are finding applications to many problems in physics[3]). To visualize the desired approach to the photon combinatorial pair problem the neural network may be transcribed into an equivalent analog circuit. This is illustrated in fig. 2 where a neural-network circuit is applied to the problem of selecting the 3 photon pairs in a 6 photon event. In this example, each possible photon pair combination, or node, inprovides input (stimulation) to an amplifier (neuron). The output of each amplifier is interconnected recursively to the other amplifiers to produce the desired response of the system. The interconnections are illustrated in detail only for a single node in fig. 2. Here it is shown that the output from the amplifier at the node involving the pair of photons number 1 and number 6 is used to provide a positive feedback (stimulation) to itself while providing a negative feedback (inhibition) to all of the other amplifiers involving possible pairs with photon number 1 or photon number 6. Each of the other possible photon pairs, or nodes, has similar interconnections which are not shown in fig. 2. Depending on the input and the relative amount of positive and negative feedback, the amplifiers are driven into saturation either *turned on* or *turned off*. Those nodes which are turned on, indicated by the lights in fig. 2, are the finally selected photon pairs. Each possible pair of photons thus tries to reinforce itself while inhibiting the selection of any of the other possibilities involving the two photons. The network, therefore, has the desired properties: it attempts to use a given photon only in one pair combination and tries to select the set of pairs with the maximum combined probability.

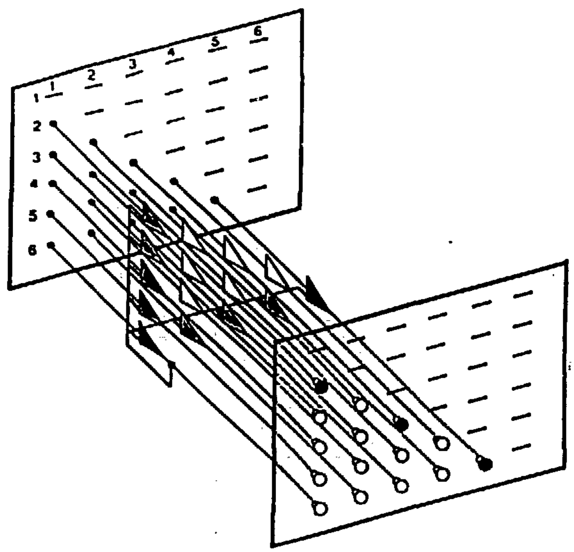

Fig. 2. A schematic view of the application of a *neural-network circuit* to the problem of selecting the 3 correct photon pairs in a 6 photon event. The circuit connections are shown in detail for only one *node* or pair of photons (1 and 6).

It is a straightforward matter to encode the neural-network circuit described above as an iterative algorithm to be used on a digital computer. The probability that the photon pair consisting of photon i and photon j is correct is given by P_{ij}^n, which corresponds to the output of the amplifiers of fig. 2 after the nth iteration. The initial probabilities P_{ij}^0 corresponding to the input, are determined from any initial information which may be used to estimate the likelihood that a given pair is correct. This information comes predominantly from the calculated invariant mass of the pair. The probability that pair ij is correct at iteration $n+1$ is given by the expression

$$P_{ij}^{n+1} = P_{ij}^{n} + g \cdot \left[f \cdot P_{ij}^{n} - \frac{1}{N_{con}} \cdot \sum_{k \neq i,j} (P_{ik}^{n} + P_{kj}^{n}) \right] , \tag{2}$$

where the parameter f is the ***relative feedback gain.*** It gives the strength of the positive self-feedback relative to the negative feedback from the other nodes. The factor $\frac{1}{N_{con}}$ normalizes the negative feedback terms by the number of nodes *connected* to node ij so that the relative feedback is essentially independent of photon multiplicity. The parameter g corresponds to the ***overall gain*** of the amplifiers and determines how quickly the probabilities saturate. The property of saturation is introduced by setting

$$P_{ij}^{n+1} = \begin{cases} 0 & if \;\; P_{ij}^{n+1} < 0 \\ 1 & if \;\; P_{ij}^{n+1} > 1 \end{cases} . \tag{3}$$

The procedure is then to iterate Eq. 2 until all probabilities have saturated either ***on*** or ***off.*** (Note: $P_{ij}^{n} = P_{ji}^{n}$ with $P_{ii}^{n} \equiv 0$).

III. Results

We now apply the neural-network algorithm of Eq. 2 to the above-considered case of photon emission in the p + Au reaction at 12 GeV with full geometrical and energy acceptance for the photons. To apply the algorithm it is necessary to provide as input the initial probability P_{ij}^{0} that the photon pair ij is a true pair. The initial probabilities are clearly a function of the invariant mass of the photon pair but also may be taken to have other dependences. For the present, we consider only the dependence on the invariant mass of the pair and calculate the initial probabilities using a simple bootstrap method which requires no prior information on where to expect the true photon pairs. The invariant mass dependence of the initial probability to be a true pair is obtained by the ratio of the invariant mass distribution using all pairs for real events to that for mixed events, that is, from the ratio of the distribution of fig. 1a to the distribution of fig. 1b. The ratio is scaled to have a maximum initial probability of 0.6 and a minimum probability of 0.1 so that the initial probabilities are not close to saturation.

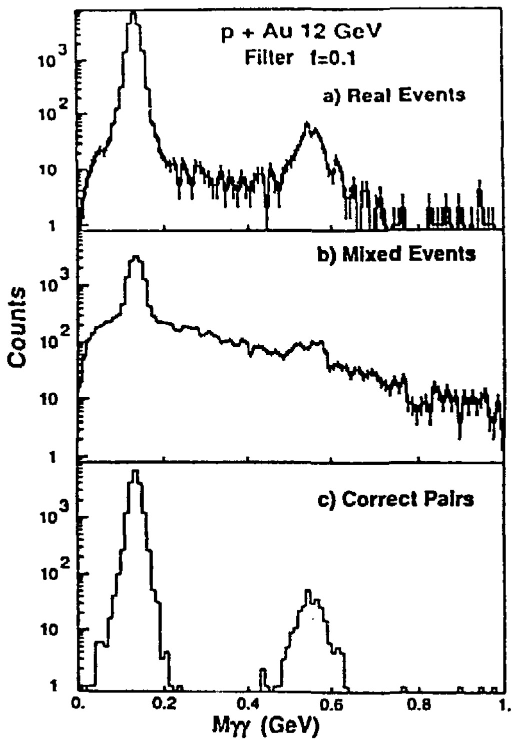

Fig. 3. The gamma-gamma invariant mass distribution for reactions of protons on Au at 12 GeV with complete photon acceptance. The invariant mass distribution is shown after filtering with the neural-net algorithm with parameters $g = 0.3$ and $f = 0.1$ for (a) real events, and (b) mixed events with the same distribution of total gamma multiplicities as the real events. The mass distribution of the true pairs of (a) are shown in (c).

The invariant mass distribution of the pairs selected by using the neural-net algorithm with initial probabilities determined with this bootstrap method with a feedback parameter of $f = 0.1$ is shown in fig. 3a. This should be compared with fig. 1a obtained by using all photon pairs. Although no information was supplied about the mass distribution of the true photon pairs, other than from the data itself, the algorithm is seen to preferentially select π^0's and η's and suppress the underlying and intervening combinatorial background. The η's are selected to some extent because of the very slight enhancement of the initial probability in the region of the η (see fig. 1a) but to a larger extent are chosen due to the fact that, after the photons which pair correctly to make π^0's are selected, the remaining photons, which pair to make an η, will be chosen simply as the only remaining possibility.

By filtering the photon pairs with the neural-net algorithm, a bias is introduced as to which pairs are selected, as expected and desired. This bias is also introduced into the combinatorial background as can be seen in fig. 3b where the neural-net algorithm has been applied to the mixed events. Rather than selecting all photon pairs to obtain a smooth combinatorial-like backgound as in fig. 1b, the algorithm preferentially selects photon pairs which have an invariant mass with the largest probability of being a true pair and hence will enhance the combinatorial background also in the region of the true pairs. Thus, it is obvious that the number of correct pairs cannot be extracted from the real events of fig. 3a by a simple subtraction of a smooth background deduced from the region outside of the mass peak of interest. Furthermore, it would not be correct to subtract a combinatorial background obtained by applying the algorithm to mixed events as in fig. 3b since the true pairs within real events have an influence on which combinatorial pairs are selected. In other words, the combinatorial background of fig. 3a is not necessarily of the same shape as the distribution of fig. 3b. The problems of how to extract the actual number of correct versus combinatorial pairs selected by the algorithm and also how to determine the efficiency or the fraction of the true pairs which pass the neural-net filter, are concerns which have been discussed elsewhere.[1]) These problems must be dealt with in order to use the algorithm to obtain absolute cross section information.

The behavior of the neural-network algorithm is dependent primarily on a single parameter[1]), the feedback gain f. At large values of the feedback gain the algorithm approaches the result obtained by taking all photon pairs. Also, for large f the efficiency or correct-to-true ratio approaches unity, as would be obtained by using all pairs. The quality is found to improve steadily as the feedback gain is decreased, but at the same time it is associated with a decrease in the efficiency for selecting the true pairs. Therefore, one must choose the feedback gain f as a compromise between increasing the quality of the selected photon pairs but at the price of decreasing the efficiency for choosing the correct pairs.

IV. Conclusion

There exists useful information in the full multi-photon event, when considered as a whole, which may be used to help suppress the selection of combinatorial background pairs — namely, that the true photon pairs use each photon in only a single pair. It was shown that by using an algorithm modeled after a recursive neural-network, it was possible to use this information to suppress the combinatorial background relative to the true photon pairs. The amount of suppression is found to vary according to a single feedback parameter. However, the background suppression is obtained at the cost of a decreased efficiency for selecting the true photon pairs. At high gamma multiplicities, this diminishing efficiency becomes the limiting consideration for application of the method described. The multiplicity limit is imposed by the increasing amount of false information from the growing number of combinatorial pairs. In order to use the neural-network algorithm to assist in the extraction of particle production cross section information, it is necessary to determine the selection efficiency of the algorithm. Whether this filter step is beneficial with its added systematic uncertainty will depend on the details of the case under consideration.

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3. See other contributions to this meeting, for example.