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MULTIAXIAL FATIGUE CRITERION FOR 2-1/4 Cr—1 Mo STEEL FOR USE IN HIGH-TEMPERATURE STRUCTURAL DESIGN

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ABSTRACT

An improved multiaxial fatigue failure criterion is described that is based on a definition of equivalent inelastic strain range incorporating the shear and normal components of inelastic strain range on the planes of maximum inelastic shear strain range. Optimum values of certain parameters contained in the formulation were obtained by the method of least squares from the results of combined axial-torsional strain cycling tests of 2-1/4 Cr—1 Mo steel conducted at 538°C (1000°F). The ability of this criterion to correlate the test results was compared with that of the Mises equivalent inelastic strain range criterion and was found to be superior.

A procedure is described for calculating the required shear and normal components of strain range under general multiaxial strain cycling conditions. An improved definition of equivalent total strain range based on these considerations is directly applicable to the method of estimating fatigue damage in ASME Code Case N-47.

1. INTRODUCTION

Several methods have been proposed to account for the interaction of creep and fatigue damage mechanisms in metals at elevated temperature. For example, variations of seven basic methods[1-7] were considered in one recent publication.[8] Summations of time and cycle fractions, together with a bilinear interaction diagram, were adopted in Appendix T of ASME Boiler and Pressure Vessel Code Case N-47 (Ref. 1), for design of certain components of nuclear power plants. The multiaxial formulation of this method, like most of the others considered in Ref. 8, is currently based on Mises definitions of equivalent normal stress and strain. Total strain is used in this method, whereas inelastic strain is used in most of the others.

An improved multiaxial creep-rupture strength criterion, based on another definition of equivalent stress, was proposed by Huddleston.[9] This criterion is currently under consideration for inclusion in Appendix T of Code Case N-47 because it was found to provide much better estimates of creep damage than the Mises equivalent stress criterion.

An improved multiaxial fatigue criterion is described in Sect. 2 of this paper. This criterion is based on a definition of equivalent inelastic strain range incorporating the shear and normal components of inelastic strain range on the planes of maximum inelastic shear strain range. The ability of this criterion to correlate the results of elevated-temperature, axial-torsional strain cycling tests of 2-1/4 Cr—1 Mo steel is compared with that of a criterion based on a Mises definition of equivalent inelastic strain range, and is found to be superior.

A procedure for calculating the required shear and normal components of strain range under general multiaxial strain cycling conditions is described in Sect. 3. The procedure is analogous to the method used to calculate equivalent total strain range in Code Case N-47. Section 4 describes the application of the improved criterion to the method of estimating fatigue damage in Code Case N-47. A summary and conclusions are contained in Sects. 5 and 6, respectively.

2. IMPROVED MULTIAXIAL FATIGUE CRITERION

Brown and Miller[10] examined multiaxial fatigue data for many materials and test conditions and concluded that crack initiation and propagation were governed by the shear and normal strains on the planes of maximum shear strain. These authors found it useful to make a distinction between two types of loading,

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based on the orientation of the planes of maximum shear strain relative to the free surface. In Case A loading, these planes are normal to the free surface, and in Case B they are at 45° to the free surface. Brown and Miller[11] conducted strain cycling tests on tubular specimens of AISI 316 and 1% Cr-Mo-V steel at room and elevated temperatures under combined axial-torsional stress. This constitutes Case A loading, with uniaxial stress being at the transition to Case B loading. To correlate the results of these tests, they used an expression of the form

$$(\Delta\gamma'/2g)^{\beta} + (\Delta\varepsilon'/e)^{\beta} = 1 \,, \qquad (1)$$

where $\Delta\gamma'$ and $\Delta\varepsilon'$ are engineering shear and normal components of strain range, respectively, on the planes of maximum shear strain range. Values of the parameters g, e, and β were determined for several values of N_f, the number of cycles to failure. For combined axial-torsional stress

$$(\Delta\gamma')^2 = (\Delta\gamma)^2 + [(1+\nu)\Delta\varepsilon]^2 \qquad (2a)$$

and

$$\Delta\varepsilon' = (1-\nu)\Delta\varepsilon/2 \,, \qquad (2b)$$

where $\Delta\gamma$ and $\Delta\varepsilon$ are engineering torsional shear and axial strain ranges, respectively, and Poisson's ratio ν typically varies from about 0.3 for small (elastic) values of $\Delta\varepsilon$ to about 0.5 for large (mostly inelastic) values of $\Delta\varepsilon$. Letting $\Delta\gamma = \Delta\gamma_0$ for $\Delta\varepsilon = 0$ and $\Delta\varepsilon = \Delta\varepsilon_0$ for $\Delta\gamma = 0$, and substituting Eq. (2) into Eq. (1), results in

$$(\Delta\gamma'/\Delta\gamma_0)^{\beta} + \{1 - [(1+\nu)\Delta\varepsilon_0/\Delta\gamma_0]^{\beta}\}[2\Delta\varepsilon'/(1-\nu)\Delta\varepsilon_0]^{\beta} = 1. \qquad (3)$$

In Appendix T of Code Case N-47, the Mises definition of equivalent total strain range is implicitly based on the inelastic value of Poisson's ratio, $\nu = 0.5$. With this value for ν, Eq. (3) may be written

$$(3\Delta\varepsilon_0/2\Delta\gamma_0)^{\beta}(2\Delta\gamma'/3\Delta\varepsilon_0)^{\beta} + [1 - (3\Delta\varepsilon_0/2\Delta\gamma_0)^{\beta}](4\Delta\varepsilon'/\Delta\varepsilon_0)^{\beta} = 1 \,, \qquad (4)$$

which is strictly applicable to inelastic components of strain.

To derive a useful expression for equivalent inelastic strain range $\overline{\Delta\varepsilon'}$ from Eq. (4) it is necessary to assume that the ratio $\Delta\gamma_0/\Delta\varepsilon_0$ and the parameter β do not depend on N_f. This idealization leads to the following equation, which is both a definition of $\overline{\Delta\varepsilon'}$ and a statement of the improved multiaxial fatigue criterion.

$$\overline{\Delta\varepsilon'} = [(2\Delta\gamma'/3)^{\beta} + (B^{\beta} - 1)(4\Delta\varepsilon')^{\beta}]^{1/\beta}/B = \Delta\varepsilon_0 \,, \qquad (5)$$

where the constant B takes the place of $2\Delta\gamma_0/3\Delta\varepsilon_0$ in Eq. (4).

By assigning appropriate values to B and β, Eq. (5) can be made to agree with several multiaxial fatigue failure criteria. A few examples are given in Table 1, including a criterion proposed by Lohr and Ellison.[12]

Brown and Miller[10] refer to graphs of $\Delta\varepsilon'$ vs $\Delta\gamma'/2$ for a given N_f as Γ-plane plots.

Table 1. Agreement of Eq. (5) with selected fatigue criteria

Fatigue criterion		B	β
Octahedral shear strain		$2/\sqrt{3}$	2
Maximum shear strain		1	1
Maximum principal strain		4/3	1
$\Delta\gamma$* (Lohr and Ellison)	Case A	2	1
	Case B	1	1

Because the parameters g, e, and β in Eq. (1) depend on N_f, the corresponding Γ-plane plots are a group of curves that vary in shape and position with N_f. By plotting the reduced values $(4\Delta\varepsilon'/\Delta\varepsilon_0)$ vs $(2\Delta\gamma'/3\Delta\varepsilon_0)$ and keeping B and β constant, a unique curve results from Eq. (5) depending only on the values chosen for B and β. Figure 1 shows such reduced Γ-plane plots for the fatigue criteria listed in Table 1, each a special case of Eq. (5), and for B = 2 with β = 2, 1, and 1/2. Note that the uniaxial stress state is represented by a point with the coordinates (1,1) and pure shear by (B,0).

To obtain values of B and β based on multiaxial fatigue test data, it is convenient to combine Eq. (5) with the Coffin-Manson Law[13,14] for inelastic strain range,

$$\Delta\varepsilon_0 = AN_f^{-\alpha} \,, \qquad (6)$$

where A and α are material parameters, to give a relationship between $\Delta\gamma'$, $\Delta\varepsilon'$, and N_f. For combined axial-torsional stress, Eq. (2) with $\nu = 0.5$ may be written

$$(2\Delta\gamma'/3)^2 = (2\Delta\gamma/3)^2 + (\Delta\varepsilon)^2 \qquad (7a)$$

and

$$4\Delta\varepsilon' = \Delta\varepsilon \,. \qquad (7b)$$

Substitution of Eqs. (6) and (7) into Eq. (5) gives

$$\overline{\Delta\varepsilon'} = \{[(2\Delta\gamma/3)^2 + (\Delta\varepsilon)^2]^{\beta/2} + (B^{\beta} - 1)(\Delta\varepsilon)^{\beta}\}^{1/\beta}/B = AN_f^{-\alpha} \,. \qquad (8)$$

Solving Eq. (8) for N_f results in

$$N_f = (AB)^{1/\alpha}/\{[(2\Delta\gamma/3)^2 + (\Delta\varepsilon)^2]^{\beta/2} + (B^{\beta} - 1)(\Delta\varepsilon)^{\beta}\}^{1/\alpha\beta} \,. \qquad (9)$$

Under these conditions, the Mises definition of equivalent inelastic strain range becomes

$$\overline{\Delta\varepsilon} = [(\Delta\gamma)^2/3 + (\Delta\varepsilon)^2]^{1/2} = AN_f^{-\alpha} \qquad (10)$$

or

$$N_f = (A)^{1/\alpha}/[(\Delta\gamma)^2/3 + (\Delta\varepsilon)^2]^{1/2\alpha} \,. \qquad (11)$$

For a given material, optimum values of A, α, B, and β can be obtained by the method of least squares, provided that suitable test data are available. The estimation of A and α can be based on uniaxial data alone. For Case A loading, however, B requires the addition of torsional data, and β requires combined

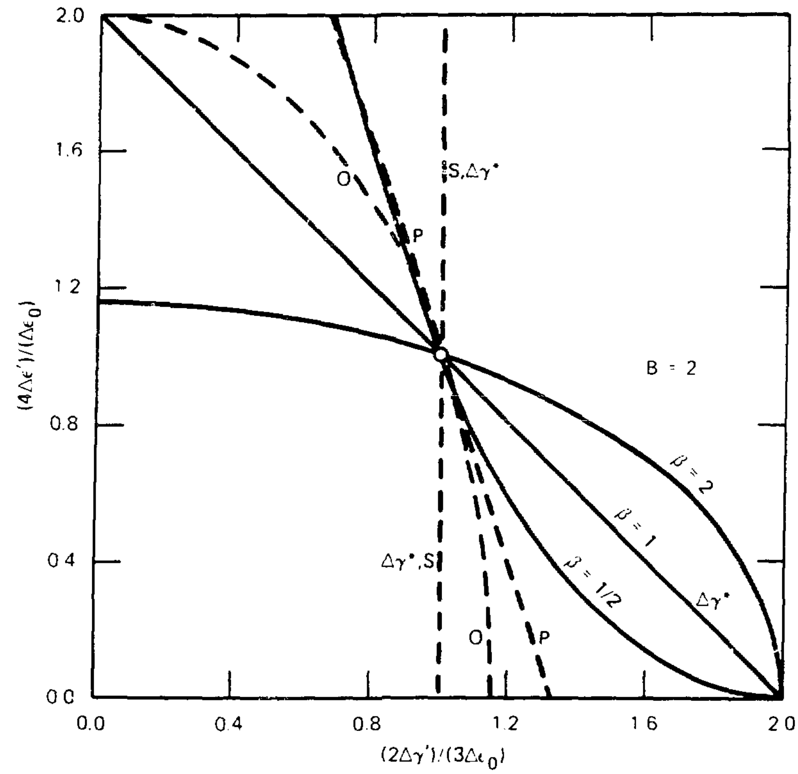

Fig. 1. Reduced Γ-plane plot of $(4\Delta\varepsilon')/(\Delta\varepsilon_0)$ vs $(2\Delta\gamma')/(3\Delta\varepsilon_0)$ for various fatigue criteria. Solid lines correspond to $\Delta\varepsilon'/\Delta\varepsilon_0 = 1$ for given values of B and β. Dashed lines correspond to simpler criteria. Curve O is for octahedral shear strain criterion, S for maximum shear strain, and P for maximum principal strain. Δγ* stands for criterion of Lohr and Ellison.

axial-torsional data. For Case B loading, pure shear obtained by 2:1 biaxial stressing takes the place of torsion.

Alternatively, A and α can be based on all of the data for Case A loading, torsional and combined axial-torsional as well as uniaxial, and B can be based on combined axial-torsional data as well as pure torsional. This approach was used in fitting Eqs. (9) and (11) to the results of axial-torsional tests of tubular specimens of 2-1/4 Cr-1 Mo steel at 538°C (1000°F) conducted at Oak Ridge National Laboratory by K. C. Liu.

By taking the logarithm, Eq. (11) can be put in a form suitable for standard linear regression analysis to obtain least-squares estimates of parameters related to A and α. However, from Eq. (9) log N_f is found to depend linearly on these parameters and nonlinearly on B and β. This class of problems is known as separable least-squares, and the most efficient method of solution is the variable projection method.[15] In obtaining estimates of A, α, B, and β for Eq. (9) and estimates of A and α for Eq. (11), individual observations were weighted so that the axial, torsional, and combined axial-torsional subsets of the data would all have the same influence on the residual sum of squares. The estimates are listed in Table 2, along with a scatter factor F. This factor is defined so that 95% of the observed lives N_f fall within the range $\hat{N}_f/F < N_f < \hat{N}_f F$, where $\hat{N}_f$ is the life estimated by either Eq. (9) or Eq. (11). In other words, this range corresponds to ±2 standard errors on log of life. Examination of the values of F contained in Table 2 reveals that the scatter about the expected value of N_f for Eq. (9) is less than half the scatter for Eq. (11).

Table 2. Results of least-squares fits of Eqs. (9) and (11) to axial-torsional (Case A) fatigue data for 2-1/4 Cr-1 Mo steel at 538°C (1000°F)

Parameter	Eq. (9)	Eq. (11)
A	263.3	742.6
α	0.8709	0.9276
B	4.160	
β	2.450	
F	2.32	4.67

Results of the exploratory tests used in this study are shown in Figs. 2 and 3 as log-log plots of $\Delta\varepsilon'$ and $\Delta\varepsilon$, respectively, vs N_f. In Fig. 2, the solid line corresponds to Eq. (9), and in Fig. 3 it corresponds to Eq. (11), with the parameter values in Table 2. The scatter about these lines associated with the factor F is indicated by dashed lines. Again the improvement in data correlation due to inclusion in the model of fitted parameters B and β is evident.

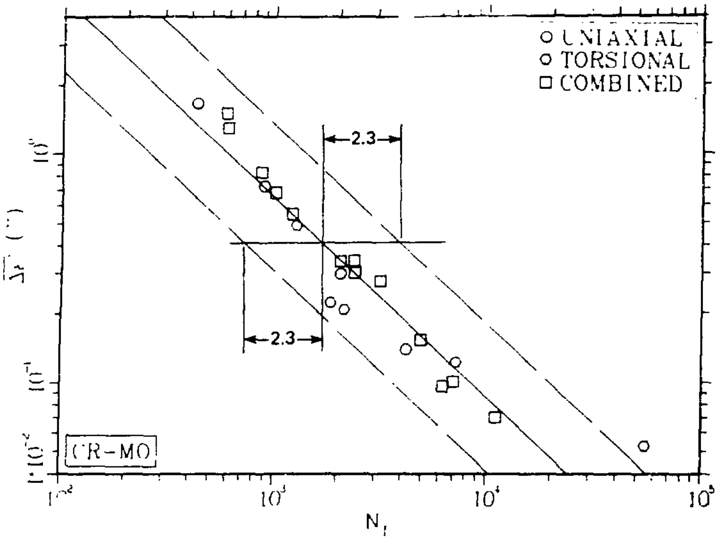

Fig. 2. Plot of $\overline{\Delta\varepsilon'} = [(2\Delta\gamma'/3)^\beta + (B^\beta - 1)(4\Delta\varepsilon')^\beta]^{1/\beta}/B$ vs N_f for axial-torsional strain cycling tests of 2-1/4 Cr—1 Mo steel at 538°C conducted by K. C. Liu.

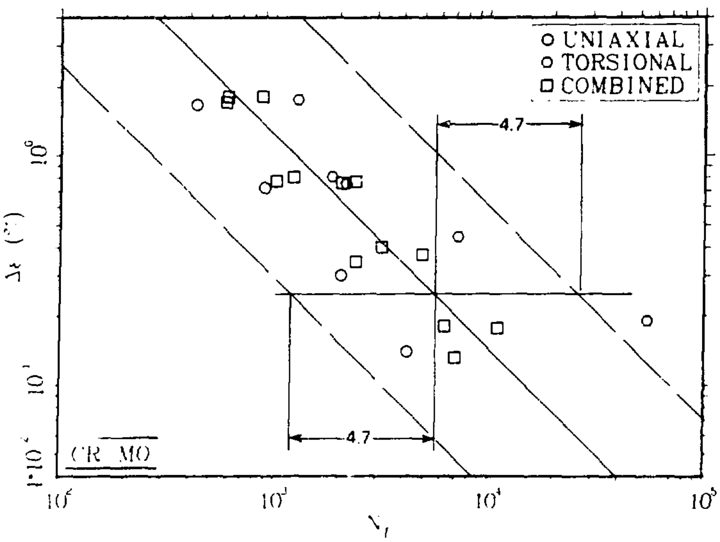

Fig. 3. Plot of (Mises) $\overline{\Delta\varepsilon}$ vs N_f for axial-torsional strain cycling tests of 2-1/4 Cr—1 Mo steel at 538°C conducted by K. C. Liu.

The test results are plotted on reduced Γ coordinates in Figs. 4 and 5. In Fig. 4, A and α from Table 2 for Eq. (9) were used in the Coffin-Manson Law, Eq. (6), to calculate the uniaxial strain range $\Delta\varepsilon_0'$ corresponding to the observed value of N_f in each test. In Fig. 5, A and α from Table 2 for Eq. (11) were used to calculate $\Delta\varepsilon_0$. Similarly, in Fig. 4 the solid curve represents Eq. (5) with the fitted values of B and β from Table 2, and in Fig. 5 it represents Eq. (5) with the fixed values of B and β corresponding to the octahedral shear criterion in Table 1. The solid curves in Figs. 4 and 5 pass through the points with coordinates (1,1) and (B,0) as they are constrained to do. However, the test data corresponding to uniaxial stress are spread out along the straight line passing through (0,0) and (1,1), and those corresponding to pure shear are spread out along the abscissa. The combined axial-torsional data tend to lie in radial bands. All of this variability is due both to the inherent scatter in N_f and to the lack of perfect agreement with the assumptions made concerning the values of B and β. Once again, there is noticeably better agreement between the data and curves for Eq. (9) than that for Eq. (11). In addition, by comparing values of B and β in Table 2 with those in Table 1, and the shape of the curve in Fig. 4 with that

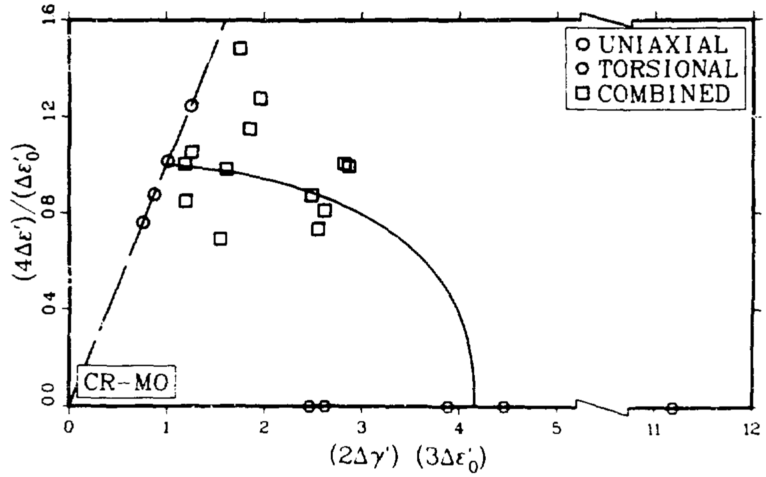

Fig. 4. Reduced Γ-plane plot for 2-1/4 Cr–1 Mo steel at 538°C. Values of A and α from Eq. (9) used to estimate inelastic axial strain range $\Delta\epsilon_0'$ for observed values of N_f.

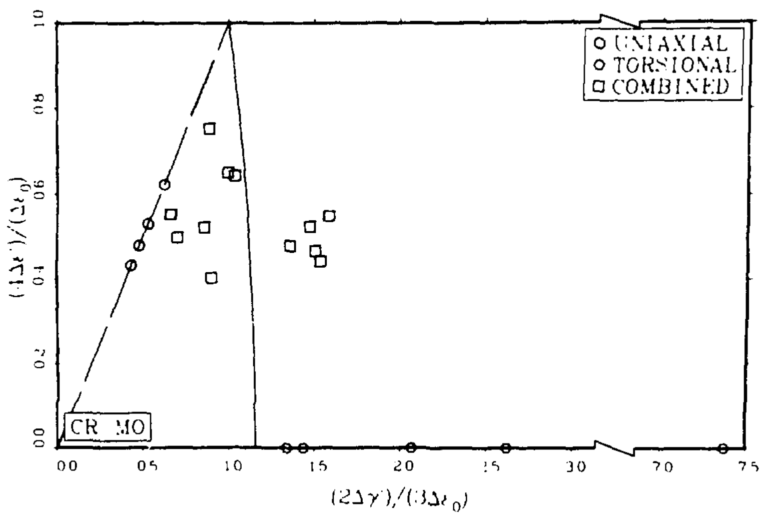

Fig. 5. Reduced Γ-plane plot for 2-1/4 Cr–1 Mo steel at 538°C. Values of A and α from Eq. (11) used to estimate inelastic axial strain range $\Delta\epsilon_0$ for observed values of N_f.

in Fig. 5, one may reasonably infer that the improved criterion of Eq. (5) provides better estimates of fatigue life than each of the criteria listed in Table 1.

In these experiments, the planes of maximum inelastic shear strain range were normal to the free surface of the specimen (Case A loading). Different apparatus is required to conduct experiments with the planes at 45° to the free surface (Case B loading); however, the available evidence[10] suggests that B and β may each be about unity.

3. CALCULATION OF SHEAR AND NORMAL COMPONENTS OF STRAIN RANGE ON THE PLANES OF MAXIMUM SHEAR STRAIN RANGE

To apply the fatigue criterion given by Eq. (5) in realistic design situations, a general procedure is required for calculating the shear $\Delta\gamma'$ and normal $\Delta\epsilon'$ components of strain range on the planes of maximum shear strain range. This section provides such a procedure.

The simple relationships used to calculate $\Delta\gamma'$ and $\Delta\varepsilon'$ for the in-phase, axial-torsional tests considered in Sect. 2 were given in Eq. (7). Formulae for variable-phase, sinusoidal histories of axial and torsional strain were presented in Ref. 16. For more general multiaxial cycling conditions, a double maximization procedure similar to one found in Appendix T of Code Case N-47 (Ref. 1) can be used to calculate $\Delta\gamma'$ and $\Delta\varepsilon'$. As outlined below, expressions containing differences in components of strain over a time interval are to be maximized with respect to both the beginning and the end of the time interval. The expressions themselves were derived from the well-known coordinate transformation rules of Continuum Mechanics.

Fatigue cracking usually originates at a free surface. If the components of strain at a point on the free surface are defined with respect to a Cartesian coordinate system oriented with its z-axis normal to the surface, then $\gamma_{xz} = \gamma_{yz} = 0$ and

$$\Delta\gamma' = \max_{m,n} (\gamma_3^{mn}, \gamma_2^{mn}, \gamma_1^{mn}) , \tag{12}$$

where

$$\gamma_3^{mn} = [(\varepsilon_x^{mn} - \varepsilon_y^{mn})^2 + (\gamma_{xy}^{mn})^2]^{1/2} \tag{13a}$$

and

$$\gamma_2^{mn}, \gamma_1^{mn} = \frac{1}{2}\left|\varepsilon_x^{mn} + \varepsilon_y^{mn} - 2\varepsilon_z^{mn} \pm \gamma_3^{mn}\right| . \tag{13b}$$

In Eq. (13), $\varepsilon_x^{mn} = \varepsilon_x(t_n) - \varepsilon_x(t_m)$, etc., where $\varepsilon_x(t_n)$, for instance, is the normal strain in the x-direction at time t_n. With t_M and t_N denoting the times corresponding to the maximum in Eq. (12), the planes of maximum shear strain range are normal to the free surface (Case A loading) if γ_3^{MN} is the largest of the three principal shear strain differences γ_3^{MN}, γ_2^{MN}, and γ_1^{MN}, and the planes of maximum shear strain range are at 45° to the free surface (Case B loading) if γ_2^{MN} or γ_1^{MN} is the largest. For Case A loading,

$$\Delta\varepsilon' = \max_{k,\ell} \frac{1}{2}\{\varepsilon_x^{k\ell} + \varepsilon_y^{k\ell} \pm [\gamma_{xy}^{k\ell}(\varepsilon_x^{MN} - \varepsilon_y^{MN}) - (\varepsilon_x^{k\ell} - \varepsilon_y^{k\ell})\gamma_{xy}^{MN}]/\gamma_3^{MN}\} , \tag{14}$$

where the ± signs correspond to mutually perpendicular planes. For Case B loading,

$$\Delta\varepsilon' = \max_{k,\ell} \frac{1}{4}\{\varepsilon_x^{k\ell} + \varepsilon_y^{k\ell} + 2\varepsilon_z^{k\ell} \pm [(\varepsilon_x^{k\ell} - \varepsilon_y^{k\ell})(\varepsilon_x^{MN} - \varepsilon_y^{MN}) + \gamma_{xy}^{k\ell}\gamma_{xy}^{MN}]/\gamma_3^{MN}\} , \tag{15}$$

where the correct sign to use is known from Eq. (13b). If the times corresponding to the maximum in Eq. (14) or (15) are denoted t_K and t_L, then for in-phase cycling $t_K = t_M$, $t_L = t_N$, and the term with the ± sign is equal to zero in Eq. (14) and to γ_3^{MN} in Eq. (15).

These equations can be simplified by substituting $-\varepsilon_z$ for the term $\varepsilon_x + \varepsilon_y$, which follows from the assumption of constant volume inelastic straining. However, in their present form these equations are also valid with the corresponding components of total strain for use in a fatigue criterion based on equivalent total strain range.

4. APPLICATION OF IMPROVED CRITERION TO CODE CASE N-47

The improved definition of equivalent inelastic strain range given in Eq. (5) incorporates the shear $\Delta\gamma'$ and normal $\Delta\varepsilon'$ components of inelastic strain range on the planes of *maximum* inelastic shear strain range. With suitable values of the parameters B and β, much better correlation of multiaxial fatigue data was obtained with this definition than with the corresponding Mises definition.

This improved definition can be made fully compatible with the total strain approach currently employed in Code Case N-47 simply by replacing $\Delta\gamma'$ and $\Delta\varepsilon'$ with the corresponding components of total strain range. A procedure for obtaining optimum values of B and β based on total-strain data would differ somewhat from the procedure described in Sect. 2 for inelastic-strain data, because a more complicated strain vs life relationship than Eq. (6) would be required. An approach used previously by the author, involving an efficient numerical inversion of the Basquin/Coffin-Manson Law[17,13,14] for total strain range,

$$\Delta\varepsilon_0 = A_1 N_f^{-\alpha_1} + A_2 N_f^{-\alpha_2} , \tag{16}$$

could be readily adapted for this purpose, however.

The procedure described in Sect. 3 for calculating $\Delta\gamma'$ and $\Delta\varepsilon'$ under general multiaxial cycling conditions can also be followed for total strain components and is analogous to the procedure for calculating Mises equivalent total strain range in Appendix T of Code Case N-47. Thus, the improved multiaxial fatigue criterion is considered directly applicable to the method of estimating fatigue damage in Code Case N-47, provided suitable design values are determined for B and β. Additional multiaxial testing is needed to provide a suitable data base for the determination of these design values and to assess the validity of this approach in concert with the improved criterion for estimating creep damage.[9]

5. SUMMARY

Summation of time and cycle fractions is the method used in ASME Code Case N-47 (Ref. 1) for estimating creep and fatigue damage. The multiaxial formulation of this method is currently based on Mises definitions of equivalent stress and equivalent total strain range. An improved multiaxial fatigue failure criterion based on a more general definition of equivalent inelastic strain range $\overline{\Delta\varepsilon'}$ was described in Sect. 2. This definition, given in Eq. (5), is based on the work of Brown and Miller[10,11] and involves the shear $\Delta\gamma'$ and normal $\Delta\varepsilon'$ components of inelastic strain range on the planes of maximum inelastic shear strain range.

The improved criterion can be made to agree with a number of other fatigue criteria by assigning appropriate values to its parameters B and β (Table 1). For example, with $B = 2/\sqrt{3}$ and $\beta = 2$, the improved criterion is the same as the one based on the Mises definition of equivalent inelastic strain range $\overline{\Delta\varepsilon}$. Using empirically determined values of B and β (Table 2), much better correlation was obtained with $\overline{\Delta\varepsilon}'$ than with $\overline{\Delta\varepsilon}$ of the results of axial-torsional strain cycling tests of 2-1/4 Cr-1 Mo steel at 538°C (1000°F). In these experiments, the planes of maximum inelastic shear strain range were normal to the free surface of the specimen (Case A loading). Different apparatus is required to conduct experiments with the planes at 45° to the free surface (Case B loading); however, the available evidence[10] suggests that B and β may each be about unity.

In Sect. 3, a procedure was described for calculating the inelastic strain range components $\Delta\gamma'$ and $\Delta\varepsilon'$ under general multiaxial strain cycling conditions. The same procedure can be followed for total strain range components and is similar to the procedure for calculating Mises equivalent total strain range in Code Case N-47.

As pointed out in Sect. 4, the improved fatigue criterion is fully compatible with the method of estimating fatigue in Code Case N-47 provided that components of total strain range rather than inelastic strain range are used in Eq. (5). Development of a suitable procedure for obtaining optimum values of B and β based on total-strain data should present little difficulty.

6. CONCLUSIONS

1. For axial-torsional strain cycling tests of 2-1/4 Cr-1 Mo steel at 538°C (1000°F), the definition of equivalent inelastic strain range given in Eq. (5), with empirically determined values for the parameters B and β, provides much more precise estimates of fatigue damage under multiaxial stress and strain conditions than the Mises definition of equivalent inelastic strain range.
2. An improved definition of equivalent total strain range, also given by Eq. (5), is directly applicable to the method of estimating fatigue damage in ASME Code Case N-47.
3. Assessment of the validity of this approach requires further multiaxial creep-fatigue testing.

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