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Transition Crossing in the Main Injector

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Abstract

This report summarizes the study of various longitudinal problems pertaining to the transition-energy crossing in the proposed Fermi Lab Main Injector. The theory indicates that the beam loss and bunch-area growth are mainly caused by the chromatic non-linear effect, which is enhanced by the space-charge force near transition. Computer simulation using the program TIBETAN shows that a "γ_T jump" of about 1.5 unit within 1 ms is adequate to achieve a "clean" crossing in the currently proposed h=588 scenario.

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1. Introduction

The transition-energy crossing is characterized by a time scale T_c during which the particle motion is non-adiabatic,[1–4]

$$T_c = \left(\frac{\pi E_s \beta_s^2 \gamma_T^3}{qe\hat{V}|\cos\phi_s|\dot{\gamma}_s h\omega_s^2} \right)^{\frac{1}{3}}, \tag{1}$$

where the subscript s represents the synchronous particle, γ_T is the transition energy, h is the harmonic number, and ϕ_s, ω_s, $\beta_s c$, $E_s = m_0 c^2 \gamma_s$, and $\dot{\gamma}_s$ are the synchronous phase, revolution frequency, velocity, energy, and acceleration rate, respectively.

Problems related to transition crossing can mainly be divided into two categories: single- and multi-particle. In the former category, we study the effect of chromatic non-linearities which impel particles of different momenta to cross transition at different times; while in the latter, we study the bunch-shape mismatch and microwave instability induced by low- and high-frequency self fields, respectively. Theoretical estimates are presented in the first part of section 2; results of computer simulation are addressed in the second part. Compensation methods and requirements are discussed in section 3.

2. Problems at Transition Energy

A. Theoretical Estimates

Chromatic non-linear effect

Particles of different momenta traverse closed orbits of different lengths L. The difference may be expressed in terms of the momentum deviation ($\delta \equiv \Delta p/p$) as

$$\frac{L}{L_s} = 1 + \frac{\delta}{\gamma_T^2}\left(1 + \alpha_1 \delta + O(\delta^2)\right). \tag{2}$$

The so-called "frequency-slip factor" η can thus be written as

$$\eta = \eta_0 + \eta_1 \delta + \cdots,$$

where

$$\eta_0 = \frac{1}{\gamma_T^2} - \frac{1}{\gamma_s^2}, \quad \text{and} \quad \eta_1 \approx \frac{2}{\gamma_s^2}\left(\alpha_1 + \frac{3\beta_s^2}{2}\right).$$

The two terms in η_1 correspond respectively to the differences in circumference and velocity for particles of different momenta at the first non-linear order. The effect of η_1 on the particle motion is important only near the transition energy ($\gamma_s = \gamma_T$) when η_0 approaches zero. Define[4-6] the "non-linear time" T_{nl} during which $\left|\eta_1\hat{\delta}(0)\right|$ is larger than $|\eta_0|$,

$$T_{nl} = \frac{|(\alpha_1 + \frac{3}{2}\beta_s^2)\hat{\delta}(0)\ \gamma_T|}{\dot{\gamma}_s}, \tag{3}$$

where

$$\hat{\delta}(0) = \frac{2^{1/2}\omega_s\left(hAqe\hat{V}|\cos\phi_s|T_c\right)^{1/2}}{3^{2/3}\pi^{1/2}\Gamma(2/3)E_s\beta_s^2}$$

is the maximum momentum spread at transition, $\Gamma(2/3) \approx 1.354$, and A is the bunch area before transition. The effective increase in the bunch area during the crossing depends on the ratio of T_{nl} to T_c (eqs. 4.25 and 4.27 in ref.4),

$$\frac{\Delta A}{A} \approx \begin{cases} 0.76\,\dfrac{T_{nl}}{T_c}, & \text{for } T_{nl} \ll T_c; \\ e^{\frac{4}{3}\left(\frac{T_{nl}}{T_c}\right)^{3/2}} - 1, & \text{for } T_{nl} \geq T_c. \end{cases} \tag{4}$$

Beam loss occurs if the effective bunch area $A + \Delta A$ after transition is larger than the bucket area.

It is assumed[7] for the Main Injector that $\gamma_T = 20.4$, $\omega_s = 566.78\text{s}^{-1}$, $h = 588$, $\hat{V} = 2.78\text{MV}$, $\phi_s = 37.6°$ and $\alpha_1 = 1$. With these parameters, it can be shown that $T_c \approx 2.0\text{ms}$, $\hat{\delta}(0) \approx 8.5 \times 10^{-3}$, and $T_{nl} \approx 2.7\text{ms}$. According to eq. 4, the phase-space area occupied by particles near the edge of the bunch is much larger than the bucket area after transition. Therefore, beam loss is expected to occur. Quantitatively, the amount of beam loss depends on the particle distribution in the phase space.

Bunch-shape mismatch

Both reactive and resistive impedances cause mismatch[8-9] in the nominal bunch shape at the time the synchronous phase is switched at transition. A reactive impedance changes the focusing force of the rf system differently below and above transition. The amount of mismatch is then proportional to the ratio of the self field to the rf field provided by the accelerating cavities. In addition to the mismatch, a resistive impedance causes energy dissipation which compensates part of the rf acceleration. Because this compensation induces a shift in the synchronous phase (ϕ_s), the amount of synchronous phase ($\pi - 2\phi_s$) to be switched at transition is changed accordingly.

Quantitatively, the amount of mismatch again depends on the particle distribution in phase space. For a parabolic distribution, the effective increase in the bunch area due to the mismatch, induced by a coupling impedance $|Z_L/n|$ at low frequency range, is (eq. 5.18 in ref.4)

$$\frac{\Delta A}{A} = \frac{2h\hat{I}\,|Z_L/n|}{\hat{V}|\cos\phi_s|\,\hat{\phi}^2(0)}, \tag{5}$$

where

$$\hat{I} = \frac{3hN_0qe\omega_s}{4\hat{\phi}(0)}$$

is the peak current, and

$$\hat{\phi}(0) = 3^{1/6}\Gamma(2/3)\left(\frac{2hA}{\pi qe\hat{V}|\cos\phi_s|T_c}\right)^{1/2}$$

is the maximum phase spread of the bunch at transition. If the resistive impedance at low frequency is $\mathcal{R}$, the shift in synchronous phase can also be shown as (eq. 5.22 in ref.4)

$$\Delta\phi_s = \frac{\hat{I}\mathcal{R}}{\hat{V}|\cos\phi_s|} \tag{6}$$

for $\Delta\phi_s$ much smaller than 1.

The effective impedance of the space charge below the cutoff frequency is

$$\frac{Z}{n} = \frac{ig_0Z_0}{2\beta_s\gamma_s^2},$$

where g_0 is a geometric factor, and $Z_0 = (\varepsilon_0 c)^{-1} = 377\Omega$. Taking $g_0 = 4.5$, this corresponds to a capacitive impedance of about 2Ω at transition. With an intensity of $N_0 = 6 \times 10^{10}$ per bunch, the increase of bunch area due to the corresponding bunch-shape mismatch is about 10%.

Microwave instability

Near the transition energy, the frequency spread which provides Landau damping vanishes along with the vanishing phase stability and the decreasing synchrotron-oscillation frequency. Both the reactive and the resistive components of the coupling are likely to induce instability. However, since particles cross transition with a non-zero acceleration rate, the synchrotron-oscillation frequency defined by the time derivative of the angle variable (canonically conjugate with the action variable J) of the system Hamiltonian, is also non-zero at transition. Consequently, the threshold for microwave instability to occur at transition becomes (eq. 5.58 in ref.4) for the parabolic distribution

$$\frac{8h\hat{I}\,|Z_H/n|}{3\hat{V}|\cos\phi_s|\,\hat{\phi}^2(0)} \geq 1, \tag{7}$$

where $|Z_H/n|$ refers to the coupling impedance at microwave frequency range. Again, the coefficient in eq. 7 may differ for different particle distribution.

With a beam intensity of 6×10^{10} per bunch, the threshold impedance is of the order of 10Ω. Since the space-charge impedance is only 2Ω, it is not likely to induce microwave instability at transition.

The theoretical estimates indicate that the primary concern at transition is due to the chromatic non-linear effect. The development of the "non-linear tails" after the synchronous-phase switch over is further enhanced by the space-charge force. Computer simulation is needed to understand more precisely the various mechanisms and to quantitatively determine the crossing efficiency.

B. Results of computer simulation

The computer program TIBETAN was originally developed in the Brookhaven National Laboratory to study the transition process in the proposed RHIC collider. The program simulates the longitudinal motion of a particle beam by tracking a collection of macro-particles in phase space. It constructs the self fields directly in the time (phase) domain. The bin length used for the construction of the self fields is chosen in accordance with the cutoff frequency of the vacuum pipe.

This program has been used to simulate the transition process in the Main Injector. Beam-induced fields are calculated every turn with 3600 macro-particles. Before transition, the bunch is assumed to have a Gaussian-like distribution in longitudinal phase space with a 95% area of 0.4eV·s. Fig. 1 shows the phase-space diagram after transition at γ_s=22. With $\alpha_1 = 1$, the chromatic non-linear effect results a beam loss of about 15%. For a bunch with 6×10^{10} protons, the enhancement of the beam loss due to longitudinal space charge is less than 1%.

The transition-crossing efficiency is defined as the ratio of the total number of particles inside the rf bucket when the synchronous energy is far above the transition energy, to the one when it is far below. The solid line in fig.2 shows the crossing efficiency as a function of the bunch area before transition. With a smaller bunch area, beam loss due to the non-linear effect is reduced. However, the effect of beam-induced fields becomes more important.

3. Compensation Methods

An effective way to cure both the beam-induced and the chromatic non-linear effect is to increase the transition-crossing rate of the beam. This can be accomplished either by temporarily adjusting the lattice to achieve a "γ_T jump", or by manipulating the synchronous phase and the voltage of the accelerating rf system.

A. Crossing by an acceleration rate increase

Particle motion is non-adiabatic during the transition time T_c. If the magnetic-field ramping rate can be adjusted or the momentum aperture is adequate at transition, the synchronous phase may be switched from ϕ_s to 90° during this period to temporarily increase the acceleration rate.

The dash line in fig 2 indicates the crossing efficiency achieved by this technique. The synchronous phase is switched from 37.6° to 90° for 4 ms at transition. Effectively, $\dot{\gamma}_s$ is increased by a factor of 1.7. With the bunch area 0.4eV·s, the beam loss is reduced from

15% to 5%.

B. Crossing by a "γ_T jump"

Compared with the method addressed above, "γ_T jump" often provides a larger crossing-rate enhancement without causing severe mismatch at transition. In the case that the non-linear effect is dominating, the amount of $\Delta\gamma_T$ needed to eliminate the un-desired beam loss and bunch-area growth is

$$\Delta\gamma_T \sim 2\ \dot{\gamma}_s\ (2T_{nl}) \sim 1.5,$$

with both $\dot{\gamma}_s$ and T_{nl} taking the original values. Fig. 3 shows the phase-space diagram after transition ($\gamma_s = 22$) with a γ_T jump of 1.5 unit in 1ms. The crossing rate is enhanced by a factor of 9. The crossing efficiency is 100%, while the bunch-area growth is negligible.

For a given α_1, the amount of $\Delta\gamma_T$ needed is proportional to the momentum spread and, therefore, the square root of the bunch area. This relation is shown in fig. 4.

Typically, the γ_T jump should be centered at the moment that the synchronous phase is switched at transition. However, in the case that the space-charge effect is appreciable, the "non-linear tails" are enhanced after this moment. Therefore, jumping with the center shifted after the transition center (say, by 0.4ms) might be helpful if the desired amount of $\Delta\gamma_T$ is not achievable.

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FIGURE CAPTIONS

Fig. 1. Longitudinal phase-space diagram of a proton bunch after transition at $\gamma_s = 22$. The crossing efficiency is about 85%.

Fig. 2. Transition-crossing efficiency as a function of the bunch area before transition.

Fig. 3. Similar to fig. 1, but with a γ_T jump of 1.5 unit in 1ms. The crossing efficiency is 100%.

Fig. 4. The amount of $\Delta\gamma_T$ needed to eliminate the un-desired effects at transition.

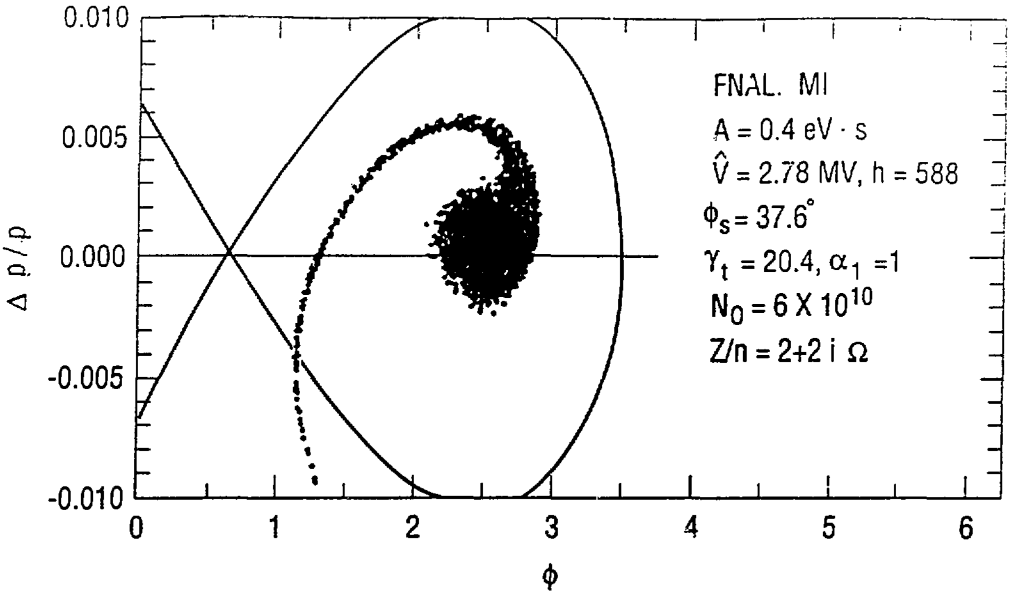

FNAL. MI
A = 0.4 eV · s
V̂ = 2.78 MV, h = 588
ϕs = 37.6°
γt = 20.4, α1 = 1
N0 = 6 X 10^10
Z/n = 2+2 i Ω
Δ p / p
ϕ

Fig. 1.

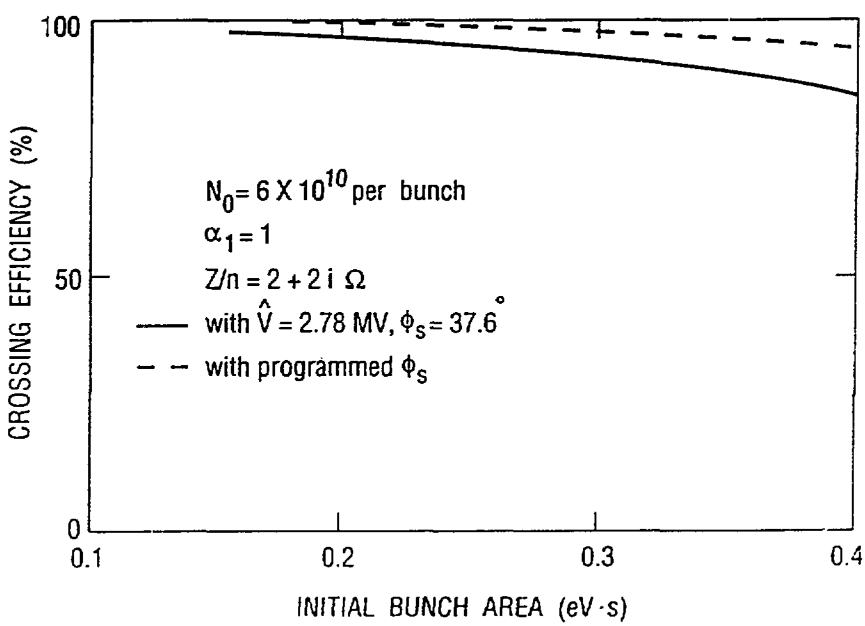
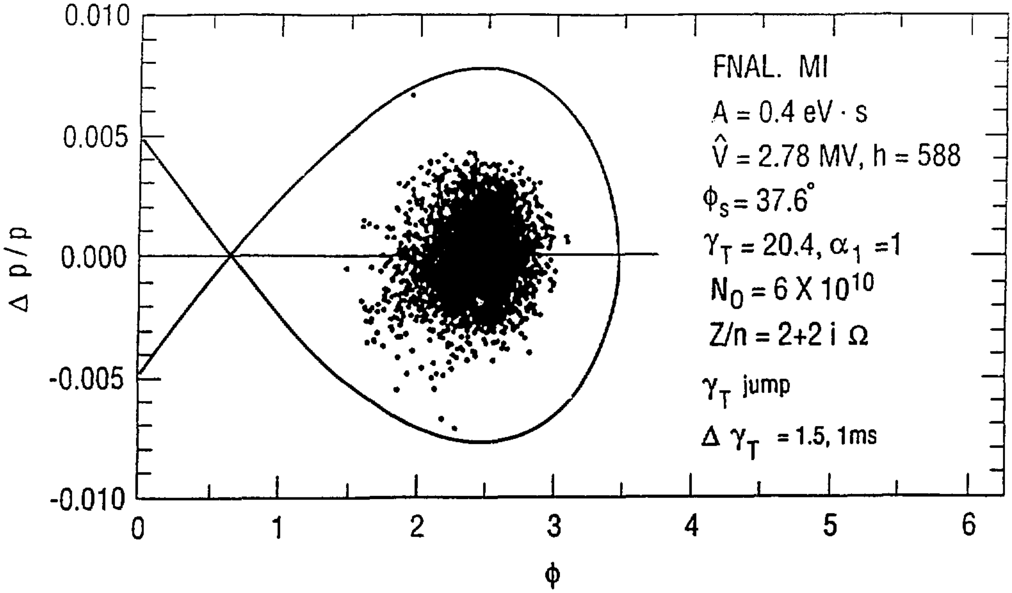
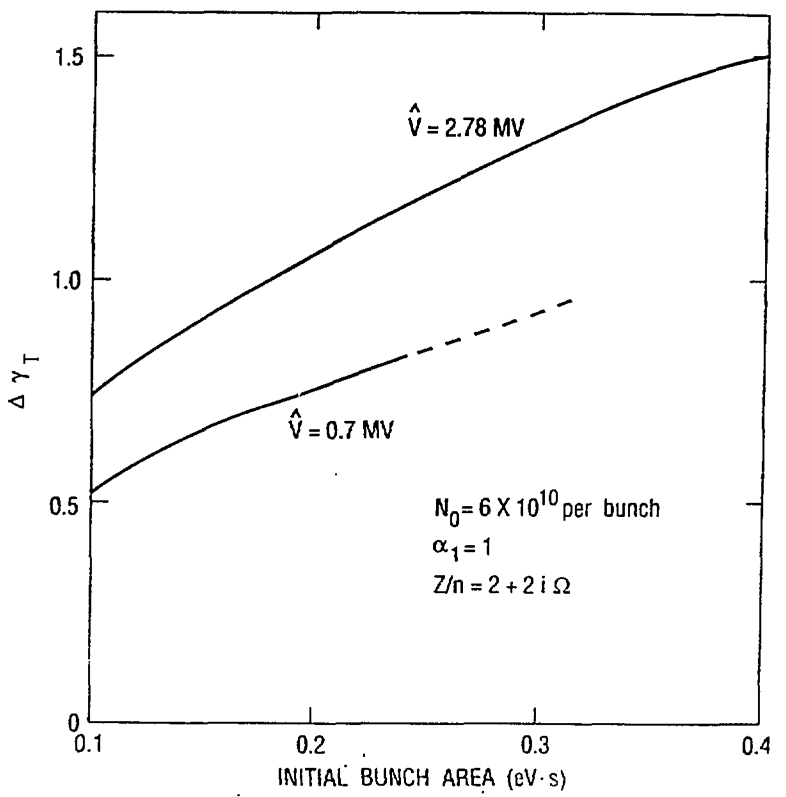

N0 = 6 X 10^10 per bunch
α1 = 1
Z/n = 2 + 2 i Ω
with V̂ = 2.78 MV, ϕs = 37.6°
with programmed ϕs
CROSSING EFFICIENCY (%)
INITIAL BUNCH AREA (eV·s)

Fig. 2.



0.010
0.005
0.000
-0.005
-0.010
Δ p / p
0
1
2
3
4
5
6
φ
FNAL. MI
A = 0.4 eV · s
V̂ = 2.78 MV, h = 588
φs = 37.6°
γT = 20.4, α1 = 1
N0 = 6 X 10^10
Z/n = 2+2 i Ω
γT jump
Δ γT = 1.5, 1ms

Fig. 3.



1.5
1.0
0.5
0
Δ γT
0.1
0.2
0.3
0.4
INITIAL BUNCH AREA (eV·s)
V̂ = 2.78 MV
V̂ = 0.7 MV
N0 = 6 X 10^10 per bunch
α1 = 1
Z/n = 2 + 2 i Ω

Fig. 4.