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THE ION-CHANNEL LASER

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A relativistic electron beam propagating through a plasma in the ion-focussed regime exhibits an electromagnetic instability at a resonant frequency $\omega \sim 2\gamma^2 \omega_\beta$. Growth is enhanced by optical guiding in the ion channel, which acts as a dielectric waveguide, with fiber parameter $V \sim 2\,(I/I_A)^{1/2}$. A 1-D theory for such an "ion-channel laser" is formulated, scaling laws are derived and numerical examples are given. Possible experimental evidence is noted.

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The transport of relativistic electron beams (REBs) in plasmas has a long history,[1] and over the last ten years the mechanism of ion-focussing has been developed and successfully employed in accelerator work.[2] In addition, the propagation in plasmas of short pulse, low emittance REBs has attracted interest in connection with the plasma lens,[3] the continuous plasma focus,[4] the plasma wakefield accelerator,[5] and the beat wave accelerator.[6]

At the same time, coherent radiation from REBs has been the subject of extensive work, in connection with the Free Electron Laser (FEL)[7] the Cyclotron Auto-Resonant Maser (CARM),[8] and other free electron devices.[9]

Of particular interest here is experimental evidence of coherent radiation from intense, relativistic electron beams in unmagnetized plasmas.[10,11] Explanations offered for the high power levels observed have included streaming instabilities, strong-turbulence, and virtual cathode oscillations, among others. Kato *et al.*,[10] remark on the possibility of an FEL analogy based on jitter motion in "large-amplitude electrostatic waves generated by instability"; however, to date, no satisfactory theory has been set down to explain the power levels observed.

In this Letter, we develop the theory of an ion channel "free-electron" laser (ICL), consisting of an intense, relativistic beam of electrons injected into a plasma less dense than the beam. The ICL makes use of ion-focussing to transport the beam, and a resonance, akin to that of the planar wiggler FEL, to produce coherent radiation. In addition, the ion channel acts as an optical fiber with step index of refraction. For high current, dielectric guiding eliminates the usual constraint that the Rayleigh length must be longer than the gain length.

The ICL consists of a tank of neutral gas, from centimeters to meters in length, through which a plasma column millimeters in width is produced by an ionizing laser pulse.[2] Within less than a recombination time, and with proper matching, as in a continuous plasma focus,[4] an REB is injected, propagating in the axial (+z) direction (Fig. 1).

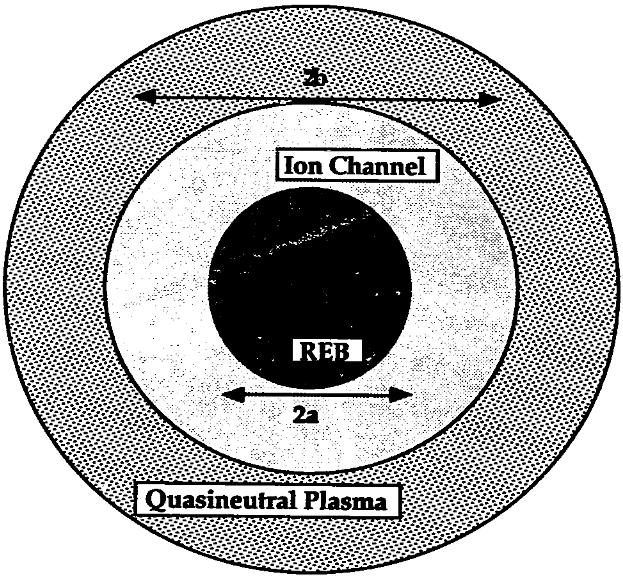

Figure 1. An REB, propagating through an underdense plasma, expels plasma electrons from the beam volume and beyond to produce an "ion-channel", which then focusses the beam, and causes it to radiate.

As the beam head propagates through the plasma, it continuously expels plasma electrons from the beam volume, leaving fixed the relatively immobile ions to provide focussing for the remainder of the beam. It is assumed that the transverse force on the beam due to self-fields is much less than the transverse electric field due to the ion charge. This requires $n_p >> n_b/\gamma^2$, where n_b is peak beam density, n_p is the plasma density prior to channel formation, and γ is the Lorentz factor. We will also assume $n_b > n_p$, later indicating the generalization to an overdense plasma. For definiteness the beam density is assumed to be a step radial profile, with radius a.

The expulsion of plasma electrons produces a cylindrical channel of radius $b \sim a(n_b/n_p)^{1/2}$ occupied by unneutralized ions and this is the "ion channel". It is assumed that the current rise time, τ_r, is long compared to a plasma period so that large radial plasma oscillations are not excited as plasma electrons are ejected from the channel and this requires $\omega_p\tau_r >> 1$, where $\omega_p{}^2 = 4\pi n_p e^2/m$, -e is the electron charge, and m is the electron mass. It is also assumed that the beam length, τ, is short compared to the time, τ_i, for the ions to collapse inward due to the radial electric field of the beam. This requires, $\tau_i \sim (b/c)(m_i/m)^{1/2}(I_A/I)^{1/2}/4 > \tau$ where m_i is the ion mass, $I = \pi a^2 n_b e c$ is the peak beam current and $I_A = mc^3/e \sim 17$ kA is the Alfven current.

The zeroth order transverse motion of a beam electron is that of a relativistic, 2-D, simple harmonic oscillator in the potential $U = m\omega_p{}^2(x^2+y^2)/4$, i.e., electrons oscillate in x and y at the betatron frequency $\omega_\beta \sim \omega_p(mc/2p_z)^{1/2}$. Energy, axial momentum, and angular momentum in the axial direction are constants of the motion. In the center of momentum frame, electrons are oscillating with upshifted frequency $\omega_1 \sim \gamma\omega_\beta$ and radiate incoherently. In the lab frame the frequency of radiation in the forward (+z) direction is $\omega \sim 2\gamma\omega_1 \sim 2\gamma^2\omega_\beta$. We will show that coherent radiation, near the frequency ω, may be amplified via an induced correlation of longitudinal and betatron phase, which corresponds to a phase-space bunching of the beam.

We consider the motion in the ion-channel of a single electron, subject to an electromagnetic wave linearly polarized in the y direction. Denote the vector potential $A_y = (mc^2/e)A\sin(\zeta)$, where $\zeta = k_z z - \omega t + \varphi$. A and φ are eikonal amplitude and phase, respectively, and vary slowly in time on the ω^{-1} scale, and in z on the $k_z{}^{-1}$ scale. The Hamiltonian is

$$H = \sqrt{m^2c^4 + p_z^2c^2 + p_x^2c^2 + \left(p_y + \frac{e}{c}A_y\right)^2 c^2} + U \quad (1)$$

where p_x, p_y, and p_z are the canonical momenta in x, y, and z, respectively. In deriving the equations of motion, we will neglect the anharmonicity in the transverse motion and second order terms in A, as well as derivatives of the slowly varying eikonal quantities, and transverse gradients.

It is convenient to introduce variables q_z, q_x, θ_x, q_y, and θ_y, such that $p_z=mcq_z$, $p_y=mcq_y\sin(\theta_y)$, and $p_x=mcq_x\sin(\theta_x)$. For A=0, q_x and q_y are constants and $d\theta_{x,y}/dt=\omega_\beta$. We also define the phase variable $\psi=\theta_y+\zeta$, averaged over the betatron period, and the detuning parameter $\Delta\omega=k_z v_z-\omega+\omega_\beta$, where v_z is the betatron-averaged drift velocity in z, $v_z/c=1-(2+q_x^2+q_y^2)/4q_z^2$. We define $a_\beta^2=(q_x^2+q_y^2)/2$, analogous to the wiggler parameter in an FEL. For the round beam of Fig. 1, $a_\beta=q_z k_\beta a/2^{1/2}=2^{3/2}\varepsilon_n/a$ and is initially the same for each particle. The rms normalized emittance is $\varepsilon_n=0.25 q_z k_\beta a^2$, where $k_\beta=\omega_\beta/c$.

With an average over the betatron period, the perturbed equations of motion derived from Eq. (1) take a form reminiscent of that found by Kroll *et al.*, for the FEL:[12]

$$\frac{d\psi}{dt}=k_z v_z-\omega+\frac{d\theta_y}{dt}+\frac{d\varphi}{dt}+\frac{1}{2}k_z c\,\frac{q_y}{q_z^2}A\cos(\psi),$$
$$\frac{d\theta_y}{dt}=\omega_\beta\left\{1-\frac{1}{2q_y}A\cos(\psi)\right\},$$
$$\frac{dq_z}{dt}=-\frac{1}{2}k_z c\,\frac{q_y}{q_z}A\sin(\psi),$$
$$\frac{dq_y}{dt}=-\frac{1}{2}\left\{\omega_\beta+\frac{1}{4}k_z c\,\frac{q_y^2}{q_z^2}\right\}A\sin(\psi),$$
$$\frac{dq_x}{dt}=-\frac{1}{8}k_z c\,\frac{q_y q_x}{q_z^2}A\sin(\psi). \tag{2}$$

It is assumed that electrons with $q_y<A$ contribute negligibly to amplification, and that $a_\beta<1$. It is also assumed that $\Delta\omega\ll\omega_\beta$, or $\omega-k_z v_z\sim\omega_\beta$. For a fast wave, with $a_\beta\ll1$, this corresponds to $\omega\sim2\gamma^2\omega_\beta$.

Maxwell's equations take the form

$$\left\{k_z\frac{\partial}{\partial z}+\frac{\omega}{c^2}\frac{\partial}{\partial t}\right\}(A\,e^{i\varphi})=i\eta\frac{\omega_b^2}{2c^2}\left\langle\frac{q_y}{q_z}\exp(-i\chi)\right\rangle, \tag{3}$$

where $\chi=\psi-\varphi$ and an average has been performed over the periods $2\pi/\omega$, $2\pi/\omega_\beta$ and over all electrons at z,t, as indicated by the brackets. The quantity ω_b is the beam-plasma frequency, $\omega_b^2=4\pi n_b e^2/m$, and η is the overlap integral of the mode and beam radial profiles. Following Bonifacio *et al.*,[13] slippage is neglected and a change of coordinates is made from z,t to $s=t-z/v_z$ and t. As for an FEL, combining Eq. (2) and Eq. (4) gives an equation for the eikonal alone. The solution for the eikonal is given by a superposition of three terms varying as $\exp(\Gamma t)$, corresponding to the three roots of the cubic gain equation, which is, in the limit of zero detuning, $\Gamma^3=i(2\rho\omega_\beta)^3$. In the fast-wave, small a_β limit, with $\eta=1$. $\rho\sim(I/32\gamma I_A)^{1/3}$. Growth is cubic for short times, and for longer times may be characterized by an exponential gain length $L_{gain}=c/\mathrm{Re}(\Gamma)$, or $L_{gain}\sim 0.3\lambda_\beta(\gamma I_A/I)^{1/3}$, taking $a_\beta<1$.

As J.S. Wurtele and E.T. Scharlemann have noted,[14] a more realistic beam profile will have an intrinsic spread in detuning, and to prevent damping of the instability, one expects that the detuning spread should be small compared to the growth rate. Indeed, a Maxwell-Vlasov treatment,[15] imposes the approximate conditions on the spreads in momenta: $\Delta a_\beta^2<2\rho$ and $\Delta p_z/p_z<\rho$.

As K. R. Chen has noted,[16] in the slow-wave limit ($k_zc<<\omega$), bunching is reduced, and occurs primarily in the azimuthal phase, in the opposite sense as for axial bunching. A similar effect was found, by Chu and Hirshfield[17] for the cyclotron maser instability. For the ICL, the transition from axial to azimuthal bunching occurs at $\omega/ck_z\sim2$. Below it will be seen that, due to dielectric guiding, azimuthal bunching dominates only for low current operation in a waveguide near cut-off.

Neglecting optical guiding[18] and diffraction, the approximation $\eta\sim1$ is adequate when the gain length is short compared to the Rayleigh length, $L_{gain}<<L_{Ray}=\pi a^2/\lambda$, where $\lambda=2\pi c/\omega$. Typically, however, diffraction is important and, in this case, the effect of the channel wall must be included. Neglecting collisions of plasma electrons, the channel serves as a cylindrically symmetric, dielectric waveguide, with step discontinuity in the dielectric constant: $\varepsilon=1$ for $r<b$ and $\varepsilon=1-\omega_p^2/\omega^2$ for $r>b$.

Such a waveguide will always have at least one guided mode, the HE_{11} mode;[19] we proceed to compute the overlap between this mode and the beam. The transverse vector potential is $A_y=(mc^2/e)AJ_0(k_pr)\sin(\zeta)$, for $r<b$, where $r^2=x^2+y^2$. The total power is $P_{tot}=P_0(\omega k_z/c)b^2A^2\Lambda$, where $P_0=m^2c^5/e^2=8.71$ GW,

$$\Lambda=\frac{\mu^2}{32}\left\{V\,J_1(V)\exp\left(\frac{1}{V}\frac{J_0(V)}{J_1(V)}\right)\right\}^2, \tag{4}$$

$\mu = \exp(\gamma_E)$, and $\gamma_E \sim 0.5772$ is Euler's constant. $V = k_p b = 2(I/I_A)^{1/2}$ is the waveguide parameter and $V \leq 1$ ($I \leq 4$ kA) is assumed. In this case, the field varies negligibly transversely, across the beam. The dispersion relation is $\omega^2 \sim c^2 k_z^2 + \omega_p^2$. The power flowing through the beam volume is $P_b = P_0(\omega k_z/c)a^2A^2/8$ and the overlap integral is $\eta = P_b/P_{tot} = a_\beta^2/(2\gamma V^2 \Lambda)$. The Pierce parameter with dielectric guiding is then $\rho' = \rho\eta^{1/3}$, and the gain length is $L'_{gain} = L_{gain}/\eta^{1/3} \sim 0.6\lambda_\beta \gamma^{2/3}\Lambda^{1/3}/a_\beta^{2/3}$. The factor $\Lambda^{1/3}$ ranges from $\Lambda^{1/3} \sim 7\ 10^{10}$ for I=0.2 kA to $\Lambda^{1/3} \sim 2.4$ for I ~ 2 kA.

Scattering with the neutral atoms and ions of the gas will increase the emittance and this has been studied by Montague *et al.*[20] Extending their result, and assuming scattering with neutrals dominates, the increase in normalized emittance in one betatron wavelength is $\Delta\varepsilon_n = 4\pi r_e Z^2 \ln(\theta_{max}/\theta_{min})/f$, where f is the ionization fraction, r_e is the classical electron radius and $\theta_{max}/\theta_{min} \sim 5.26\ 10^4/(AZ)^{1/3}$. Z is the atomic number, and A is the atomic weight. For the examples, below, we will take Z~50, A~100 and f ~ 10%, corresponding to $\Delta\varepsilon_n \sim 10^{-6}$ cm-rad.

Most beam-plasma instabilities will be rather benign for typical parameters; however, as K. Takayama and S. Hiramatsu have noted,[21] growth of the ion-hose instability is not always negligible.[22] In the rigid beam model, growth varies as $\exp(z/L_h)^{1/3}$ with $L_h \sim 1.26\lambda_\beta A(a/1\text{cm})^2(1\text{ns}/\tau)^2(1\text{kA}/I)$; a few e-folds are probably tolerable.

The efficiency, ε, may be estimated from the power at the onset of non-linearity and particle trapping; this gives $\varepsilon \sim \rho$, and an output power $P_{out} \sim \varepsilon P_{beam}$, where $P_{beam} \sim mc^2\gamma I$ is the initial beam power. Numerical studies indicate that efficiency may be increased significantly by tapering the plasma density in z, near saturation.

These scaling laws have been applied to four numerical examples and parameters are given in Table I. The results have been checked with a many-particle simulation based on Eqs. (2) and (3). The first example was also checked with a simulation following the full equations of motion derived from Eq. (1). The most severe constraint was found to be the condition $\Delta a_\beta^2 < 2\rho$, which is marginally satisified in the first three examples. For a monotone decreasing radial profile, this constrains the emittance to a low value: $\varepsilon_n \sim \gamma\lambda\rho/\pi$. To exhibit the consequences of this constraint, the fourth example was designed with a large a_β. It should be emphasized, however, that such an X-Ray laser could not be realized without a sharp distribution in transverse energy, corresponding to a step radial profile, or perhaps a spinning beam. In the first three examples, the plasma densities required are not out of the ordinary. For the fourth example, the plasma density is high;

however, it need only be maintained over a few centimeters. Ion-hose growth is significant only for the X-Ray example, where it is potentially severe and requires further study. Ion motion appears as a non-negligible constraint on beam length.

Table I: Examples of Ion Channel Laser Scalings.

	Microwave	Sub-Millimeter	Infrared	X-Ray
λ(cm)	1.75	$5\ 10^{-2}$	$1\ 10^{-3}$	$1\ 10^{-6}$
E(MeV)	2	4	10	100
I(kA)	4	4	4	4
ε_n(cm-rad)	0.25	$1\ 10^{-2}$	$5\ 10^{-4}$	$3\ 10^{-5}$
n_p(cm^{-3})	$6\ 10^{10}$	$8\ 10^{12}$	$1\ 10^{15}$	$2\ 10^{19}$
L'_{gain} (cm)	70	16	4	0.2
λ_β(cm)	36	5	0.6	$2\ 10^{-2}$
τ_i(ns)	15	1	0.1	$1\ 10^{-3}$
a_β	0.6	0.5	0.4	2
a(cm)	1	$7\ 10^{-2}$	$3\ 10^{-3}$	$4\ 10^{-5}$
ρ(%)	5	3	1	0.7
P_{out}(GW)	0.4	0.5	0.6	3

This limit on beam length due to ion-motion motivates the study of the analogous instability of a magnetically self-focussed beam ($n_b<n_p$), the regime in which the experimental work of Refs. 10 and 11 was performed. Simple estimates may be made by identifying, $k_\beta \sim (2I_{net}/\gamma I_A)^{1/2}/a$ and $a_\beta \sim (\gamma I_{net}/I_A)^{1/2}$. The experimental results are characterized by efficiencies of a few percent and a broad-band spectrum extending far above ω_p. Such efficiencies are somewhat lower than predicted by the scalings given here, probably due to nonlinear focussing and spreads in momenta. The spectrum may be understood from the result for the resonant frequency: $\omega - k_z v_z \sim \omega_\beta$. An electron with small transverse energy has $a_\beta \sim 0$, and is resonant with $\omega \sim 2\gamma^2 \omega_\beta$, while electrons with large transverse energies are resonant with $\omega \sim 2\gamma^2 \omega_\beta / a_\beta^2$.

In conclusion, we have presented the concept of an ion-channel laser, together with a first theory. The phenomenon of dielectric guiding allows short gain lengths for wavelengths ranging from the microwave to the X-Ray.

We wish to thank K.R. Chen for discussions regarding the analogy with CARMs and the relativistic mass effect. Further work on this subject is forthcoming

by K. R. Chen, *et al.*[23] We also wish to thank William A. Barletta, Shigenori Hiramatsu, Ernst T. Scharlemann, Ken Takayama, and Jonathan S. Wurtele for many helpful comments and discussions.

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