Pre-tuning of TRISTAN Superconducting RF Cavities

Tsuyoshi Tajima, Takaaki Furuya, Toshiji Suzuki[1]

National Laboratory for High Energy Physics
1-1 Oho, Tsukuba-shi, Ibaraki-ken, 305 Japan

and

Yohsuke Iino

Mitsubishi Heavy Industries, Ltd., Nagoya Aircraft works
10, Oye-cho, Minato-ku, Nagoya, 455 Japan

Abstract

Pre-tuning of thirty-two TRISTAN superconducting cavities has been done. In this paper are described the pre-tuning system and the results of all the cavities. The average field flatness was 1.4 % after pre-tuning. From our experience, the followings are important, 1) to evacuate the cavity during the process of the pre-tuning to avoid the uncertainty in evacuation, 2) pre-tuning is needed after annealing because it causes changes of the cell length and the field profile and 3) field flatness sometimes changes when expanded and 4) cells should not be expanded more than 1.5 mm after pre-tuning since inelastic deformation occurs.

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[1]On leave from Tohoku University, Mikamine, Sendai 982, Japan.

1. Introduction

Thirty-two 508MHz 5-cell superconducting RF cavities have been produced for TRISTAN main ring. Half of them were already installed in the tunnel in the summer of 1988 and have been successfully operated [1]. The remaining half is scheduled to be installed in the summer of 1989.

There are about seven main processes in producing an SC cavity from niobium ingot: forging, rolling, forming, welding, annealing, pre-tuning and surface treatment [2]. Among these, pre-tuning is done 1) to tune the resonant frequency of the cavity into the frequency range of piezo-electric tuner [3] and 2) to improve the field flatness of as-received cavity.

The tuner which may cause thermal breakdown such as the retractable tuner used for normal conducting cavities [4] should be avoided for superconducting cavities. Therefore, most of the laboratories which have produced multi-cell RF cavities employ an inelastic deformation of each cell for the pre-tuning [5-7].

When we tried the first pre-tuning for the two 5-cell cavities which were used in TRISTAN accumulation ring, we used a pair of hydlauric press. However, we found it difficult to control the deformation precisely with the hydlauric system. Thus we decided to use a mechanical gear system driven by four independent motors. The controllability improved to be 0.01mm and it became much easier to handle.

Most of our efforts during the construction period were focused on simplifying the system and reducing the working time to meet hard schedule. We presently consume about five hours for setting and pre-tuning one cavity. Devices for measuring overall cavity length at four points and for expanding a cavity at two beam-tubes were introduced from cavity 7a, which improved the precision of length measurement and enabled us to perform the expansion test, what we call "8 mm expansion test", which will be described later

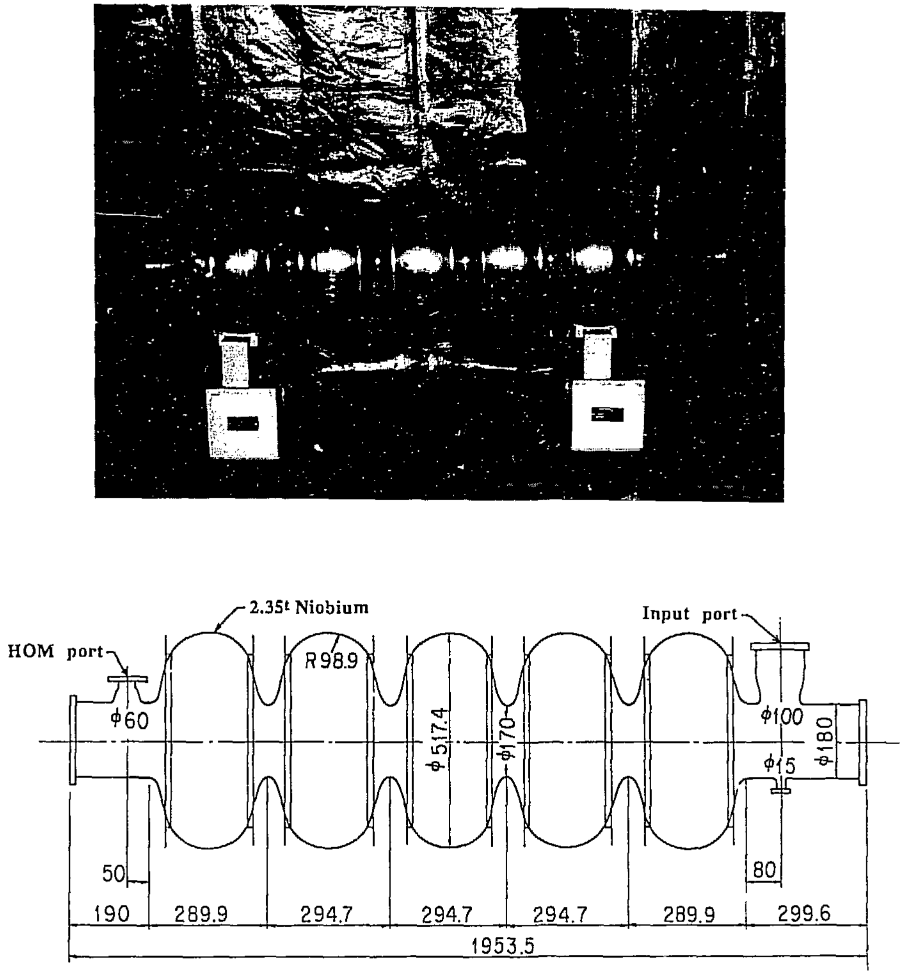


Fig.1 Photo of TRISTAN superconducting cavity and its nominal dimension.

As for the field profile measurement, we use the familiar bead-perturbation method. The difference from other laboratories is that we do the measurement after evacuating the cavity since we knew that the field profile after evacuation differs from the one at 1 atm when we tested the first cavity.

As for the deformation of cells, we do not use expansion much by the fact that expanding a cell affects the shapes of neighbor cells, which makes the pre-tuning complicated. When we have to expand a cell, we expand it slightly too much at first so that we can start squeezing.

The parameters used for our pre-tuning, effects of annealing and evacuation as well as deformation characteristics will be discussed. In addition, the results of 8 mm expansion test will be shown.

2. System

A photograph of an SC cavity and its nominal dimension in operation are shown in Fig.1. The corrugate-shaped cavity is made by welding hydroformed half cells of 2.35mm-thick niobium sheet [2].

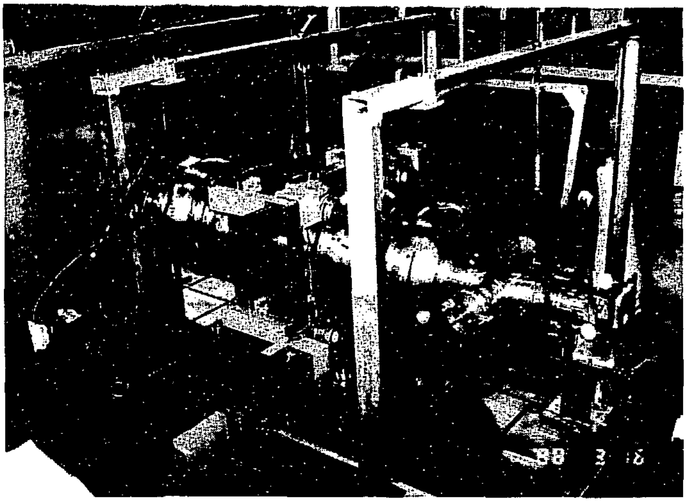
2.1 Pre-tuning Instrument

The pre-tuning instrument which has been used for the TRISTAN SC cavities is shown in Fig.2. It consists of a sturdy iron table, three bars with spring balances, a deformation device and four dial gauges set at both ends.

The spring balances are used to eliminate unnecessary friction with attaching devices, which causes the uncertainty of the measurement and to cancel gravity since the cavity bends by about 4.5 mm at the center when supported only at both two beam-tubes, which will cause the deviation of the passage of the bead in field profile measurement.

The deformation device is made up of two pairs of thick (48mm) steel plates which horizontally slide to hold the irises of one cell. The device also slides parallel to the cavity axis to move to another cell. Since the diameter of beam-tube differs from the iris of cells by 10 mm, the parts of the plates which surround the iris are half-ring-shape demountable pieces to hold the beam-tube.

The dial gauges are used to measure the overall length of the cavity, the displacement when the cavity is evacuated, etc.. Four



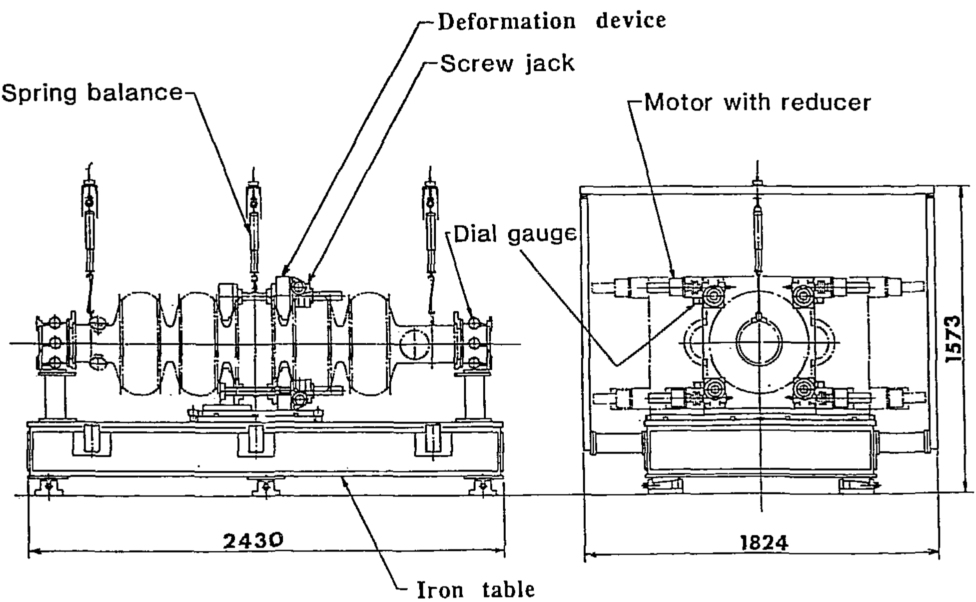


Fig.2 Pre-tuning instrument.

dial gauges are used to improve the accuracy since the two beam-tube flanges are not always parallel. We employ an average of the values measured by the four dial gauges. Usually reproducibility of the measurements is less than 0.05 mm.

As for the motor control of the deformation device, it has two modes. One is the mode to drive the four motors independently and the other is to drive all the motors simultaneously. We use the latter mode for the ordinary pre-tuning and the former mode is used only for adjusting the spacing of four segments of the plates occasionally. There are four digital dial gauges set on the deformation device to measure the displacement of its four segments. Once one sets a value and presses a button on the control panel, the deformation device moves until each digital dial gauge reaches the set value. The last digit is 0.01 mm, i.e., one can control the deformation with a precision of 0.01 mm.

2.2 RF measurement

As mentioned above, the measurements of the resonant frequency and field profile of a cavity is done after evacuating the cavity. We made special beam-tube flanges so that the bead measurements can be performed without breaking the vacuum. A cross-sectional view of the flange is shown in Fig.3. The vacuum is sealed with a rubber sheet of about 2 mm in thickness. A nylon string with a 12mm-diam. aluminum sphere (bead) is set through the cavity axis, maintaining its tension with a spring.

The movement of the bead is controlled by a personal computer. A typical pattern of the movement of the bead is that it runs through the cavity axis from HOM-side beam-tube to INPUT-side beam-tube at a speed of about 3 cm/s. On reaching the limit switch attached on the inner surface of the flange, it returns to the startpoint at doubled speed.

Configuration of the RF measurement system is shown in Fig.4. It consists of a signal generator (SG), a power divider, RF amplifiers, a power meter, a frequency counter, a phase detector, a phase shifter, a digital thermometer and a personal computer.

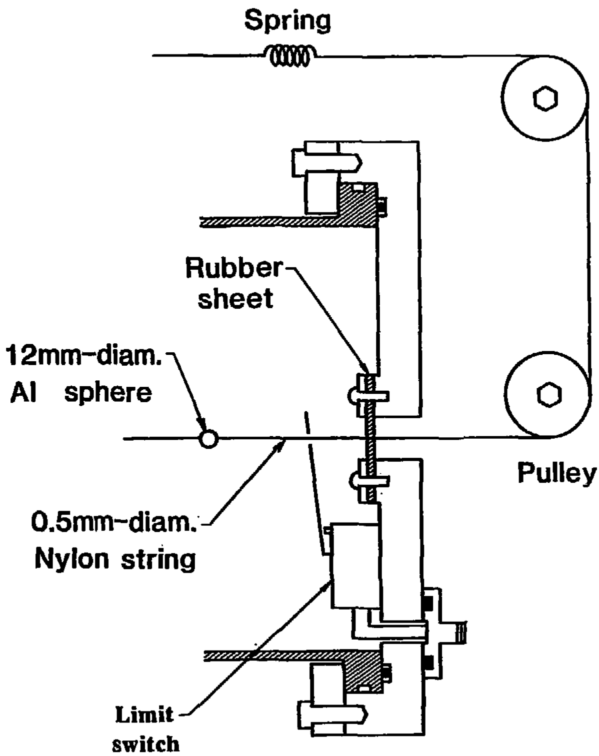


Fig.3 Cross-sectional view of the end beam-tube flange.

The SG output signal is divided and fed into the cavity through the input port and also into the phase-locked loop (PLL) together with the signal picked up through one of the HOM ports.

The resonant frequency is searched and the phase difference between input and pick-up signals is corrected manually. Before running the bead, the phase is locked and FM is performed using the phase signal during the passage of the bead.

The computer takes the data on the frequency deviation as the bead passes the cavity axis, showing them on the screen at the same time, as well as taking the data on the temperature of the cavity. This frequency deviation is proportional to the square of the field where the bead passes. The temperature data is used to predict the frequency of other states, e.g., at 0°C or at 4.2K.

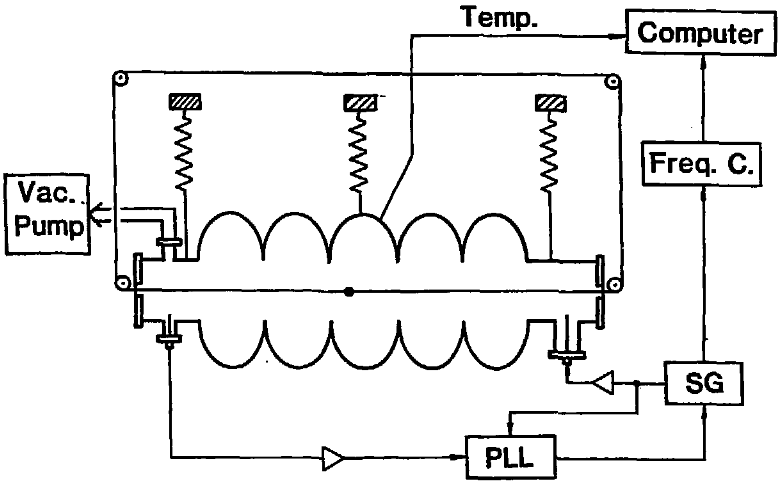


Fig.4 Configuration of bead-perturbation measurement system.

After the bead reaches the end, the computer starts calculating the field flatness, the deformation length required and predicts the frequencies at 4.2K and 1 atm using the parameters shown in Table I. A part of this calculation is based on the simple

Table I Present pre-tuning parameters.

Coupling costant $k=\frac{\omega_\pi-\omega_0}{\omega_\pi+\omega_0}$	0.00773
Freq. shift by tuning	415 kHz/mm
by temperature	-3.56kHz/K (R.T.-273K)
	656kHz (273K-4.2K)
by ε of air	112kHz
Freq. detune	-480kHz
Target freq.	508.101MHz
Operation Freq.	508.581MHz

method of solving an equivalent circuit, which was previously treated by Fernandes, et al. [8].

As for the target frequency of pre-tuning at room temperature, one shoud set it to meet the operational frequency in the TRISTAN main ring, 508.58MHz at 4.2K. The target frequency is determined as follows,

$f_{target} = f_{operation} - f_{detune} - 656kHz - 3.56kHz/°C * (Room Temp.).$

where f_{detune} is the bias for the pre-load of piezo-tuner, which is 480 kHz at present, and the third and the fourth terms are due to the temperature difference from 0°C to 4.2K and from room temperature to 0°C, respectively. When the cavity is cooled down, the resonant frequency rises since the cavity shrinks. Using the above equation, the target frequency at 20°C becomes 507.373MHz. In addition, to predict the frequency at 1 atm, the effect of the difference of the medium in the cavity, i.e. the vacuum and the air, whose empirical value is 112kHz (denoted as ε in Table I), is considered as well as the change of overall length in evacuation.

3. Procedure

A flow chart of the sequence of pre-tuning is shown in Fig.5. The procedure is as follows.

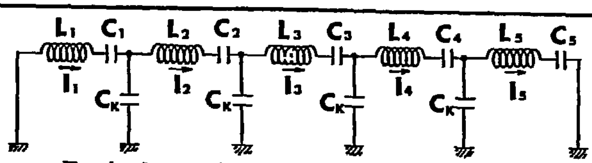
(1) The cavity is set on the 2430 by 1200 mm iron table, being supported at both beam-tubes, and with all openings sealed by blind flanges, some of which have RF ports, vacuum ports or bead driving equipment.

(2) The cavity is suspended at three points, the two end beam-tubes and the center cell, using spring balances.

(3) The cavity is evacuated through one of the HOM ports by a rotary pump to about 20 mTorr for about 20 minutes.

(4) The change of overall length in evacuation is measured with four dial gauges manually. Its average value is input to the computer as the parameter used to calculate the frequency at 1 atm.

(5) Manual frequency search and the automated bead measurement are performed. The display on the screen is like the one in Fig.16. Comparing the level of five peaks, one can feel the field flatness visually.



Equivalent circuit of 5-cell sc cavity.

⇓

See Appendix

$$\omega_1/\omega = 1/\sqrt{1 - k\cdot I_2/I_1},$$
$$\omega_2/\omega = 1/\sqrt{1 - k\cdot (I_1+I_3)/I_2},$$
$$\omega_3/\omega = 1/\sqrt{1 - k\cdot (I_2+I_4)/I_3},$$
$$\omega_4/\omega = 1/\sqrt{1 - k\cdot (I_3+I_5)/I_4},$$
$$\omega_5/\omega = 1/\sqrt{1 - k\cdot I_4/I_5}.$$

$$k = \frac{\omega_\pi - \omega_0}{\omega_\pi + \omega_0} \quad (1)$$

⇓ I = E

For ideal flat π-mode, i.e. $I_1 = -I_2 = I_3 = -I_4 = I_5$

$$\omega_1/\omega = 1/\sqrt{1+k},$$
$$\omega_2/\omega = 1/\sqrt{1+2k},$$
$$\omega_3/\omega = 1/\sqrt{1+2k},$$
$$\omega_4/\omega = 1/\sqrt{1+2k},$$
$$\omega_5/\omega = 1/\sqrt{1+k}.$$

Ideal state (2)

⇓

Bead measurement → $\Delta f \propto E^2$

⇓

Substitution of $\sqrt{\Delta f}$ in eq.(1) → Real state $\omega_1/\omega, \cdots, \omega_5/\omega$

⇓

Comparison of real state and ideal state

⇓

Calculation of field flatness and necessary deformation length

⇓

Deformation to approach the ideal state

Fig.5 Equivalent circuit for a 5-cell cavity and the flow of pre-tuning.

(6) The computer calculates the field flatness and the necessary length of the inelastic deformation for each cell, based on the Eqs. (1) and (2) in Fig.5.

(7) Set the deformation device to the cell which needs a deformation and the spacing of the plates is set to touch the cell softly not to cause any pre-deformation.

(8) The cell is given a deformation using the data in Fig.12 so that an inelastic deformation of about 80 percent of the required length can be obtained, i.e. the tuning plates move by the correspondent amount. We expand cells as few times as possible since, as mentioned before, we found that the expansion of one cell affects the shape of neighbor cells, which makes the pre-tuning complicated and increase the number of iterations of the deformation.

(9) (5) and (6) are repeated, the calculated remaining deformation length is added to the set value on the controller of the deformation device and another deformation is given. A few iterations of this process are done until the remaining length becomes less than 0.05 mm, which corresponds to a frequency difference of about 20 kHz.

(10) The pre-tuning device is moved to another cell and (7) - (9) are repeated until the field flatness becomes within 2%.

4. Results and discussion

The data on the lengths, the frequencies and the field flatness of all thirty-two cavities before and after pre-tuning are shown in Table II and Figs. 6-8. The lengths of the cavities after pre-tuning range from 1943 to 1954 mm after evacuation. Most of the inelastic deformations given were squeezing, i.e. lowering the frequency as is shown in Fig.7. The maximum total length of inelastic deformation was about 13 mm of the cavity 9a.

As one can see in the Fig.7, the frequency before pre-tuning changes drastically at cavity 8a. This is due to the fact that the die for sizing before E-beam welding was changed mistakenly at the factory without notice. When we found it at the pre-tuning of cavity 8a, cavities up to 11b had been already welded. Except those cavities and first few cavities, good reproducibility has been obtained. The discrepancy between the frequency after pre-tuning (4.2K, prediction) and that of the horizontal measurement, which is the RF measurement after horizontal assembly, is due to the several forces during the assembly. We lowered the target frequency to compensate the discrepancy from the cavity 11a.

Table II Overall length, resonant frequency and field flatness of each cavity before and after pre-tuning.

Cav. No.	Length[1] (mm) bef.	aft.	Frequency[1] (MHz) bef.	aft.	Field flatness (%) bef.	aft.
1a	1946.27	1944.63	507.610	507.394	30.7	1.7
1b	1947.17	1943.04	507.808	507.371	39.7	1.5
2a	1950.32	1945.81	507.797	507.383	20.9	0.5
2b	1951.52	1947.57	507.779	507.412	21.6	3.4
3a	1954.30	1949.30	507.841	507.412	10.4	0.2
3b	1952.80	1947.49	507.936	507.444	16.3	0.5
4a	1952.00	1953.50	507.441	507.433	13.8	0.6
4b	1951.55	1952.30	507.477	507.430	19.0	1.5
5a	1954.10	1954.00	507.492	507.455	9.8	0.9
5b	1954.50	1954.30	507.488	507.445	6.3	1.9
6a	1953.12	1953.90	507.451	507.464	8.1	1.3
6b	1954.83	1952.50	507.593	507.433	24.6	3.3
7a	1952.90	1950.90	507.600	507.424	9.8	1.3
7b	1952.20	1949.40	507.716	507.414	16.0	1.6
8a	1959.90	1949.60	508.378	507.433	28.9	0.6
8b	1958.90	1950.80	508.313	507.418	46.6	1.6
9a	1959.31	1947.57	508.565	507.409	39.3	0.8
9b	1958.54	1948.15	508.447	507.412	37.0	2.5
10a	1956.79	1947.53	508.244	507.397	23.6	0.8
10b	1956.13	1948.24	508.160	507.426	24.9	1.6
11a	1953.84	1945.26	508.179	507.366	19.6	2.7
11b	1954.58	1945.17	508.230	507.343	23.9	1.4
12a	1953.37	1952.08	507.478	507.365	9.5	2.2
12b	1952.74	1951.24	507.485	507.355	13.6	2.0
13a	1953.70	1952.57	507.517	507.398	6.2	1.5
13b	1953.61	1950.62	507.640	507.390	17.4	1.5
14a	1953.78	1952.11	507.494	507.356	12.7	0.4
14b	1954.13	1952.71	507.527	507.379	4.3	0.4
15a	1953.61	1952.38	507.510	507.382	8.0	0.8
15b	1954.78	1953.53	507.499	507.379	18.1	1.6
16a	1953.47	1953.16	507.417	507.344	46.1	1.5
16b	1954.65	1953.14	507.557	507.397	32.5	1.8

1) Evacuated at room temperature.

The average field flatness after pre-tuning was 1.4%, which was 20.6% before pre-tuning. The field flatness, however, sometimes becomes worse when the cavity is axially expanded as will be discussed later.

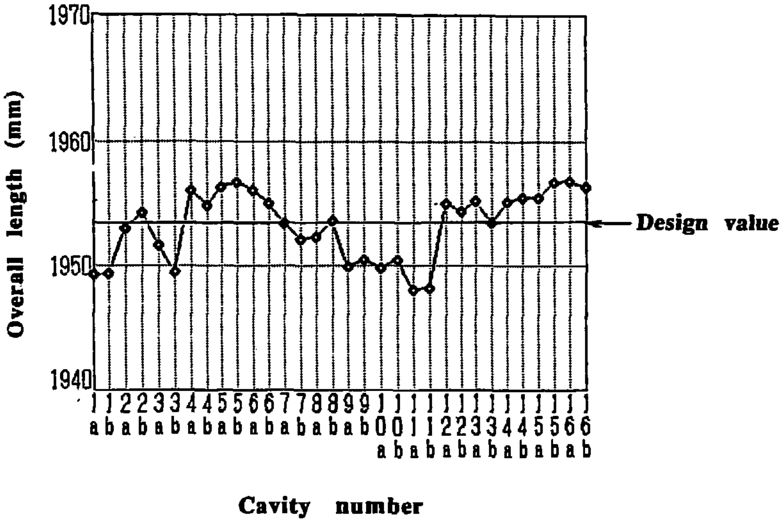


Fig.6 Distribution of cavity lengths after pre-tuning. (converted to 4.2K and 508.58MHz).

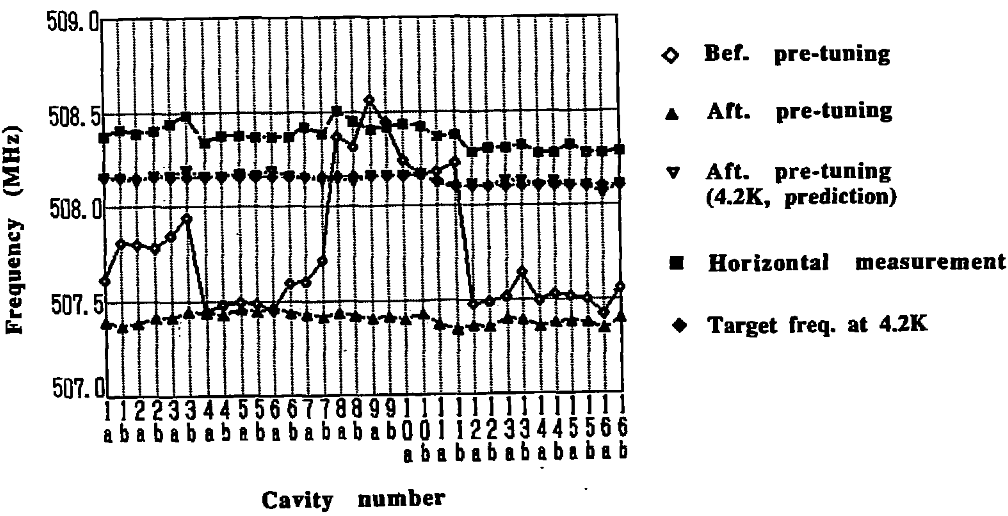


Fig.7 Resonant frequencies of the cavities before and after pre-tuning as well as that at horizontal measurements.

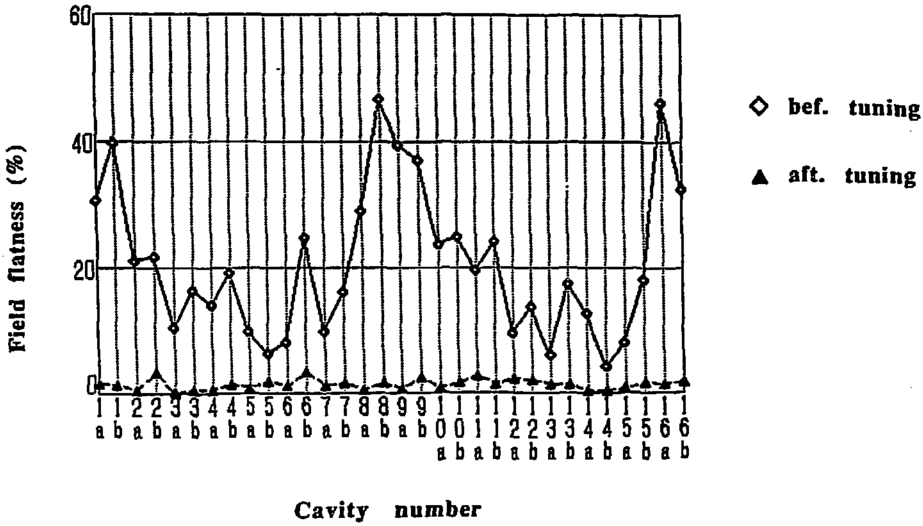


Fig.8 Field flatness of the cavities before and after pre-tuning.

Parameters

As is shown in Table I, the parameters used for pre-tuning are the coupling constant and the frequency shifts caused by several reasons, e.g., evacuation, temperature change, etc..

The coupling constant was calculated by the familiar computer code SUPERFISH. The frequency shift per unit length was measured at the first few cavities and corrected later to the most proper value statistically. The frequency shift by temperature was calculated using the expansion coefficient of niobium, 0.129% integrated from 0°C to 4.2K, and 7.0×10^{-3} /deg from room temperature to 0°C. These values are in good agreement with experimental values.

Frequency shift by electro-polishing (5-10 μm) was measured to be negligibly small and that by HOM couplers was less than 30 kHz, thus we neglected these parameters.

Effect of annealing

SC cavities are annealed after the welding of half cells at a temperature of 700°C for 90 minutes in a titanium box for degassing and relieving the stress caused by welding, etc. [2].

To know if it is possible to pre-tune before annealing and neglect it after annealing, we measured the length, resonant frequency and field profile after annealing the pre-tuned cavity. The result showed that the pre-tuning after annealing is indispensable because the frequency and the field profile change by an intolerable amount. Figure 9 shows the changes of the cell lengths of the pre-tuned cavity due to annealing as a function of

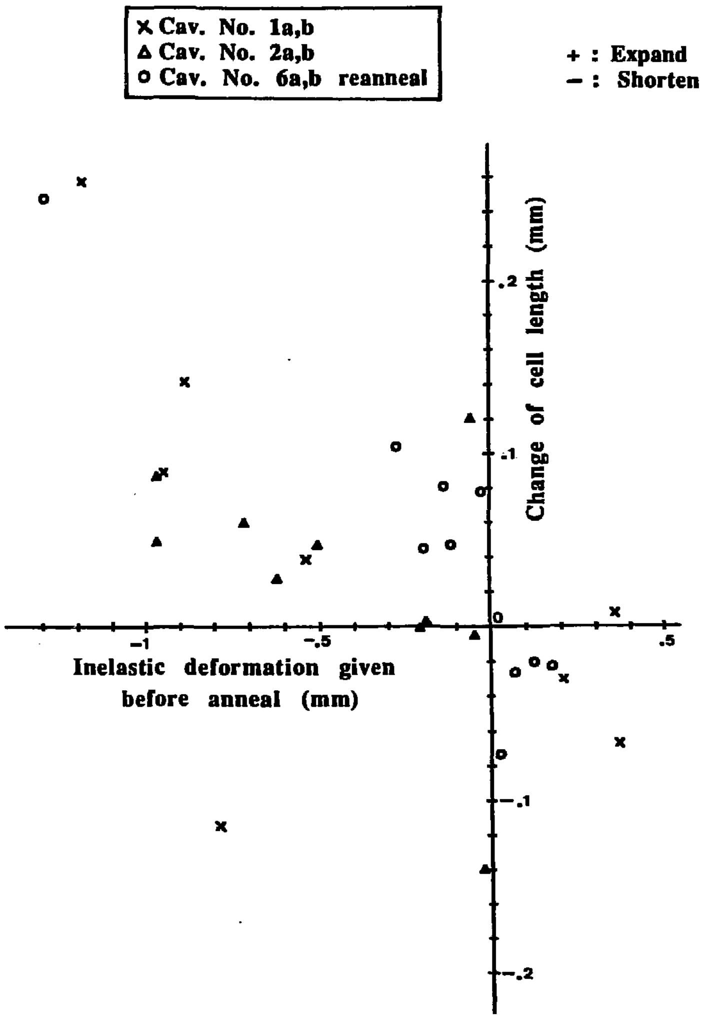


Fig.9 Changes of the cell lengths after the annealing of the pre-tuned cavities as a function of the given inelastic deformation lengths.

the length of the given inelastic deformation. It indicates that 1) in most cases, the cells that were given inelastic deformation toward squeezing tend to expand and vice versa. It is probably because the stress acquired in the process of inelastic deformation is partially released, 2) in a few cases, the squeezed cells shortened as opposed to the case above, which must be due to the stress accumulated before the pre-tuning.

<u>Effect of evacuation</u>

As mentioned before, we evacuate the cavity before the pre-tuning to eliminate the uncertainty caused by evacuation. Figure 10 shows the distribution of the squeezed length in evacuation. It clearly shows the uncertainty. The difference of the squeezed length amounts to as much as about 3 mm, which corresponds to 250 kHz. In Fig.10, the change of the length when leaked to 1 atm is shown as well. We started measuring this value from cavity 9a

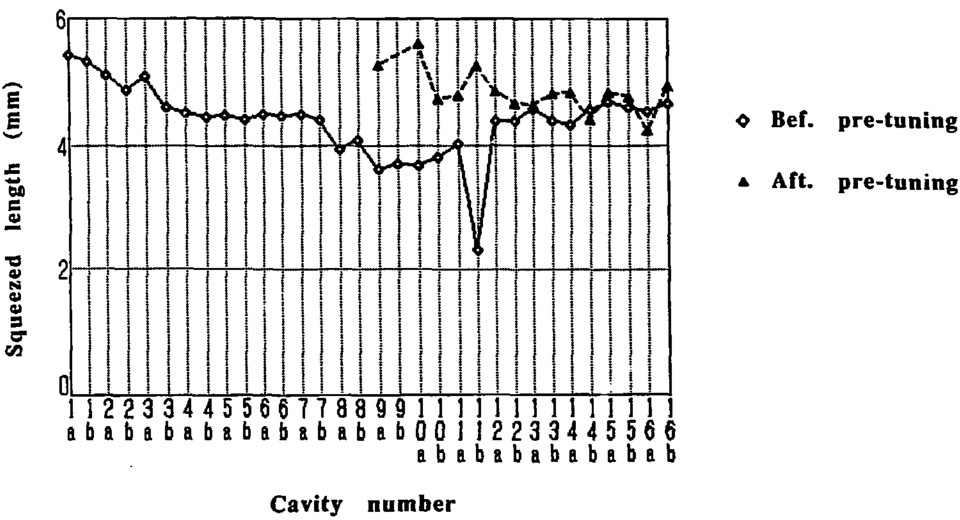


Fig.10 Squeezed length of the cavity by evacuation before and after pre-tuning.

since the squeezed length had become much different from the previous values. As one can see in the figure, the length measured after pre-tuning differ from the ones before. To see what makes the difference, we tried to plot the difference as a function of the given inelastic deformation length (Fig. 11).

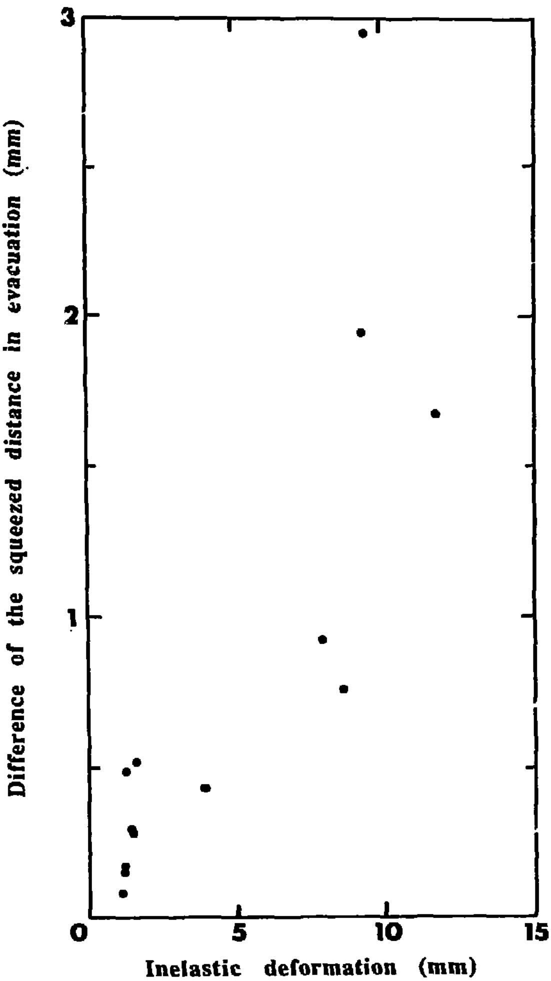


Fig.11 Difference of the squeezed distance in evacuation between before and after pre-tuning as a function of the total inelastic deformation length of each cavity.

Although it is a collective effect of each cell, one can see the positive correlation. It might mean that a part of the inelastic deformation recovered, but when the cavity is evacuated again, the properties do not change from the previous ones.

Table III shows the changes of the field flatness in evacuation. Some cavities improvedthe field flatness and others

Table III Field flatness before and after evacuation.

Cav. No.	Field flatness (%) bef. evac.	aft. evac.
1 a	23	26
2 a	4	1>
9 a	6	1>
11 a	18	20
11 a-2nd	3	2
11 a-3rd	3	3
13 a	7	7
13 b	3	2
15 a	9	9
15 b	13	18
16 a	40	46
16 b	31	32
6 b-2nd	8	5

did not change or degrade. They are unpredictable without evacuation.

Deformation characteristics

Figure 12 shows the inelastic deformation obtained as a function of the displacement of the deformation device, i.e., the change of the spacing between the device plates facing each other. It was found that the displacement of the deformation device is different from the deformation length given to the cavity.

Figure 13 shows the relationship between the displacement of the cavity and that of the deformation device. It is probably because the point the device touches is not the iris itself but a few cm away radially. Taking this fact into account, the inelastic deformation length as a function of the given deformation is shown in Fig.14. We use Fig.12 for our routine pre-tuning instead of Fig.14 since it is much easier and faster to read the value of the digital dial gauges displayed on the controller panel than reading eight analog dial gauges.

We tried to decrease the number of iteration, knowing the deformation characteristics in detail, but what we knew was it is quite difficult to get the same curve as Fig.12 for each cell because the points the tuner plates touch are not always same mechanically, which causes horizontal shift of the curve in the

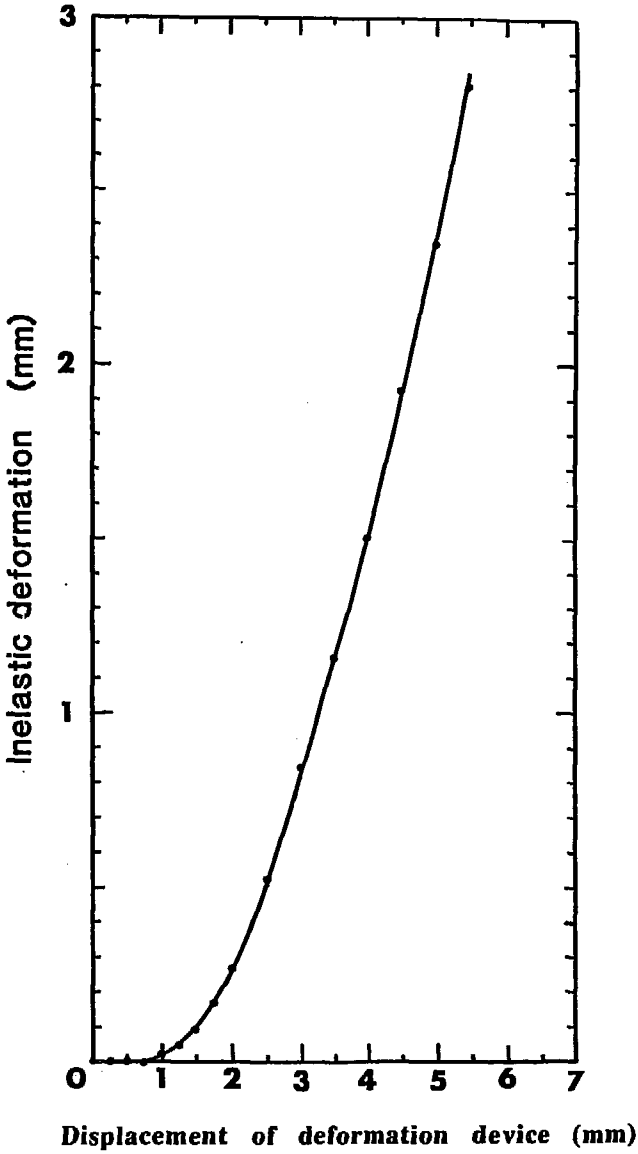


Fig.12 Inelastic deformation (squeezed) as a function of the displacement of the deformation device.

figure. An additional device to know the mechanically same touching point will be needed for a more automated system.

8 mm expansion test

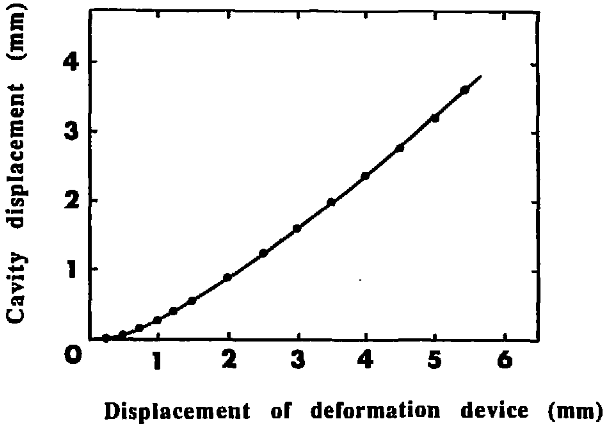


Fig.13 Cavity displacement as a function of the displacement of the deformation device.

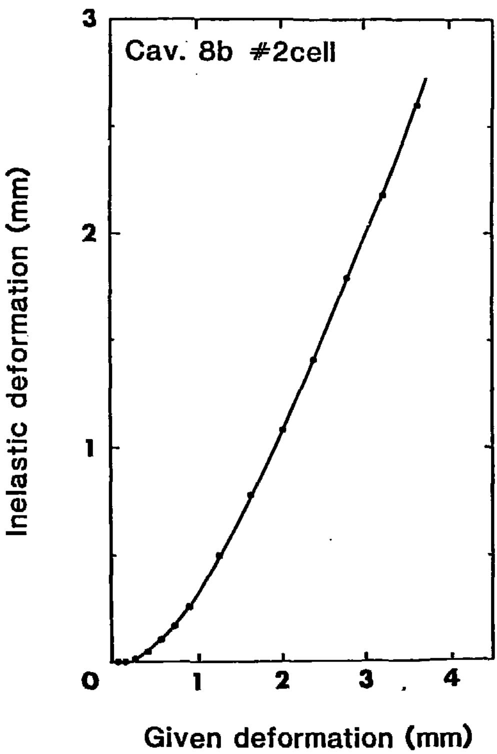


Fig.14 Inelastic deformaion as a function of the given deformation.

This test started from the cavity 9a. It is a sort of simulation for an operation in the TRISTAN ring, i.e. the cavity is expanded up to the frequency whose predicted value at 4.2K is slightly over the TRISTAN operation frequency, 508.58MHz.

To expand the cavity, two steel plates are set at the both ends of the cavity with four short bars as shown in Fig.15. The cavity is expanded while measuring the overall length of the cavity. The length, 8 mm, was decided so that no hazardous inelastic deformation may occur.

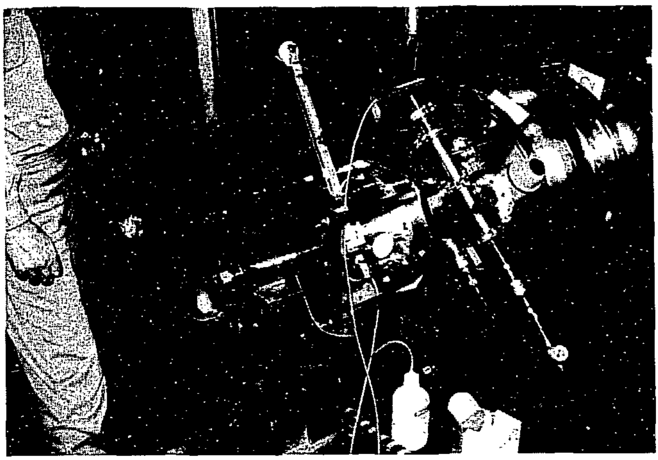


Fig.15 Attachment for 8 mm expansion test.

Through this test, one can know the field profile in the actual operation where the cavity is expanded by the piezo-tuner with a mechanical motor. The results of all the measured cavities are shown in Table IV. This test is conducted after a moderate field flatness is obtained. As shown in Table IV, field flatness often changes in expansion. There was no systematic change among the cavities. An example of the change of the field flatness during this test is shown in Fig. 16. The change is considered to be dependent on the elastic feature of each cell, i.e. the elastic constant differs from one cell to another in some cavities.

Table IV 8 mm expansion test.

Cav. No.	Expanded length(mm)	Frequency (MHz) bef.	Expand.	4.2K[1]	Field flatness (%) bef.	aft.
9a	8.29	507.384	508.079	508.823	0.4	10.2
9b	10.10	507.490	508.248	508.999	6.7	9.8
10b	7.94	507.432	508.089	508.833	2.7	3.4
11a	7.68	507.322	507.962	508.709	1.3	3.5
11b	8.31	507.285	507.980	508.731	5.7	3.5
12a	7.43	507.342	507.967	508.713	5.7	2.3
12b	7.79	507.346	507.987	508.726	3.3	3.1
13a	7.94	507.392	508.007	508.741	1.6	0.6
13b	7.84	507.361	508.030	508.760	1.7	4.4
14a	7.93	507.351	508.010	508.752	1.1	7.2
14b	8.22	507.355	508.032	508.778	3.2	4.4
15a	7.91	507.369	508.025	508.746	1.2	4.7
15b	7.82	507.377	508.017	508.736	1.4	2.7
16a	7.70	507.347	507.977	508.694	3.2	7.1
16b	7.91	507.397	508.053	508.759	5.4	7.4

1) Calculated

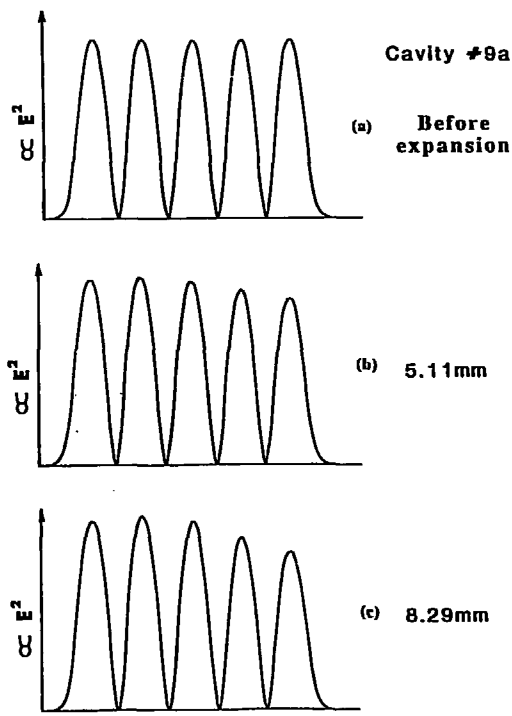


Fig.16 An example of the change of field flatness during the expansion test.

Figure 17 shows the number of cells which were deformed elastically or inelastically through this 8 mm expansion test. It

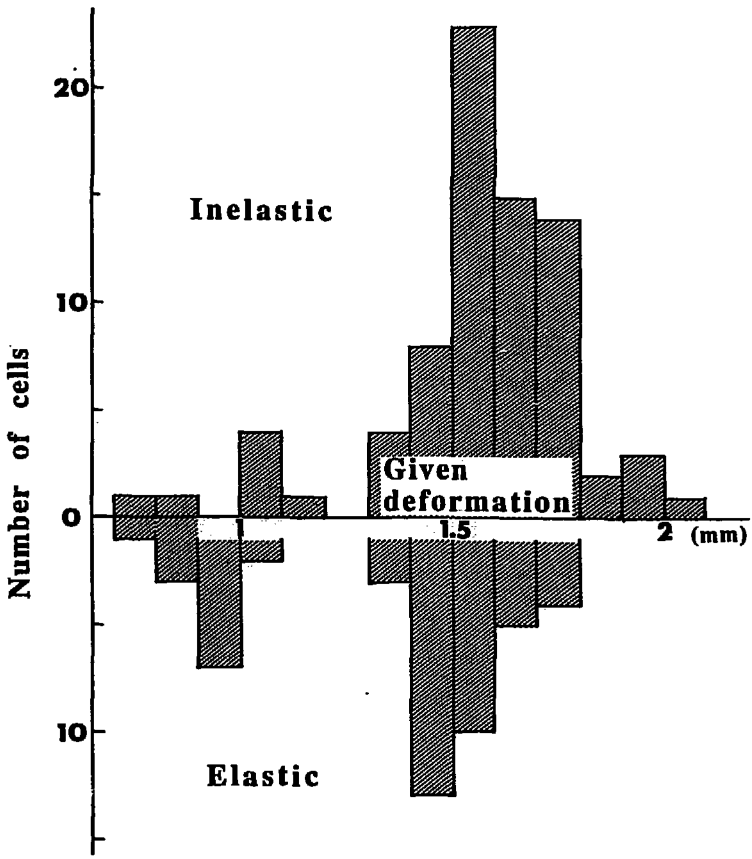


Fig.17 Number of the cells deformed elastically or inelastically in expansion test as a function of the deformation given to each cell.

indicates that one should not expand cells more than about 1.5 mm since the number of the cells deformed inelastically increase rapidly. Moreover, it also indicates that some cells happen to be deformed inelastically even after expanded about 1 mm, which may be due to the difference of the strength of the material.

Conclusion

Thirty-two 508MHz 5-cell superconducting cavities for TRISTAN were pre-tuned. After the pre-tuning, the overall lengths of the cavities were 1943-1954 mm, the π-mode frequencies varied from 507.343 to 507.464 MHz, and the field unflatness reduced from the as-received value 20.6% to1.4% on average. From our experience, we learned the followings.

(1) It is better to evacuate the cavity before pre-tuning to eliminate the uncertainty in evacuation.

(2) Suspending the cavity with spring balances is useful to make the length measurements accurate since it eliminates frictions.

(3) Annealing the cavity makes its shape different, therefore pre-tuning should be done after annealing, whereas the changes by electropolishing and HOM couplers are negligible.

(4) Expanded by about 1.5 mm each, cells start suffering inelastic deformation.

At present, it takes approximately five hours to pre-tune one cavity with three or four people. About four iterations of deformation for each cavity are needed presently. To make the system more automated, the sensors to detect mechanical pressure should be added to the deformation device.

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Appendix Derivation of the equations (1) and (2) in Fig.5.

To solve the equivalent circuit in Fig.5, we assume that there is only nearest neighbor coupling (denoted as C_k), the circuit equation is as follows,

$$\begin{aligned} &L_1\ddot{I}_1+(1/C_1+1/C_k)\cdot I_1-1/C_k\cdot I_2=0,\\ &-I_1/C_k+L_2\ddot{I}_2+(1/C_2+2/C_k)\cdot I_2-1/C_k\cdot I_3=0,\\ &-I_2/C_k+L_3\ddot{I}_3+(1/C_3+2/C_k)\cdot I_3-1/C_k\cdot I_4=0,\\ &-I_3/C_k+L_4\ddot{I}_4+(1/C_4+2/C_k)\cdot I_4-1/C_k\cdot I_5=0,\\ &-I_4/C_k+L_5\ddot{I}_5+(1/C_5+1/C_k)\cdot I_5=0. \end{aligned} \tag{A.1}$$

To make the equation simple the conversion of variables is performed as,

$$\begin{aligned} &1/C_1+1/C_k=1/C_1',\\ &1/C_2+2/C_k=1/C_2',\\ &1/C_3+2/C_k=1/C_3',\\ &1/C_4+2/C_k=1/C_4',\\ &1/C_5+1/C_k=1/C_5'. \end{aligned} \tag{A.2}$$

Then Eq.(A.1) can be rewritten as,

$$\begin{aligned} &L_1\ddot{I}_1+1/C_1'\cdot I_1-1/C_k\cdot I_2=0,\\ &-1/C_k\cdot I_1+L_2\ddot{I}_2+1/C_2'\cdot I_2-1/C_k\cdot I_3=0,\\ &-1/C_k\cdot I_2+L_3\ddot{I}_3+1/C_3'\cdot I_3-1/C_k\cdot I_4=0,\\ &-1/C_k\cdot I_3+L_4\ddot{I}_4+1/C_4'\cdot I_4-1/C_k\cdot I_5=0,\\ &-1/C_k\cdot I_4+L_5\ddot{I}_5+1/C_5'\cdot I_5=0. \end{aligned} \tag{A.3}$$

Dividing each equation by L_i, setting $1/L_iC_i'=\omega_i^2$ and $I_i=I_ie^{i\omega t}$, Eq.(A.3) becomes as,

$$\begin{aligned} &(-\omega^2+\omega_1^2)I_1-\omega_1^2\cdot C_1'/C_k\cdot I_2=0,\\ &-\omega_2^2\cdot C_2'/C_k\cdot I_1+(-\omega^2+\omega_2^2)I_2-\omega_2^2\cdot C_2'/C_k\cdot I_3=0,\\ &-\omega_3^2\cdot C_3'/C_k\cdot I_2+(-\omega^2+\omega_3^2)I_3-\omega_3^2\cdot C_3'/C_k\cdot I_4=0,\\ &-\omega_4^2\cdot C_4'/C_k\cdot I_3+(-\omega^2+\omega_4^2)I_4-\omega_4^2\cdot C_4'/C_k\cdot I_5=0,\\ &-\omega_5^2\cdot C_5'/C_k\cdot I_4+(-\omega^2+\omega_5^2)I_5=0. \end{aligned} \tag{A.4}$$

Dividing each equation by ω_i^2 respectively, Eq.(A.4) is rewritten as,

$$\begin{aligned} &\{1-(\omega/\omega_1)^2\}I_1-C_1'/C_k\cdot I_2=0,\\ &-C_2'/C_k\cdot I_1+\{1-(\omega/\omega_2)^2\}I_2-C_2'/C_k\cdot I_3=0,\\ &-C_3'/C_k\cdot I_2+\{1-(\omega/\omega_3)^2\}I_3-C_3'/C_k\cdot I_4=0,\\ &-C_4'/C_k\cdot I_3+\{1-(\omega/\omega_4)^2\}I_4-C_4'/C_k\cdot I_5=0,\\ &-C_5'/C_k\cdot I_4+\{1-(\omega/\omega_5)^2\}I_5=0. \end{aligned} \tag{A.5}$$

Practically, the differences among the frequencies of each cell is less than 1%, therefore the difference of $\omega_i^2(=1/LC_i')$ is less than 10^{-4}. Thus we can set C_i'/C_k as a particular constant k, which is approximately the same as a half of the coupling constant, i.e. $(\omega_\pi-\omega_0)/(\omega_\pi+\omega_0)$. Then the above equation is simplified as,

$$
\begin{aligned}
&\{1-(\omega/\omega_1)^2\}I_1-k\cdot I_2=0,\\
&-k\cdot I_1+\{1-(\omega/\omega_2)^2\}I_2-k\cdot I_3=0,\\
&-k\cdot I_2+\{1-(\omega/\omega_3)^2\}I_3-k\cdot I_4=0,\\
&-k\cdot I_3+\{1-(\omega/\omega_4)^2\}I_4-k\cdot I_5=0,\\
&-k\cdot I_4+\{1-(\omega/\omega_5)^2\}I_5=0.
\end{aligned}
\qquad (A.6)
$$

Solving the above equation for ω_i/ω, one obtains

$$
\begin{aligned}
&\omega_1/\omega=1/\sqrt{1-k\cdot I_2/I_1},\\
&\omega_2/\omega=1/\sqrt{1-k\cdot(I_1+I_3)/I_2},\\
&\omega_3/\omega=1/\sqrt{1-k\cdot(I_2+I_4)/I_3},\\
&\omega_4/\omega=1/\sqrt{1-k\cdot(I_3+I_5)/I_4},\\
&\omega_5/\omega=1/\sqrt{1-k\cdot I_4/I_5}.
\end{aligned}
\qquad (A.7)
$$

For ideal flat π-mode, i.e. $I_1=-I_2=I_3=-I_4=I_5$, Eq.(A.7) becomes a simple form as,

$$
\begin{aligned}
&\omega_1/\omega=1/\sqrt{1+k},\\
&\omega_2/\omega=1/\sqrt{1+2k},\\
&\omega_3/\omega=1/\sqrt{1+2k},\\
&\omega_4/\omega=1/\sqrt{1+2k},\\
&\omega_5/\omega=1/\sqrt{1+k}.
\end{aligned}
\qquad (A.8)
$$