ON GAUGED BARYON AND LEPTON NUMBERS

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ABSTRACT

The observation that Baryon number and Lepton number are conserved in nature provides strong motivation for associating gauge symmetries to these conserved numbers. This endeavor requires that the gauge group of electroweak interactions be extended from $SU(2)_L \times U(1)_Y$ to $SU(2)_L \times U(1)_R \times U(1)_{Baryon} \times U(1)_{Lepton}$ where $U(1)_R$ couples only to the right-handed quarks and leptons. If it further postulated that right-handed currents exist in par with the left-handed ones, then the full electroweak symmetry is $SU(2)_L \times SU(2)_R \times U(1)_{Baryon} \times U(1)_{Lepton}$. The $SU(2)_L \times SU(2)_R \times U(1)_{Baryon} \times U(1)_{Lepton}$ model is described in some detail. The triangle anomalies of the three families of quarks and leptons in the model are cancelled by invoking leptoquark matter which is new fermionic matter that carries baryon as well as lepton numbers. In addition to the standard neutral boson ($Z^0$), the theory predicts two neutral gauge bosons with mass lower bounds of 120 GeV and 210 GeV which makes these particles prospective candidates for production at LEP, the TEVATRON and the SSC.

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Empirical evidence at the present time suggests that at the level of the strengths of the strong, weak and electromagnetic interactions Baryon number and Lepton number are strictly preserved in Nature. Here we take lepton number to be the sum of the individual lepton numbers associated with the electron and its neutrino \( L_e \), the muon and its neutrino \( L_\mu \) and the tau and its neutrino \( L_\tau \) i.e

\[
L = L_e + L_\mu + L_\tau
\]

(1)

In the standard model of weak and electromagnetic interactions\(^{(1)}\) based on the gauge group \( SU(2)_L \times U(1)_Y \), Baryon and Lepton numbers are separately conserved at the global level. Since these symmetries are anomalous they are violated by instanton effects. However, it has been shown that these Baryon and Lepton number violating instanton effects are small\(^{(2)}\). One may combine Baryon number and Lepton number into Baryon minus Lepton number \( (B - L) \) and Baryon plus Lepton number \( (B + L) \). In this case the former combination is free from anomalies while the latter combination is now anomalous and is violated by instanton effects. Here we consider Baryon and Lepton numbers as being conserved locally. Since no photon-like particles have been discovered to date, the symmetries associated with Baryon number and lepton number must be broken. We examine the implications of such an adventure for the SLC, the LEP and the SSC colliders.

We start by assigning abelian gauge symmetries to Baryon number and lepton number conservation. Conventional quarks are assigned baryon number equal to a third and conventional leptons are assigned Lepton number equal to one. Also weak interactions are taken to be universal for both the quarks and leptons. With this requirement of universality the conventional left handed \( L = (1 + \gamma_5)/2 \), fermions are weak isospin doublets and the right handed \( R = (1 - \gamma_5)/2 \), fermions are weak isospin singlets. It follows that the underlying gauge symmetry of electroweak interactions is extended\(^{(3)}\) from \( SU(2)_L \times U(1)_Y \) to \( SU(2)_L \times U(1)_R \times U(1)_B \times U(1)_L \) where \( U(1)_B \) is \( U(1)_{\text{Baryon}} \) and \( U(1)_L \) is \( U(1)_{\text{Lepton}} \). The current associated with \( U(1)_R \) consists of only the right handed quarks and leptons. If right handed neutrinos are added to accompany the left handed neutrinos, then the structure of the \( U(1)_R \) current is identical to the \( U(1)_B \) current of \( SU(2)_L \) weak isospin currents. If it is further assumed that right-handed charged currents exist on par with left-handed ones\(^{(4)}\), which will be taken to the case in what follows, the underlying symmetry turns out to be more symmetrical between left and right and is\(^{(3)}\) \( SU(2)_L \times SU(2)_R \times U(1)_B \times U(1)_L \).
The full symmetry of elementary particle interactions is taken to be $G$ where

$$G = SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_B \times U(1)_L$$

(2)

Under this symmetry the conventional quarks and leptons transform as follows,

**Quarks**

$$
\begin{align*}
& \left( \begin{array}{c} u_i \\ d_i \\ c_i \\ s_i \\ t_i \\ b_i \\ L \\ R 
\end{array} \right) \sim (3,2,1,1/3,0); \\
& \left( \begin{array}{c} u_i \\ d_i \\ c_i \\ s_i \\ t_i \\ b_i \\ L \\ R 
\end{array} \right) \sim (3,1,2,1/3,0)
\end{align*}
$$

(3)

**Leptons**

$$
\begin{align*}
& \left( \begin{array}{c} \nu_e^o \\ e^- \\ \nu_\mu^o \\ \mu^- \\ \nu_\tau^o \\ \tau^- \\ L \\ R 
\end{array} \right) \sim (1,2,1,0,1); \\
& \left( \begin{array}{c} \nu_e^o \\ e^- \\ \nu_\mu^o \\ \mu^- \\ \nu_\tau^o \\ \tau^- \\ L \\ R 
\end{array} \right) \sim (1,1,2,0,1)
\end{align*}
$$

(4)

Unlike the case of the standard model, the triangle anomalies no longer cancel between the conventional quarks and leptons. To see this, let the gauge fields of $SU(3), SU(2)_L, SU(2)_R, U(1)_B, U(1)_L$ gauge groups generically be denoted by $G, W_L, W_R, B^o, L^o$. Also let the anomaly coefficients with three gauge fields at the vertices of the triangle be denoted by $A(G^3), A(GW^o_L^2), \ldots \ldots$ etc. All the anomaly coefficients vanish except for $A(B^oW^o_L^2), A(B^oW^o_R^2), A(L^oW^o_L^2)$ and $A(L^oW^o_R^2)$. The values of these coefficients for one family of conventional quarks and leptons are

$$A(B^oW^o_L^2) = 1/2, A(B^oW^o_R^2) = -1/2, A(L^oW^o_L^2) = 1/2, A(L^oW^o_R^2) = -1/2$$

Freedom from anomalies can be achieved by introducing exotic quark and lepton representations of $SU(2)$. One solution is to use mirror fermions. Under $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_B \times U(1)_L$, mirror fermions transform in exactly the same way as conventional quarks and leptons but carry opposite chiralities,

$$
(3,2,1,1/3,0)_R + (3,1,2,1/3,0)_L + (1,2,1,0,1)_R + (1,1,2,0,1)_L
$$

(5)
This solution of anomaly cancellation is by no means trivial since mirror fermions occur naturally in schemes of grand unification with at least three families of conventional quarks and leptons i.e if $G$ descends from a grand unifying symmetry like $SO(4N+2) \ (N > 2)$, it will most inevitably be accompanied by mirror fermions. In the model under consideration, three mirror fermion families are required to cancel the anomalies of three families of conventional quarks and leptons. The anomalies may also be cancelled by invoking leptoquark matter into the theory. Leptoquark matter represents fermions that simultaneously carry baryon number as well as lepton number. In choosing the leptoquark representations the following guidelines are implemented:

- The electric charge carried by the leptoquark fermions are multiples of $\pm \frac{1}{3}$. This choice ensures that there are no unconventional charges in the theory.

- The members of the leptoquark representation are fractionally charged if the representation transforms under $SU(3)$ of colour. The low dimensionality representations are taken to be $(3,2,1,a,b) + (3,1,2,a,b)$ and will be referred to as quarklike leptoquarks since these representations resemble the representation structure to which the conventional quarks belong in the standard model.

- The members of the leptoquark representation are integrally charged if the leptoquark representation is a singlet under $SU(3)$ of colour. The low dimensionality representation are taken to be $(1,2,1,x,y) + (1,1,2,x,y)$ and will be referred to as leptonlike leptoquarks since these representations resemble the representations to which the conventional leptons belong in the standard model.

Leptoquark matter representations that cancel the anomalies of the three families of conventional quarks and leptons are

$$(3, 2, 1, 4/3, 1) + (3, 1, 2, 4/3, 1) + (1, 2, 1, -7, -6) + (1, 1, 2, -7, -6)$$  (6)

The quarklike leptoquarks carry charges $(2/3, -1/3)$ and the leptonlike leptoquarks carry charges $(0, -1)$. Clearly leptoquark representations carrying electric charges other than the conventional ones are also possible. In assigning the electric charges to the representations, the electric charge formula used is

$$Q_{en} = T_L^e + T_R^e + \frac{1}{2} T_B^e - \frac{1}{2} T_l^e$$  (7)

where $T_L^e, T_R^e, T_B^e, T_l^e$ are the diagonal generators of $SU(2)_L, SU(2)_R, U(1)_B, U(1)_l$. 
The gauge bosons and the fermions of the theory acquire masses through higgs mechanism. The relevant higgs scalar representations are a neutral singlet \( S^0 \), two doublets \( \Phi_L, \Phi_R \) and a rank two tensor \( \Phi_{LR} = \Phi \). Under \( G \) their transformation properties are

\[
S^0 \sim (1, 1, 1, 1) \\
\Phi_R \sim (1, 1, 2, -1/2, 1/2) \\
\Phi_L \sim (1, 2, 1, -1/2, 1/2) \\
\Phi \sim (1, 2, 2, 0, 0)
\] (8)

Note that the higgs field \( \Phi \) transforming as \( (1,2,2,0,0) \) not only gives masses to the conventional quarks and leptons but also to the leptoquark fermions required to cancel the anomalies of the three families. A discrete symmetry, left \( \Longleftrightarrow \) right, is imposed on the theory to limit the number of free parameters. In particular, the number of gauge couplings reduce from four to three. These are \( g \) for \( SU(2)_L \) and \( SU(2)_R \), \( g_B \) for \( U(1)_B \) and \( g_t \) for \( U(1)_t \). The discrete symmetry implies that left handed neutrinos are accompanied by their right handed counterparts. The vacuum expectation values of the scalars are taken to be the following,

\[
< S^0 > = \xi \\
< \Phi_R > = \begin{pmatrix} 0 \\ \eta_R \end{pmatrix} \\
< \Phi_L > = \begin{pmatrix} 0 \\ \eta_L \end{pmatrix} \\
< \Phi > = \begin{pmatrix} \kappa_1 & 0 \\ 0 & \kappa_2 \end{pmatrix}
\] (9)

After spontaneous symmetry breaking the mass matrices for the charged and neutral gauge fields are the following,

\[
\begin{pmatrix}
W_L^+ \\
W_R^-
\end{pmatrix} = \begin{pmatrix}
\frac{1}{2} g^2 (\kappa_1^2 + \kappa_2^2 + \eta_L^2) & -\frac{1}{2} g^2 \kappa_1 \kappa_2 \\
-\frac{1}{2} g^2 \kappa_1 \kappa_2 & \frac{1}{2} g^2 (\kappa_1^2 + \kappa_2^2 + \eta_R^2)
\end{pmatrix}
\] (10)

and
where $W^\pm_{L(R)}$, $B^0$, $L^0$ are the gauge fields associated with the generators of $SU(2)_L$, $U(1)_B$, $U(1)_R$. The mass of the charged right-handed gauge bosons are constrained by their contribution to the various $\beta$-decay processes. In order to conform to the highly successful standard electroweak theory as much as possible, the mixing between the charged $W_L$ and $W_R$ gauge bosons is eliminated by taking either $\kappa_1$ or $\kappa_2$ to be zero. In what follows $\kappa_2$ will be set equal to zero.

The most stringent constraints on the mass $M_{W_R}$ come from the $\beta$-decay of the muon and the polarisation of the outgoing electrons. At small momentum transfers, the effective interaction lagrangian in the model for muon decay is

$$L = -\frac{G_F}{\sqrt{2}} \bar{\mu}\gamma^\nu(1 + \gamma_5)\bar{\nu}_\mu \gamma^\lambda(1 + \gamma_5)\nu_e - \frac{G_F}{\sqrt{2}} \frac{M_{W_R}^2}{M_W^2} \bar{\mu}\gamma^\nu(1 - \gamma_5)\bar{\nu}_\mu \gamma^\lambda(1 - \gamma_5)\nu_e$$

(12)

It is found that if $M_{W_R} \geq 3M_{W_L}$ the theoretical prediction agrees with the experimentally measured value within one standard deviation. Thus in this effective theory based on the electroweak gauge symmetry $SU(2)_L \times SU(2)_R \times U(1)_B \times U(1)_R$, the right-handed charged gauge bosons can be light, i.e. masses in the less than one TeV range.

In the neutral boson sector the physical fields consist of the massless photon $A$ with electromagnetic coupling $e$, 

$$\frac{A}{e} = \frac{W^0_L}{g} + \frac{W^0_R}{g} + \frac{B^0}{g_B} - \frac{L^0}{g_l}$$

$$e^{-2} = 2g^{-2} + g_B^{-2} + g_l^{-2}$$

(13)

and three massive neutral gauge bosons with states and the corresponding square masses given by the following expressions in which $\kappa_2$ has been set equal to zero in the light of the discussions in the charged current sector;

$$Z_1 = Z^0 \cos \chi - H^0 \sin \chi$$
\[ M^2_{Z_1} = M_{ZZ} \cos^2 \chi + M_{HH} \sin^2 \chi - M_{ZH} \sin 2 \chi \]  
(14)

\[ Z_2 = \frac{g_B B^0 + g_l L^0}{\sqrt{g_B^2 + g_l^2}} \]

\[ M^2_{Z_2} = \frac{1}{2} (g_B^2 + g_l^2) \xi^2 \]  
(15)

\[ Z_3 = H^0 \cos \chi + Z^0 \sin \chi \]

\[ M^2_{Z_3} = M_{HH} \cos^2 \chi + M_{ZZ} \sin^2 \chi + M_{ZH} \sin 2 \chi \]  
(16)

where the mixing angle and the elements of the \( Z, H \) mass matrix are defined as follows,

\[ \tan 2 \chi = \frac{2 M_{ZH}}{M_{HH} - M_{ZZ}} \]  
(17)

\[ M_{ZZ} = M_{Z^0}^2 \]  
(18)

\[ M_{HH} = M_{Z^0}^2 \sin^2 \theta_w \left[ \frac{\kappa_1^2 \cos^2 \theta_w}{\sin^2 \theta_w (\kappa_1^2 + \eta_L^2)} \right] \]

\[ + \frac{\cos 4 \theta_w \eta_R^2}{\cos 2 \theta_w \sin^2 \theta_w (\kappa_1^2 + \eta_L^2)} + \frac{\sin^2 \theta_w \eta_L^2}{\cos 2 \theta_w (\kappa_1^2 + \eta_L^2)} \]  
(19)

\[ M_{ZH} = M_{Z^0}^2 \left[ \frac{\sin^2 \theta_w \eta_L^2}{\sqrt{\cos 2 \theta_w (\kappa_1^2 + \eta_L^2)}} - \frac{\sqrt{\cos 2 \theta_w \kappa_1^2}}{(\kappa_1^2 + \eta_L^2)} \right] \]  
(20)

\[ M_{ZZ} = M_{Z^0}^2 \]  
(21)

In working out the neutral massive eigenstates the gauge couplings \( g_B \) and \( g_l \) are taken to be of the same order of magnitude, i.e. \( g_l \approx g_B \approx e \sqrt{\frac{2}{\cos 2 \theta_w}} \). The gauge field \( Z_2 \) couples to fermion number which we define to be Baryon number plus Lepton number. The fields in the definitions of \( Z_1, Z_3 \) are the massive neutral boson \( Z^0 \) of \( SU(2)_L \times U(1)_Y \). The weak hypercharge \( Y^0 \) of \( SU(2)_L \times U(1)_Y \) and \( H^0 \) that is orthogonal to \( Y^0 \) are defined as follows.

\[ \frac{Y^0}{g_Y} = \frac{W_R^0}{g} + \frac{B^0}{g_B} + \frac{L^0}{g_l} \]  
(22)

\[ Z^0 = W_L^0 \sin \theta_w + Y^0 \cos \theta_w \]  
(23)

\[ H^0 = -S_{B-l} \sin \omega + W_R^0 \cos \omega \]  
(24)

where \( S_{B-l} \) is the Baryon minus lepton number gauge field of the left-right symmetric model\(^{(9)}\),

\[ S_{B-l} = \frac{g_l B^0 - g_B L^0}{\sqrt{g_l^2 + g_B^2}} \]  
(25)

The weak mixing angle is \( \sin \theta_w = e/g \), \( \sin \omega = \sec \theta_w \sqrt{\cos 2 \theta_w} \) and \( M_{Z^0} \) is the mass of the neutral boson of the standard \( SU(2)_L \times U(1)_Y \) model.
Out of these, $Z_1$ is constrained to be almost the $Z^0$ gauge boson of the standard $SU(2)_L \times U(1)_Y$ model with mass of 91 GeV\(^{(10)}\). Constraints on the masses of $Z_2$ and $Z_3$ come from parity violating neutral current interactions involving the neutrinos and the electrons. The interaction Lagrangian for the various processes is parameterised in terms of the effective couplings as follows,

$$\sqrt{2}G_F^{-1}L = \sum_{q=u,d} \bar{\nu}_\mu \gamma^\mu (1 - \gamma_5) \nu_\mu [G^u_L \bar{q} \gamma_\mu (1 - \gamma_5) q + G^d_R \bar{q} \gamma_\mu (1 + \gamma_5) q + \bar{e} \gamma_\mu (G^u_V - G^d_A) e + \delta_{\mu e} \bar{e} \gamma_\mu (1 - \gamma_5) e] + C_{1q} \bar{e} \gamma^\mu \gamma_5 e \bar{q} \gamma_\mu q + C_{2q} \bar{e} \gamma^\mu \gamma_5 q$$

The experimentally measured values of the various couplings\(^{(11)}\) are

$$G^u_L = 0.339 \pm 0.017 \quad G^d_R = -0.172 \pm 0.014 \quad G^d_L = -0.429 \pm 0.014$$
$$G^u_R = -0.011 \pm 0.037 \quad G^d_V = -0.044 \pm 0.036 \quad G^u_A = -0.498 \pm 0.027$$
$$C_{1u} = -0.249 \pm 0.071 \quad C_{1d} = 0.381 \pm 0.064 \quad C_{2u} - \frac{1}{2} C_{2d} = 0.19 \pm 0.37$$

For comparison the value of the weak mixing angle used is $\sin^2 \theta_w = 0.22$. For the predictions of the model to fall within two standard deviations of the experimentally measured values of the couplings, the lower bounds on the masses of $Z_2$ and $Z_3$ gauge bosons are 120 GeV and 210 GeV. These particles fall within the energy regimes of LEP, the TEVATRON and the SSC. All fermions derive masses and mixing angles from Yukawa couplings involving only the higgs field $\Phi \sim (1, 2, 2, 0, 0)$. The Yukawa interaction Lagrangian is

$$L_{\text{Yukawa}} = \sum_{u, \nu} y_{f^u f^{\nu'} \Phi} f^u L f^{\nu'} R + \sum_{d, e} y_{f^d f^{e'} \Phi} \tilde{f}^d L \tilde{f}^{e'} R + h.c$$

where $\Phi = \sigma_2 \Phi^* \sigma_2$. After spontaneous symmetry breaking, the generic form of the mass matrices in the fermion-leptoquark fermion bases ($f, F$) is

$$M^u = \left( \begin{array}{ccc} y_{f^u f^{\nu'} \Phi} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_{F^u F^{\nu'} \Phi} \end{array} \right)$$

where $f^u = u, c, t$ and $F^u = U, N$ correspond to the u-type leptoquark fermions. There are similar mass matrices $M^d$, $M^{\nu}$, $M^e$ for the fermions $f^d = d, s, b$; $f^{\nu} = e, \mu, \tau$; $f^e = \nu_e, \nu_\mu, \nu_\tau$ and the d-type leptoquark fermions $F^d = D, E$. The mass matrices are diagonalised by the bi-unitary transformations $U^u_L, U^u_R$.
The Kobayashi-Maskawa type mixing matrices are given by the usual formulas, $V_L^{KM} = k_L^u k_R^d$ and $V_R^{KM} = k_R^u k_L^d$ and each matrix consists of three angles and one phase since there is no mixing between the $W^+_L$ and $W^+_R$ gauge bosons. The source of CP violation in this model is intrinsic just as in the standard model.

At present the lower bound on the masses of the leptoquark fermions come from the reaction $e^+ e^- \rightarrow FF$ at TRISTAN\(^{(12)}\). The model proposed here has characteristic features that are different from other models\(^{(13)}\) like the $E_6$ superstring inspired models also with neutral gauge bosons in the 100 GeV to 1 TeV range. For instance, the neutrinos of the $Z_2$ boson couple to conventional leptonic and hadronic matter with vectorlike couplings and $Z_3$ has no full strength couplings to the conventional left handed neutrinos.

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References


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