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REPRESENTATION OF SYMMETRIC METRIC CONNECTION VIA RIEMANN–CHRISTOFFEL CURVATURE TENSOR

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Bivector $\bar{\sigma}_{\mu}^{\nu'}$ which is the Jacoby matrix of the transformation to the Riemanian coordinates is considered in the paper. Basing on the dual nature of $\bar{\sigma}_{\mu}^{\nu'}$ we i) have obtained the representation of metric connection (Christoffel symbols) at the Riemanian coordinates via Riemann-Christoffel curvature tensor; ii) have constructed the covariant conserved four-momentum in the general theory of relativity.

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1. Introduction

In the paper [1] the gauge-covariant conserved charge in the Yang-Mills theory with groups SU(N) has been built up. The suggested construction is based on the dual nature of parallel displacement operator. On the one hand, it is a bivector, on the other hand, it can be considered as gauge transformation function leading to the contour gauge, which is characterized by the representation of vector potential via field strength tensor [2] (see also ref. [1]).

In the present paper we consider the analogous approach to the determination of the energy-momentum integral conservation law in general relativity. In Section 2 we study the object which existence is the corollary of the fundamental geometrical notions which, on the one hand, is a bivector and, on the other hand, can be considered as a Jacoby matrix of coordinates transformation. In Section 3 the derivation of covariant conserved four-momentum in general relativity is given.

2. g - and σ-displacement

Let us begin with the fixed coordinate system x and consider the transformation: $x^{\mu} \to \bar{x}^{\bar{\mu}}$. The quantity $a_{\mu}^{\nu'}(x,x')$ (we assume that $a_{\mu\nu'}(x,x') = a_{\nu'\mu}(x',x)$) is a bivector if under the transformation of coordinates it changes as vector at each point x and x':

$$a_{\bar{\mu}}^{\bar{\nu}'}(\bar{x}(x), \bar{x}'(x')) = \frac{\partial x^{\alpha}}{\partial \bar{x}^{\bar{\mu}}} \, a_{\alpha}^{\gamma'}(x,x') \, \frac{\partial \bar{x}^{\bar{\nu}'}}{\partial x^{\gamma'}} \quad . \tag{1}$$

Let us fix point x'. In order the invertible matrix $a_{\mu}^{\nu'}(x,x')$ ($\det(a_{\mu}^{\nu'}) \neq 0$) could be considered as the Jacoby matrix of the transformation $x^{\mu} \to a^{\mu'}$:

$$\frac{\partial a^{\nu'}}{\partial x^{\mu}} = a^{\nu'}_{\mu}(x,x') \quad , \tag{2}$$

it is necessary and sufficient to fulfill the condition of the system (2) integrability [3]

$$\partial_{\mu} a^{\nu'}_{\nu}(x,x') = \partial_{\nu} a^{\nu'}_{\mu}(x,x') \quad ,$$

which due to the symmetry of metric connection $\Gamma^{\alpha}_{\mu\nu} = \Gamma^{\alpha}_{\nu\mu}$ can be written down in the covariant form

$$a^{\nu'}_{\nu;\mu} = a^{\nu'}_{\mu;\nu} \quad . \tag{3}$$

Semicolon denotes the covariant derivative

$$\varphi_{\mu;\nu} = \partial_{\nu}\varphi_{\mu} - \Gamma^{\alpha}_{\mu\nu}\varphi_{\alpha} \quad ,$$

$$\varphi^{\mu}{}_{;\nu} = \partial_{\nu}\varphi^{\mu} + \Gamma^{\mu}_{\alpha\nu}\varphi^{\alpha} \quad .$$

From the bivector transformation law (1) it follows that in the coordinate system a

$$a^{\nu'}_{\mu'}(a(x),a'(x')) = a^{\nu'}_{\mu'}(x',x') .$$

The dependence on the choice of boundary condition - $a^{\nu'}_{\mu'}(x',x')$ reduces to the affine (linear) transformation of coordinate system a. Further we shall always assume

$$a^{\nu'}_{\mu'}(x',x') = \delta^{\nu'}_{\mu'} \quad . \tag{4}$$

It means that coordinate system a touches system x at the point x' so that at this point components of any tensors coincide. The affine transformation of system a corresponds to the arbitrary transformation of system x.

Bivector $a^{\nu'}_{\mu}(x,x')$ sets the rule of a-displacement of contravariant vector φ^{μ} from point x to point x':

$$\varphi^{\nu'} = \varphi^{\mu} a_{\mu}^{\nu'}(x,x') \quad .$$

From the condition of conservation of vector norm $\varphi_{\mu}\varphi^{\mu}$ follows the displacement rule of covariant vector

$$\varphi_{\nu'} = a^{-1}{}^{\mu}_{\nu'}\,\varphi_{\mu} \quad ,$$

where $a^{-1}{}^{\mu}_{\nu'}$ is inverse matrix to $a_{\mu}^{\nu'}$. The notion of a-displacement is extended to the arbitrary tensors according to the rule: every contravariant index is transferred from point x to point x' by matrix $a_{\mu}^{\nu'}$, and covariant ones - by matrix $a^{-1}{}^{\mu}_{\nu'}$.

The algebraic operation (addition, direct product, convolution) is extended to the tensors defined at different space-time points according to the rule: in order to apply the algebraic operations it is necessary at first to place the tensors at one point x with the help of a-displacement. Basing on this rule by analogy with ref. [1] one can define the integral of vector

$$\int dx\, g^{1/2}\, \varphi^{\mu}(x)\, a_{\mu}^{\nu'}(x,x') \quad ,$$

where $g \equiv -\det(g_{\mu\nu})$, and covariant differential with respect to a distant point

$$dx^{\mu}\partial_{\mu}\left(\varphi^{\alpha}(x)\, a_{\alpha}^{\nu'}(x,x')\right) \quad . \tag{5}$$

These definitions are trivially generalized on the arbitrary tensors.

Eq. (5) in the limit $x \to x'$ must coincide with the ordinary covariant differential due to uniqueness of the last one. From this it follows that

$$a_{\mu}^{\nu'}(x,x') \xrightarrow[x \to x']{} \delta_{\mu'}^{\nu'} + \Gamma^{\nu'}_{\mu'\alpha'}(x')(x-x')^{\alpha'} + \ldots \tag{6}$$

From the connection transformation law

$$\Gamma^{\bar{\alpha}}_{\bar{\mu}\bar{\nu}} = - \frac{\partial x^{\gamma}}{\partial \bar{x}^{\bar{\mu}}} \frac{\partial x^{\rho}}{\partial \bar{x}^{\bar{\nu}}} \left(\frac{\partial \bar{x}^{\bar{\alpha}}}{\partial x^{\rho}} \right)_{;\gamma}$$

it follows that in the system a the connection has the form

$$\Gamma^{\alpha'}_{\mu'\nu'} = - a^{-1\,\gamma}_{\ \ \mu'}\, a^{-1\,\rho}_{\ \ \nu'}\, a^{\alpha'}_{\ \rho;\gamma} \quad . \tag{7}$$

Substituting eq. (6) into eq. (7) we obtain in accordance with ref. [4]

$$\Gamma^{\alpha'}_{\mu'\nu'}(a'(x')) = 0$$

Thus, the coordinate system a is geodesical at point x'.

We shall consider the bivectors defined on the family of geodesics starting at point x'. Let us assume that point x' has not conjugate points (i.e. geodesics intersect only at point x') in the considered region of space-time. Point x is connected with point x' by geodesic which is given in parametric representation $z^{\mu}(\tau)$, where τ is a canonical parameter, by means of the equation

$$\frac{D}{d\tau}\frac{dz^{\mu}}{d\tau} \equiv \frac{dz^{\nu}}{d\tau}\left(\frac{dz^{\mu}}{d\tau}\right)_{;\nu} = 0 \tag{8}$$

and boundary conditions $z(\tau) = x,\ z(\tau') = x'$.

World function - biscalar $\sigma(x,x')$ is defined on the family of geodesics [5,6]

$$\sigma(x,x') = \frac{1}{2}(\tau-\tau')\int_{\tau'}^{\tau} d\tau''\, g_{\mu''\nu''}\frac{dz^{\mu''}}{d\tau''}\frac{dz^{\nu''}}{d\tau''} \quad , \tag{9}$$

$$\bar{\sigma}(x,x') = -\sigma(x,x') \quad .$$

Covariant derivatives of $\sigma(x,x')$ with respect to x are denoted by simple indices, and with respect to x' - by prime indices. The following commutation relations are valid [5]

$$\sigma_{\dots\mu'\nu\dots} = \sigma_{\dots\nu\mu'\dots} \quad , \tag{10}$$

$$\sigma_{\mu\nu\dots} = \sigma_{\nu\mu\dots} \quad , \quad \sigma_{\mu'\nu'\dots} = \sigma_{\nu'\mu'\dots} \quad .$$

The world function satisfies the fundamental equations

$$\sigma_{\mu}\sigma^{\mu} = 2\sigma \quad , \quad \sigma_{\mu'}\sigma^{\mu'} = 2\sigma \quad , \tag{11}$$

where

$$\sigma^{\mu} = g^{\mu\nu}\sigma_{\nu} = (\tau-\tau')\frac{dz^{\mu}}{d\tau} \quad , \tag{12}$$

$$\sigma^{\mu'} = g^{\mu'\nu'}\sigma_{\nu'} = -(\tau-\tau')\frac{dz^{\mu'}}{d\tau} \quad .$$

The existence of parallel displacement bivector - $g_{\mu}^{\nu'}$ is based on the fundamental geometrical notion - the connection. It is determined by the differential equation [6]

$$\sigma^{\gamma} g_{\mu;\gamma}^{\nu'} = 0 \tag{13}$$

with boundary condition (4). The bivector of parallel displacement has a group property [7]

$$g_{\mu}^{\nu''} g_{\nu''}^{\alpha'} = g_{\mu}^{\alpha'} \quad ,$$

where all three points x, x' , x'' belong to the same geodesic. In order to bring the possibility of consideration of $g_{\mu}^{\nu'}$ as Jacoby matrix to light let us obtain the expression for $g_{\mu;\rho}^{\nu'}$ (see also ref. [5]). We use the method proposed in refs. [2,7]. As the result of covariant differentiation with respect to ρ with the help of the commutation rule

$$\varphi_{\alpha;\beta\gamma} - \varphi_{\alpha;\gamma\beta} = R^{\mu}{}_{\alpha\beta\gamma}\,\varphi_{\mu} \quad ,$$

where

$$R^{\mu}{}_{\nu\alpha\beta} = \partial_{\alpha}\Gamma^{\mu}_{\nu\beta} - \partial_{\beta}\Gamma^{\mu}_{\nu\alpha} + \Gamma^{\rho}_{\nu\beta}\Gamma^{\mu}_{\rho\alpha} - \Gamma^{\rho}_{\nu\alpha}\Gamma^{\mu}_{\rho\beta} \quad ,$$

one obtains

$$\sigma^{\gamma} g^{\nu'}_{\mu;\rho\gamma} + \sigma^{\gamma}_{\rho}\, g^{\nu'}_{\mu;\gamma} + R^{\alpha}{}_{\mu\gamma\rho}\, g^{\nu'}_{\alpha}\sigma^{\gamma} = 0 \, . \tag{14}$$

Let us convolve eq. (14) with vector of deviation of geodesics δz^{ρ}, which satisfies the equation

$$\left(\frac{D}{d\tau}\right)^{2}\delta z^{\nu} - R^{\nu}{}_{\alpha\mu\rho}\frac{dz^{\alpha}}{d\tau}\frac{dz^{\mu}}{d\tau}\,\delta z^{\rho} = 0$$

with boundary conditions $\delta z^{\alpha}(\tau) = \delta x^{\alpha}, \ \delta z^{\alpha}(\tau') = 0$. Taking account of [5]

$$\sigma^{\mu}_{\nu}\,\delta z^{\nu} = \sigma^{\nu}\,\delta z^{\mu}{}_{;\nu} \quad , \tag{15}$$

one obtains

$$\sigma^{\gamma}\left(g^{\nu'}_{\mu;\rho}\,\delta z^{\rho}\right)_{;\gamma} = -R^{\alpha}{}_{\mu\gamma\rho}\, g^{\nu'}_{\alpha}\sigma^{\gamma}\delta z^{\rho} \, .$$

Integrating this inhomogeneous equation with boundary condition $g^{\nu'}_{\mu;\rho}\,\delta z^{\rho}\big|_{x'} = 0$, we obtain [5]

$$g^{\nu'}_{\mu;\rho} = -\int_{\tau'}^{\tau}\frac{d\tau''}{\tau''-\tau'}\, g^{\beta''}_{\mu}\, g^{\nu'}_{\alpha''}\, R^{\alpha''}{}_{\beta''\gamma''\rho''}\,\sigma^{\gamma''}\frac{\partial z^{\rho''}}{\partial x^{\rho}} \quad , \tag{16}$$

where $\delta z^{\rho''} = (\partial z^{\rho''}/\partial x^{\rho})\,\delta x^{\rho}$.

Let us show now with the help of eq. (16) that the integrability condition (3)

$$g^{\nu'}_{\nu;\mu} = g^{\nu'}_{\mu;\nu} \tag{17}$$

is equivalent to the equation

$$R_{\mu\nu\alpha\beta} = 0 \quad . \tag{18}$$

Actually, if eq. (18) is satisfied, it follows from eq. (16) that

$$g^{\nu'}_{\nu;\mu} = g^{\nu'}_{\mu;\nu} = 0 \quad .$$

Let us prove the inverse statement. If eq. (17) is satisfied, $g^{\nu'}_{\mu}$ can be considered as Jacoby matrix. Substituting $g^{\nu'}_{\mu}$ in eq. (7) and using eq. (16), we obtain

$$\Gamma^{\gamma'}_{\mu'\nu'} = \int_{\tau'}^{\tau} \frac{d\tau''}{\tau''-\tau'} \, R^{\gamma'}{}_{\nu'\alpha'\beta'} \sigma^{\alpha'} \frac{\partial z^{\beta'}}{\partial y^{\mu'}} \quad . \tag{19}$$

The metric connection ($g_{\mu\nu;\alpha} = 0$) is symmetric

$$\Gamma^{\gamma'}_{\mu'\nu'} = \Gamma^{\gamma'}_{\nu'\mu'} \quad , \tag{20}$$

and Riemann-Christoffel tensor for metric connection satisfies the identity

$$R_{\gamma'\nu'\alpha'\beta'} = - R_{\nu'\gamma'\alpha'\beta'} \quad . \tag{21}$$

From eqs. (19), (21) follows

$$g_{\gamma'\alpha'} \Gamma^{\alpha'}_{\mu'\nu'} \equiv \Gamma_{\gamma'\mu'\nu'} = - \Gamma_{\mu'\gamma'\nu'} \quad . \tag{22}$$

Using eqs. (20), (22) one obtains

$$\Gamma_{\gamma'\mu'\nu'} = -\Gamma_{\mu'\gamma'\nu'} = \Gamma_{\nu'\gamma'\mu'} = -\Gamma_{\gamma'\nu'\mu'} \quad .$$

Therefore $\Gamma^{\gamma'}_{\mu'\nu'} = 0$. As point x is arbitrary, $R_{\mu'\nu'\alpha'\beta'} = 0$, and consequently $R_{\mu\nu\alpha\beta} = 0$.

Thus, the parallel displacement bivector $g^{\nu'}_{\mu}$ can be considered as Jacoby matrix only in the case $R_{\mu\nu\alpha\beta} = 0$. Under this condition coordinate system g will be affine, i.e. $\Gamma^{\gamma'}_{\mu'\nu'} = 0$ at any point x.

The existence of $\bar{\sigma}$-displacement bivector $\bar{\sigma}^{\nu'}_{\mu} = -\sigma^{\nu'}_{\mu}$ is based on the fundamental geometrical notion - metric. It is determined by the differential equation [6]

$$\sigma^{\mu}\bar{\sigma}^{\nu'}_{\rho\mu} = (\delta^{\mu}_{\rho} - \sigma^{\mu}_{\rho})\bar{\sigma}^{\nu'}_{\mu} \tag{23}$$

with boundary condition (4). Eq. (23) is obtained as a result of covariant differentiations of fundamental eq. (11). Due to commutation relations (10) we have

$$\bar{\sigma}^{\nu'}_{\mu\nu} = \bar{\sigma}^{\nu'}_{\nu\mu} \quad ,$$

i.e. integrability condition (3) is fulfilled. Bivector $\bar{\sigma}^{\nu'}_{\mu}$ is the Jacoby matrix of transformation from coordinates x^{μ} , which define geodesic by its endpoint to coordinates $\bar{\sigma}^{\nu'} = -\sigma^{\nu'}$ (12), which define geodesic by tangent vector at initial point x', having the length that equals the length of geodesic and is oriented in direction $x' \to x$ [5,6]. Coordinates $\bar{\sigma}^{\nu'}$ are Riemanian coordinates. Actually, convolving eq. (23) with σ^{ρ} one obtains

$$\sigma^{\rho}\sigma^{\mu}\bar{\sigma}^{\nu'}_{\rho\mu} = \sigma^{\rho}(\delta^{\mu}_{\rho} - \sigma^{\mu}_{\rho})\bar{\sigma}^{\nu'}_{\mu} = 0 \quad . \tag{24}$$

The last equality follows from equation

$$\sigma^{\mu}(\sigma_{\mu\nu}-g_{\mu\nu})=0 \quad ,$$

which is obtained by differentiation of eq. (11). Analogously the following equation is obtained

$$\sigma^{\mu}\sigma_{\mu\nu'}=\sigma_{\nu'}$$

with the help of which we rewrite eq. (24) in the following form

$$0=\sigma^{\rho}\sigma^{\mu}\bar{\sigma}^{\nu'}_{\rho\mu}=\sigma^{\rho'}\sigma^{\mu'}\sigma^{-1}{}^{\rho}_{\rho'}\sigma^{-1}{}^{\mu}_{\mu'}\bar{\sigma}^{\nu'}_{\rho\mu} \tag{25}$$

Since point x' has not conjugate points, then $\det(\sigma^{\nu'}_{\mu})\neq 0$ [6]. Using eq. (7) let us rewrite eq. (25) in the form

$$\bar{\sigma}^{\rho'}\bar{\sigma}^{\mu'}\Gamma^{\nu'}_{\rho'\mu'}=0 \quad . \tag{26}$$

Condition (26) characterizes Riemanian coordinates.

We obtain representations of $\sigma_{\nu\rho}$ and $\bar{\sigma}^{\rho'}_{\nu}$ (see also ref. [5]). Differentiating eq. (11) we come to

$$\sigma^{\mu}\sigma_{\nu\rho\mu}+\sigma^{\mu}_{\rho}\sigma_{\mu\nu}-\sigma_{\nu\rho}=R_{\nu\gamma\mu\rho}\,\sigma^{\gamma}\sigma^{\mu} \quad . \tag{27}$$

We convolve eq. (27) with the vector of geodesics deviation δz^{ρ} and use eq. (15), as a result we obtain

$$\sigma^{\mu}((\sigma_{\nu\rho}-g_{\nu\rho})\delta z^{\rho})_{;\mu}=R_{\nu\gamma\mu\rho}\,\sigma^{\gamma}\sigma^{\mu}\delta z^{\rho} \quad . \tag{28}$$

Integrating eq. (28) with boundary condition $(\sigma_{\nu\rho}-g_{\nu\rho})\delta z^{\rho}\big|_{x'}=0$ one obtains

$$\sigma_{\nu\rho}-g_{\nu\rho}=\int_{\tau'}^{\tau}\frac{d\tau''}{\tau''-\tau'}\,g^{\alpha''}_{\nu}R_{\alpha''\gamma''\mu''\rho''}\sigma^{\gamma''}\sigma^{\mu''}\frac{\partial z^{\rho''}}{\partial x^{\rho}} \quad . \tag{29}$$

From the geometrical sense $g^{\nu'}_{\mu}$ and the fact that the tangent vector is transferred in parallel along geodesic (see eq.

(8)) follows equation [6]

$$g^{\nu}_{\mu'}\sigma_{\nu} = -\sigma_{\mu'} \quad . \tag{30}$$

Differentiating eq. (30) one obtains

$$-\sigma_{\mu'\alpha} = g^{\nu}_{\mu'}\sigma_{\nu\alpha} + g^{\nu}_{\mu';\alpha}\sigma_{\nu} \quad . \tag{31}$$

Substituting eqs. (16), (29) in eq. (31) we come to

$$\bar{\sigma}^{\mu'}_{\alpha} = g^{\mu'}_{\alpha} - \int_{\tau'}^{\tau} d\tau'' \frac{\tau-\tau''}{(\tau''-\tau')^2} g^{\mu'}_{\alpha''} R^{\alpha''}{}_{\gamma''\mu''\rho''}\sigma^{\gamma''}\sigma^{\mu''}\frac{\partial z^{\rho''}}{\partial x^{\alpha}} . \tag{32}$$

Let us note that

$$\frac{\partial z^{\rho''}}{\partial x^{\rho}} = \frac{\tau''-\tau'}{\tau-\tau'}\bar{\sigma}^{-1}{}^{\rho''}_{\alpha'}\bar{\sigma}^{\alpha'}_{\rho} \quad . \tag{33}$$

To be sure in the validity of eq. (33) one can use Riemanian coordinates, where $z^{\rho''} = \bar{\sigma}^{\rho'}(\tau'')$ and

$$\bar{\sigma}^{\rho'}(\tau'') = \bar{\sigma}^{\rho'}(\tau)\frac{\tau''-\tau'}{\tau-\tau'} \quad . \tag{34}$$

We substitute eq. (33) in eq. (32). By simple manipilations we get integral equation

$$\bar{\sigma}^{-1}{}^{\mu}_{\alpha'} = g^{\mu}_{\alpha'} + \frac{1}{\tau-\tau'}\int_{\tau'}^{\tau} d\tau'' \frac{\tau-\tau''}{\tau''-\tau'} g^{\mu}_{\alpha''} R^{\alpha''}{}_{\gamma''\mu''\rho''}\sigma^{\gamma''}\sigma^{\mu''}\bar{\sigma}^{-1}{}^{\rho''}_{\alpha'} , \tag{35}$$

from which by iterations one can obtain $\bar{\sigma}^{-1}{}^{\mu}_{\alpha'}$ with the desired accuracy. Namely

$$\bar{\sigma}^{-1}{}^{\mu}_{\nu'} = g^{\mu}_{\eta'}\Big(\delta^{\eta'}_{\nu'} + \int_{\tau'}^{\tau} d\tau'' C^{\eta'}_{\nu'}(\tau,\tau'') + \tag{36}$$

$$+ \int_{\tau'}^{\tau} d\tau'' C^{\eta'}_{\beta'}(\tau,\tau'')\int_{\tau'}^{\tau''} d\tau''' C^{\beta'}_{\nu'}(\tau'',\tau''') + \dots\Big) \quad ,$$

where

$$C^{\eta'}_{\nu'}(\tau,\tau'') = \frac{1}{\tau-\tau'}\,\frac{\tau-\tau''}{\tau''-\tau'}\, g^{\eta'}_{\alpha''} g^{\gamma''}_{\gamma'} g^{\mu''}_{\mu'} g^{\rho''}_{\nu'} R^{\alpha''}{}_{\gamma''\mu''\rho''}\bar{\sigma}^{\gamma'}(\tau'')\bar{\sigma}^{\mu'}(\tau'') .$$

Using formulae [8]

$$g^{\eta'}_{\alpha''} g^{\gamma''}_{\gamma'} g^{\mu''}_{\mu'} g^{\rho''}_{\nu'} R^{\alpha''}{}_{\gamma''\mu''\rho''} =$$
$$= \sum_{K=0}^{\infty} \frac{1}{K!}\, \bar{\sigma}^{\alpha'_1}(\tau'')\ldots\bar{\sigma}^{\alpha'_K}(\tau'')\, R^{\eta'}{}_{\gamma'\mu'\nu';\alpha'_1\ldots\alpha'_K}(\tau') ,$$

and taking into account eq. (34), we rewrite eq. (36) in the form

$$\bar{\sigma}^{-1\mu}{}_{\nu'} = g^{\mu}_{\eta'}\Big(\delta^{\eta'}_{\nu'} + \tag{37}$$
$$+ \sum_{K=0}^{\infty} \frac{(K+1)}{(K+3)!}\,\bar{\sigma}^{\alpha'_1}(\tau)\ldots\bar{\sigma}^{\alpha'_K}(\tau)\, R^{\eta'}{}_{\gamma'\mu'\nu';\alpha'_1\ldots\alpha'_K}(\tau')\bar{\sigma}^{\gamma'}(\tau)\bar{\sigma}^{\mu'}(\tau)$$

$$+ \sum_{m=0}^{\infty} \frac{\bar{\sigma}^{\alpha'_1}(\tau)\ldots\bar{\sigma}^{\alpha'_m}(\tau)}{(m+4)(m+5)} \sum_{n=0}^{m} \frac{(n+1)}{(m-n)!\,(n+3)!}\, R^{\eta'}{}_{\gamma'\mu'\rho';\alpha'_{n+1}\ldots\alpha'_m}(\tau') \times$$

$$\times R^{\rho'}{}_{\alpha'\beta'\nu';\alpha'_1\ldots\alpha'_n}(\tau')\bar{\sigma}^{\gamma'}(\tau)\bar{\sigma}^{\mu'}(\tau)\bar{\sigma}^{\alpha'}(\tau)\bar{\sigma}^{\beta'}(\tau) + \ldots\Big) .$$

Formulae (37) should be applied together with the following formulae [8]

$$g^{\mu}_{\eta'}\varphi_{\mu} = \sum_{K=0}^{\infty} \frac{1}{K!}\,\bar{\sigma}^{\alpha'_1}(\tau)\ldots\bar{\sigma}^{\alpha'_K}(\tau)\,\varphi_{\eta';\alpha'_1\ldots\alpha'_K}(\tau') .$$

The expression for connection in Riemanian coordinates due to eq. (7) has the form

$$\Gamma^{\alpha'}_{\mu'\nu'} = -\bar{\sigma}^{-1\mu}{}_{\mu'}\bar{\sigma}^{-1\nu}{}_{\nu'}\bar{\sigma}^{\alpha'}_{\mu\nu} = \bar{\sigma}^{\alpha'}_{\rho}\bar{\sigma}^{-1\nu}{}_{\nu'}\big(\bar{\sigma}^{-1\rho}{}_{\mu'}\big)_{;\nu} . \tag{38}$$

Substituting eq. (35) in eq. (38) one obtains

$$\Gamma^{\alpha'}_{\mu'\nu'}(\tau)=\bar{\sigma}^{\alpha'}_{\rho}\bar{\sigma}^{-1\nu}{}_{\nu'}\,g^{\rho}_{\eta';\nu}\left(\delta^{\eta'}_{\mu'}+\frac{1}{\tau-\tau'}\int_{\tau'}^{\tau}d\tau''\,\frac{\tau-\tau''}{\tau''-\tau'}A^{\eta'}_{\rho''}\bar{\sigma}^{-1\rho''}{}_{\mu'}\right)+$$

$$+\bar{\sigma}^{\alpha'}_{\rho}\,g^{\rho}_{\eta'}\int_{\tau'}^{\tau}d\tau''\,\frac{\tau-\tau''}{(\tau-\tau')^{2}}\left\{A^{\eta'}_{\rho'';\nu''}\bar{\sigma}^{-1\rho''}{}_{\mu'}\bar{\sigma}^{-1\nu''}{}_{\nu'}+\right. \tag{39}$$

$$\left.+A^{\eta'}_{\rho''}\bar{\sigma}^{-1\rho''}{}_{\gamma'}\Gamma^{\gamma'}_{\mu'\nu'}(\tau'')\right\} ,$$

where

$$A^{\eta'}_{\rho''}=g^{\eta'}_{\alpha''}R^{\alpha''}{}_{\gamma''\mu''\rho''}\sigma^{\gamma''}\sigma^{\mu''} .$$

Eq. (39) has the simplest form in Riemanian coordinates where $\bar{\sigma}^{\nu'}_{\mu}=\delta^{\nu'}_{\mu}$. Integral equation (39) gives the representation of metric connection in Riemanian coordinates via Riemann-Christoffel curvature tensor. This representation can be done explicit by means of iterations. This representation emphasizes the analogy noted in ref. [9] between Riemanian coordinates (26) and Fock-Schwinger gauge $x^{\mu}A_{\mu}(x)=0$ for which there is the representation of vector potential via field strength tensor.

Metric tensor at Riemanian coordinates is given by

$$g_{\mu'\nu'}=\bar{\sigma}^{-1\mu}{}_{\mu'}\bar{\sigma}^{-1\nu}{}_{\nu'}\,g_{\mu\nu} . \tag{40}$$

Substituting eq. (37) into eq. (40) one obtains the known representation of metric tensor at Riemanian coordinates via Riemann-Christoffel curvature tensor [10]

$$g_{\mu'\nu'}(\tau)=g_{\mu'\nu'}(\tau')+\frac{1}{3}R_{\mu'\alpha'\beta'\nu'}(\tau')\bar{\sigma}^{\alpha'}(\tau)\bar{\sigma}^{\beta'}(\tau)+$$

$$+\frac{1}{3!}R_{\mu'\alpha'\beta'\nu';\gamma'}(\tau')\bar{\sigma}^{\alpha'}(\tau)\bar{\sigma}^{\beta'}(\tau)\bar{\sigma}^{\gamma'}(\tau)+$$

$$+\frac{1}{5!}\left\{R_{\mu'\alpha'\beta'\nu';\gamma'\eta'}+\frac{16}{3}R_{\mu'\alpha'\beta'\rho'}R^{\rho'}{}_{\gamma'\eta'\nu'}\right\}\bar{\sigma}^{\alpha'}(\tau)\bar{\sigma}^{\beta'}(\tau)\bar{\sigma}^{\gamma'}(\tau)\bar{\sigma}^{\eta'}(\tau)+.$$

For arbitrary tensor analogous representation includes the terms with covariant derivatives of given tensor at point x' (see also ref. [10]).

3. Energy-momentum integral conservation law

Using dual nature of $\bar{\sigma}^{\nu'}_{\mu}$ let us construct covariant four-momentum conserved on mass shell

$$G^{\mu\nu}-T^{\mu\nu}=0 \quad , \tag{41}$$

where

$$G^{\mu\nu}=R^{\mu\nu}-\frac{1}{2}g^{\mu\nu}R \quad ,$$

$$R_{\mu\nu}=R^{\alpha}{}_{\mu\alpha\nu} \quad , \quad R=g^{\mu\nu}R_{\mu\nu} \quad ,$$

and $T^{\mu\nu}=-2g^{-1/2}\delta S_m/\delta g_{\mu\nu}$ is the energy-momentum tensor of matter fields (we choose units such that $c=8\pi G=1$, where G is gravitational constant).

Equation

$$g^{-1/2}\partial_{\mu}\left(g^{1/2}(G^{\mu\nu}-T^{\mu\nu})\bar{\sigma}^{\nu'}_{\nu}\right)=0 \tag{42}$$

is the consequence of the equations of motion and covariant conservation law. Indeed, the l.h.s. of eq. (42) can be written as follows

$$\left(G^{\mu\nu}-T^{\mu\nu}\right)_{;\mu}\bar{\sigma}^{\nu'}_{\nu}+\left(G^{\mu\nu}-T^{\mu\nu}\right)\bar{\sigma}^{\nu'}_{\nu\mu} \quad .$$

Taking into account that $T^{\mu\nu}$ and $G^{\mu\nu}$ are tensors, we rewrite eq. (42) in the form

$$g^{-1/2}\partial_\mu\left(g^{1/2}\bar{\sigma}^{-1\,\mu}{}_{\mu'}\left(G^{\mu'\nu'}-T^{\mu'\nu'}\right)\right)=0\quad, \tag{43}$$

where

$$G^{\mu'\nu'}=\bar{\sigma}^{\mu'}_{\mu}\,\bar{\sigma}^{\nu'}_{\nu}\,G^{\mu\nu} \tag{44}$$

is the tensor in Riemanian coordinates. $T^{\mu'\nu'}$ is defined analogously.

Denote the operator of covariant divergence with respect to a distant point by

$$D_{\alpha'}=g^{-1/2}\partial_\alpha\left(g^{1/2}\bar{\sigma}^{-1\,\alpha}{}_{\alpha'}\ldots\right)=(g')^{-1/2}\partial_{\alpha'}\left((g')^{1/2}\ldots\right)\quad.$$

The last equality is obtained with allowance for

$$\Gamma^{\alpha'}_{\mu'\alpha'}=g^{-1/2}\partial_\mu\left(g^{1/2}\bar{\sigma}^{-1\,\mu}{}_{\mu'}\right)\quad.$$

The operators $D_{\alpha'}$ and $D_{\beta'}$ commute

$$D_{\alpha'}D_{\beta'}\ldots=D_{\beta'}D_{\alpha'}\ldots\quad. \tag{45}$$

At the Riemanian coordinates we have

$$G^{\mu'\nu'}+t^{\mu'\nu'}=\frac{1}{2}\,D_{\alpha'}D_{\beta'}\,g^{\mu'\alpha'\nu'\beta'}\quad, \tag{46}$$

where

$$g^{\mu'\alpha'\nu'\beta'}=g^{\mu'\nu'}g^{\alpha'\beta'}-g^{\mu'\beta'}g^{\alpha'\nu'}\quad. \tag{47}$$

Eq. (46) can be considered as definition of quantity $t^{\mu'\nu'}$ by analogy with definition of Landau-Lifshitz pseudotensor [5]. In the Riemanian coordinates $t^{\mu'\nu'}$ is given by

$$2\,t^{\mu'\alpha'}=-\partial_{\beta'}\Gamma^{\nu'}_{\nu'\eta'}g^{\mu'\eta'\alpha'\beta'}+\Gamma^{\beta'}_{\rho'\eta'}\Gamma^{\rho'}_{\beta'\nu'}g^{\mu'\nu'\alpha'\eta'}+ \tag{48}$$

$$+\Gamma^{\alpha'}_{\nu'\eta'}\Gamma^{\mu'}_{\beta'\rho'}g^{\rho'\nu'\eta'\beta'}+\Gamma^{\beta'}_{\eta'\nu'}\left(\Gamma^{\alpha'}_{\beta'\rho'}g^{\mu'\nu'\rho'\eta'}+\Gamma^{\mu'}_{\beta'\rho'}g^{\alpha'\nu'\rho'\eta'}\right).$$

Eq. (46) due to the duality of $\bar{\sigma}^{\nu'}_{\mu}$ is valid in an arbitrary coordinate system, if in it all entering quantities are expressed via $\bar{\sigma}^{\nu'}_{\mu}$ in accordance with their own transformation laws. Tensor $G^{\mu'\nu'}$ is defined by eq. (44), $t^{\mu'\nu'}$ in arbitrary coordinates is given by

$$2t^{\mu'\alpha'}=g^{\mu\eta\alpha\beta}\bar{\sigma}^{\mu'}_{\mu}\bar{\sigma}^{\alpha'}_{\alpha}\left\{\bar{\sigma}^{-1}{}^{\rho}_{\nu'}\bar{\sigma}^{\nu'}_{\rho\eta\beta}-\right.$$
$$\left.-\bar{\sigma}^{-1}{}^{\nu}_{\beta'}\bar{\sigma}^{-1}{}^{\rho}_{\nu'}\bar{\sigma}^{\beta'}_{\eta\beta}\bar{\sigma}^{\nu'}_{\rho\nu}\right\}+g^{\rho\nu\eta\beta}\bar{\sigma}^{\alpha'}_{\nu\eta}\bar{\sigma}^{\mu'}_{\beta\rho}+$$
$$+g^{\mu\nu\rho\eta}\bar{\sigma}^{-1}{}^{\gamma}_{\beta'}\bar{\sigma}^{\beta'}_{\eta\nu}\left(\bar{\sigma}^{\mu'}_{\mu}\bar{\sigma}^{\alpha'}_{\gamma\rho}+\bar{\sigma}^{\alpha'}_{\mu}\bar{\sigma}^{\mu'}_{\gamma\rho}\right)\quad. \tag{49}$$

Eq. (49) is symmetric over indices μ' and α' due to eq. (46). Note that

$$\bar{\sigma}^{-1}{}^{\rho}_{\nu'}\left(\bar{\sigma}^{\nu'}_{\rho\eta\beta}-\bar{\sigma}^{\nu'}_{\rho\beta\eta}\right)=R^{\alpha}{}_{\alpha\eta\beta}=0\quad.$$

Using eq. (46), we rewrite eq. (43) as follows

$$g^{-1/2}\partial_{\mu}\left(g^{1/2}\bar{\sigma}^{-1}{}^{\mu}_{\mu'}\left(T^{\mu'\nu'}+t^{\mu'\nu'}\right)\right)=$$
$$=\frac{1}{2}D_{\mu'}D_{\alpha'}D_{\beta'}g^{\mu'\alpha'\beta'\nu'}=0\quad. \tag{50}$$

The last equality is the obvious consequence of eqs. (45), (47).

Let us use eq. (50) to obtain integral conservation law of energy-momentum of the system consisting of matter fields and gravitational field. Integrating eq. (50) over finite four dimensional volume and using the Stocks theorem one obtains

$$\int dx\, g^{1/2} g^{-1/2} \partial_\mu \left(g^{1/2} \bar{\sigma}^{-1\mu}{}_{\mu'} \left(T^{\mu'\nu'} + t^{\mu'\nu'} \right) \right) = \tag{51}$$

$$= \oint d\Sigma_\mu\, g^{1/2} \bar{\sigma}^{-1\mu}{}_{\mu'} \left(T^{\mu'\nu'} + t^{\mu'\nu'} \right) = 0 \quad ,$$

where $d\Sigma_\mu = \frac{1}{3!} \varepsilon_{\mu\nu\alpha\beta}\, dx^\nu \wedge dx^\alpha \wedge dx^\beta$. Choose as four-volume the cylinder which is obtained by cutting of world tube by two three-surfaces, which belong to the family of noncrossing three-surfaces ordering by parameter s. From eq. (51) in the limit of coincidence of these two surfaces we obtain

$$\frac{d}{ds} \int d\Sigma_\mu\, g^{1/2} \bar{\sigma}^{-1\mu}{}_{\mu'} \left(T^{\mu'\nu'} + t^{\mu'\nu'} \right) = \tag{52}$$

$$= - \oint d\Sigma_{\mu\nu}\, g^{1/2} \lambda^\nu \bar{\sigma}^{-1\mu}{}_{\mu'} \left(T^{\mu'\nu'} + t^{\mu'\nu'} \right) ,$$

where $d\Sigma_{\mu\nu} = \frac{1}{2!} \varepsilon_{\mu\nu\alpha\beta}\, dx^\alpha \wedge dx^\beta$, λ^β is unit vector normal to three-surface on their boundary (*for surface* $t = const$ $\lambda^\beta = (1,0,0,0)$).

Basing on the analogy of electrodynamic the following physical interpretation of eq. (52) can be given. The integral in the l.h.s. of eq. (52) is energy-momentum of the system (the term containing $T^{\mu'\nu'}$ is energy-momentum of matter fields and the term including $t^{\mu'\nu'}$ is energy-momentum of gravitational field); the integral in the r.h.s. of eq. (52) is a sum of energy-momentum flows of matter fields (the term containing $T^{\mu'\nu'}$) and gravitational field (the term including $t^{\mu'\nu'}$). The physical interpretation of eq. (52) is the following: the change of energy-momentum into three-volume equals energy-momentum flow through two-space surrounding the volume. In the absence of energy-momentum flows through two-surface one obtains the conservation law of energy-momentum

$$P^{\nu'} = \int d\Sigma_{\mu}\, g^{1/2}\, \bar{\sigma}^{-1\mu}{}_{\mu'} \left(T^{\mu'\nu'} + t^{\mu'\nu'} \right) = const \quad . \tag{53}$$

Eq. (53) can be rewritten with the help of eq. (46) and Stocks theorem as follows

$$P^{\nu'} = \frac{1}{2} \oint d\Sigma_{\rho\alpha}\, g^{1/2}\, g^{\mu\alpha\beta\nu}\, \bar{\sigma}^{-1\rho}{}_{\mu'}\, \bar{\sigma}^{\mu'}_{\mu\beta}\, \bar{\sigma}^{\nu'}_{\nu} \quad .$$

Let us consider the case $R_{\mu\nu\alpha\beta} = 0$. In virtue of eqs.(16) and (32) we have

$$\bar{\sigma}^{\nu'}_{\mu} = g^{\nu'}_{\mu} \quad , \quad \bar{\sigma}^{\nu'}_{\mu\nu} = 0 \quad . \tag{54}$$

Eq. (42) takes the form

$$g^{-1/2}\, \partial_{\mu} \left(g^{1/2}\, T^{\mu\nu} g^{\nu'}_{\nu} \right) = 0 \quad . \tag{55}$$

It is the consequence of eq. (54) and covariant conservation law. Indeed, l.h.s. of eq. (55) can be written in the following way

$$T^{\mu\nu}{}_{;\mu}\, g^{\nu'}_{\nu} + T^{\mu\nu} g^{\nu'}_{\nu;\mu} \quad .$$

Integrating eq. (55), using Stocks theorem and assuming the absence of energy-momentum flow through surrounding surface, we obtain the conserved covariant four-momentum

$$P^{\nu'} = \int d\Sigma_{\mu}\, g^{1/2}\, T^{\mu\nu} g^{\nu'}_{\nu} \quad . \tag{56}$$

In this case $(R_{\mu\nu\alpha\beta} = 0)$ $g^{\nu'}_{\nu}$ can be considered as Jacoby matrix of transformation to the affine (including Minkowskian) coordinates and also as the set of four Killing vectors

$$\varphi^{(\nu')}_{\nu} = g^{\nu'}_{\nu} \quad ,$$

as in virtue of eq. (54) the equations defining Killing vectors are hold:

$$\varphi^{(\nu')}{}_{\nu;\mu} + \varphi^{(\nu')}{}_{\mu;\nu} = 0 \quad .$$

Thus, eq. (56) agrees with the known results (see ref. [4]) and is the limiting case of eq. (53).

4. Discussion

We have considered quantities $g_{\mu}^{\nu'}$ and $\bar{\sigma}_{\mu}^{\nu'}$ defined on the family of geodesics. Bivectors $g_{\mu}^{\nu'}$ and $\bar{\sigma}_{\mu}^{\nu'}$ can be defined on more general paths families P [1] satisfying the following conditions:

i) family paths meet only at point x';

ii) for every point x there is only one path connecting it with point x';

iii) infinitely close points x and $x+\delta x$ are connected with point x' by infinitely close paths.

In general case bivector $g_{\mu}^{\nu'}$ is defined by equation [7]

$$\frac{dz^{\alpha}}{d\tau} g_{\mu;\alpha}^{\nu'} = 0 \quad ,$$

where $dz^{\alpha}/d\tau$ is the tangent vector to path of family P, and boundary condition (4). The statement that bivector $g_{\mu}^{\nu'}$ can be considered as Jacoby matrix only in the case of $R_{\mu\nu\alpha\beta}=0$ holds true for arbitrary paths family P. In the proof given in Sec. 2 the trivial changes would be done, for instance, the usage instead of eq. (15) the correct in general case equation

$$\delta z^{\mu}\left(\frac{dz^{\nu}}{d\tau}\right)_{;\mu} = \frac{D}{d\tau}\delta z^{\nu}$$

etc.

Relative to an arbitrary family P biscalar $\sigma(x,x')$ is defined by eq. (9). Eqs. (11) and (12) in general do not hold true. Bivector $\bar{\sigma}_{\mu\nu'} \equiv -\sigma_{;\mu\nu'}$ can be considered as Jacoby matrix of the transformation to coordinates $\bar{\sigma}_{\nu'} \equiv -\sigma_{;\nu'}$ for arbitrary paths family P. However as it follows from eq. (56) one would restrict oneself by those families for which bivector $\bar{\sigma}_{\mu}^{\ \nu'}$ has a right limit

$$\bar{\sigma}_{\mu}^{\ \nu'} \xrightarrow[R_{\mu\nu\alpha\beta}\to 0]{} g_{\mu}^{\ \nu'} \quad .$$

Therefore we restrict ourselves by the consideration of geodesics which curvature is caused only by the curvature of space-time.

We have chosen identity (46) for the definition of quantity $t^{\mu'\nu'}$ having been influenced by following considerations. The variation of action functional of gravitational field $2S_g = \int dx\, g^{1/2} R$ on mass shell (41) under infinitesimal variation of coordinate system $x^\mu \to \bar{x}^\mu = x^\mu + \delta\xi^\mu$ can be written down as

$$\bar{\delta}\int dx\, g^{1/2} R = -\int dx\, g^{1/2} T^{\mu\nu}\delta g_{\mu\nu} + \int dx\, \partial_\alpha\left(g^{1/2}\left(R\delta\xi^\alpha + W^\alpha\right)\right), \tag{57}$$

where

$$W^\alpha = g^{\mu\nu}\delta\Gamma^\alpha_{\mu\nu} - g^{\mu\alpha}\delta\Gamma^\nu_{\mu\nu} \quad .$$

Here δ is Lie variation [6]

$$\delta g_{\mu\nu} = -\delta\xi_{\mu;\nu} - \delta\xi_{\nu;\mu} \quad , \tag{58}$$

$$\delta\Gamma^\alpha_{\mu\nu} = \frac{1}{2} g^{\alpha\beta}\left(\delta g_{\beta\mu;\nu} + \delta g_{\beta\nu;\mu} - \delta g_{\mu\nu;\beta}\right) ,$$

$$\delta R^\alpha_{\ \mu\beta\nu} = \delta\Gamma^\alpha_{\mu\nu;\beta} - \delta\Gamma^\alpha_{\mu\beta;\nu} \quad .$$

Let us express all variations in eq. (57) in terms of $\delta\xi^\mu$ with the help of eqs. (58) and use covariant conservation law:

$T^{\mu\nu}{}_{;\nu} = 0$ From condition $\bar{\delta}\int dx\, g^{1/2} R = 0$ it follows that

$$\partial_\mu\left(g^{1/2}\left\{\left(2T^{\mu\nu} + g^{\mu\nu}R - \frac{3}{2}R^{\mu\nu}\right)\delta\xi_\nu + \right.\right. \tag{59}$$

$$\left.\left. + \frac{1}{2}\, g^{\alpha\nu\mu\beta}\left(\delta\xi_{\nu;\alpha\beta} + \delta\xi_{\nu;\beta\alpha}\right)\right\}\right) = 0 \quad .$$

Eq. (59) is obviously covariant. Using equality

$$\frac{1}{2}\, g^{\alpha\nu\mu\beta}\left(\delta\xi_{\nu;\alpha\beta} + \delta\xi_{\nu;\beta\alpha}\right) =$$

$$= -\frac{1}{2}R^{\mu\nu}\delta\xi_\nu + B^{\alpha\rho\mu}\partial_\alpha\delta\xi_\rho + g^{\alpha\nu\mu\beta}\,\partial_\beta\partial_\alpha\delta\xi_\nu \quad ,$$

where

$$B^{\alpha\rho\mu} = \Gamma^{\alpha}_{\beta\nu}\, g^{\nu\rho\beta\mu} + \Gamma^{\rho}_{\beta\nu}\, g^{\alpha\nu\beta\mu} \quad ,$$

we transform eq. (59). Due to arbitrariness of $\delta\xi_\nu$ the coefficients of $\delta\xi_\nu$ and its derivatives in eq. (59) are equal to zero

$$\partial_\mu\left(2g^{1/2}\left(T^{\mu\nu} - G^{\mu\nu}\right)\right) = 0 \qquad (\delta\xi_\nu) \quad , \tag{60}$$

$$2g^{1/2}\left(T^{\rho\alpha} - G^{\rho\alpha}\right) + \partial_\mu\left(g^{1/2}B^{\alpha\rho\mu}\right) = 0 \qquad (\partial_\alpha\delta\xi_\rho) \quad , \tag{61}$$

$$g^{1/2}B^{\alpha\rho\mu} + \partial_\gamma\left(g^{1/2}g^{\alpha\rho\gamma\mu}\right) = 0 \qquad (\partial_\mu\partial_\alpha\delta\xi_\rho) \, , \tag{62}$$

$$g^{1/2}g^{\alpha\nu\mu\beta} + \text{cyclic permutation of } \alpha\mu\beta = 0 \quad (\partial_\mu\partial_\beta\partial_\alpha\delta\xi_\nu) \, . \tag{63}$$

As known, tensor $g^{\alpha\nu\mu\beta}$ has the same symmetries as

Riemann-Christoffel tensor and in particular satisfies eq. (63). One can make sure in the validity of eq. (62) by explicit calculations. The identities (60) and (61) are trivial and does not lead to constructive consequences. From the covariant expressions of divergential term of variation of functional $\int dx\, g^{1/2} R$ does not follow the relation analogous to eq. (46). Instead of identities (60) and (61) we have used identities (46) and (50) which together with identities (62) and (63) form a closed system leading to the constructive consequences.

Identity (46) is valid in arbitrary coordinate system. However, if we define quantity $t^{\mu'\nu'}$ by eq. (48) as it is usually made in the theory of pseudotensor, $t^{\mu'\nu'}$ is not a tensor. Usin- dual nature of $\bar{\sigma}^{\nu'}_{\mu}$ we have defined the quantity $t^{\mu'\nu'}$ in general case by eq. (49) which takes the form of eq. (48) only in Riemanian coordinates. Quantity $t^{\mu'\nu'}$ defined by eq. (49) is nonlocal: it depends not only on the value of space-time curvature at point x, but also on the values of space-time curvature at all points of geodesic connecting the points x and x'. Note that at the very point x' the quantity $t^{\mu'\nu'}$ is equal to zero, i.e. the energy-momentum density of gravitational field is equal to zero at point x'.

We have constructed the covariant, and therefore the physically significant expressions of energy-momentum of system involving matter and gravitational fields confined into a finite three-volume. Before attaching a physical meaning to this construction a physical meaning would be attached to the notion of σ-displacement and, in fact, to the Riemanian coordinate system. As it was noted in ref. [11], in which Riemanian coordinates for the construction of a covariant approximation to the mean energy-momentum pseudotensor in the small region of space-time have been used, "the physical interpretation of normal (Riemanian with the special fixation of affine transformation - author) coordinates come out of their exact correspondence to Minkowskian coordinates in one particular respect, namely the measurement of interval". Actually, as it follows from the results of Sec. 2, all the quantities referred to the Riemanian coordinate system can be obtained by means of covariant differentiation of world function (9) which is a one half of the interval measured between points x and x' along geodesic. Thus, by means of measurement along geodesics of intervals and its variations due to variations of endpoints an observer can determine the energy-momentum in three-volume and the energy-momentum flow through boundary surface.

The covariant derivatives of world function, beginning in the third (which appearing in the expression of energy-momentum of gravitational field), differ from zero only, if the space-time is not flat. This agrees with the fact that the gravitational field exists or not depending on the fact whether Riemann-Christoffel tensor differs from zero or not [5,6] in a global but not a local sence. Due to the nonlocality of the defined energy-momentum density of gravitational field there is the accordance for space-time region including all points of geodesics connecting points x and x' and not for neighbourhood of point x.

REFERENCES

1. A.V.Selikhov, Preprint IAE-4733/1, Moscow, 1988; submitted to "Annals of Physics".
2. S.V.Ivanov and G.P.Korchemsky, Phys.Lett. B154 (1985), 197; S.V.Ivanov, G.P.Korchemsky and A.V.Radyushkin, Yad.Fiz. 44 (1986), 420 (Sov.J.Nucl.Phys. 44 (1986), 145); G.P.Korchemsky, private communication.
3. W.Pauli, "Theory of relativity", Pergamon Press, London, 1958.
4. V.Fock, "The Theory of Space, Time and Gravitation", Pergamon Press, London, 1959.
5. J.L.Synge, "Relativity: The General Theory", North-Holland, Amsterdam, 1960.
6. B.S.De Witt, "Dynamical Theory of Groups and Fields", Gordon and Breach, New York, 1965.
7. J.Winter, J. de Phys. (Paris), 45, Supp. C6 (1984), 53; Phys.Rev. D32 (1985), 1871; and unpublished.
8. A.O.Barvinsky and G.A.Vilkovisky, Phys.Reports 119 (1985), 1.
9. S.N.Nikolaev and A.V.Radyushkin, Nucl.Phys. B213 (1983), 285.
10. A.Z.Petrov, "Einstein Spaces", Fizmatgiz, Moscow, 1961.
11. F.A.E.Pirani, Phys.Rev. 105 (1957), 1089.

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