Beam-Driven Currents in the $1/\nu$ Regime in a Helical System

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Beam currents driven by a neutral particle injection in a helical system (stellarator, heliotron/torsatron) are studied in the 1/ν collisionality regime. The general expression for the beam driven current is obtained for arbitrary magnetic field configurations by solving the drift kinetic equation for electrons. It is found that $F = J_{net}/J_b$ (J_{net} is the net current and J_b is the fast ion beam current) increases as f_t and Z_{eff} where f_t is the fraction of trapped electrons and Z_{eff} is the effective ionic charge number. Especially, for $Z_{eff} \simeq 1$ the effect of trapped electrons is large and F is roughly proportional to f_t. On the other hand, if $Z_{eff} \gtrsim 3$ the effect of trapped electrons becomes small.

KEYWORDS: beam-driven current, Ohkawa current, helical system, stellarator, heliotron, torsatron, neoclassical theory

§1. Introduction

Recently, new helical devices such as CHS[1] and ATF[2] are operated in which the parallel injections of neutral beams are being carried out to heat the plasma. The neutral beam injection parallel to the magnetic field lines easily produces a beam-driven current in the toroidal direction. If a large toroidal current is driven in the helical system (stellarator, heliotron torsatron), the rotational transform changes significantly, and the equilibrium, stability, and the plasma transport might be altered. Therefore, it is very important to estimate the beam-driven toroidal current in the presence of a beam-driven momentum source in helical systems.

In tokamaks the toroidal currents driven by the neutral particle injection have been investigated theoretically by many authors[3-8]. Ohkawa[3] pointed out that the electron return currents due to the collision with fast ions produced by the injection of fast neutrals play an essential role to determine the net toroidal current driven by the neutral beam injection. It is thus important to obtain the electron distribution function in the velocity space in the presence of fast ions. The original Ohkawa theory[3] and together with other calculation[4] assumed a displaced Maxwellian distribution for electrons. This displacement is then determined by balancing the rate at which momentum is gained by the electrons from collisions with fast ions against the rate of loss to the background thermal ions. However, electron electron collisions and the velocity dependence of frictional force between fast ions and electrons lead to a distorted electron distribution different from a simple displaced Maxwellian distribution. Another important effect is the presence of trapped electrons due to the toroidicity which have an effect analogous to their modification of the conductivity of a toroidal plasmas. The effects of trapped particles were first investigated by Connor and Cordey[5] based on the drift kinetic equation with model collision operators. Cordey et al.[6] have developed a full Fokker-Planck kinetic theory which both allowed for distortions in the electron distributions due to fast ion-electron collisions and incorporated electron-electron collisions in the same way as the Spitzer-Härm theory of plasma resistivity. This theory was developed to include the trapped electrons by Start, Cordey, and Jones[7]. The beam driven current was calculated including the trapped electrons for tokamaks of arbitrary aspect ratio by Start and Cordey[8]. The effect of plasma rotation on the beam-driven current is discussed by Hirshman and Sigmar[9]. A beam-driven steady state tokamak reactor is investigated by Mikkelsen and Singer[10].

In the present paper, we formulate the expression for the beam-driven current in the $1/\nu$ collisionality regime of a plasma in a helical device such as stellarator, heliotron and torsatron and investigate the dependence of the beam-driven current on the magnetic field configuration. In §2, we solve a drift kinetic equation for electrons with a model collision operator[11,12] including the effects of trapped electrons. It is assumed that $v_{Ti} \ll v_b \ll v_{Te}$, where v_{Te} and v_{Ti} are electron and ion thermal velocity and v_b is the beam velocity, since this relation holds for experimental conditions. It is shown that the influences of the magnetic field configuration on beam driven current appear through the fraction of trapped electrons. It should be noted that the formulation which will be developed in §2 is valid for arbitrary equilibrium magnetic field configurations in the helical system. The influences of the magnetic field configuration on the beam-driven current are examined in §3 using various vacuum magnetic field configurations. The fraction of trapped particles is dependent on the shift of the magnetic axis. Hence, the ratio of the net current to the fast ion beam current depends on the magnetic axis shift if Z_{eff} is small. Conclusions and discussion are given in §4.

§2. Formulation of the Beam-Driven Current in a Helical System

Neutral atoms injected parallel to the magnetic field lines are ionized in the plasma to produce fast ions which make a beam current colliding with electrons if the beam velocity v_b exceeds a critical velocity. The electron distribution function f_e is then distorted due to the collisions with fast ions and deviates from a local Maxwellian distribution f_{e0}. This deviation of the electron distribution function produces an electron return current. Thus the net beam-driven current is created as the sum of the fast ion beam current and the electron return current. It is the key point to obtain the exact electron distribution function in the presence of fast beam ions to calculate the net beam-driven current.

For small distortions the perturbed distribution function $f_{e1} = f_e - f_{e0}$ satisfies the following linearized drift kinetic equation:

$$v_{\parallel}\frac{\mathbf{B}}{B}\cdot\nabla f_{e1} = C_{ee}(\ f_{e1}\ ) + C_{ei}(\ f_{e1}\ ) + C_{eb}(\ f_{e0}\ ), \qquad 1$$

where $v_{\parallel}$ is the parallel velocity, and C_{ee}, C_{ei}, and C_{eb} are the linearized electron electron, electron-thermal ion, and electron-fast ion collision operators, respectively. The reference frame is used such that the thermal ions are at rest. The perturbed distribution f_{e1} is split into two parts:

$$f_{e1} = f_{e1}^{b} + \hat{f}_{e1}, \tag{2}$$

where f_{e1}^{b} is the distribution function for circulating electrons and satisfies the following equation

$$0 = C_{ee}(f_{e1}^{b}) + C_{ei}(f_{e1}^{b}) + C_{eb}(f_{e0}). \tag{3}$$

For high electron temperatures where the electron thermal velocity v_{Te} greatly exceeds the fast ion velocity v_b, the solution f_{e1}^{b} in Eq.(3) is given in Ref.6 as follows:

$$f_{e1}^{b} = \frac{2Z_b n_b v_b v_\parallel f_{e0}}{Z_{eff} n_e v_{Te}^2}\left[1 + \frac{6}{5}\left(\frac{v_b}{v_{Te}}\right)^2 \right], \tag{4}$$

where Z_b is the fast ion charge number, n_b and n_e are the fast ion and electron densities, respectively, and $Z_{eff} = \sum n_i Z_i^2/n_e$ is the effective ionic charge number. Substitution of Eqs.(2) and (3) into Eq.(1) yields

$$v_\parallel \frac{\mathbf{B}}{B}\cdot\nabla \hat{f}_{e1} = C_{ee}(\hat{f}_{e1}) + C_{ei}(\hat{f}_{e1}). \tag{5}$$

Here, we use the following model collision operators[11,12]:

$$C_{ee}(\hat{f}_{e1}) = \nu_{ee} m_e v_\parallel \frac{\partial}{\partial\mu}\left(\frac{\mu v_\parallel}{B}\frac{\partial \hat{f}_{e1}}{\partial\mu}\right) + \nu_{ee} v_\parallel f_{e0} \frac{\int \nu_{ee} v_\parallel \hat{f}_{e1} d^3v}{\int \nu_{ee} v_\parallel^2 f_{e0} d^3v}, \tag{6}$$

and

$$C_{ei}(\hat{f}_{e1}) = \nu_{ei} m_e v_\parallel \frac{\partial}{\partial\mu}\left(\frac{\mu v_\parallel}{B}\frac{\partial \hat{f}_{e1}}{\partial\mu}\right), \tag{7}$$

where

$$\nu_{ee} = \nu x^{-3/2} h(x), \tag{8}$$

$$\nu_{ei} = \nu Z_{eff} x^{-3/2}, \tag{9}$$

$$\mu = \frac{\frac{1}{2} m_e v_\perp^2}{B}, \tag{10}$$

$$x = \left(\frac{v}{v_{Te}}\right)^2, \tag{11}$$

with

$$\nu = \frac{4\pi n_e e^4 \ln\Lambda}{m_e^2 v_{Te}^3}, \tag{12}$$

$$h(x) = \left(1 - \frac{1}{2x}\right)\Phi(\sqrt{x}) + \frac{1}{\sqrt{\pi x}} e^{-x}, \tag{13}$$

$$\Phi(t) = \frac{2}{\sqrt{\pi}} \int_0^t e^{-y^2} dy. \tag{14}$$

Note that the model collision operator given by Eq.(6) ensures the momentum conservation in the electron-electron collisions. We solve Eq.(5) in the $1/\nu$ collisionality regime using a technique similar to that in a tokamak neoclassical theory, i.e., the expansion of f_{e1} with respect to $\nu_{eff}/\omega_b \ll 1$

$$f_{e1} = f_{e1}^{(0)} + \frac{\nu_{eff}}{\omega_b} f_{e1}^{(1)} + \cdots, \tag{15}$$

where ν_{eff} and ω_b are the effective collision frequency and the bounce frequency, respectively. Substituting Eq.(15) into Eq.(5) gives the fact that f_{e0} is constant along $\boldsymbol{B}$. Operating $\langle \frac{B}{v_\parallel} \cdots \rangle$ to the next order equation where $\langle\ \rangle$ indicates a surface average, we have

$$\left\langle \frac{B}{v_\parallel} C_{ee}(f_{e1}^{(0)}) \right\rangle + \left\langle \frac{B}{v_\parallel} C_{ei}(f_{e1}^{(0)}) \right\rangle = \left\langle \frac{B}{v_\parallel} C_{ee}(f_{e1}^{b}) \right\rangle + \left\langle \frac{B}{v_\parallel} C_{ee}(f_{e1}^{b}) \right\rangle, \tag{16}$$

from which the distribution function $f_{e1}^{(0)}$ for the circulating particles is obtained:

$$\frac{\partial f_{e1}^{(0)}}{\partial \mu} = - \frac{\nu_{ee} f_{e0}}{m_e(\nu_{ee} + \nu_{ei}) \langle v_\parallel \rangle} \frac{1}{\int d^3 v \nu_{ee} v_\parallel^2 f_{e0}}$$

$$\times \left\{ \left\langle B\int_c d^3 v \nu_{ee} v_\parallel f_e^{(0)} \right\rangle - \left\langle B\int d^3 v \nu_{ee} v_\parallel f_{e1}^b \right\rangle \right\} + \frac{1}{<v_\parallel>}\left\langle v_\parallel \frac{\partial f_{e1}^b}{\partial \mu} \right\rangle, \qquad (17)$$

where

$$\left\langle B\int_c d^3 v \nu_{ee} v_\parallel f_e^{(0)} \right\rangle$$

$$= - \left\{ \left\langle B\int_c d^3 v \nu_{ee} \mu \frac{v_\parallel}{<v_\parallel>} \left\langle v_\parallel \frac{\partial f_{e1}^b}{\partial \mu} \right\rangle \right\rangle \right.$$

$$\left. + \left\langle B\int_c d^3 v \frac{\nu_{ee}^2 \mu v_\parallel f_{e0}}{m_e(\nu_{ee} + \nu_{ei})<v_\parallel>} \right\rangle \left\langle B\int d^3 v \nu_{ee} v_\parallel f_{e1}^b \right\rangle \left(\int d^3 v \nu_{ee} v_\parallel^2 f_{e0} \right)^{-1} \right\}$$

$$\times \left\{ 1 - \left\langle B\int_c d^3 v \frac{\nu_{ee}^2 \mu v_\parallel f_{e0}}{m_e(\nu_{ee} + \nu_{ei})<v_\parallel>} \right\rangle \left(\int d^3 v \nu_{ee} v_\parallel^2 f_{e0} \right)^{-1} \right\}^{-1}. \qquad (18)$$

In Eqs.(17) and (18), $\int d^3 v$ and $\int_c d^3 v$ denote the integration in the whole velocity space and in the velocity space only for circulating particles, respectively. Using Eqs.(17) and (18), we can estimate the electron return current $J_{\parallel e} = -\int_c d^3 v f_e^{(0)} v_\parallel$, hence the surface averaged net current is given by $J_{net} = J_b + <J_{\parallel e}>$, where $J_b = Z_b e n_b v_b$ is the fast ion beam current. Thus, the ratio of the net current to the fast ion beam current is written by

$$F = \frac{J_b + <J_{\parallel e}>}{J_b} = 1 - A, \qquad (19)$$

where

$$A = \frac{Z_b}{Z_{eff}} \left[1 + \frac{6}{5}\left(\frac{v_b}{v_{Te}}\right)^2 \right] \frac{<B>^2}{<B^2>} f_c$$

$$\times \left\{ 1 - \frac{4}{3\sqrt{\pi}} \frac{I_1(Z_{eff})}{1 + Z_{eff} \frac{f_c}{f_t} \frac{I_2(Z_{eff})}{I_3}} \right\}, \qquad (20)$$

which expresses the effects of the electron return current and

$$I_1(Z_{eff}) = \int_0^\infty \frac{x^{3/2} h(x) e^{-x}}{h(x) + Z_{eff}} dx, \qquad (21)$$

$$I_2(Z_{eff}) = \int_0^\infty \frac{h(x)e^{-x}}{h(x) + Z_{eff}}\, dx, \tag{22}$$

$$I_3 = \int_0^\infty h(x)e^{-x}\, dx = 0.532840, \tag{23}$$

$$f_c = \frac{4}{3}\frac{<B^2>}{B_{max}^2}\int_0^1 \frac{\lambda d\lambda}{<(1 - \lambda B/B_{max})^{1/2}>},$$

$$f_t = 1 - f_c. \tag{24}$$

Here, f_c and f_t are the fractions of circulating and trapped electrons, respectively. Equations (19) and (20) show that the influences of the magnetic field configuration on the net current appear through the fraction of trapped or circulating electrons. Table I indicates the values of $I_1(Z_{eff})$ and $I_2(Z_{eff})$ for $Z_{eff} = 1 \sim 5$. From Table I and a straightforward calculation, it is shown that the term in the brace in Eq. (20) is not so strongly dependent on Z_{eff} that it is order unity for any value of f_c/f_t. Hence, the effect of electron return current given by A depends mainly on $f_c Z_b/Z_{eff}$. As collisions between electrons and thermal ions increases, the electron return current indicated by A decreases. This fact is expressed as $1/Z_{eff}$ dependence of A. Also, the electron return current increases as the fraction of circulating electrons f_c. However, this is effective only if Z_{eff} is small. Then, the ratio of the net current to the fast ion beam current F increases as f_t and Z_{eff} increase. Especially it increases in roughly proportional to the fraction of trapped electrons f_t when Z_{eff} is small. On the other hand, for large Z_{eff} the effect of trapped electrons becomes small.

§3. Effects of the Magnetic Field Configuration on the Beam Driven Current

As is shown in §2, influences of the magnetic field configuration on the ratio of the net current to the fast ion beam current F appear through the fraction of trapped electrons f_t (or circulating electrons f_c). We examine the dependence of F on the vacuum magnetic field configuration. As an example we take a typical large helical device with the coil number L = 2, the toroidal pitch number M = 10, the major radius at the coil centre R_c = 4 m, the magnetic field strength at the coil centre B_c = 4 T. The magnetic field configuration can be controlled by the vertical and quadrupole fields due to the poloidal coils. The vertical field control is expressed by the shift of the magnetic axis $\delta_a = R_{axis} - R_c$ where R_{axis} is the major radius at which the magnetic axis exists. The profile of the magnetic surface averaged in the toroidal direction is controlled by the quadrupole field.

The fraction of trapped particles f_t does not change so much for the quadrupole field control by the poloidal coils[13]. However, large shift of the magnetic axis changes it considerably. Then, in the following, the quadrupole field is fixed such that the quadrupole filed created by the helical coils is almost cancelled by the one due to the poloidal coils. Figure 1 shows the dependence of f_t on the averaged minor radius ρ for various magnetic axis shifts. For the outward axis shift, the fraction of trapped electrons is large near the magnetic axis and does not depend on the averaged minor radius so much. In contrast with it, for the inward axis shift, the fraction of trapped electrons is small near the axis and increases as the minor radius. These dependences roughly result from the change in the difference between B_{max} and B_{min}. For the large difference, the fraction of trapped electrons is large. The ratio of the net current to the fast ion beam current F vs the averaged minor radius ρ is shown in Fig.2 for various effective ionic charge numbers Z_{eff} and $Z_b = 1$. Figs.2 a and 2 b correspond to the inward shifted magnetic field configuration : $\delta_a = -0.2$ m and to the outward shifted magnetic field configuration : $\delta_a = 0.2$ m, respectively. As is mentioned in §2, F increases as f_t and Z_{eff}. When Z_{eff} is enough small ($Z_{eff} \simeq 1$) $F \simeq f_t$. On the other hand, as Z_{eff} increases F becomes roughly independent of f_t or ρ.

§4. Conclusions and Discussion

The formulation of the beam-driven current in a general helical system is obtained in the $1/\nu$ collisionality regime under the assumption that $v_b/v_{T_e} \ll 1$. The ratio of the net current to the fast ion beam current F depends on the magnetic field configuration through the fraction of circulating electrons f_c or trapped electrons f_t. The leading term of F is $F \simeq 1 - f_c Z_b/Z_{eff}$, which means that the electron return current is proportional to the fraction of circulating electrons and is inversely proportional to the effective ionic charge number Z_{eff}. If $Z_b/Z_{eff} \simeq 1$, F is roughly proportional to the fraction of trapped electrons f_t. However, for the large values of Z_{eff}, F is roughly independent of f_t. It should be noted that although these results are similar to ones in takamaks, in helical plasma the fraction and the distribution of trapped particles change so much by the control of the magnetic field configuration, for example, due to the change of the magnetic axis shift.

In this paper, the rotation of ions is neglected by choosing the reference frame where ions are at rest and the model collision operator ensuring the momentum conservation is used for electron-electron collisions. A unified form of the parallel current including bootstrap current, Spitzer current, and beam-driven current will be reported using more exact collision operator and taking account of the rotation of ions.

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References

1) K.Matsuoka, S.Kubo, M.Hosokawa, Y.Takita, S.Okamura, N.Noda, H.Yamada, H.Iguchi, K.Masai, S.Morita, K.Ida, H.Idei, C.Takahashi, K.Nishimura, T.Shoji, H.Sanuki, M.Fujiwara, Y.Abe, T.Amano, A.Ando, T.Aoki, D.-G.Bi, J.Fujita, S.Hidekuma, T.Kamimura, O.Kaneko, T.Kawamoto, A.Mohri, A.Nishizawa, S.Tanahashi, J.Todoroki, K.Tsuzuki, K.Yamazaki: in *Proc.Twelfth Int.Conf.on Plasma Physics and Controlled Nuclear Fusion Research, Nice, 1988* (IAEA, Vienna) Vol.2 (1989) 411.

2) J.F.Lyon, B.A.Carreras, K.K.Chipley, M.J.Cole, J.H.Harris, T.C.Jernigan, R.L.Johnson, V.E.Lynch, B.E.Nelson, J.A.Rome, J.Sheffield, and P.B.Thompson: Fusion Technol. 10 (1986) 179.

3) T.Ohkawa: Nucl.Fusion 10 (1970) 185.

4) D.F.H.Start, P.R.Collins, E.M.Jones, A.C.Riviere, and D.R.Sweetman: Phys.Rev.Letters · (1980) 1479.

5) J.W.Connor and J.G.Cordey: Nucl.Fusion 14 (1974) 185.

6) J.G.Cordey, E.M.Jones, D.F.H.Start, A.D.Curtis, and I.P.Jones: Nucl.Fusion 19 (1979) 249.

7) D.F.H.Start, J.G.Cordey, and E.M.Jones: Plasma Phys. 22 (1980) 303.

8) D.F.H.Start and J.G.Cordey: Phys.Fluids 23 (1980) 1477.

9) S.P.Hirshman and D.J.Sigmar: Nucl.Fusion 21 (1981) 1079.

10) D.R.Mikkelsen and C.E.Singer: Nucl.Tech.Fusion 4 (1983) 237.

11) L.M.Kovrizhnykh: Report of International Centre for Theoretical Physics, Trieste, IC/70/124 (1970).

12) M.N.Rosenbluth, R.D.Hazeltine, and F.L.Hinton: Phys.Fluids 15 (1972) 116.

13) N.Nakajima, M.Okamoto, J.Todoroki, Y.Nakamura, and M.Wakatani: Nucl.Fusion 29 (1989) 605.

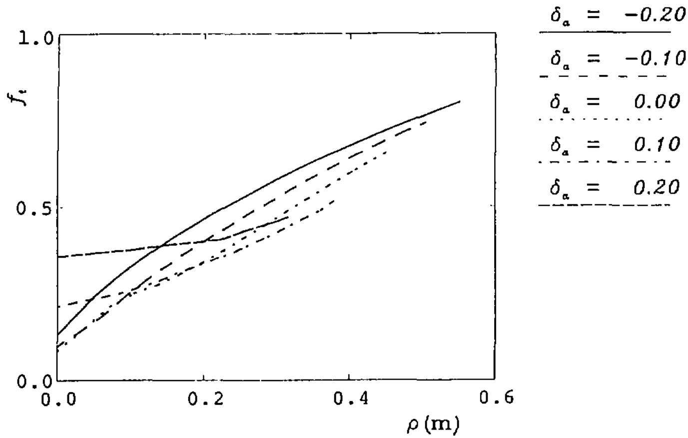
Figure Captions

Fig.1 The fraction of trapped particles f_t given by Eq.(24) vs the averaged minor radius ρ for various magnetic field axis shifts.

Fig.2 The ratio of the net current to the fast ion beam current given by Eq.(19) vs the averaged minor radius ρ for various effective ionic charge number Z_{eff}. (a) : the inward shifted magnetic field configuration with $\delta_a = -0.2$ m, (b) : the outward shifted magnetic field configuration with $\delta_a = 0.2$ m.

Table I Values of the integrations given by Eqs.(21) and (22).

Z_{eff}	I_1 (Z_{eff})	I_2 (Z_{eff})
1.0	0.568855	0.334838
2.0	0.363170	0.204953
3.0	0.266840	0.147872
4.0	0.210924	0.115700
5.0	0.174391	0.095037



1.0
0.5
0.0
f_t
0.0
0.2
0.4
0.6
ρ (m)
δ_a = −0.20
δ_a = −0.10
δ_a = 0.00
δ_a = 0.10
δ_a = 0.20

Fig.1

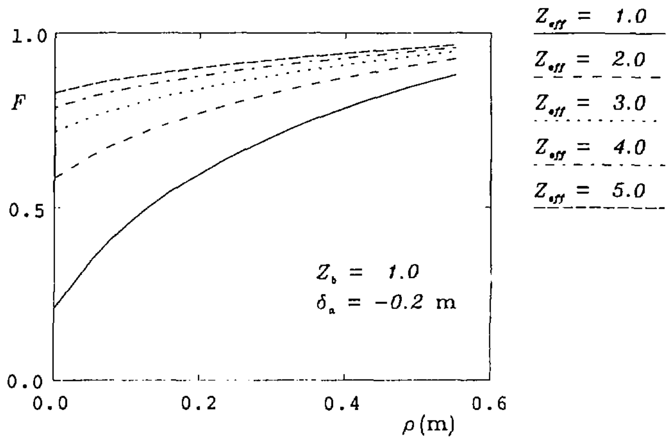
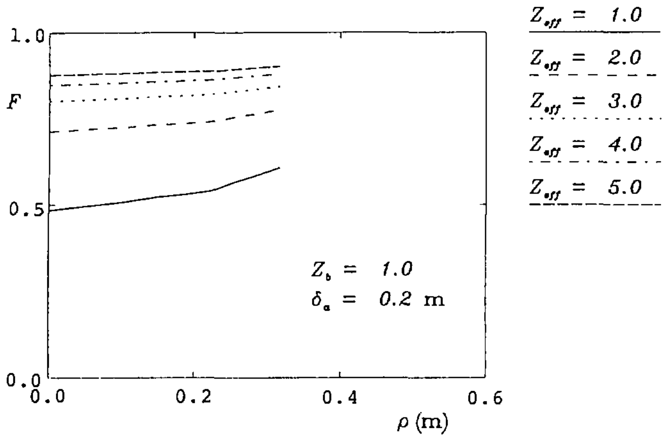

1.0
0.5
0.0
F
0.0
0.2
0.4
0.6
ρ (m)
Z_b = 1.0
δ_a = −0.2 m
Z_eff = 1.0
Z_eff = 2.0
Z_eff = 3.0
Z_eff = 4.0
Z_eff = 5.0

Fig.2-(a)



Z_{eff} = 1.0
Z_{eff} = 2.0
Z_{eff} = 3.0
Z_{eff} = 4.0
Z_{eff} = 5.0
1.0
F
0.5
0.0
Z_b = 1.0
δ_a = 0.2 m
0.0
0.2
0.4
0.6
ρ (m)

Fig.2-(b)