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The inclusive spectra of hadrons (mainly pions) produced in the semileptonic and nonleptonic decays of B-mesons are calculated. Parameters of spectra for different types of "hard" $q\bar{q}$-strings, appearing in the B-meson decays, are determined using the data on e+e- annihilation. Numerical results for B-meson decay induced by both b-->c and b-->u transitions are presented.

Fig. - 5, ref. - 10

The experimental data on B-meson decays ($\bar{B} = (b\bar{q})$) and $B\bar{B}$, $B_s\bar{B}_s$ oscillations [I,2] provide us with unique information on V_{ub}, V_{tb}, V_{ts} elements of quark mixing matrix. Theoretical predictions for the inclusive spectra of secondary hadrons (pions, mainly) in B-meson decay together with the data on hadronic multiplicity distributions obtained previously [3] can be of importance for the experimental data analysis.

Our approach is based on I/N_f expansion [4]. Leading I/N-terms contributing into B-meson decay amplitude are represented by the diagrams of Fig.I a,b,c. For description of final hadronic states we use fragmentation functions [5] of Quark Gluon String Model (QGSM) [6], appropriately modified to describe hard (or semihard) processes like W boson decay, or e+e- annihilation. The data on the later will be extensively used below.

I. The case of $\bar{B}^0 \to D^+h^-$ decay

We start with the consideration of the decay $\bar{B}^0 \to D^+h^-$ (fig.Ia) induced by b ---> c-transition. Here h^- - is a hadronic system with the mass $M=\sqrt{q^2}$ produced in the W^--decay via formation of $\bar{u}d$-string in the intermediate state. ($q=(E,\vec{q})$ - is the 4-momentum of W^--boson.)

The inclusive spectrum of these hadrons $h = \pi$, K-mesons in the B meson rest frame $dN = f(x)d\varepsilon/\varepsilon$, $x = \varepsilon/\varepsilon_{max}$, $\varepsilon_{max} = (M_B^2 - M_D^2)/2M_B$ can be expressed in terms of inclusive spectra $dN_0 = \varphi_h(M, x_0)d\varepsilon_0/\varepsilon_0$, $x_0 = 2\varepsilon_0/M_B$, $\varepsilon_0 = \sqrt{p_0^2+\mu^2}$ in $\bar{u}d$-string rest frame - H_0 and distribution of effective mass $M=\sqrt{q^2}$, $q=k_{\bar{u}}+k_d$ of $\bar{u}d$-string:

$$dW_{\bar{B}^0\to D^+h^-} = F_D(M^2,\cos\theta_0)\,dM^2/M_B^2 \cdot \frac{1}{2}\,d\cos\theta_0$$

$$F_D(M^2, \cos\theta_0) = C_0\left[(2|\vec{q}|/M_B)^3/(1-M^2/M_0^{*2})^2\right]\frac{3}{2}\cdot\sin^2\theta_0 \qquad \text{(I)}$$

In Eq.(I)

$$|\vec{q}| = \left[(M_B^2 - M^2 - M_D^2)^2 - 4M^2M_D^2\right]^{1/2}/2M_B \qquad (2)$$

M_B=5.27 GeV, M_D=I.8 GeV; θ_0 – is the angle between vector $\vec{q}$ and $\bar{u}d$-string direction, i.e. $\vec{k}_d = -\vec{k}_{\bar{u}}$, in H_0-frame. At large M_0^*=6.37 GeV [8] the probability dW in Eq.(I) depends rather weakly on transition formfactor. The constant C_0 is fixed by the normalizati condition $\int dW_{\bar{B}^0 \to D^+ h^-} = 1$.

Terms proportional to the constituent masses of light quarks $\mu \ll M_B$, which are of the order of $\mu^2/M^2, \mu^2/|\vec{q}|^2$ (see Appendix I) do not affect the shape of spectra and omitted in (I).

The upper block in Fig.I contains the "hard" $\bar{u}d$-string of the type produced in e^+e^--annihilation. The data available show that at small energies $\sqrt{S} \sim 2\div4$ GeV, relevant to B-meson decay, string fragmentation must not be strongly anisotropic. However, having no reliable data on the fragmentation angular distribution we consider two opposite cases in the following:

(a) the isotropic string fragmentation, when $dN_0 \simeq \varphi_h(x_0)\frac{d\varepsilon_0}{\varepsilon_0}\cdot\frac{1}{2}d\cos\theta_{\vec{p}_0\vec{q}}$ $\theta_{\vec{p}_0\vec{q}}$ – the angle between produced hadron momentum $\vec{p}_0$ and the directio of the vector $\vec{q}$ in H_0-frame. In this case only the average

$$\bar{F}_D(M^2) = \int_{-1}^{1}\frac{1}{2}d\cos\theta_0\, F_D(M^2, \cos\theta_0) = C_0(2|\vec{q}|/M_B)^3/(1-M^2/M_0^{*2})^2$$

of the distribution (I) will be essential below;

(b) the anisotropic fragmentation of the string (as in the case of soft hadronic interactions at high energies), where hadronic momentum is almost parallel to $\pm\vec{k}_d = \mp\vec{k}_u$; in this case $\theta_0 \simeq \theta_{\vec{p}_0\vec{q}}$ has to be put in (I).

After Lorenz transformation, from H_o to Lab.system we get for the pion energy $\mathcal{E}$:

$$\mathcal{E} = (E/M)(\mathcal{E}_o + \vec{q}\vec{P}_o/E) = \frac{x_o}{2}\left(E + \beta_o |\vec{q}| \cos\theta_o\right) \qquad (3)$$

where $\beta_o = |\vec{P}_o|/\mathcal{E}_o$, $\mathcal{E}_o = \sqrt{\vec{P}_o^2 + \mu^2}$ and $\theta_o = \theta_{\vec{P}_o\vec{q}}$.

Eq. (3) implies that $d\mathcal{E} = \frac{1}{2} x_o \beta_o |\vec{q}| \, d\cos\theta_o$ and $x_o\mathcal{E}_2 > \mathcal{E} > x_o\mathcal{E}_1$

where $\mathcal{E}_{1,2} = \frac{1}{2}(E \pm \beta_o |\vec{q}|) \simeq \frac{1}{2}(E \pm \vec{q})$.

The inclusive spectrum of hadrons $dN/d\mathcal{E}$ in Lab. frame is obtained by averaging of the number $dN_o = \varphi_h(M, x_o)\frac{dx_o}{x_o}$ (or $dN_o \frac{1}{2} d\cos\theta_{\vec{P}_o\vec{q}}$ in the case (a)) of produced hadrons in H_o-frame with the weight (I) and dividing the result by $d\mathcal{E}$:

$$dN/d\mathcal{E} = \int\limits_{M_1^2}^{M_2^2} \frac{dM^2}{M_B^2} \int\limits_{\mathcal{E}/\mathcal{E}_2}^{\mathcal{E}/\mathcal{E}_1} F_D(M^2, \cos\theta_o)\,\varphi_h(M, x_o)\,dx_o/x_o^2|\vec{q}| \qquad (4)$$

Here in accord with Eq.(3) $\cos\theta_o = (2\mathcal{E}/x_o - E)/|\vec{q}|\beta_o$.

The dependences of $\mathcal{E}_1(M)$ and $\mathcal{E}_2(M)$ on M are shown in Fig.2. One can see, that maximum energy $\mathcal{E} \le \mathcal{E}_2(M)$ of hadrons acquires at x --> I and M --> 0:

It exceeds slightly the characteristic energy $\bar{\mathcal{E}}_o = (M_B - M_D)/2$, which is equal to $\mathcal{E}_1 = \mathcal{E}_2$ at $M = M_{max} = M_B - M_D$.

The upper integration limit in (4) at $\mathcal{E} < \mathcal{E}_o$ is $M_2 = M_B - M_D$ however at $\mathcal{E} < \mathcal{E}_2(M)$ it depends on $\mathcal{E}$ and is fixed by the condition $\mathcal{E} < \mathcal{E}_2(M)$ (see Fig.2). Similarly, the lower limit $M = M_1$ of integration in (4) is fixed by condition $\mathcal{E} > \mathcal{E}_1(M)$ However in the region $M < M_o$, $M_o \simeq$ I GeV only π and ρ -mesons are taken into account below separately. This means that one can put practically $M_1 = M_o \simeq$ I GeV in (4) and also put

$$\beta_o = |\vec{P}_o|/\mathcal{E} \simeq \left[1 + 2\mu/Mx_o\right]^{-1/2} \simeq 1.$$

Eq.(4) corresponds to the case (b) of anisotropic string fragmentation, the case (a) is obtained simply by substitution: $F_D \to \bar{F}_D$.

The prediction of QGSM for inclusive spectra of $\pi^{\pm}$-mesons in $\bar{u}d$-string fragmentation is [3,5,6]:

$$\varphi_{\pi^-}(M,x_0)=2C_\pi(1-x_0)^\gamma(1+\lambda x)\,;\ \varphi_{\pi^+}(M,x_0)=2C_\pi(1-x_0)^{\gamma+1}(1+\lambda x) \tag{5}$$

where $(1+\lambda x)$ is the correction factor which ensures energy conservation. In high energy "soft" hadronic interactions (where virtualities of all particles are small: $k_u^2 \sim k_d^2 \sim \langle p_\perp^2 \rangle \ll M^2$) all experimental data are well described by parametrization (5) at $\gamma=0$ and $C_\pi=4/9$ (for kaons $C_K=0.1C_\pi$).

Fragmentation of $u\bar{u}$, $d\bar{d}$ strings produced in e^+e^--annihilation would be described similarly by

$$\Phi^0_{\pi^\pm}(M,x_0)=C_\pi\left[(1-x_0)^\gamma+(1-x_0)^{\gamma+1}\right](1+\lambda x) \tag{6}$$

where two terms (as well as factor 2 in (5)) correspond to the production of hadrons from two ends of the string.

In the case of W decay the produced $q\,\bar{q}$ - strings are "hard", i.e. virtualities of quarks are large $k_u^2 \sim k_d^2 \sim p_\bullet^2 \sim \alpha_s M^2$. Their fragmentation is accompanied by emission of hard gluon, which initiate the cascade of the new strings with smaller masses $M_2 < M_1 < M$. As a result the decay spectrum of these strings becomes softer, in the variable x, with increasing of the energy $\sqrt{s} = M$. The corresponding rise of the exponent $\gamma=\gamma(M)$ in Eq. (5) is very slow one for asymptotically large $M \gg M_\bullet$ [7] where $\gamma(M) \simeq (16/3b)\ln\ln(M/\Lambda)+const$, according to QCD, with $b=9$ and $\Lambda = 0.2$ GeV (see dashed line in Fig.3b). However, in the energy range $M\sim 2$-5 GeV relevant to B meson decay, $\gamma=\gamma(M)$

can change rapidly from zero at $M = M_0 \simeq$ I-2 GeV up to its asymptotic QCD value at large M (as Fig.3 shows). Let us determine $\gamma = \gamma(M)$, and also $\lambda = \lambda(M)$ fitting with Eq.(6) the experimental data [9] on $\pi^{\pm}$-spectra in e+e- annihilation for M < 5.2 GeV. C_π = 4/9 in accord with the data on particle multiplicities [3] . Fig.3 shows that Eq.(6) with linearly rising $\gamma(M) = (M - M_0')/M_1$, M_0' = (I.7±0.I) GeV $M_1 \simeq 0.7 \pm 0.1$ GeV produces good fits of all data. The energy $E_{\pi^+\pi^-}$ carried by π^+ and π^--mesons in e+e-annihilation must be close to 2M/3: $E_{\pi^+\pi} = 2\int_{x_{min}}^{1} \varepsilon \cdot \varphi^0_{\pi^\pm}(M, x_0)\, dx_0/x_0 \simeq \frac{2}{3}M$ with $x_{min} = \frac{2M_\pi}{M}$; $\varepsilon = \frac{x_0 M}{2}$ since the contribution of K mesons and $N\bar{N}$ pairs is negligible. Parameter $\lambda(M)$ is fixed by this condition.

Mass spectrum of $\bar{u}d$-string in $\bar{B}^0$-decay steeply falls at large M and has characteristic resonance structure due to π^- and ρ^--mesons at M < I GeV. This contribution should be added to Eq.(4):

$$\frac{dN_{res}}{d\varepsilon} = C_\rho \int_0^{\sim} \frac{dM^2}{|\vec{q}|\beta_0} \cdot \frac{m_\rho \Gamma_\rho \cdot 3\cos^2\theta_\rho}{(M^2 - m_\rho^2)^2 + m_\rho^2 \Gamma_\rho^2} + C_\pi \cdot \delta(\varepsilon - E_\pi); \tag{7}$$

where $\cos\theta_\rho = (2\varepsilon - E_\rho)/|\vec{q}|\beta_0$, $E_\rho = (M_B^2 + M^2 - M_D^2)/2M_B$, $E_\pi = (M_B^2 - M_D^2)/2M_B$. and constants $C_\rho = 2\cdot 4\pi^2 g_\rho^2/m_\rho^2 M_B^2$ =0.I6

$C_\pi = 2\cdot 4\pi^2 f_\pi^2/M_B^2$ $\simeq$ 0.06 are the probabilities of ρ and π-meson production in the upper block of the diagram - Ia. Here $g_\rho = \frac{\sqrt{2}\, m_\rho^2}{\gamma}$; $\gamma^2/4\pi \simeq 2.36$; f_π =I33 MeV. In our approach resonances with $M > M_0$ = I GeV like A_1 etc. are included in the continuum.

The spectra of charged pions $dN_{\pi^+}/d\varepsilon$; $dN_{\pi^-}/d\varepsilon$ are presented by solid lines in Fig.5a, for the case a) of anisotropic string fragmentation; dashed-dotted lines represent the spectrum of $\pi^{\pm}$-mesons for the anisotropic fragmentation - case b) while the dashed line represents π^--meson spectrum without the contribution of π^--meson from $\bar{\rho}$-decay ($\bar{B}^0 \to D^+ \rho^-$).

2. $\bar{B}^0 \to h^+ h^-$ decay.

The decay $\bar{B}^0 \to h^+ h^-$ associated with $b \to u$ transition is of particular interest; the mean number of hadrons $\langle n_{b\to u} \rangle \simeq 8.3$ is about twice larger [3] than in the $b \to c$ transition case, where $\langle n_{b\to c} \rangle = 4.4$.

The effective mass of hadrons in h^+ channel (the lower block, in Fig.Ib) is defined as

$$M'^2 = (p_u + p')^2 = 2\mu^2 + 2\varepsilon_u \varepsilon' + 2(\vec{q} + \vec{p}')\vec{p}' \simeq 2\varepsilon'^2 + y m_b \varepsilon' + 2|\vec{q}||\vec{p}'|\cos\theta_{\vec{q}\vec{p}'} \tag{8}$$

where $y = I - M^2/M_b^2$, p' and $\varepsilon' = \sqrt{\vec{p}'^2 + \mu^2}$ are momentum and energy of $\bar{d}$-quark in the B meson, $\mu \simeq M_B - m_b$ its constituent mass, $\theta_{\vec{p}'\vec{q}}$ – the angle between $\vec{q}$ and $\vec{p}'$ in B-meson rest frame and $\varepsilon_u \simeq y \cdot m_b/2$ – as in the case of free b-quark decay. The distribution of the momentum $\vec{p}'$ is determined by wave function of internal motion in B-meson:

$$dW(\vec{p}') = |\chi_B(p')|^2 \frac{d^3\vec{p}'}{2\varepsilon(2\pi)^3} \simeq \exp(-p'^2/\mu_0^2)\frac{dp'^2}{\mu_0^2} \tag{9}$$

and correspond to small $\langle \vec{p}'^2 \rangle = \mu_0^2$ with [IO] $\mu_0 \cong 0.3 \div 0.4$ GeV.

Similar to (I), the probability of the decay $\bar{B}^0 \to h^+ h^-$ has the form:

$$dW_{\bar{B}^0 \to h^+ h^-} = F_{p'}(M^2, \cos\theta_0)\frac{dM^2}{M_B^2} dW(p') \cdot \frac{1}{2} d\cos\theta_0 ;$$

$$F_{p'} = C_1 (2|\vec{q}|/M_B) \cdot \frac{3}{2}(M_1^2 - M^2) \cdot [M_1^2 \sin^2\theta_0 + M^2(1-\cos\theta_0)^2]/M_B^4 \tag{IO}$$

where $M_1^2 = M_B^2(I - 2\varepsilon'/M_B)$ and C_1 is fixed by normalization condition

$\int dw_{\bar{B}^0\to h^+h^-}$ =I.

Terms of the order of μ^2 were omitted here - as in Eq.(I) (see Appendix II). The spectrum of hadrons in $\bar{B}^0\to h^+h^-$ decay is the sum of contributions of upper and lower blocks in Fig.Ib:

$$\frac{dN^{tot}}{d\varepsilon} = \frac{dN^{up}}{d\varepsilon} + \frac{dN^{low}}{d\varepsilon} \tag{II}$$

where the upper block contribution is quite similar to (4),(7) and in the case (b) has the form:

$$\frac{dN^{up}}{d\varepsilon} = \int_{M_1^2}^{M_2^2}\frac{dM^2}{M_B^2}\int_{\varepsilon/\varepsilon_2}^{\varepsilon/\varepsilon_1}\frac{dx_0}{|\vec{q}|x_0^2}\int_{p'} dw(p')F_{p'}(M^2, \cos\theta_0)\,\varphi_h^-(M,x_0) + \\ + dN^{up}_{res}/d\varepsilon; \tag{I2}$$

which differs from (4), (7) only by substitution $\int_{p'}dw(p')F_{p'}\to F_D$.

For consistency we omit in the following all terms of the order of $p'\sim\mu$, i.e. we use $dw(p')$ in the form of Eq.(9) with $\mu_0\simeq\mu\to 0$. Note, that in this approximation the integral of $F_{p'}$ over $\cos\theta_0$:

$$\bar{F}_{p'}(M^2) = \frac{1}{2}\int_{-1}^{1} d\cos\theta_0\, F_{p'} \simeq C_1\left(\frac{2|\vec{q}|}{M_B}\right)(M_1^2 - M^2)\cdot(M_1^2+2M^2)/M_B^4 \tag{I3}$$

which has to be put in (I2) in place of $\int_{p'} dw_{p'}F_{p'}$ in the case (a), reduces,at $M_1^2=m_b^2$ to the well known probability of the free b-quark decay: $\bar{F}(M^2)\simeq f_b(y)$ with $f_b(y)=2y^2(3-2y)$, where y=I-M^2/M_B^2, $C_1=2$, $2|\vec{q}|/M_B\simeq y$.

The contribution $dN^{up}_{res}/d\varepsilon$ in Eq.(I2) has exactly the form of Eq.(7) with the single substitution $M_D^2\to M'^2$(in Eqs. for E_ρ, E_π).

Consider now the quantity $dN^{low}/d\varepsilon$. The $u\bar{d}$-string in the lower block in Fig.Ib is directed along the vector $\vec{q}$ because momentum of u-quark, $\vec{p}_u=-\vec{q}-\vec{p}\,'\approx-\vec{q}$, is large in comparison with the momentum $\vec{p}\,'$ of the spectator antiquark $\bar{d}$. It means that the angular distribution of the string is close to $\delta(1-\cos\theta_{\vec{p}_0\vec{q}})$, where $\theta_{\vec{p}_0\vec{q}}$ is the angle between $-\vec{q}$ and $\vec{p}_0$ in the ud-string rest frame H_0. The corresponding contribution of the lower block in the case (b) of the anisotropic string fragmentation is:

$$\frac{dN^{low}}{d\varepsilon} = \frac{1}{2\varepsilon}\int_{M_1^2}^{M_2^2}\frac{dM^2}{M_B^2}\int_{p'} dw(p')\bar{F}_{p'}(M^2)\left[\varphi_h^+(M',x_2')+\varphi_h^+(M',x_1')\right] + dN_{res}^{low}/d\varepsilon ; \quad (I4)$$

where $x_1 = \varepsilon/\varepsilon_1'$ and $x_2 = \varepsilon/\varepsilon_2'$, $\varepsilon_{12}' = (\varepsilon' \pm |\vec{q}|)/2$, are associated with hadrons produced with $\vec{p}_0$ parallel to $\vec{q}$, $E' = \sqrt{|\vec{q}'|^2 + M'^2} = M_B - E$, M' is the mass (8) of the string.

In the case (a) of string isotropic decay the first term here must have the form close to (I2):

$$\frac{dN^{low}}{d\varepsilon} = \int_{M_1^2}^{M_2^2}\frac{dM^2}{M_B^2}\int_{\varepsilon/\varepsilon_1'}^{\varepsilon/\varepsilon_2'}\left(\frac{dx_0'}{|\vec{q}|x_0'^2}\right)\int_{p'} dw(p')\bar{F}_{p'}(M^2)\varphi_h^+(M',x_0') + \frac{dN_{res}}{d\varepsilon} ; \quad (I5)$$

where, in both cases $\bar{F}_{p'}(M^2)$ - is the function given by Eq. (I3) $\bar{F}_{p'} \simeq f_2(y) = 2y^2(3-2y)$ and the contribution of the resonance states is

$$\frac{dN_{res}^{low}}{d\varepsilon} = C_\rho' \int_{M_0^2}^{M_B^2-m_\rho^2} F_\rho(M^2)\frac{dM^2}{M_B^2}\theta(\varepsilon-\varepsilon_1')\theta(\varepsilon_2'-\varepsilon)/|\vec{q}|p_0 + C_\pi'\frac{4\varepsilon^3}{(M_B/2)^4} ;$$

Here $F_\rho(M^2) = 4\cdot(2|\vec{q}|/M_B)^3$ is the normalized spectrum of ρ , or π mesons produced in the lower block in Fig.I, $dW(|\vec{q}|) = F_\rho(|\vec{q}|)\frac{2d|\vec{q}|}{M_B}$ with $|\vec{q}| \simeq (M_B^2 - m_\rho^2 - M^2)/2M_B$, $\frac{2d|\vec{q}|}{M_B} = \frac{dM^2}{M_B^2}$, for ρ-meson, and $|\vec{q}| \simeq \varepsilon$ for the π-meson case; $C_\rho' \simeq 0.2$, $C_\pi' \simeq 0.1$ (see Appendix II). The angular distribution in $\rho \to 2\pi$ decay in the lower block in Fig.Ib is taken to be isotropic.

The results for the charged pions spectra $dN_{\pi^+}/d\varepsilon$, $dN_{\pi^-}/d\varepsilon$ produced in both channels in the decay $\bar{B}^0 \longrightarrow h^+h^-$, and calculated using Eqs.(7)-(I5), (in the case (b) of isotropic string fragmentation) are shown in Fig.5b. The bumps at $\varepsilon \sim \varepsilon_{max}$ are formed by π^- coming from ρ^--decay and by single π^+-mesons from lower block of the diagram in Fig.Ib. More realistic calculations with bound b-quark would lead to "smearing" of $\pi^{\pm}$-spectrum near $\varepsilon \sim M_B/2$.

The charged pions spectra $dN/d\varepsilon = dN^{low}/d\varepsilon$ (also for the case a)) in the semileptonic decay $\bar{B}^0 \to \ell \tilde{\nu} h^+$ (the diagram of Fig.Ic) are presented in Fig.5c. $\pi^{\pm}$-spectra for hadronic and semileptonic decays of B^--meson are shown in Fig.5d,e.

All numerical results were obtained in the limit $p', \mu \to 0$ with quantity M'^2 taken in the simplest form $M'^2 \simeq 2\mu^2 + y \cdot \mu \cdot M_B$. Thus, the fluctuations of M' connected with oscillations of $\cos\theta_{\vec{p}\vec{q}}$ were not taken into account (it would be much better to retain in (8) only terms linear in p' and μ , i.e. to use M'^2 in the form $M'^2 \simeq y (\varepsilon' m_\ell + |\vec{p}'| |\vec{q}| \cos\theta_{\vec{p}'\vec{q}})$. However, it slightly changes only hadronic spectrum in semileptonic decay in Fig.5c, and practically has no other consequences.

The approach developed here could be applied directly to the calculations of lepton spectra in $B \to \ell \nu + h$ -decay, in any order of accuracy, since $\cos\theta_0$ -distribution in Eqs.(I),(IO) determines directly the distribution of lepton energy ε .

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Appendix I

a) In standard parametrization of the vertex $\overline{B^0}D^+W^-$ the matrix element of the process in Fig.Ia ($\overline{B^0} \longrightarrow D^+h^-$, or $\overline{B^0} \longrightarrow D^+d\bar{u}$) is

$$M_D = (G/\sqrt{2})\cdot V_{bc}\,(\bar{d}\gamma_\alpha(1+\gamma_5)u)\left[f_+(P_B+P_D)_\alpha + f_-(P_B-P_D)_\alpha\right] =$$
$$= \sqrt{2}\,G\,V_{bc}\left[f_+ P_{D\alpha}(\bar{d}\gamma_\alpha(1+\gamma_5)u) + \mu(f_+ + f_-)(\bar{d}u)\right] \qquad \text{(I.I)}$$

where $f_\pm(M)$ are form-factors of the vertex $\overline{B^0}D^+W^-$ of the dipole form $f_\pm(0)/(I - M^2/M_0^{*2})$. Accounting for the constituent quark masses, μ=0.4 GeV one gets:

$$\langle|M_D|^2\rangle = 2G^2|V_{bc}|^2 M_B^2\Big\{4f_+^2\Big[\vec{q}^{\,2}\sin^2\theta_0 + 4\mu^2(\vec{q}^{\,2}/M^2)\cos^2\theta_0 +$$
$$+ 2\mu^2(1 + M_D^2/M_B^2 - \mu^2/2M_B^2)\Big] + 2f_-^2 M^2\mu^2/M_B^2 + 4\mu^2 f_+ f_-(1 - \frac{M_D^2}{M_B^2})\Big\} \qquad \text{(I.2)}$$

where $M^2 = q^2 = E^2 - \vec{q}^{\,2}$, $\theta_0 = \theta_{\vec{q}\vec{k}_u}$ -is the angle between $\vec{k}_u = -\vec{k}_d$ and the momentum $\vec{q} = \vec{k}_u + \vec{k}_d$ of W^- boson in the string (or W^-) rest frame H_0. As $f_- \ll f_+$ one often omits the last two terms in (I.2). Comparison with Eq.(I) shows that:

$$F_D(M, \cos\theta_0) = \frac{3}{2}C_0|\vec{q}|/G^2 M_B^5\,|V_{bc}|^2 f_+^2(0) \qquad \text{(I.3)}$$

Note, that in the limit $M^2 = q^2 \longrightarrow 4\mu^2 \longrightarrow 0$, the quantity $\langle|M_D|^2\rangle$ does not depend on $\cos\theta_0$: $\langle|M_D^2|\rangle_{M^2\to\mu^2\to 0} \longrightarrow 4G^2|V_{bc}|^2 f_+^2(0)\cdot M_B^2(M_B^2-M_D^2)$.

b) In the case of $\overline{B^0} \longrightarrow h^+h^-$ decay to continious spectrum (Fig.Ib, at $b \longrightarrow u$ transition) the element of S-matrix is

$$M_{h^+h^-} = \left(\frac{G}{\sqrt{2}}\right) V_{bu} \left(\bar{d}\gamma_\alpha(1+\gamma_5)u\right)\cdot\left(\bar{q}_1\gamma_\alpha(1+\gamma_5)q_2\right)$$

while

$$\langle |M_{h^+h^-}|^2\rangle = 128 G^2 |V_{bu}|^2 (p_b\cdot q_1)(p_u q_2) = 8 G^2 |V_{bu}|^2 (M_1^2 - M^2)\times$$
$$\times\Big\{M_1^2 \sin^2\theta_0 + M^2(1-\cos\theta_0)^2 - 16\mu^2\big[M_1^2 - \left(3(M^2-M_1^2)+2M_1^4/M_B^2\right)\cos^2\theta_0 -$$
$$- \left(M_1^2 + 2M^4/(M_1^2-M^2)\right)\cos\theta_0\big]\Big\}\cdot|\chi_B(p')|^2 .$$

The function (IO) $F_{p'}$ is connected to $\langle |M_{h^+h^-}|^2\rangle$ by

$$F_{p'} = \frac{3}{2} C_1 \left(|\vec{q}|/M_B^5 |V_{bu}|^2 G^2\right)\cdot\langle |M_{h^+h^-}|^2\rangle/\lambda;\ \lambda^{-1} = \left(1 - \frac{\varepsilon'}{M_B} + \frac{E}{|\vec{q}|}\frac{|\vec{p}'|}{M_B}\cos\theta_{\vec{q}\vec{p}'}\right);$$

since the final state phase space is equal to $d\tau_4 = \frac{dM^2}{2\pi} d\tau_2(k_u, k_d)\, d\tau_3$ where $d\tau_3 = \frac{|\vec{q}|}{4\pi M_B \lambda}\frac{d^3p'}{2\varepsilon'(2\pi)^3}$; $d\tau_2 = \frac{1}{8\pi}\cdot\frac{1}{2}\, d\cos\theta_0$

Appendix 2

The probability of production of one π-meson in the lower block in Fig.Ib,c is $C_\pi = W_{B\to\pi e\nu}/W_{B\to u e\nu} \simeq W_{B\to\pi e\nu}/W_{b\to u e\nu}$, where $W_{b\to u e\nu} = G^2 m_b^5/192\pi^3$ is the probability of free b quark decay. Taking $W_{B\to\pi e\nu} = (G^2 M_B^5 a_0^2/384\pi^3)\cdot(M_B^*/M_B)^6$ with [IO] $M_B^* \simeq 7.3$ GeV and [8,IO] $a_0 \simeq 0.33$–0.4 one obtains:

$$C_\pi' = (a_0^2/2)(M_B/m_b)^5 (M_B^*/M_B)^6 \simeq 0.1$$

For the probability of one ρ^- meson production one obtains similarly:

$$C_\rho = W_{B\to\rho e\nu}/W_{B\to h e\nu} = C_\pi W_{B\to\rho e\nu}/W_{B\to\pi e\nu} \simeq$$
$$\simeq C_\pi W_{B\to\rho\pi}/W_{B\to\pi\pi} = (A_0/a_0)^2(2M_B+m_\rho)/(M_B+m_\rho) \simeq 0.2$$

as

$$W_{B\to\rho\pi} = (G^2 f_\pi^2/16\pi)(M_B/m_\rho)^2\left[A_1(0)(M_B+m_\rho) - A_2(0)\frac{M_B^2}{M_B+m_\rho}\right]^2,$$

and [10] $A_1(0)=A_2(0) = A_0 \simeq 0.28$, while $W_{B\to\pi\pi} = (G^2 f_\pi^2/16)a_0^2\cdot m_b^3$.

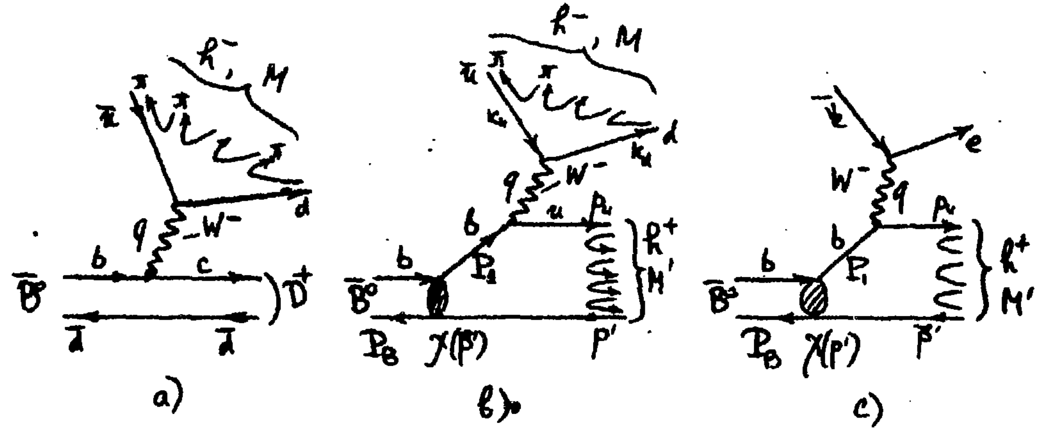

Fig.I. Diagrams corresponding to leading terms of I/N_c - expansion for the decay processes: a) $\overline{B^0} \longrightarrow D^+h^-$, b) $\overline{B^0} \longrightarrow h^+h^-$, c) $\overline{B^0} \longrightarrow e\tilde{\nu}\, h^+$.

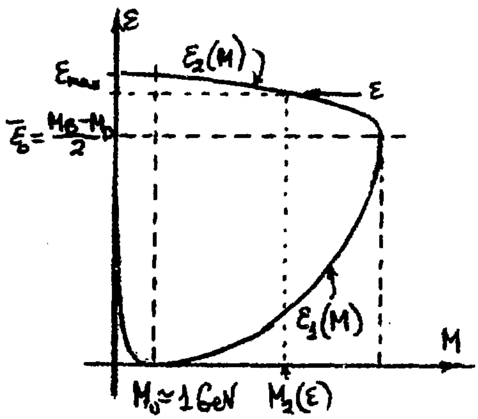

Fig.2. M- dependence of integration limits $\mathcal{E}_1 = 1/2\,(E+|\vec{q}|)$ and $\mathcal{E}_2 = \frac{1}{2}(E-|\vec{q}|)$ in Eq.(4).

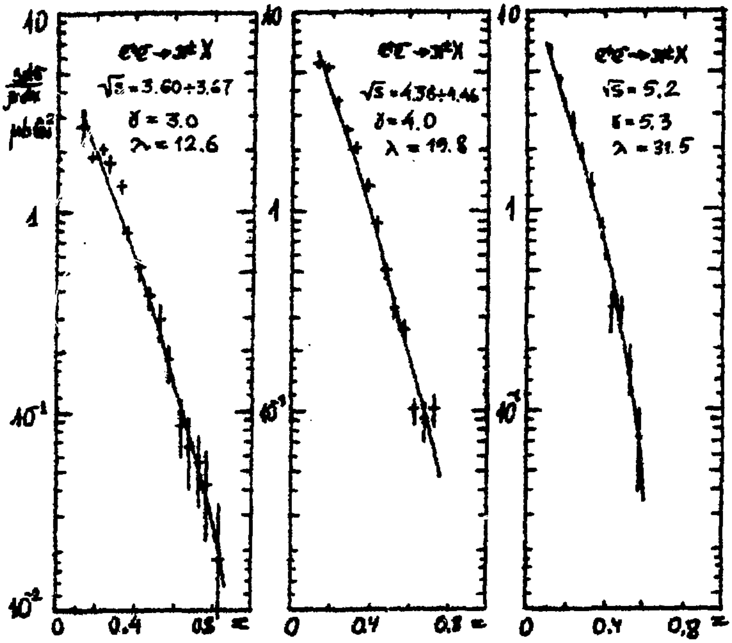

Fig. 3a)

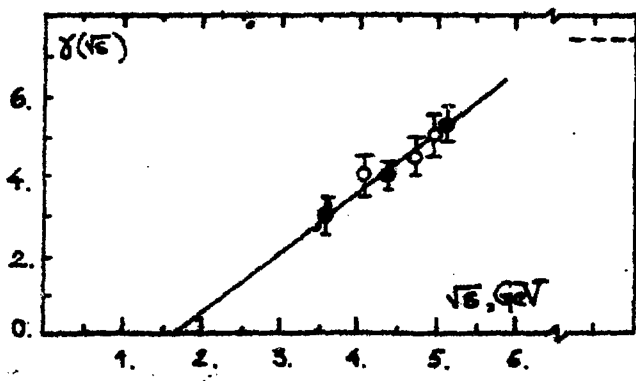

Fig.3b

Fig.3. a) Charged pions spectra obtained with Eq.(6) versus the data of DASP-Collab. [8] at: $\sqrt{s}$ = 3.6–3.7 GeV, $\sqrt{s}$ = 4.36–4.46 GeV and $\sqrt{s}$ = 5.2 GeV.

b) The slopes $\gamma(\sqrt{s})$ of spectra producing the best fit of the data. Black circles correspond to the energies – shown in Fig.3a. Dashed line is the asymptotic QCD prediction [7].

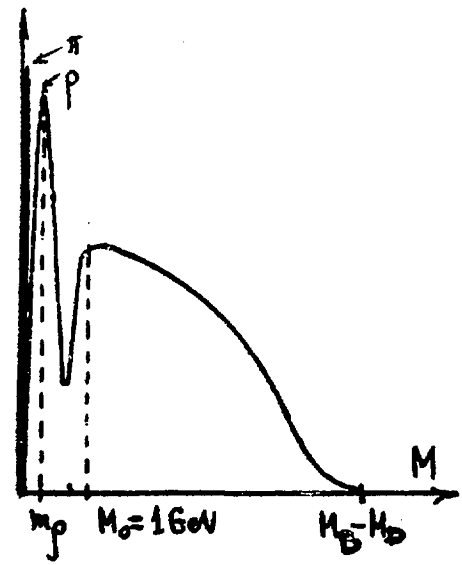

Fig.4. Mass spectrum of $q\bar{q}$-string of the upper block of the diagram in Fig.Ia.

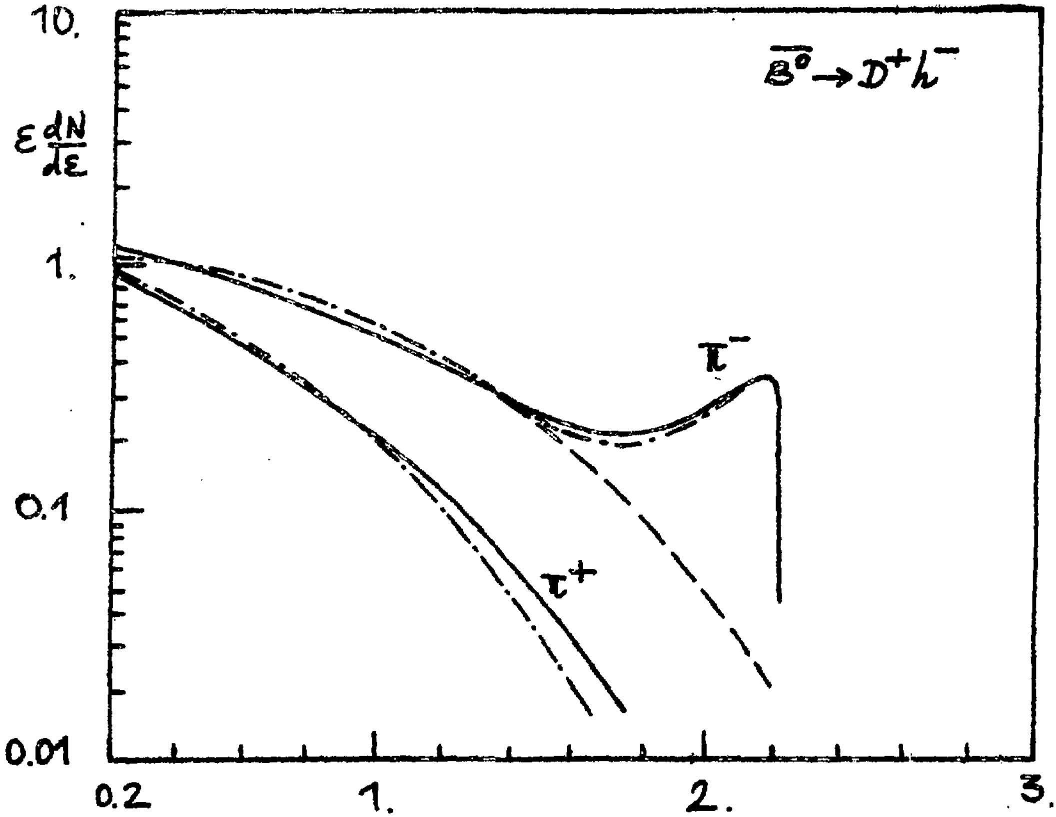

Fig.5. a) Inclusive spectra of charged π^-, and π^+-mesons in the case (a) of isotropic string fragmentation produced in $\overline{B^0} \to D^+h^-$ decay. Dashed-dotted lines correspond to $\pi^\pm$-spectra in case (b) of anisotropic fragmentation. Dashed line represents π^+-spectrum when contribution of the process $\overline{B^0} \to D^+\rho^-$ is excluded.

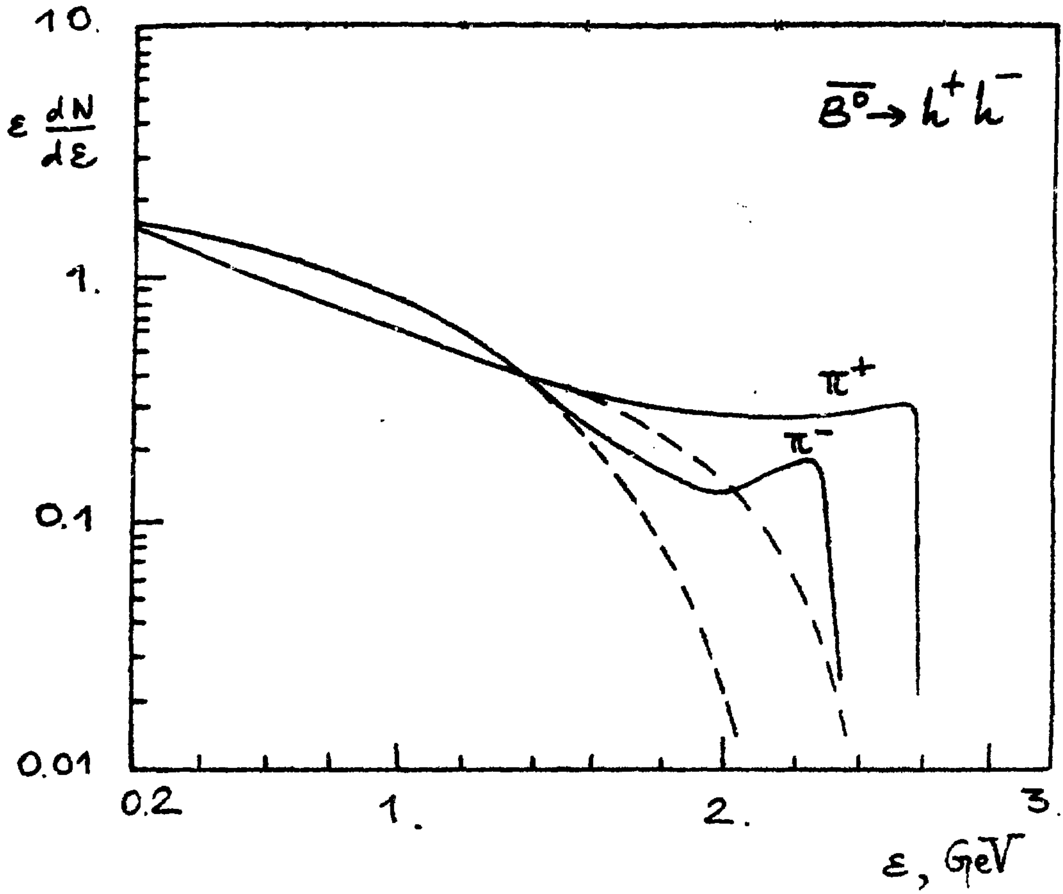

Fig.5b) $\pi^{\pm}$-spectra produced in upper and lower blocks (in the case (a) of the diagram of Fig.Ib for $\overline{B^0} \longrightarrow h^+h^-$-decay. $\pi^{\pm}$-spectra without single π-, ρ-contibution are shown by dashed lines.

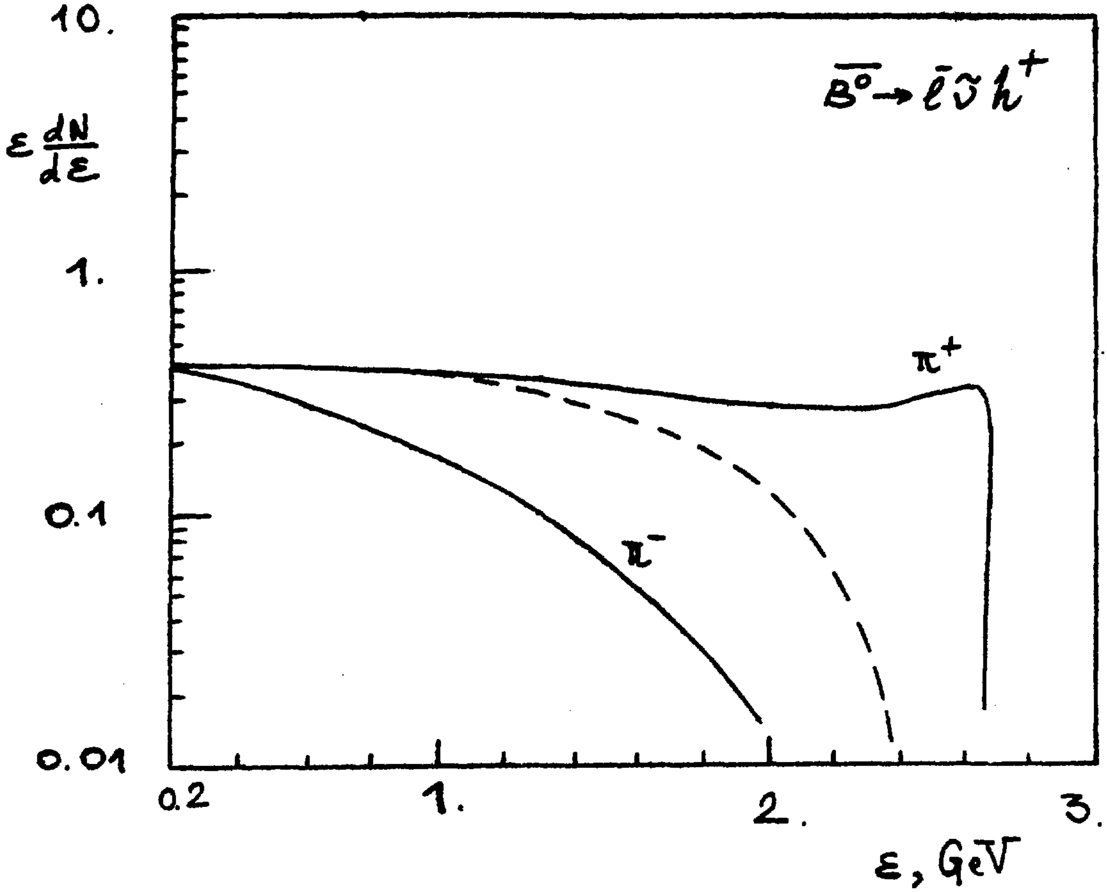

Fig.5c) $\pi^{\pm}$-spectra produced (in the case (a))in lower block of the diagram of Fig.Ic for $\overline{B^0} \to e^- \tilde{\nu}\ h^+$-decay. Dashed line corresponds to π^+-spectrum when contribution of the process $\overline{B^o} \to \ell^- \tilde{\nu} \pi^+$ is excluded.

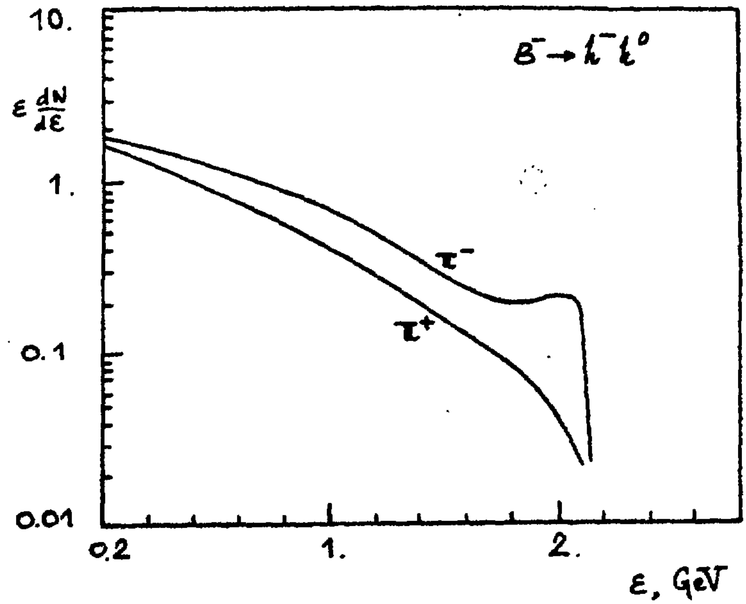

Fig.5d) $\pi^{\pm}$-spectra for $B^- \to h^- h^0$-decay (case (a)).

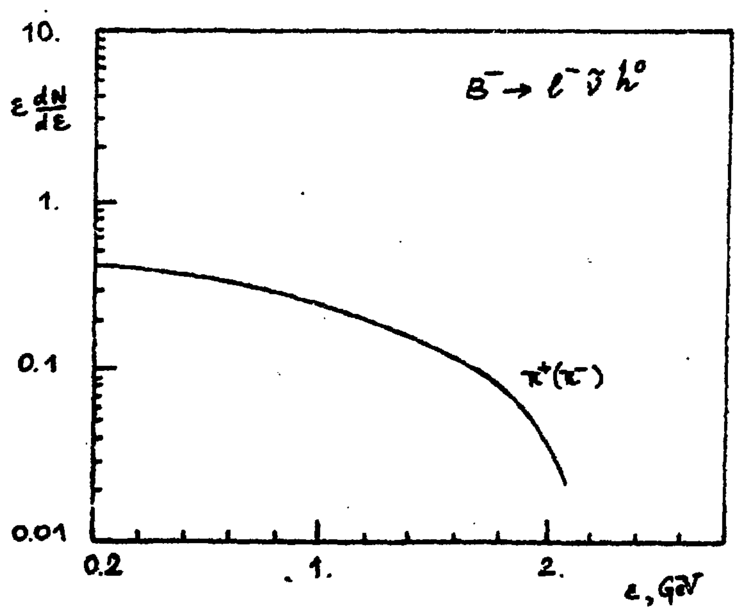

Fig.5e) $\pi^{\pm}$-spectra for $B^- \to \ell^- \tilde{\nu} h^0$-decay (case (a)).

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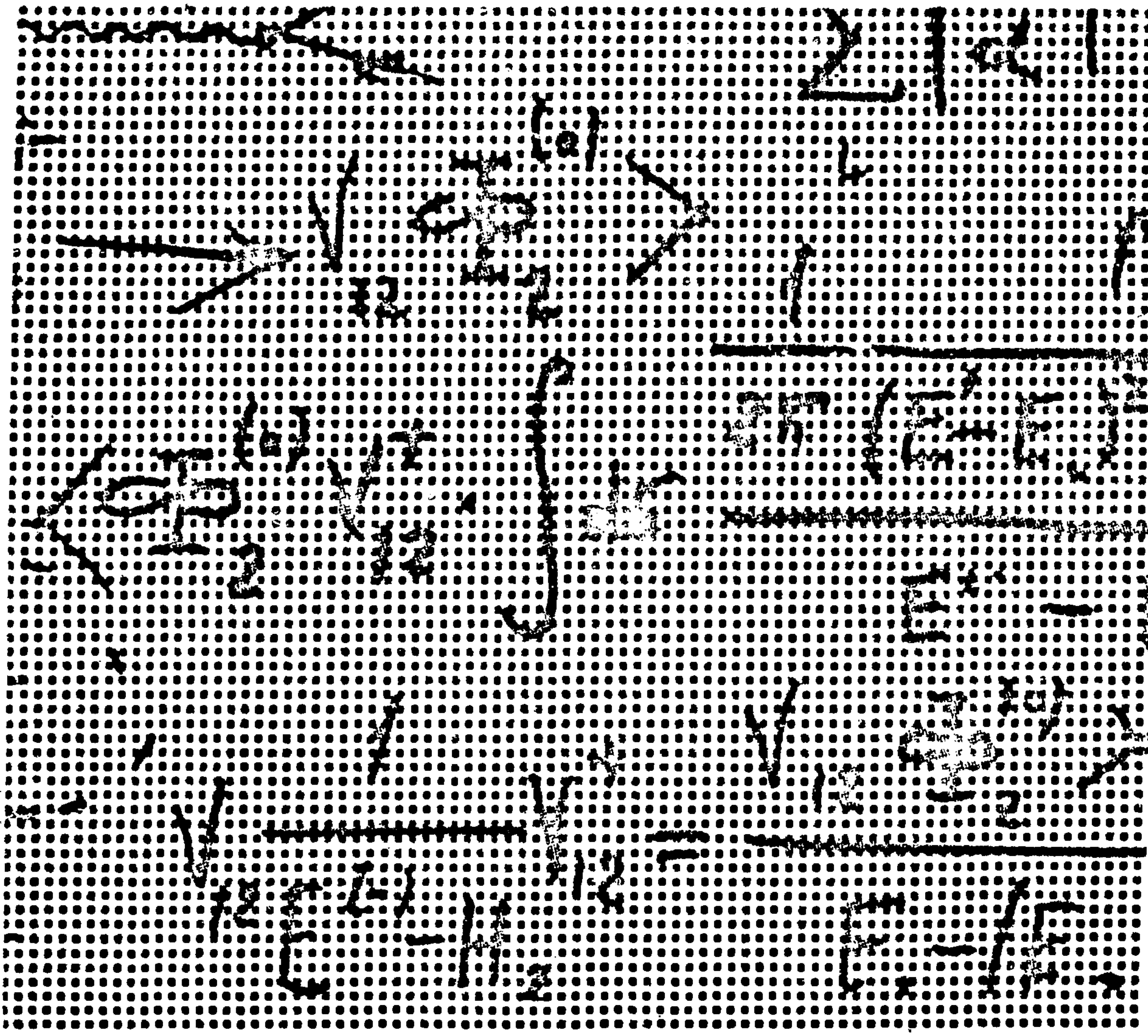

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