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THE DUAL DESCRIPTION OF LONG-DISTANCE QCD (DUAL QCD)

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ABSTRACT

We construct and solve a local field theory which describes in terms of dual variables a system having an A_μ propagator behaving like $\frac{M^2}{q^4}$ in the infrared and discuss how this theory can be used as a starting point for describing long-distance QCD.

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I. Motivation for Dual QCD

Dual QCD is QCD expressed in terms of dual potentials C_μ. In the first part of this talk I will present the motivation for dual QCD. In the second part I will construct $\mathcal{L}^{(0)}$, the quadratic part of the dual QCD Lagrangian, and solve the resulting Abelian gauge theory to obtain the particle spectrum. This Lagrangian $\mathcal{L}^{(0)}$ is the starting point for dual QCD.

The goal of dual QCD is to study long-distance Yang-Mills theory. In this talk I will not discuss the inclusion of quarks. The Yang-Mills Lagrangian is

$$\mathcal{L} = -\frac{1}{4}(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu + e f_{abc} A^b_\mu A^c_\nu)^2 . \tag{1}$$

We have denoted the Yang-Mills coupling constant by e; ie., $\alpha_s = \frac{e^2}{4\pi}$.

Long-distance Yang-Mills theory is a strongly coupled gauge theory. The coupling constant increases and the matrix elements of $A^a_\mu(x)$ become large at long range. At the same time, in a theory with confinement, physical quantities exhibit a smooth and non singular long-distance behavior. There is, however, the alternate possibility of using dual potentials C_μ to describe gauge theories[1]. The coupling constant g for the dual potential is the inverse of the Yang-Mills coupling constant e, ie. $g = \frac{2\pi}{e}$. The dual potentials are weakly coupled at long range and hence should be the appropriate variables for describing long-distance physics.

Let us first recall how dual variables are used to describe the electrodynamics of a relativistic dielectric medium characterized by a momentum dependent dielectric constant $\epsilon(q)$ and magnetic permeability $\mu(q)$, where

$$\epsilon(q)\mu(q) = 1 . \tag{2}$$

The equations of motion are the source free Maxwell's equations:

$$\vec{\nabla} \cdot \vec{\mathbf{D}} = 0 \quad , \quad \vec{\nabla} \times \vec{\mathbf{H}} = \frac{\partial \vec{\mathbf{D}}}{\partial t} , \tag{3}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad , \quad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} , \tag{4}$$

and the constitutive equations are

$$\vec{\mathbf{D}} = \epsilon \vec{E} \quad , \quad \vec{B} = \mu \vec{\mathbf{H}} , \tag{5}$$

which relate the electric displacement vector $\vec{\mathbf{D}}$ and the magnetic $\vec{\mathbf{H}}$ vector to $\vec{E}$ and $\vec{B}$. We introduce dual potentials C_μ to solve Eq. (3) by writing

$$\vec{\mathbf{D}} = -\vec{\nabla} \times \vec{C} \quad , \quad \vec{\mathbf{H}} = -\frac{\partial \vec{C}}{\partial t} - \vec{\nabla} C_0 . \tag{6}$$

Eqs. (2), (5) and (6) yield

$$-\vec{\nabla} \times \vec{C} = \frac{1}{\mu}\vec{E} \quad , \quad -\left(\vec{\nabla} C_0 + \frac{\partial \vec{C}}{\partial t}\right) = \frac{1}{\mu}\vec{B} . \tag{7}$$

Inserting Eq. (7) into Eq. (4) then gives the equations of motion for C_μ, These are generated by the Lagrangian,

$$\mathcal{L} = -\frac{1}{4} G^{\mu\nu} \mu(q) G_{\mu\nu} , \tag{8}$$

where

$$G_{\mu\nu} \equiv \partial_\mu C_\nu - \partial_\nu C_\mu . \tag{9}$$

In the Landau gauge the C_μ propagator $\Delta_{C\mu\nu} =< C_\mu C_\nu >$ generated from $\mathcal{L}$ is

$$\Delta_{C\mu\nu} = \frac{1}{q^2\mu(q)} \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) . \tag{10}$$

The propagator Δ_C, Eq. (10), describes exactly the same physics ((Eqs. (3), (4) and (5)) as the ordinary A_μ propagator Δ_A, where

$$\Delta_{A\mu\nu} = \frac{1}{q^2\epsilon(q)} \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) . \tag{11}$$

In Yang-Mills theory Mandelstam [1] showed how to define dual potentials C^a_μ. However, the transformation $A^a_\mu \rightarrow C^a_\mu$ is not explicitly known. Consequently, in contrast with electrodynamics, the Yang-Mills Lagrangian as a function of the dual potentials C_μ cannot be explicitly written down. We only know [1] that it is invariant under a non Abelian gauge transformation of the potentials C^a_μ, which in matrix notation is given by

$$C_\mu \rightarrow \Omega^{-1} C_\mu \Omega + \frac{i}{g} \Omega^{-1} \partial_\mu \Omega , \tag{12}$$

where Ω is a SU(3) matrix.

We are interested, however, in solving Yang-Mills theory only at long distances. For this purpose we must find only the Lagrangian $\mathcal{L}(C)$ describing the long-distance regime of Yang-Mills theory. Since we expect that the C_μ fields interact weakly at long distances, $\mathcal{L}(C)$ should be a minimal gauge invariant extension of a quadratic Lagrangian $\mathcal{L}^{(0)}(C)$; non minimal additions to $\mathcal{L}(C)$ should not be relevant at long distances. Thus although the full dual QCD Lagrangian is not explicitly known, its long-distance part $\mathcal{L}(C)$ necessary to describe long-distance QCD can be determined.

The first step is to construct the quadratic Lagrangian $\mathcal{L}^{(0)}(C)$. It describes an Abelian gauge theory and hence must be of the form of Eq. (8). In order to specify $\mu(q)$ we must have some information about long-distance Yang-Mills dynamics. During the past 10 years a number of authors [2] have calculated the gluon propagator Δ_A in the simplest self-consistent truncation of the Schwinger Dyson equations of Yang-Mills theory compatible with the requirements of gauge invariance. These calculations have been carried out in different gauges and differ in technical details; nevertheless they all yield a solution for $\Delta_A(q)$ which has the behavior

$$\Delta_A(q) \sim -\frac{M^2}{(q^2)^2} \quad , \quad q^2 \rightarrow 0 , \tag{13}$$

where M^2 is an undetermined parameter. From Eq. (11) we see that the solution (13), describes a dielectric medium for which $\epsilon(q) \to -q^2/M^2$, as $q^2 \to 0$. The corresponding magnetic permeability $\mu(q)$ is then given by

$$\mu(q) = -\frac{M^2}{q^2} + 1 \tag{14}$$

at small q^2. The choice of the constant 1 for the non-leading low momentum contribution to $\mu(q)$ in Eq. (14) represents a choice of normalization of the fields. Inserting Eq. (14) for $\mu(q)$ into Eq. (8) then gives $\mathcal{L}^{(0)}(C)$.

Using Eqs. (10) and (14) we see that the C_μ propagator Δ_C corresponding to the A_μ propagator (Eq. (13)) is

$$\Delta_{C\mu\nu}(q) = \frac{1}{q^2 - M^2}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right). \tag{15}$$

(Eq. (15)) indicates that $\mathcal{L}^{(0)}(C)$ contains a massive vector meson. Calculation of the long-distance corrections to the singular A_μ propagator (Eq. (13)) via the Schwinger Dyson equations is essentially impossible. On the other hand, the long-distance corrections to the physically equivalent Δ_C (Eq. (15)) should be calculable. This is the goal of dual QCD.

In the coordinate representation Eqs. (8) and (14) give

$$\mathcal{L}^{(0)}(C) = -\frac{1}{4}G_{\mu\nu}\left(\frac{M^2}{\partial^2}\right)G^{\mu\nu} - \frac{1}{4}G_{\mu\nu}G^{\mu\nu}. \tag{16}$$

We now introduce an auxiliary antisymmetric tensor $\tilde{F}_{\mu\nu} = -\tilde{F}_{\nu\mu}$ in order to write $\mathcal{L}^{(0)}(C)$ in local form. (We omit color indices since $\mathcal{L}^{(0)}$ is just a sum of terms corresponding to each of the colors). We integrate the identity

$$\exp\left\{i\left[-\frac{1}{4}G_{\mu\nu}\frac{M^2}{\partial^2}G^{\mu\nu}\right]\right\}\exp\left\{i\left[\frac{1}{4}\left(\tilde{F}_{\mu\nu} + G_{\mu\nu}\frac{M}{\partial^2}\right)\partial^2\left(\tilde{F}^{\mu\nu} + \frac{M}{\partial^2}G^{\mu\nu}\right)\right]\right\} =$$

$$\exp\left\{i\left[\frac{M}{2}\tilde{F}_{\mu\nu}G^{\mu\nu} + \frac{1}{4}\tilde{F}_{\mu\nu}\partial^2\tilde{F}^{\mu\nu}\right]\right\} \tag{17}$$

over $\tilde{F}_{\mu\nu}$ and evaluate the integral over the left-hand side of Eq. (17) by translating variables

$$\tilde{F}_{\mu\nu} \to \tilde{F}_{\mu\nu} + \frac{M}{\partial^2}G_{\mu\nu}. \tag{18}$$

However, since $G_{\mu\nu}$ satisfies the kinematic identity

$$\partial^\mu \epsilon_{\mu\nu\alpha\beta}G^{\alpha\beta} = 0, \tag{19}$$

we need only integrate over fields $\tilde{F}_{\mu\nu}$ satisfying the same identity:

$$\partial^\mu \epsilon_{\mu\nu\alpha\beta}\tilde{F}^{\alpha\beta} = 0. \tag{20}$$

Using Eqs. (16), (17) and (20), we can then replace $\mathcal{L}^{(0)}$ by the local Lagrangian

$$\mathcal{L}^{(0)}(C, \tilde{F}, H) = \frac{M}{2}\tilde{F}_{\mu\nu}G^{\mu\nu} + \frac{1}{4}\tilde{F}_{\mu\nu}\partial^2\tilde{F}^{\mu\nu} - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} - \frac{1}{2}H^{\mu}\partial^{\nu}\epsilon_{\mu\nu\alpha\beta}\tilde{F}^{\alpha\beta}, \quad (21)$$

where H^{μ} is a Lagrange multiplier field. The last term in Eq. (21) accounts for the constraint, Eq. (20).

II. Solution of Zero Order Dual QCD

Varying H^{μ} in the Lagrangian (Eq. (21)) then gives Eq. (20). Varying $\tilde{F}_{\mu\nu}$ gives

$$MG_{\mu\nu} = -\partial^2\tilde{F}_{\mu\nu} - \epsilon_{\mu\nu\lambda\sigma}\partial^{\lambda}H^{\sigma}, \quad (22)$$

while varying C^{ν} gives

$$\partial_{\mu}G^{\mu\nu} = M\partial_{\mu}\tilde{F}^{\mu\nu}. \quad (23)$$

From Eq. (23) we see that $M\partial_{\mu}\tilde{F}^{\mu\nu}$ has the interpretation of a monopole current and hence the auxiliary fields $-M\tilde{F}^{\mu\nu}$ are the components of the magnetization tensor. These appear as fundamental dynamical variables since the dielectric constant is a differential operator, and as a consequence the "constitutive equations" (Eq. (22)) (analogous to Eq. (7)) are equations of motion.

In order to clarify the physical meaning of the equations of motion (20), (22) and (23), it is convenient to introduce the following three dimensional notation for the components of $\tilde{F}_{\mu\nu}$:

$$B_k \equiv -M\tilde{F}_{0k} \quad , \quad E_k = -\frac{M}{2}\epsilon_{klm}\tilde{F}_{lm}, \quad (24)$$

We have used the notation $\vec{B}$ and $\vec{E}$ for the components of the magnetization since they represent contributions to the magnetic and electric fields which depend upon the state of the dielectric medium. Using Eq. (24) we can write Eq. (20) as

$$\vec{\nabla}\cdot\vec{E} = 0,$$

$$\vec{\nabla}\times\vec{B} - \frac{\partial\vec{E}}{\partial t} = 0. \quad (25)$$

Eq. (25) makes evident the physical meaning of the constraint ((Eq. (20)); namely, there is no *electric* current in the medium. Next define

$$Z^{\nu} = \frac{1}{M}\partial_{\mu}\tilde{F}^{\mu\nu}, \quad (26)$$

we then have

$$\partial_{\nu}Z^{\nu} = 0.$$

From Eqs. (23) and (26) we see that $M^2 Z^\nu$ is the *monopole* current. Using Eqs. (24) we can write Eq. (26) as

$$-\vec{\nabla}\cdot\vec{B} = M^2 Z^0 ,$$

$$\vec{\nabla}\times\vec{E} + \frac{\partial\vec{B}}{\partial t} = M^2 \vec{Z} . \tag{27}$$

The monopole current then determines $\vec{E}$ and $\vec{B}$ via Eqs. (25) and (27).

To solve Eqs. (20), (22) and (23) it is convenient to eliminate the potentials C_μ. Taking the divergence of Eq. (22) and using Eq. (23) we obtain

$$(\partial^2 + M^2) Z^\mu = 0 . \tag{28}$$

Hence Z^μ is a free massive vector field of mass M. Eqs. (20) and (26) are just the Maxwell equations for $\tilde{F}_{\mu\nu}$ in terms of its monopole sources Z^μ. The general solution to these equations can then be written as

$$\tilde{F}_{\mu\nu} = -\frac{(\partial_\mu Z_\nu - \partial_\nu Z_\mu)}{M} - \frac{(\partial_\mu \psi_\nu - \partial_\nu \psi_\mu)}{M} , \tag{29}$$

where ψ_μ satisfies

$$\partial^2 \psi_\mu = 0 \quad , \quad \vec{\nabla}\cdot\vec{\psi} = 0 . \tag{30}$$

Eq. (29) then gives $\tilde{F}_{\mu\nu}$ as a sum of a massive vector field contribution (a particular solution to inhomogeneous equations (20) and (26)), and a massless vector field contribution (the general solution of the corresponding homogeneous equations). The constraint (Eq. (20)) eliminates additional redundant degrees of freedom in $\tilde{F}_{\mu\nu}$.

To determine the field H^μ we perform the operation $\epsilon^{\alpha\beta\mu\nu}\frac{\partial}{\partial x^\alpha}$ on Eq. (22). Using Eq. (20) and the identity

$$\epsilon^{\alpha\beta\mu\nu}\epsilon_{\mu\nu\lambda\sigma} = -2(\delta^\alpha_\lambda \delta^\beta_\sigma - \delta^\alpha_\sigma \delta^\beta_\lambda) ,$$

we obtain

$$\partial^2 H_\beta - \partial_\beta \partial_\alpha H^\alpha = 0 . \tag{31}$$

Thus H^μ is a free massless field. The invariance of Eq. (31) under a gauge transformation of the Lagrange multiplier field, H_μ,

$$H_\mu \to H_\mu + \partial_\mu \Lambda_H(x) , \tag{32}$$

reflects the corresponding invariance of $\mathcal{L}^{(0)}$, Eq. ((21)).

The Lagrangian $\mathcal{L}^{(0)}$ then describes a massive vector field Z^μ and two massless vector fields ψ^μ and H^μ corresponding to seven physical degrees of freedom. Inserting Eq. (29) into (22) and using Eqs. (28) and (30) we obtain the following explicit expression for $G_{\mu\nu}$:

$$G_{\mu\nu} = -(\partial_\mu Z_\nu - \partial_\nu Z_\mu) - \frac{1}{M}\epsilon_{\mu\nu\lambda\sigma}\partial^\lambda H^\sigma . \tag{33}$$

We see that $G_{\mu\nu}$ depends only upon Z_μ and H_μ, and does not involve the massless ψ_μ degree of freedom. Eq. (33) then determines C_μ up to a gauge transformation,

$$C_\mu \to C_\mu + \partial_\mu \Lambda_C(x)\,, \tag{34}$$

which reflects the invariance of $\mathcal{L}^{(0)}$ under the transformation (34).

We now proceed with the canonical quantization of the theory. Defining canonical momenta

$$\vec{\pi}_C \equiv \frac{\delta \mathcal{L}^{(0)}}{\delta \dot{\vec{C}}} = \dot{\vec{C}} + \vec{\nabla} C_0 - \vec{B}\,,$$

$$\vec{\pi}_B \equiv \frac{\delta \mathcal{L}^{(0)}}{\delta \dot{\vec{B}}} = \frac{\dot{\vec{B}}}{M^2}\,,$$

$$\vec{\pi}_E \equiv \frac{\delta \mathcal{L}^{(0)}}{\delta \dot{\vec{E}}} = \frac{\vec{H}}{M} - \frac{\dot{\vec{E}}}{M^2}\,, \tag{35}$$

we can then write $\mathcal{L}^{(0)}$ in Hamiltonian form,

$$\mathcal{L}^{(0)} = \vec{\pi}_C \cdot \dot{\vec{C}} + \vec{\pi}_B \cdot \dot{\vec{B}} + \vec{\pi}_E \cdot \dot{\vec{E}} - \mathcal{H}\,, \tag{36}$$

where

$$\mathcal{H} = \frac{1}{2}(\vec{\pi}_C + \vec{B})^2 + \frac{\vec{\pi}_B^2}{2M^2} - \frac{1}{2}\left(\vec{H} - \frac{\vec{\pi}_E}{M}\right)^2 + \frac{1}{2}(\vec{\nabla}\times\vec{C})^2 - (\vec{\nabla}\times\vec{C})\cdot\vec{E}$$
$$- \frac{\vec{B}\nabla^2\vec{B}}{2M^2} + \frac{\vec{E}\nabla^2\vec{E}}{2M^2} + \vec{H}\cdot\frac{\vec{\nabla}\times\vec{B}}{M} + C_0\vec{\nabla}\cdot\vec{\pi}_C + \frac{H_0}{M}(-\vec{\nabla}\cdot\vec{E})\,. \tag{37}$$

Varying $\vec{\pi}_C$, $\vec{\pi}_B$ and $\vec{\pi}_E$ in $\mathcal{L}^{(0)}$ gives Eqs. (35), while varying $\vec{H}$ yields

$$\vec{H} - \frac{\vec{\pi}_E}{M} = \frac{\vec{\nabla}\times\vec{B}}{M}\,, \tag{38}$$

which can be used to eliminate $\vec{H}$. Varying C_0 and H_0 in $\mathcal{L}^{(0)}$ gives the equations of constraint

$$G_1 \equiv \vec{\nabla}\cdot\vec{\pi}_C = 0 \quad , \quad G_2 \equiv \vec{\nabla}\cdot\vec{E} = 0\,. \tag{39}$$

The quantities G_1 and G_2 are the generators of the gauge transformations Eqs. (32) and (34) and hence the constraints of Eq. (39) are preserved by the equations of motion. The fields H_0 and C_0 are not determined by the dynamics. We can then impose two more conditions

$$\vec{\nabla}\cdot\vec{H} = 0 \quad , \quad \vec{\nabla}\cdot\vec{C} = 0\,, \tag{40}$$

which fix the gauge. Eqs. (39) and (40) then reduce the number pairs of canonical variables from 9 to 7 which is just what is necessary to describe a massive vector particle and two massless vector particles. This gauge choice (Eq. (40)) gives $H_0 = 0$, and hence H_μ satisfies

$$\partial^2 H_\mu = 0\,. \tag{41}$$

Next we insert the solution of the equations of motion (Eqs. (29) and (33)) into Eq. (37) for the Hamiltonian density $\mathcal{H}$. Using Eqs. (28), (30) and (40) we obtain the following expression for the Hamiltonian in the momentum representation:

$$\int d\vec{x}\mathcal{H}(\vec{x}) = \int d\vec{k}\left\{\omega\left[-Z^*_\mu(\vec{k})Z^\mu(\vec{k})\right] + k[\vec{\phi}^*(\vec{k})\cdot\vec{\phi}(\vec{k}) - \vec{\psi}^*(\vec{k})\cdot\vec{\psi}(\vec{k})]\right\}, \tag{42}$$

where

$$\omega = \sqrt{\vec{k}^{\,2} + M^2}\,, \tag{43}$$

and

$$\vec{\phi}(\vec{x}) \equiv \vec{\psi}(\vec{x}) + \frac{\vec{H}(\vec{x})}{M}\,. \tag{44}$$

From Eqs. (30), (40), (41) and (44) we see that

$$\partial^2\vec{\phi} = 0 \quad , \quad \vec{\nabla}\cdot\vec{\phi} = 0\,, \tag{45}$$

ie., $\vec{\phi}$ is also a massless transverse vector field. Let $\psi(\vec{k}\,,\lambda)$ and $\phi(\vec{k}\,,\lambda)$, $\lambda = 1,2$, be the two independent transverse components of $\vec{\psi}(\vec{k})$ and $\vec{\phi}(\vec{k})$. Then using the canonical commutation relations for the fields $\vec{C}$, $\vec{B}$, and $\vec{E}$ and their conjugate momenta we obtain

$$\left[\phi(\vec{k}\,,\lambda)\,,\phi^\dagger(\vec{k}\,'\,,\lambda')\right] = \delta(\vec{k} - \vec{k}\,')\delta_{\lambda\lambda'}\,,$$

$$\left[\psi(\vec{k}\,,\lambda)\,,\psi^\dagger(\vec{k}\,'\,,\lambda)\right] = -\delta(\vec{k} - \vec{k}\,')\delta_{\lambda\lambda'}\,,$$

$$\left[\phi(\vec{k}\,,\lambda)\,,\psi^\dagger(\vec{k}'\,,\lambda')\right] = \left[\psi(\vec{k}\,,\lambda)\,,\phi^\dagger(\vec{k}'\,,\lambda')\right] = 0\,. \tag{46}$$

Thus $\phi^\dagger(\vec{k}\,,\lambda)$ creates "massless particles" of positive norm and $\psi^\dagger(\vec{k}\,,\lambda)$ creates "massless particles" of negative norm. We thus conclude that the dual field theory determined by $\mathcal{L}^{(0)}(C)$, (Eq. (21)), describing a system having an A_μ propagator, $\Delta_A(q) \sim \frac{M^2}{(q^2)^2}$ is a theory of non-interacting particles having the following characteristics: a positive norm vector meson $\vec{Z}$ of mass M, a positive norm massless vector meson $\vec{\phi}$, and a negative norm massless vector meson $\vec{\psi}$.

Next note that the differences

$$\frac{H(\vec{k}\,,\lambda)}{M} = \phi(\vec{k}\,,\lambda) - \psi(\vec{k}\,,\lambda)\,, \quad \lambda = 1,2\,, \tag{47}$$

satisfy

$$H(\vec{k}\,,\lambda)H^\dagger(\vec{k}'\,,\lambda') = H^\dagger(\vec{k}'\,,\lambda')H(\vec{k}\,,\lambda)\,, \tag{48}$$

and hence $H^\dagger$ creates zero norm states. Now let us impose the following subsidiary condition on physical states $|\Psi\rangle$:

$$H(\vec{k}, \lambda)|\Psi\rangle = 0 \,. \qquad (49)$$

Then the massless ψ and ϕ excitations give no contribution to the energy of physical states just as scalar and longitudinal photons give no contribution to the energy of physical states in covariantly quantized electrodynamics. The matrix elements of the Hamiltonian between states $|\Psi\rangle$ satisfying Eq. (49) are then given by

$$\langle\Psi| \int d\vec{x} \mathcal{H}(\vec{x}) |\Psi\rangle = \langle\Psi| \int d\vec{k}\omega \left[-Z^*_\mu(\vec{k}) Z^\mu(\vec{k})\right] |\Psi\rangle \,. \qquad (50)$$

Thus in the space of states satisfying Eq. (49) the Hamiltonian is positive definite and describes a system of non-interacting vector mesons of mass M . This space includes the C_μ sector of the theory, since it follows from Eqs. (33) and (48) that Eq. (49) is valid for any state $|\Psi\rangle$ of the structure

$$|\Psi\rangle = (C_\mu(x))^N \, |0\rangle \,.$$

As an alternate to canonical quantization we can calculate the propagator for the C_μ, $\tilde{F}_{\alpha\beta}$ and H_μ fields by inverting the matrix defined by the quadratic Lagrangian $\mathcal{L}^{(0)}(C)$ (Eq. (21)), supplemented by the gauge fixing term

$$-\frac{1}{2\alpha}(\partial_\mu C^\mu)^2 - \frac{1}{2\beta}(\partial_\mu H^\mu)^2 \,. \qquad (51)$$

In the Landau gauge $\alpha \to 0$, the C propagator is given by Eq. (15), while the mixed $C\tilde{F}$ propagator, $\Delta_{\mu,\alpha\beta} = \langle C_\mu \tilde{F}_{\alpha\beta}\rangle$, and the $\tilde{F}$ propagator, $\Delta_{\alpha\beta,\gamma\delta} = \langle \tilde{F}_{\alpha\beta}\tilde{F}_{\gamma\delta}\rangle$, are given by

$$\Delta_{\mu,\alpha\beta}(q) = \frac{2iM}{(q^2 - M^2)} \frac{q_\lambda S_{\lambda\mu,\alpha\beta}}{q^2} \,, \qquad (52)$$

and

$$\Delta_{\alpha\beta,\gamma\delta}(q) = -\frac{4}{(q^2 - M^2)} \frac{q_\lambda S_{\lambda\sigma,\alpha\beta} q_{\lambda'} S_{\lambda'\sigma,\gamma\delta}}{q^2} \,, \qquad (53)$$

where

$$S_{\alpha\beta,\gamma\delta} = \frac{g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\delta}g_{\beta\gamma}}{2} \,.$$

The poles at $q^2 = M^2$ in Eqs. (52) and (53) represent the contribution of Z intermediate states, while the poles at $q^2 = 0$ are the contributions of ψ intermediate states. The additional factors of momenta in the numerator are a consequence of the derivatives appearing in Eq. (29).

III. Discussion

The final step is the construction of $\mathcal{L}(C)$, the extension of $\mathcal{L}^{(0)}(C)$, which is invariant under non-Abelian gauge transformations (Eq. (12)) of the dual potentials

C_μ^a. We have shown [3] that such a theory possesses many of the properties of a dual superconductor and could form the basis for describing long-distance QCD. However, in [3] we did not impose the essential constraint (Eq. (20)) and the resulting quadratic Lagrangian $\mathcal{L}_{old}^{(0)}(C)$ contained redundant degrees of freedom. As a consequence, the long-distance Lagrangian $\mathcal{L}_{old}(C)$ (obtained essentially by making the replacement $\partial_\mu f \rightarrow \partial_\mu f - ig[C_\mu, f]$ in $\mathcal{L}_{old}^{(0)}(C)$) did not lead to a unitary S matrix. This problem should be eliminated, if $\mathcal{L}(C)$ is generated from the quadratic Lagrangian $\mathcal{L}^{(0)}(C)$ ((Eq. (21)), which takes the constraint (Eq. (20)) into account. However, the modifications of $\mathcal{L}_{old}(C)$ necessitated by the constraint are not straightforward. They require further symmetry (non-Abelian versions of both (Eqs. (32) and (34))) and they have not yet been found. If these modifications can be found, then the unitarity contributions from the massless ψ and ϕ degrees of freedom should cancel and the resulting Lagrangian $\mathcal{L}(C)$ would describe a consistent unitary theory of long-distance QCD.

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