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UNIVERSIDADE DE SÃO PAULO

INSTITUTO DE FÍSICA
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J. Gamboa
Instituto de Física, Universidade de São Paulo and
Centro de Estudios Cientificos de Santiago, Casilla 16443,
Santiago 9, Chile

V.O Rivelles
Instituto de Física, Universidade de São Paulo

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THE BRST FORMALISM AND THE QUANTIZATION OF HAMILTONIAN SYSTEMS WITH FIRST CLASS CONTRAINTS

J.GAMBOA
Instituto de Fisica ,Universidade de São Paulo,C.Postal 20516 , CEP 01498 ,S.Paulo , SP ,Brasil and Centro de Estudios Cientificos de Santiago . Casilla 16443 , Santiago 9 ,Chile.

V.O.RIVELLES*
Instituto de Fisica ,Universidade De São Paulo , C.Postal 20516 ,CEP 01498, S.Paulo , SP ,Brasil.

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INTRODUCTION

Gauge invariance plays an important role in the present theoretical physics .In the past gauge invariance had permited to solve important problems in quantum field theory and particle physics[1].

Along the hamiltonian lines the gauge symmetry appears when the theory under study has first class constraints[2].

We suppose that some theory is given with first class constraints $\phi_a(p,q) = 0$,then we say that $\phi_a(p,q)$ is a first class contraint iff,

$$[\phi_a, \phi_b] = C^c_{ab}\phi_c \tag{1.1}$$

here [,] means Poisson braket.(1.1) is usually called in the physical literature "gauge open algebra" because generally (1.1) is not a closed algebra(in the sense of ordinary Lie algebras).Here C^c_{ab} in general is not constant.

Algebras like (1.1) describe systems such as the relativistic particle,strings ,membranes,gravitation,etc(and of course, their supersymmetric relatives).

The quantization of these theories is plagued with difficulties, for this reason it is necessary to study new quantization methods for wich these systems can be studied.

In the last ten years it has been discovered a general quantization method which permits to study these systems. This method, called Batalin-Fradkin-Vilkovisky(BFV) formalism is reviewed briefly in section 2. Section 3, is devoted to study simple applications . Here the quantization of the relativistic particle and the relativistic spinning particle is worked out in detail. Several points not discussed in the literature are point out and we find the correct expression for the Feynman propagator in both cases.Section 4 contains conclusions and an outlook.

2.- The BFV Formalism : Review

In this section we review the BFV formalism[3]. As it was explained in the introduction , this method is a procedure for quantizing systems with first class constraints and is the most general method know today to treat this class of systems.

We consider a dynamical system described by a phase space F_1 whose coordinates are (p_i, q^i),i=1,2,3,...,N; the canonical hamiltonian is H_0, and the dynamical system is subject to M first class constraints ϕ_a satisfying the algebra (1.1).

The action for this system taken be:

$$S = \int_{t_1}^{t_2} dt(p_i\dot{q}^i - H_0 - \lambda^a\phi_a), \tag{2.1}$$

where theλ^a are lagrange multipliers. Then,in the BFV formulation, we consider that the lagrange multipliers can be treated in the same foot as the canonical variables (p,q).This oblige us to introduce conjugate canonical momenta to λ_a,say π_a, such as:

$$[\lambda_a, \pi^b] = \delta^b_a. \tag{2.2}$$

Then , in order that the dynamics of the theory does not change they must be imposed as new constraints.i.e.

$$\pi^a = 0. \tag{2.3}$$

In the BFV notation,the set of 2M constraints (ϕ_a, π^a) it denote by G_a and they obviously satisfy the gauge algebra

$$[G_a, G_b] = K^c_{ab} G_c. \tag{2.4}$$

In the algebraic sense.the procedure of treating the Lagranges multiplier in the same foot that the coordinates (p,q). is equivalent to replace the old phase space F_1 by an other phase space F_2 ,such that :

$$F_1 \longrightarrow F_2 : (p,q) \oplus (\pi_a, \lambda_a) \tag{2.5}$$

The next step in BFV construction consists in incorporating a pair of canonically conjugated ghosts $(\eta_a, \mathcal{P}^a)$ for each constrained (with opuest statistic),i.e,

$$\{\eta_a, \eta_b\} = 0, \{\mathcal{P}^a, P^b\} = 0,$$

$$\{\eta_a, \mathcal{P}^b\} = -\delta^b_a \tag{2.6}$$

thus, the phase space is replaced by :

$$(p,q) \longrightarrow (p,q) + (\pi_a, \lambda) + (\mathcal{P}_a, \eta_a) \tag{2.7}$$

The hamiltonian structure (2.7) has remarkable properties .We would like to enumerate some of them:

a) In (2.7) .we replaced the local gauge invariance by a global symmetry. This name is due to Becchi,Rouet,Stora and Tyutin who discovered a similar symmetry in the context of Yang-Mills theory [4,5].

The BRST symmetry is a name given by the physicists to a symmetry deeply rooted in cohomology theory[6].

b) the symmetry generator Q (usually called called BRST charge) for a theory with the gauge algebra (2.4) ,has the form:

$$Q = \eta^a G_a + \frac{1}{2} \mathcal{P}_a K^a_{bc} \eta^b \eta^c + \cdots \tag{2.8}$$

(2.8) is anticonmutative and is ,by construction, nilpotent,i.e.

$$\{Q, Q\} = 0 \tag{2.9}$$

c) At quantum level, in the extended phase space (2.7) , there exists the following theorem proved by Fradkin and Vilkovisky [3].

Theorem

Let a hamiltonian system with G_a constraints be described by the effective action S_{eff} given by

$$S_{eff} = \int_{t_1}^{t_2} dt(p_i q^i + \eta_a \mathcal{P}^a + \pi_a \lambda^a - H_0 - \{Q, \psi\}). \qquad 2.10)$$

where Q is the BRST charge and Ψ is an arbitrary function (gauge fixing function). Then the path integral:

$$Z_\Psi = \int D\mu \exp[iS_{eff}], \qquad (2.11)$$

where$D\mu$ is a Liouville measure ,is independent of the choice of Ψ ,i.e.

$$Z_\Psi = Z'_\Psi.$$

This remarkable theorem is useful to prove the unitary of theories and permit to calculate off-shell propagators (generally a complicate problem). For a demostration of the theorem , see ref.[3].

3.-APLICATIONS

The massive relativistic particle is described by the following action:

$$S = -m \int_{t_1}^{t_2} d\tau \sqrt{-\dot{x}^2}. \qquad (3.1)$$

In the hamiltonian formalism it is easy to verify that there exist the following constraint :

$$\mathcal{H} = \frac{1}{2}(p^2 + m^2) = 0 \qquad (3.2)$$

and the canonical hamiltonian $H_0 = p_\mu \dot{x}^\mu - L$ is identically zero.

This is a general caracteristic of generally covariant systems .It is easy to verify using

$$[x_\mu, x_\nu] = 0 = [p_\mu, p_\nu],$$
$$[x_\mu, p^\nu] = \delta^\nu_\mu, \qquad (3.3)$$

that the constraint algebra (3.2) is

$$[\mathcal{H}, \mathcal{H}] = 0 \qquad (3.4),$$

and by consequence , (3.4) is a first class algebra. Thus , to quantize the relativistic particle, we can use the BFV formalism developed in the section 2. The extended phase space (2.7) in this case is :

$$(p_\mu, x^\mu) \oplus (\pi_N, N) \oplus (\bar{\mathcal{P}}, \eta, \mathcal{P}, \bar{\eta})$$

where N is a Lagrange multiplier , π_N their canonical momenta and the$\mathcal{P}'s$ and $\eta's$ are the anticonmutative ghosts that in this case satisfy :

$$\{\eta,\bar{\mathcal{P}}\} = -1 = \{\bar{\eta},\mathcal{P}\}.$$
$$\{\eta,\bar{\eta}\} = \{\mathcal{P},\bar{\mathcal{P}}\}$$

The action in the extended phase space now is :

$$S = \int_{t_1}^{t_2} d\tau(\pi_N\dot{N} + \dot{\eta}\bar{\mathcal{P}} + \dot{\bar{\eta}}\mathcal{P} + p^\mu\dot{x}_\mu + \{Q,\Psi\}) \tag{3.5}$$

using the (2.8) prescription , the BRST charge is:

$$Q = \eta\mathcal{H} + \mathcal{P}\pi \tag{3.6}$$

and the fixing gauge function is chosen in the form:

$$\Psi = \bar{\mathcal{P}}N \tag{3.7}$$

The choice of Ψ , according to the Fradkin-Vilkovisky theorem ,is arbitrary , nevertheless here it is convenient the election (3.7) because it is equivalent to choose the proper time gauge $\dot{N} = 0$. This gauge choice is consistent with the reparametrization invariance. Using the Fradkin-Vilkovisky theorem , we obtain :

$$Z_\Psi = \int DND\pi D\eta D\bar{\mathcal{P}}D\bar{\eta}D\mathcal{P}Dp^\mu Dx_\mu.$$
$$.\exp[i\int_{t_1}^{t_2} d\tau(\pi_N\dot{N} + \dot{\eta}\bar{\mathcal{P}} + \dot{\bar{\eta}}\mathcal{P}p_\mu\dot{x}^\mu + N\mathcal{H} + \mathcal{P}\bar{\mathcal{P}})]. \tag{3.8}$$

The (3.8) integrals can be calculated imposing the following BRST invariant boundary conditions :

$$x(t_1) = x_1, x(t_2) = x_2,$$
$$\eta(t_1) = 0 = \eta(t_2), \bar{\eta}(t_1) = \bar{\eta}(t_2) = 0,$$
$$\pi(t_1) = \pi(t_2) = 0 \tag{3.9}$$

Integrating π_N ,we obtain the $\delta[\dot{N}]$ factor and the integration in ghosts momenta give the usual expresion for the transition amplitude in the proper time gauge [7].

To integrate in x_μ and p_μ it is convenient to eliminate the zero mode associate to $N(t)$,then we write :

$$N(t) = N(0) + M(t) \tag{3.10}$$

where we have the following boundary condition for M(t)

$$M(0) = 0. \tag{3.11}$$

Using (3.10) the $\delta[\dot{N}]$ factor can be writen as:

$$\delta[\dot{M}] = \int dN(0)\delta[M(t) - N(0)]det(\partial_\tau)^{-1}. \tag{3.12}$$

thus, (3.8) becomes:

$$Z_\Psi = \int dN(0) \int D\eta d\bar{\eta} Dx_\mu Dp^\mu det(\partial_\tau)^{-1}$$

$$\exp[i \int_{t_1}^{t_2} d\tau(p^\mu \dot{x}_\mu + N(0)\mathcal{H} + \dot{\eta}\bar{\eta})] \tag{3.13}$$

The determinant that appears in (3.13) is indetermined and it can be taken out of the path integral as a factor absorbed by an overall normalization.

Following Teitelboim arguments [7], the integral in N(0) can not be taken in the range$(-\infty, \infty)$ because we are obliged to choose only a classical trajectory. This observation is physically very satisfactory and it is crucial to obtain the correct result.

Integrating on η and $\bar{\eta}$,we obtain $det(-\partial_\tau^2)$ This expression can be calculated using the boundary conditions (3.9) and ζ-function regularization. The result is $(t_2 - t_1)$ and the integral (3.12) is :

$$Z_\Psi = \mathcal{N}' \int_0^\infty dT \int Dx^\mu Dp_\mu \exp[i \int_{t_1}^{t_2} d\tau(p^\mu \dot{x}_\mu + N(0)\mathcal{H})] \tag{3.14}$$

where $T = N(0)(t_2 - t_1)$ and $\mathcal{N}$ is a normalization constant. The integration on P is gives:

$$Z_\Psi = \mathcal{N}' \int_0^\infty dT \int Dx^\mu \exp[i \int_{t_1}^{t_2} d\tau(\frac{1}{2N(0)}\dot{x}^2 + \frac{1}{2}m^2 N(0))] \tag{3.15}$$

note that the effective action in (3.15) is precisaly the einbein version of the relativistic particle.To integrate in (3.15) we make the following change of variables:

$$x^\mu(t) = x_1^\mu + \frac{\Delta x^\mu}{\Delta t}(t - t_1) + y^\mu(t) \tag{3.16}$$

(3.16) is consistent with (3.8) iff:

$$y^\mu(t_1) = 0 = y^\mu(t_2). \tag{3.17}$$

Using (3.16), (3.15) yields:

$$Z_\Psi = \mathcal{N}' \int_0^\infty dT det(\frac{-\partial_\tau^2}{N(0)})^{\frac{-D}{2}} \exp[i(\frac{(\Delta x)^2}{2T} + m^2 T)] \tag{3.18}$$

The determinant in (3.18) can be calculated using ζ-function regularization and the boundary condition (3.17) the result is :

$$det(\frac{-\partial_\tau^2}{N(0)}) = T$$

Thus, (3.18) is:

$$Z_\Phi = \mathcal{N}' \int_0^\infty dT T^{\frac{-D}{2}} \exp[i(\frac{\Delta x^2}{2T} + \frac{m^2 T}{2})] =$$
$$= \mathcal{N}' \int \frac{d^D p}{(2\pi)^D} \frac{\exp ip.(x_2 - x_1)}{p^2 + m^2 - i\epsilon} =$$
$$= G(x_2 - x_1; m^2).$$

This expression is the Feynman propagator for the relativistic particle. Recently, two diferent derivations of this result has been obtained in the literature [8,9]. Also Giannakis,Ordonez,Rubin and Zucchini have obtained similar results using the lagrangian formalism [10].

SPINNING PARTICLE

The massive spinning particle is described by the following constraints[11]:

$$\mathcal{H} = \frac{1}{2}(p^2 + m^2) = 0,$$
$$\mathcal{S} = \theta^\mu p_\mu + m\theta_5 = 0 \qquad (3.19)$$

where θ_μ and θ_5 are grassmanian variables that obey the following algebra:

$$\{\theta_\mu, \theta^\nu\} = i\delta_\mu^\nu,$$
$$\{\theta_5, \theta_5\} = i \qquad (3.20),$$

and the even variables, satisfy the algebra (3.3).
Using (3.19) and (3.3), it is easy to verify that the constraints algebra is:

$$[\mathcal{H}, \mathcal{H}] = 0,$$
$$[\mathcal{H}, \mathcal{S}] = 0,$$
$$\{\mathcal{S}, \mathcal{S}\} = 2i\mathcal{H}. \qquad (3.21)$$

It is easy to see using (2.8) that the BRST charge is:

$$Q = \eta\mathcal{H} + \mathcal{P}\pi_N + cS + \pi_\lambda\mathcal{P}_c + i\bar{\mathcal{P}}cc \qquad (3.22)$$

where $(\eta, \bar{\eta}, \bar{\mathcal{P}}, \mathcal{P})$ are the ghosts coordinates and the ghosts momenta (anticonmutative) associated to $\mathcal{H}$ and $(c, \bar{c}, \bar{\mathcal{P}}_c, \mathcal{P}_c)$ are the coordinates and the ghost momenta (conmutative) associated to S. The conmutative ghost algebra is :

$$[c, \bar{\mathcal{P}}_c] = 1 = [\bar{c}, \mathcal{P}] \qquad (3.23)$$

and zero in the other cases. π_λ is the canonical momenta of the fermionic Lagrange multiplier λ.

The fixing gauge function Ψ is choosen as :

$$\Psi = \bar{\mathcal{P}}N + \lambda\bar{\mathcal{P}}_c. \qquad (3.23)$$

Using the Fradkin-Vilkovisky theorem, we obtain:

$$Z_\Psi = \int DN D\pi_N D\lambda D\pi_\lambda D\eta D\bar{\mathcal{P}} D\bar{\eta} D\mathcal{P} D\mathcal{P}_c D\bar{c} D\bar{\mathcal{P}}_c Dc D\theta_\mu D\theta_5 Dp^\mu Dx_\mu$$

$$\exp[i\int_{t_1}^{t_2} d\tau(\pi_N\dot{N} + \dot{\lambda}\pi_\lambda + \dot{\eta}\bar{\mathcal{P}} + \dot{\bar{\eta}}\mathcal{P} + \bar{\mathcal{P}}_c\dot{c} + \mathcal{P}_c\dot{\bar{c}} - \frac{i}{2}\dot{\theta}^\mu\theta_\mu - \frac{i}{2}\dot{\theta}_5\theta_5 +$$

$$p_\mu\dot{x}^\mu + N\mathcal{H} + \lambda S - \mathcal{P}\bar{\mathcal{P}} + \mathcal{P}_c\bar{\mathcal{P}}_c - 2i\bar{\mathcal{P}}c\lambda)] \qquad (3.24)$$

In order to calculate (3.24) we impose the following BRST invariant boundary conditions:

$$x(t_1) = x_1, x(t_2) = x_2,$$
$$\eta(t_1) = \eta(t_2) = c(t_1) = c(t_2) = 0,$$
$$\bar{\eta}(t_1) = \bar{\eta}(t_2) = \bar{c}(t_1) = \bar{c}(t_2) = 0,$$
$$\pi_N(t_1) = \pi_N(t_2) = \pi_\lambda(t_1) = \pi_\lambda(t_2) = 0$$
$$\frac{1}{2}(\theta^\mu(t_1) + \theta^\mu(t_2)) = \zeta^\mu,$$
$$\frac{1}{2}(\theta_5(t_1) + \theta_5(t_2)) = \zeta_5 \qquad (3.24).$$

Integrating over π_N, π_λ, $\mathcal{P}$, $\bar{\mathcal{P}}$, $\mathcal{P}_c$, $\bar{\mathcal{P}}_c$ and $\mathcal{P}_c$, we obtain:

$$Z_\Psi = \int DN N D\lambda D\eta D\bar{\eta} D\bar{c} Dc D\theta_\mu D\theta_5 Dp^\mu Dx_\mu$$

$$\delta[\dot{N}]\delta[\dot{\lambda}]\exp[i\int_{t_1}^{t_2} d\tau(p_\mu\dot{x}^\mu - \frac{i}{2}\dot{\theta}^\mu\theta_\mu - \frac{i}{2}\dot{\theta}_5\theta_5 +$$

$$+\mathcal{N}\mathcal{H} + \lambda S + \eta\dot{\bar{\eta}} + 2ic\lambda\dot{\bar{\eta}} - \dot{c}\bar{c})] \tag{3.27}$$

(3.27) is the hamiltonian expression for the path integral in the proper time gauge. As in the relativistic particle case. we would like to eliminate the zero modes. For this reason we write the analogous of (3.9).

$$\mathcal{N}(t) = \mathcal{N}(0) + M(t),$$
$$\lambda(t) = \lambda(0) + \zeta(t), \tag{3.28}$$

where we have the following "boundary conditions":

$$M(0) = 0,$$
$$\zeta(0) = 0. \tag{3.29}$$

The equivalent of the equation (3.10) is:

$$\delta[\dot{M}] = \int d\mathcal{N}(0)\delta[M(t) - \mathcal{N}(0)]det(\partial_\tau)^{-1},$$

$$\delta[\dot{\zeta}] = \int d\lambda(0)\delta[\zeta(t) - \lambda(0)]det(\partial_\tau)^{+1} \tag{3.30}$$

Such as in the relativistic particle case, the determinants that appears in (3.29) are indetermined because we have not sufficient boundary conditions , nevertheless, in this case the bosonic and fermionic determinants are precisely cancelled. Replacing (3.29) in (3.26) and using Teitelboim arguments to choose one classical trajectory, we obtain:

$$Z_\Psi = \int_0^\infty d\mathcal{N}(0)\int d\lambda(0)\int D\eta D\bar{\eta}D\bar{c}DcD\theta_\mu D\theta_5 Dp^\mu Dx_\mu$$

$$\exp[i\int_{t_1}^{t_2} d\tau(p_\mu\dot{x}^\mu - \frac{i}{2}\dot{\theta}^\mu\theta_\mu - \frac{i}{2}\dot{\theta}_5\theta_5 + +\mathcal{N}\mathcal{H} + \lambda S + \eta\dot{\bar{\eta}} + 2ic\lambda\dot{\bar{\eta}} - \dot{c}\bar{c})] \tag{3.31}$$

(for the integration in $\lambda(0)$ we do not write the integration range because such concept does not exist for the Berezin integral).

Using the boundary conditions (3.25), the ghosts can be explicitly calculated. Integrating in P_μ:

$$Z_\Psi = \int_0^\infty d\mathcal{N}(0)\int d\lambda(0)\int D\theta_\mu D\theta_5 Dx_\mu$$

$$\exp[i\int_{t_1}^{t_2} d\tau(\frac{\dot{x}^2}{2\mathcal{N}(0)} - \frac{i}{2}\dot{\theta}^\mu\theta_\mu - \frac{i}{2}\dot{\theta}_5\theta_5 + \frac{\lambda(0)\theta_\mu\dot{x}^\mu}{\mathcal{N}(0)} + \frac{m\lambda(0)\theta_5}{\mathcal{N}(0)})] \tag{3.32}$$

In (3.32) the effective action is the one-dimensional supergravity action if $\mathcal{N}(0)$ and $\lambda(0)$ are interpreted as the graviton and the gravitino respectively.

Making the change of variables:

$$x^\mu(t) = x_1^\mu + \frac{\Delta x^\mu}{\Delta t}(t - t_1) + y^\mu(t),$$

$$\theta^\mu(t) = \gamma_5\gamma^\mu + \psi^\mu(t),$$

$$\theta_5(t) = \gamma_5 + \psi(t), \quad (3.33)$$

and using (3.25), the consistency imply:

$$y^\mu(t_1) = 0 = y^\mu(t_2). \quad (3.34)$$

Using (3.33), (3.31) gives:

$$Z_\Psi = \int_0^\infty dN(0) \int d\lambda(0) \exp[i(\frac{(\Delta x)^2}{2T} + \frac{m^2 T}{2} + \frac{\lambda(0)\gamma_5\gamma_\mu \Delta x^\mu}{N(0)} + \frac{m\lambda(0)\gamma_5}{N(0)})]$$

$$\int Dy^\mu \exp[i \int_{t1}^{t_2} d\tau (\frac{\dot{y}^2}{2N(0)} - \frac{i}{2}\dot{\psi}\psi - \frac{i}{2}\dot{\psi}_5\psi_5)]$$

Integrating in $y(t)$ and $\lambda(0)$, we obtain :

$$Z_\Psi = \int_0^\infty \frac{dT}{T^{\frac{D+1}{2}}}(\gamma_5\gamma_\mu \Delta x^\mu + m\gamma_5)\exp[i(\frac{\Delta x^2}{2T} + \frac{m^2 T}{2} - i\epsilon)]$$

$$= \int \frac{d^D p}{(2\pi)^D} \frac{\exp ip.(x_2 - x_1)}{p^2 + m^2 - i\epsilon}(\gamma_5\gamma_\mu p_\mu + m\gamma_5). \quad (3.35)$$

(3.34) is the Dirac propagator [8].

CONCLUSIONS

In this paper we have studied the quantization of hamiltonian systems with first class constraints using the BFV formalism.

Using the two examples studied above , we see that the BFV formalism is a powerful method for quantizing theories with gauge freedom.

For most complicated theories,such as strings and membranes (that are minimal surfaces in the proceeding sense) , the problem is not solved .The main difficulty is that at the quantum level there are anomalies .This problem is complicated and it is a very important one .Using the BFV formalism this problem is not understood at the path integral level .

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references

[1] see e.g. E. Abers and B. Lee, Phys.Rep.9C(1973)1.
[2] P.A.M. Dirac ,Lectures On Quantum Mechanics. Yeshiva University , 1964.
A.J. Hanson , T. Regge, And C.Teitelboim, Constrained Hamiltonian Systems. Accademia Dei Lincei,Roma, 1976.
[3] E.S.Fradkin and G.Vilkovisky. Phys. Lett.55B(1975)224;
I.A Batalin and G.Vilkovisky, Phys. Lett.69(1977)309;
M. Henneaux , Phys. Rep.126(1985)1.
[4] C.Becchi , A. Rouet , R. Stora. Phys. Let.52B(1974)344.
[5] I.V. Tyutin , Lebedev report.(unpublished)1975.
[6] see e.g. B.Doubronine , S. Novikov and A. Fomenko ,Geometrie Contemporaine.vol.3 Mir, Moscou 1984.
[7] C.Teitelboim , Phys.Rev.D25(1982)3152.
[8] V.Fainberg and A.Marshakov , Nucl.Phys.B306(1988)659.
[9] A.Cohen,G.Moore,P.Nelson and J.Polshinski Nucl.Phys.B267(1986)143.
[10] I.Giannakis , C.R.Ordonez, M.A.Rubin and R.Zucchini ,
Rockefeller preprint RU88/b1/28.
[11] C.Teitelboim ,Phys.Rev.Lett.38(1977)1106.