Magnetic Field Structure near the Plasma Boundary in Helical Systems and Divertor Tokamaks

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Abstract

Magnetic field structure of the scrape off layer (SOL) region in both helical systems and divertor tokamaks is studied numerically by using model fields. The connection length of the field line to the wall is calculated. In helical systems, the connection length, L, has a logarithmic dependence on the distance from the outermost magnetic surface or that from the residual magnetic islands. The effect of axisymmetric fields on the field structure is also determined. In divertor tokamaks, the connection length also has logarithmic properties near the separatrix. Even when the perturbations, which resonate to rational surfaces near the plasma boundary, are added, logarithmic properties still remain. We compare the connection length of torsatron/helical-heliotron systems with that of divertor tokamaks. It is found that the former is shorter than the latter by one order magnitude with similar aspect ratio.

1 Introduction

In straight helical systems and in axisymmetric toroidal systems like tokamaks, the magnetic surfaces are perfectly closed within the separatrix, provided that there are no perturbations such as error fields and MHD fluctuations. In toroidal helical systems, the closed magnetic surfaces can be partly destroyed so that an ergodic region appears in the scrape off layer (SOL) region. This occurs because magnetic islands are constructed on rational surfaces by the toroidal effect; consequently, islands overlap once they reach a certain size (Rosenbluth *et al.*,1966; Filonenko *et al.*,1967; Hamzeh,1972). Destruction of the magnetic surface is generally believed to be one source of the anomalous diffusion in toroidal helical systems (Wobig and Fowler,1988). Cary and Hanson(1986) indicated that the ergodicity is systematically reduced by the variation of the parameters of systems. Though this method enlarges the closed region, the ergodicity still remains outside of the closed region. When the field is ergodic, it is necessary to analyze its statistical properties.

On the other hand, in tokamaks, when we add the perturbations which resonate to boundary rational surfaces, the ergodic region appears near the separatrix and the separatrix disappears. This ergodic magnetic field is known as a concept of a ergodic magnetic limiter (EML)(Neuhauser*et al.*,1989). The edge plasma behavior is modified by EML. The role of the ergodic field on the plasma confinement has been investigated experimentally (Samain *et al.*, 1984; Ohyabu and Degrassie, 1987; McCool, 1989; Shen, *et al.*, 1989; Shoji,*et al.*,1989) and theoretically (Martin and Taylor, 1984). The motivation of these investigations is that particles and high heat flux can be handled and impurity is controlled. The modification of the edge plasma also affects the core plasma confinement.

Divertor function has an important role in controlling particles and heat in toroidal plasmas. It also improves the confinement of the core plasma. Several types of improved confinement, including H-mode (Wagner *et al.*,1982), improved ohmic confinement (Söldner *et al.*,1988) and improved divertor confinement (Tsuji *et al.*,1988), have been observed in tokamaks. The separatrix has a divertor function in axisymmetric systems. The question of whether the ergodic region plays the role of the divertor function or not merits further study. The problem of the SOL plasma in toroidal helical systems is discussed by authors briefly in a preceding paper (Itoh *et al.*,1989). This ergodic SOL region was also studied in tokamaks with small error fields (Neuhauser *et al.*, 1989).

We in this article study the magnetic field structure of the SOL region in both helical systems and divertor tokamaks. On the outside of the outermost magnetic surface (OMS), the field line reaches the wall. We define connection length as the distance from the initial point to the wall. The connection length of the magnetic field is used to investigate the field structure. To construct a model of vacuum magnetic fields, we use two models, i.e., a simple toroidal harmonic function in helical systems and a circular current which concentrates near the magnetic axis in divertor tokamaks. The connection length has an infinite value within the OMS, but rapidly decreases away from it. The connection length has a logarithmic dependence on δ, where δ is the distance from the OMS. On the outside of the OMS, residual magnetic islands, which are isolated from the closed magnetic surface, exist. The connection length has an infinite value on these islands and also has a logarithmic dependence in the vicinity of them.

In helical systems, a characteristic value in regard to logarithmic properties exists in the SOL region. Expanding on results in the previous article, we study the dependence on both the toroidal pitch number and the axisymmetric fields. The effect of the residual magnetic islands in the SOL is also studied.

Helical systems and divertor tokamaks are compared from the viewpoint of the absolute value of the connection length. From numerical results, the connection length of torsatron/helical-heliotron systems is found to be less than that of tokamaks by one order magnitude.

2 Connection Length in Helical Systems

2.1 Model

The vacuum magnetic field of toroidal helical systems can be expressed by the magnetic scalar potential as

$$\mathbf{B} = -\nabla\Phi. \tag{2.1}$$

The potential, Φ, is represented in terms of the toroidal harmonic functions as (Morse and Feshbach, 1953)

$$\Phi = \sum_{\ell,m} \alpha_{\ell m} f_{\ell m}(y)\sqrt{1 - y\cos\psi}\cos(\ell\psi + m\zeta) \tag{2.2}$$

where ℓ and m are the poloidal and toroidal pitch numbers, respectively, $\alpha_{\ell m}$ is a numerical coefficient, and (y, ψ, ζ) are the toroidal coordinates. The relation between the toroidal coordinates and the quasi-toroidal coordinates (r, ζ, θ) are

$$\left.\begin{aligned} \frac{r}{R}\cos\theta &= \frac{\sqrt{1-y^2}}{1 - y\cos\psi} - 1, \\ \frac{r}{R}\sin\theta &= \frac{y\sin\psi}{1 - y\cos\psi}. \end{aligned}\right\} \tag{2.3}$$

where r is minor radius, R is major radius, θ is poloidal angle and ζ is toroidal angle. The equation which determines $f_{\ell m}$ is given in Morse and Feshbach (1953).

By taking a simple (ℓ, m)-Fourier component and superimposing the axisymmetric toroidal and vertical fields, $\mathbf{B}_\zeta$ and $\mathbf{B}_v$, the total magnetic field is expressed as

$$\mathbf{B} = -\nabla\{\alpha_{\ell m} f_{\ell m}(y)\sqrt{1 - y\cos\psi}\cos(\ell\psi + m\zeta)\} + \mathbf{B}_\zeta + \mathbf{B}_v. \tag{2.4}$$

The equation of field line,

$$\frac{d\rho}{B_\rho} = \frac{\rho d\theta}{B_\theta} = \frac{dz}{B_z}, \tag{2.5}$$

is solved to determine the structure of the field, where (ρ, θ, z) are the cylindrical coordinates. Eqn.(2.4) has been used to study magnetic surfaces of $\ell = 2$ helical systems (Yoshikawa,1983; Nagasaki *et al.*,1988).

In axisymmetric systems such as tokamaks and in straight helical configurations, the magnetic surfaces are perfectly closed within the separatrix, provided that there are no field perturbations. In toroidal helical systems, however, the magnetic surfaces near the separatrix are partly destroyed, resulting in the disappearance of the separatrix. Therefore, the ergodic region appears at the plasma boundary. Since particles and heat are mainly carried along the field line, the connection length, which has a finite value in the ergodic region, is an important factor in evaluating the qualities of the SOL region.

An example of closed magnetic surfaces is shown in Fig.1, simulating Heliotron-E device $(\ell = 2, m = 19)$ at (a)$\zeta = 0$ and (b)$\zeta = \pi/2m$. Values of characteristic quantities of the magnetic surface are $\iota(0) = 0.5, \iota(a) = 2.44, a/R = 0.07$, where a is average minor radius.

2.2 Properties of Connection Length in Ergodic Region

Connection length is calculated as follows. The initial point of the field line is given. By solving Eqn.(2.5), we follow the field line in the direction of increasing ζ until it reaches the wall. The connection length, L_+, is defined as the distance along the field line from the initial point to the wall. L_- is also given by following the field line in the opposite direction. $L_\pm$ is a function of the initial point coordinates.

Figure 2 shows the example of the radial profile of connection length L_+. Parameters are the same as in Fig.1 Initial points of the field line are (r, θ, ζ) =($r, 0, \pi/2m$). Since magnetic surfaces are perfectly closed within the OMS, the connection length has an infinite value within it. Away from it, the connection length decreases rapidly. Magnetic islands exist on rational surfaces and still remain even in the SOL region. In the SOL region, islands are isolated and self-enclosed and have their own inner structure; they have similarly small islands around them. This is called fractal structure. An example is shown in Fig.2(b). The connection length has an infinite value on the islands and decreases away from the islands' surfaces. Figure 3 shows the asymptotic behavior near the OMS and residual islands' surfaces. The asymptotic behavior near the OMS is important in view of the divertor function (Wagner and Lackner,1984). The connection length $L_\pm$ has a logarithmic dependence on δ, where δ is the distance from the OMS or from residual islands' surfaces. This dependence holds up within a range, $\delta/R \sim 10^{-3}$.

The value of the connection length is slightly different between the inside and outside of the torus. The logarithmic dependence is common, i.e., the coefficient, $L/\ln(R/\delta)$, is same, but the constant term is different. The connection length on the side where the O-points of residual islands exists is longer than the other side. A step structure in the radial profile can be seen in Fig.2(a). This is because residual islands remain in the SOL region. The O-points of $\iota = m/(\ell + j)(j = 1, 3, 5, \cdots)$ magnetic islands appear on the outside of torus, $\theta = 0$. The field line near the 19/7 surface must rotate one and a half helical pitches on its journey to the vicinity of the 19/5 surface. In the case of 19/5 surface, the field line must rotate one helical pitch to reach the 19/3 surface.

It is also found that the connection length for different values of (θ, ζ) has the same dependence except for a constant term. Figure 4 shows the θ and ζ dependence of $L_\pm$ for $\delta/R = 10^{-3}$. From these results, the connection length can be approximated as

$$L_\pm(\theta, \zeta) \sim L_0 \mp (\frac{\ell}{m}\theta + \zeta)R. \qquad (2.1)$$

Dotted lines in Fig.4(a) do not form a rhom because the connection length on the inside of the torus is slightly shorter than on the outside by the toroidal effect. The second term in Eq.(2.1) approximates the distance that the field line rotates to reach the low field sides, $\ell\theta + m\zeta = \pm\pi$, i.e., X-point. The distance that the field line travels to reach the low field sides is approximated by $\pi/m \mp (\ell\theta/m + \zeta)$, because the rotational transform near the OMS is much smaller than the geometrical pitch, m/ℓ. This means that the field line moves in a laminar manner and obeys the ergodic law in radial directions near the X-point. For this reason, the connection length near the OMS can be written as

$$L_{\pm}(\delta,\theta,\zeta) \sim L(\delta) + \frac{\ell}{m}\{\frac{\pi}{\ell} \mp (\theta + \frac{m\zeta}{\ell})\}R \tag{2.2}$$

where the toroidal and poloidal angle is measured in a single pitch, $0 \leq \theta \lesssim 2\pi/\ell$ and $-\pi/m < \zeta < \pi/m$. There is periodicity in both θ and ζ.

$L(\delta)$ is a logarithmic function of δ,

$$L(\delta) = \frac{\ell}{m}\frac{R}{\lambda_1}\ln\frac{b-a}{\delta\lambda_2} \tag{2.3}$$

where a is a plasma radius and b is a wall radius. The coefficients λ_1 and λ_2 are approximately 2.0 and 0.67 respectively in the case of Fig.1.

The value of λ_1 is a key parameter because the absolute value of the connection length depends on it. Figure 5 shows the m dependence of λ_1 for given values of B_ζ and B_v. The value of λ_1 is weakly dependent on m. It is confirmed that its value is independent of the wall distance. Dependence on the axisymmetric fields is studied in the next section. The logarithmic property in straight systems are also studied. (See Appendix.) The value of λ_1 in straight systems is proportional to $(a/R)^{1/2}$ and is close to the value in toroidal systems. That is, helical systems have logarithmic properties regardless of the existence of the separatrix.

2.3 Effect of axisymmetric fields on L

Characteristic quantities of the magnetic surface, such as rotational transform, plasma volume and specific volume, are changed by axisymmetric fields. The effect of axisymmetric fields has been evaluated in view of the plasma confinement, for example, MHD activity and particle confinement. Here we will analyze the effects on the field structure of the SOL region.

We first study the effect of the toroidal field. With the increase of the toroidal field, the plasma volume increases, while the rotational transform is lowered and the shear is weakened. In the $\ell = 2$ case, the rotational transform at the magnetic axis is analytically given as

$$\iota(0) \simeq \frac{4}{m}\left(\frac{\alpha_{\ell m}}{B_\zeta R}\right)^2. \tag{2.4}$$

From Eq.(2.4), the toroidal field which satisfies the condition $\iota(0) = m/(\ell + j)$ can be written as

$$\frac{B_\zeta R}{\alpha_{\ell m}} \simeq \frac{2}{m}\sqrt{\ell + j} \tag{2.5}$$

In the vicinity of the magnetic axis, the toroidal effect is weak and the analysis of straight helical systems can be applied. In straight helical systems, the short minor radius of the magnetic surface, r_s, is written as (Soloveev and Shafranov,1966)

$$r_s = r_0\sqrt{1 - \frac{4}{m} \mid \frac{\alpha_{\ell m}}{B_\zeta R} \mid}. \quad (2.6)$$

This determines the lower limit of the toroidal field,

$$\mid \frac{B_\zeta R}{\alpha_{\ell m}} \mid > \frac{4}{m}. \quad (2.7)$$

When the toroidal field is lower than this limit, the confinement region surrounding the major radius $\rho = R$ cannot be constructed.

Figure 6 shows the relationship between the toroidal field and the positions of both the OMS and the O-points of the magnetic islands on the outside of torus, $\theta = 0$. The toroidal pitch number is $m = 19$ and the vertical field is not added. The values of the toroidal field, at which the central rotational transform is equal to $m/(\ell + j)(j = 1, 3, 5, \cdots)$ or the confinement region disappears, agree with the analytic values given by Eqns.(2.5) and (2.7). With the increase of the toroidal field, the rotational transform at the plasma radius becomes smaller. If it falls bellow $m/(\ell + j)$, the O-points of $\iota = m/(\ell + j)$ islands appear on the outside of the OMS, $\theta = 0$.

The toroidal field changes λ_1 as shown in Fig.7. From the numerical result, we find that λ_1 is proportional to $(a/R)^{1/2}$. The inverse aspect ratio a/R is proportional to $(B_\zeta - B_\zeta^*)^{1/2}$, where B_ζ^* is the critical toroidal field at which the confinement region disappears (Nagasaki *et al.*,1988). Therefore, except in the vicinity of $B_\zeta \sim B_\zeta^*$, the dependence of λ_1 on the toroidal field is weak $\lambda_1 \propto (B_\zeta - B_\zeta^*)^{1/4}$.

We next study the effect of the vertical field. Application of the vertical field influences the plasma volume. The magnetic surfaces are shifted and the boundary region is affected. Not only the OMS but the O-points are shifted. Figure 8 illustrates the shift of the OMS and the O-points of the magnetic islands on both the inside and outside of the torus. Parameters are the same as in Fig.1. Since the shift of the magnetic surface is proportional to B_v/ι, the inner side of the OMS, where ι is smaller, moves faster than the outer side. In the case of Fig.8, the O-points move from the outside of the torus ($\theta = 0$) to the inside ($\theta = \pi$) with the increase of the vertical field. The confinement region is optimized at $B_v = B_v^*$. The edge rotational transform is also optimized as shown in Fig.9. When B_v is less than B_v^*, the O-point and the X-point of $m/(\ell + j)$ magnetic islands exist at $(\theta, \zeta) = (0, 0)$ and $(\pi, 0)$, respectively. Locations of the O-point and X-point are reversed on opposite sides. The helical resonant component $\tilde{B}_{\ell+j,m}$ changes its sign from positive to negative, and vice versa at $B_v = B_v^*$. This implies that $\tilde{B}_{\ell+j,m}$ is expanded near B_v^* as

$$\tilde{B}_{\ell+j,m} \simeq \gamma(B_v - B_v^*) + \cdots \tag{2.8}$$

where γ is a numerical coefficient. The width of resonant islands is proportional to $\sqrt{\tilde{B}_{\ell+j,m}}$ (Gourdon *et al.*,1968; Matsuda and Yoshikawa,1975). This suggests that the width of natural islands is proportional to $\sqrt{| B_v - B_v^* |}$ near the optimum B_v^*. Consequently the change of both the plasma radius and the rotational transform is proportional to $\sqrt{| B_v - B_v^* |}$ in the vicinity of B_v^*. This is confirmed by numerical calculation.

From numerical results, it is found that the behavior of the $\iota = 19/3$ rational surface differs from those of the others. The shift of this rational surface is very small, less than 5% of the shift of 19/5 surface, and the OMS never becomes larger than the 19/3 surface by the change of B_v. This property is also confirmed in the case of other toroidal pitch numbers. It is presumed that $\iota = m/(\ell + 1)$ is the upper limit of the edge rotational transform. If more optimization is needed, the other resonan' ields, which makes the width of $\iota = m/(\ell + 1)$ islands narrower, must be added.

Control of the field perturbation $\tilde{B}_{\ell',m}$ would reduce the ergodicity. The ergodicity, however, remains on the outside of the OMS so that the connection length still has a logarithmic dependence in the SOL region. The dependence of λ_1 on the vertical field is very weak. Within the range, $-0.005 \leq B_v R/\alpha_{lm} \leq 0.01$, λ_1 changes no more than $\pm$ 3%, while $\iota(a)$ approximately doubles. The increase of the plasma volume by B_v does not cause an increase in λ_1. This is in contrast to the toroidal field case.

3 Connection Length in Divertor Tokamaks

3.1 Logarithmic Dependence near Separatrix

In the same way as helical systems, the connection length can be used to investigate the field structure near the separatrix in tokamaks equipped with poloidal divertor. We consider the model in which the plasma current concentrates near the magnetic axis. Though there can be finite plasma current around the separatrix in real plasmas, we study the case where the boundary current is almost negligible.

We introduce a magnetic flux function as

$$\Psi = \rho A_\zeta \tag{3.1}$$

where A_ζ is the toroidal component of the vector potential $\mathbf{A}$. By using the complete elliptic integrals $K(k)$ and $E(k)$, the component A_ζ which is constructed by the circular current is described as

$$A_\zeta = \frac{\mu_0 I}{\pi k}\sqrt{\frac{R}{\rho}}\{(1-\frac{k^2}{2})K(k) - E(k)\} \tag{3.2}$$

where

$$k^2 = \frac{4\rho R}{(\rho + R)^2 + z^2}. \tag{3.3}$$

It is known that the magnetic surface near the magnetic axis is approximately circular and far away from the circular current, a dipole field is constructed.

The equation of the magnetic surface in divertor configurations is written by

$$\Psi = \Psi_p + \Psi_d + \Psi_v \tag{3.4}$$

where Ψ_p, Ψ_d and Ψ_v correspond to the flux functions by the plasma current, divertor current and the vertical field. The divertor currents are located at $(\rho, z) = (R, \pm z_d)$. The function Ψ_v is given by $(1/2)\rho^2 B_{v0}$. In this paper, the vertical field is added so that the X-point of the separatrix may be located at $\rho = R$. The location of the X-point also has a relation to the ratio of the plasma current I_p to the divertor current I_d. The X-point divides the distance z_d between the plasma current and the divertor current in the ratio $I_p : I_d$.

Since the poloidal field disappears at the X-point of the separatrix, the effective q value is introduced. The effective q value is defined as

$$q_I = \left| \frac{(\rho_s - R) B_{\zeta 0}}{R B_z(\rho_s)} \right| \tag{3.5}$$

where ρ_s is the radius of the separatrix magnetic surface at $z = 0$ on the outside of the torus and $B_z(\rho_s)$ is the z-component of the magnetic field at the separatrix.

Figure 10 illustrates one example of the magnetic surfaces in double null configurations. Parameters are chosen as $q_I = 3$, $z_d/R = 0.45$ and $I_a/I_p = 1/2$. We calculate the connection length in such a configuration. Like in helical systems, the connection length is defined as the distance from the initial point to the divertor plate which is set up at $\rho = R, | z | \geq z_d$. The connection length has an infinite value within the separatrix and decreases away from it. The asymptotic behavior of the connection length is shown in Fig. 11. The parameter δ is defined as the distance from the separatrix on the outside of the torus. The connection length also has the logarithmic dependence on δ like in helical systems. The coefficient C of the logarithmic function is affected by parameters of configurations. Figure 12 shows the dependence of C on parameters, q_l, z_d and I_d/I_p. From these numerical results, the connection length in divertor tokamaks can be fitted in the following formula.

$$\left.\begin{aligned} L &= C \ln \frac{R}{\delta} \qquad (\delta \to 0) \\ C &\sim 1.3 q_l R (1 + 0.42 \frac{z_d}{R}) \frac{I_d}{I_p}. \end{aligned}\right\} \tag{3.6}$$

It is confirmed that the connection length on the inside of the torus also has the same properties. However, its value is a little larger than the outside one. This is because the inside toroidal field is stronger than the outside.

3.2 Effects of Resonant Perturbing Fields

When we add the perturbations which resonate to rational surfaces near the separatrix, the separatrix disappears and the SOL region becomes ergodic once the magnetic island on the rational surface reaches a certain size. The ergodic layer width depends on the mode numbers and magnitude of the perturbations. In this section, the effect of the perturbations on the connection length is discussed.

Assume that the perturbation field $\tilde{\mathbf{B}}$, which satisfies $\nabla \cdot \tilde{\mathbf{B}} = 0$, is added to divertor tokamak configurations in the following form,

$$\left.\begin{aligned} \tilde{B}_r &= \frac{\tilde{b}R}{R + r\cos\theta}\frac{\mu_0 I_p}{2\pi R}(\frac{r}{R})^{m-1}\sin(m\theta - n\zeta) \\ \tilde{B}_\theta &= \frac{\tilde{b}R}{R + r\cos\theta}\frac{\mu_0 I_p}{2\pi R}(\frac{r}{R})^{m-1}\cos(m\theta - n\zeta) \end{aligned}\right\} \tag{3.7}$$

where (r, ζ, θ) are the quasi toroidal coordinates and m, n are the poloidal and toroidal mode numbers, respectively. This perturbation simulates the resonant helical field.

Figure 13 shows one example of radial profiles of both the connection length and the safety factor. The safety factor q in the SOL region is defined as $\int d\theta / \int d\zeta$ where the integral is taken from the initial point of the field line to the wall. The perturbation is chosen as $m/n = 5/1$ and $\tilde{b} = 0.1$. Residual islands are constructed on the rational surfaces. In this case, $m/n = 5/1$ magnetic island is completely broken and $m/n = 4/1$ magnetic island, which is induced by the toroidal effect, is seen. Sharp corrugations of the safety factor profile are seen in the SOL region. Like in toroidal helical systems, the connection length decreases rapidly away from residual islands surfaces. Figure 14 shows the asymptotic behavior at A and B in Fig. 13(a). The logarithmic dependence is seen. The coefficient C of the logarithmic function has nearly the same value as the symmetric case and the dependence of C on the geometrical parameters is similar. C has almost no dependence on $\tilde{b}$ within the range $\tilde{b} \leq 1$. It is also confirmed that C does not depend on the mode numbers of the perturbing fields. This means that logarithmic properties of the connection length in the SOL region persist no matter how the perturbation is added. In other words, the logarithmic nature of the connection length is not destroyed even if the idealized poloidal separatrix configuration is not realized. This would be the reason that the divertor functioning has been observed in many tokamaks with poloidal divertor under various operation conditions.

4 Comparison between Helical Systems and Divertor Tokamaks

Both helical systems and divertor tokamaks have logarithmic properties regardless of the preservation of symmetry. Their origins, however, seem to be different between symmetric and asymmetric systems. In symmetric systems, the properties have the relation to characteristics of the X-point of magnetic surface of the separatrix. The connection length near the X-point can be written as follows.

$$L \sim \int \frac{B}{B_p} ds \tag{4.1}$$

where B and B_p are total and relative poloidal fields, respectively and s is a distance from the X-point in the poloidal direction. The relative poloidal field B_p vanishes at the X-point, and it is approximately given as

$$B_p \sim B_{p0} x \tag{4.2}$$

where x is a distance from the X-point. If the minimum distance to the X-point is given by Δ, x may be approximated by $x \sim \sqrt{s^2 + \Delta^2}$. Substituting Eqn.(4.2) into Eqn.(4.1), logarithmic properties of the connection length, $L \propto \ln R/\Delta$ is found.

On the other hand, in broken symmetric systems, the destruction of the separatrix is caused by the overlapping of the magnetic islands. The fractal structure of many islands is confirmed by numerical calculations (See Fig.2(b)). Divergence of nearby field lines are expected in the state of positive Lyapunov exponent (Benettin and Galgani,1979; Rechester *et al.*,1979). We confirm that the connection length decays with the logarithmic form in the SOL region and that the coefficient of the logarithmic function is almost preserved in the symmetric limit.

Though the characteristics of the SOL region is similar both in helical systems and divertor tokamaks, the absolute value of the connection length is quite different. We compare the connection length value of helical systems and that of divertor tokamaks. By using the Eqns. (2.3) and (3.6), The ratio for the former L_h to the latter L_t is given by

$$\frac{L_h}{L_t} \simeq \frac{\ell}{m} \frac{1}{q_I} \frac{0.76}{\lambda_1} \frac{1}{1 + z_d/R} \frac{I_p}{I_d}. \tag{4.3}$$

For typical parameters ($\ell = 2, m = 12, q_I = 3, z_d/R = 0.45, I_d/I_p = 1/2$), this ratio is less than 1/10. In general, the connection length of torsatron/helical-heliotron (T/H) systems is shorter than that of tokamaks by one order magnitude. This indicates that particles and heat would reach the wall faster in T/H systems than in tokamaks with similar aspect ratio.

The dependence on the toroidal field is also different. In T/H systems, when the toroidal field increases, the boundary state slightly gets worse because the connection length is proportional to $B_\zeta^{-1/4}$. On the other hand, in divertor tokamaks, the plasma boundary is improved with the increase of the toroidal field. We note that the dependence of the field property on the toroidal field is opposite.

5 Summary and Discussions

Field structure of the SOL regions in both toroidal helical systems and divertor tokamaks was analyzed by calculating the connection length of the field line to the wall. The model vacuum magnetic fields are used. It was confirmed that the connection length has the logarithmic dependence in the SOL region of toroidal systems with the separatrix. We compare the connection length of T/H systems with that of divertor tokamaks. The former is shorter than the latter by one order magnitude.

The role of residual islands in helical systems was discussed. Although the logarithmic dependence is common for all values of (θ, ζ), the difference appears on the constant term. The connection length on the side where the O-point does not exist is shorter than the other side. This implies that heat preferentially follows on the side where the O-point does not exist.

In the SOL region, λ_1 has a characteristic value, by which the ergodic property can be estimated. The dependence of λ_1 on the pitch number and the axisymmetric fields was numerically calculated. The m-dependence of λ_1 is weak ; i.e., the asymptotic form of the connection length ($\delta \to 0$) is approximately given as $L \sim \ell R/(2m)\ln(a/\delta)$. The value of λ_1 increases according to the formula, $\lambda_1 \propto B_\zeta^{1/4}$. Increasing B_ζ results in an incremental increase in the inverse aspect ratio, but also brings about an undesirable reduction in the connection length. The plasma volume is also affected by B_v, but λ_1 is nearly independent of B_v. Adjustment in the vertical field can increase the volume without a reduction in the connection length. Therefore, optimization of the plasma volume by B_v is better than optimization by B_ζ in terms of the SOL plasma confinement.

From numerical results, it was found that $\iota = m/(\ell + 1)$ rational surface is hardly shifted by the vertical field and the rotational transform can not be beyond $\iota = m/(\ell + 1)$. If we need more improvement of the SOL region, the other resonant fields, which makes the width of $\iota = m/(\ell + 1)$ island small, must be added to systems. Todoroki(1989) has pointed that the component of the magnetic field in $\ell = 2$ systems, which destroys the magnetic surface near periphery, is persistent under the shift of the magnetic surfaces. The behavior of the $(\ell + 1, m)$ island may be related to this residual magnetic field. This comparison requires further research.

The value of λ_1 would be modified in real devices, because the single harmonic model field is too simple to describe the fields generated by winding currents. Although higher harmonics should be taken into account in order to apply the result to real experiments, the result remains a good first step in the SOL plasma analysis. The logarithmic dependence has been verified in a real coil system, Heliotron-E (Mizuuchi *et al.*,1984) and λ_1 is nearly 1.1. This value differs from that of the simple model field by less than a factor of two. Since the reduction of λ_1 makes the connection length longer, it will improve the plasma confinement. Analysis including higher harmonics is left for future study.

Since this dependence of L on δ is similar to that of tokamaks, analysis of the SOL plasma in tokamaks using the fluid model is applicable. Edge temperature and width of the heat channel have been estimated (Itoh *et al.*, 1989). The edge temperature scales as $P^{0.364}$ and the half width of the temperature profile scales as $P^{-0.273}$, where P is the total heat flux out of the plasma surface. These dependencies are the same as in tokamaks. It is noted, however, that the edge temperature is lower than in tokamaks because the connection length is shorter by one order of magnitude, resulting in a set of conditions that make it difficult to operate a divertor function.

We finally note that the logarithmic dependence and the coefficient C in Eqn.(3.6) in divertor tokamaks are not affected by the resonant perturbations which destroy the separatrix. This would be the reason that the divertor functioning is observed in many devices which may have error fields and edge magnetic turbulences.

6 Acknowledgment

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7 Appendix

The scalar potential which describes straight helical systems is written as (Morozov and Soloveev, 1966)

$$\Phi = \sum_{\ell,m} \alpha^{s}_{\ell m} I_\ell (m\frac{\rho}{R}) \cos(\ell\theta + m\frac{z}{R}) + B_z z \tag{A.1}$$

where I_ℓ is modified Bessel function of the first kind, and B_z is a uniform axisymmetric field. Like toroidal helical systems, the field is calculated by using a single harmonic and the connection length can be estimated. The separatrix exists in straight systems and the connection length has a finite value away from the separatrix. Then the connection length can be written as

$$\frac{L}{R} \sim \frac{\ell}{m}\frac{1}{\lambda^{s}_{1}} \ln\frac{R}{\delta} \tag{A.2}$$

where δ is the distance in the radial direction from the X-point of the separatrix. Fig.A1 shows the numerical result of B_z dependence of λ^{s}_{1}. The value of λ^{s}_{1} is approximated by $\lambda^{s}_{1} \propto (a/R)^{1/2}$. This dependence is the same as in toroidal systems. When parameters are those in Heliotron-E ($\iota(0) = 0.5, m = 19$), we obtain $\lambda^{s}_{1} \sim 2.2$. This value is close to that of toroidal systems. As a result, helical systems have a logarithmic property regardless of the existence of the separatrix and have the common dependence on the axisymmetric field.

8 References

Benettin G. and Galgani L., (1979) in Intrinsic Stochasticity in Plasmas, G. Laval and D. Gresillon Editors, Edition de Physique, 93

Cary J. R. and Hanson J. D., (1986) Phys. Fluids **29**, 2464

Filonenko N. N., (1967) Nucl. Fusion **7**, 253

Gourdon C., Marty D., Maschke E. K. and Dumont J. P., (1968) Proc. of IAEA Conf. on *Plasma Physics and Controlled Fusion Research*, 847

Hamzeh F. M., (1972) Phys. Rev. Lett. **29**, 1492

Itoh K., Nagasaki K., Itoh S. I. and Fukuyama A., (1989) Nucl. Fusion **29**, 1299

Martin T. J. and Taylor J. B., (1984) Plasma Phys. Cont. Fusion **26**, 321

Matsuda S. and Yoshikawa M., (1975) Jap. J. Applied Phys., **14**, 87

McCool S. C., et al., (1989) Nucl. Fusion **29**, 547

Mizuuchi T., Motojima O., Besshou S., Iiyoshi A. and Uo K., (1984) J. Nucl. Mat. **121**, 3

Morozov A. I. and Soloveev L. S., (1966) Review of Plasma Phys., (ed. Leontvich M. A., Consultants Bureau, New York), Vol.2, p1

Morse P. M. and Feshbach H., (1953) *Methods of Theoretical Physics*, McGraw-Hill, New York

Neuhauser J., et al, (1989) Plasma Phys. Cont. Fusion **31** 1551.

Nagasaki K., Itoh K., Wakatani M. and Iiyoshi A., (1988) J. Phys. Soc. Japan **57** 2000

Ohyabu N. and DeGrassie J. S., (1987) Nucl. Fusion **27** 2171

Rechester A. B., Rosenbluth M. N. and White R. B., (1979) in Intrinsic Stochasticity in Plasmas, G. Laval and D. Gresillon Editors, Edition de Physique, 239

Rosenbluth M. N., Sagdeev R. Z., Taylor J. B. and Zaslavski G. M., (1966) Nucl. Fusion **6**, 297

Samain A., et al., (1984) J. Nucl. Material **128 & 129**, 395

Shen Y., Miyake M., Takamura S., Kuroda T. and Okuda T., (1989) J. Nucl. Material **168**, 295

Shoji A. *et al.*, (1989) presented in Meeting of Phys. Soc. Jpn. (Kagoshima) paper 5aJ13

Söldner F. X. and ASDEX Team, (1988) Phys. Rev. Lett. **61**, 1105

Soloveev L. S. and Shafranov V. D., (1970) Review of Plasma Phys., (ed. Leontvich M. A., Consultants Bureau, New York), Vol.5, p1

Todoroki J., (1989) "Extension of Stellarator Approximation in Magnetohydrodynamic Equilibrium and Stability of Toroidal Helical Systems", IPPJ-905 (Nagoya Univ.)

Tsuji S. and JT-60 Team, (1988) presented in 12th International Conference on *Plasma Physics and Controlled Nuclear Fusion Research*

Wagner F. and ASDEX Team, (1982) Phys. Rev. Lett. **49**, 1408

Wagner F. and Lackner K., (1984) in *Physics of the Plasma Wall Interactions in Controlled Fusion* (ed. Post D.E. and Behrisch H., NATO ASI Series B131, Plenum Press) 931

Wobig H. and Fowler R. H., (1988) Plasma Phys. Cont. Fusion **30**, 721

Yoshikawa S., (1983) "Analytic Representation of Three-Dimensional Stellarator Field", Research Report PPPL-2038

9 Figure Captions

Fig. 1 Poloidal cross-sections of magnetic surfaces at toroidal angle (a) $\zeta = 0$, (b) $\zeta = \pi/2m$. Parameters are $\ell = 2, m = 19, B_\zeta R/\alpha_{\ell m} = 0.65$ and $B_v = 0.0$. Characteristic quantities of the magnetic surface are $\iota = 0.5, \iota = 2.44$, $R = 0.07$.

Fig. 2 Connection length $L_\pm$ as a function of ρ for the case of Fig.1(b)($\theta = 0, \zeta = \pi/2m$). (a) a whole figure (b) a magnified figure near $\iota = 19/7$.

Fig. 3 Logarithmic dependence of connection length on δ in the case of Fig.1(b). Symbols $\circ, \Delta$, $\square$ and $\bullet$ denotes L_+ at the points $\iota = 19/7, 19/5, 19/3$ and the OMS on the inside of the torus, respectively.

Fig. 4 Dependence of $L_\pm$ on both θ and ζ at $\delta/R \sim 10^{-3}$ near the OMS. Symbols $\circ$ and Δ denote for L_+ and L_-, respectively. (a) θ dependence at $\zeta = 0$, (b) ζ dependence at $\theta = 0$.

Fig. 5 Dependence of λ_1 on m. Parameters are $\ell = 2$, $B_\zeta R/\alpha_{\ell m} = 1.0$ and $B_v = 0.0$.

Fig. 6 Positions of the OMS and the O-points of magnetic islands as a function of the toroidal field. Symbols $\circ, \Delta$, $\square$ and $\bullet$ denote for the O-points of $\iota = 19/3, 19/5, 19/7$ and the OMS, respectively.

Fig. 7 a/R dependence of λ_1. Parameters are $\ell = 2, m = 19, B_v = 0$.

Fig. 8 Shift of the OMS and the O-points by the vertical field at $m = 19, B_\zeta R/\alpha_{\ell m} = 0.65$. Symbols are the same as in Fig.6.

Fig. 9 Edge rotational transform $\iota(a)$ as a function of the vertical field. $\iota(a)$ is optimized at $B_v R/\alpha_{\ell m} = 0.0027$.

Fig. 10 Magnetic surface of double null divertor tokamaks. Parameters are $q_I = 3, z_d/R = 0.45$ and $I_d/I_p = 1/2$.

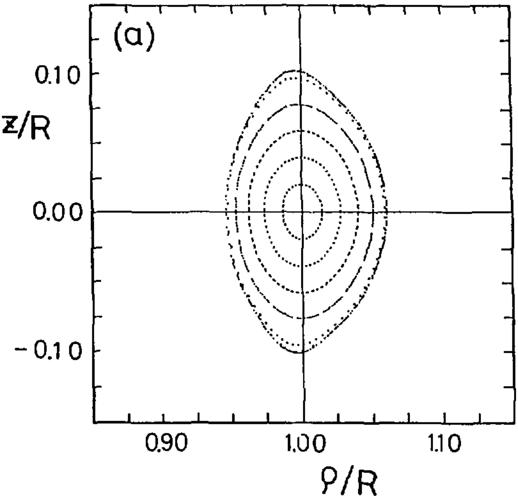
Fig. 11 Asymptotic behavior of the connection length on δ in the case of Fig. 10.

Fig. 12 Dependence of the coefficient C on geometrical parameters. (a)q_I dependence, (b)z_d dependence and (c)I_d/I_p dependence.

Fig. 13 Radial profiles of (a)the connection length and (b)the safety factor. The resonant perturbation is $m/n = 5/1, \tilde{b} = 0.1$.

Fig. 14 Logarithmic property of the connection length in the ergodic region. Symbols o and • correspond to L at the points A and B in Fig. 13, respectively. Small dips near $\delta/R \sim 10^{-5}$ and 10^{-4} correspond to the integer q-numbers.

Fig.A1 a/R dependence of λ_1^s in straight helical systems. Parameters are $\ell = 2, m = 19$.



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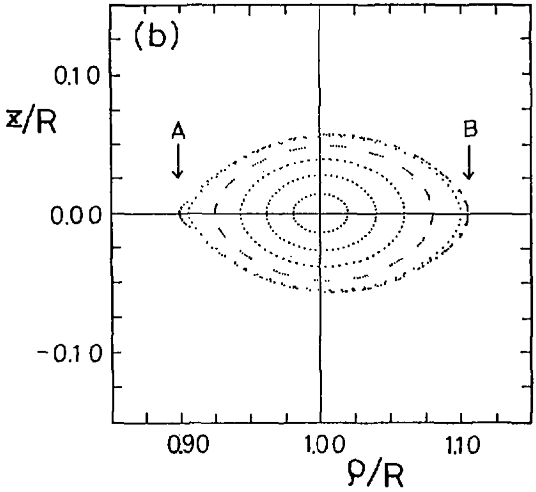

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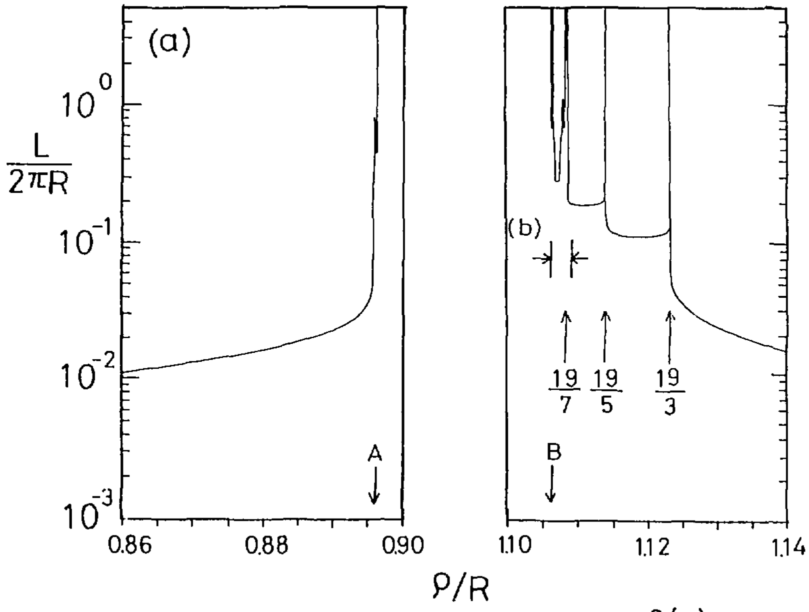

(a)
10^0
10^{-1}
10^{-2}
10^{-3}
$\frac{L}{2\pi R}$
(b)
19/7
19/5
19/3
A
B
0.86
0.88
0.90
1.10
1.12
1.14
ρ/R

Fig. 2(a)

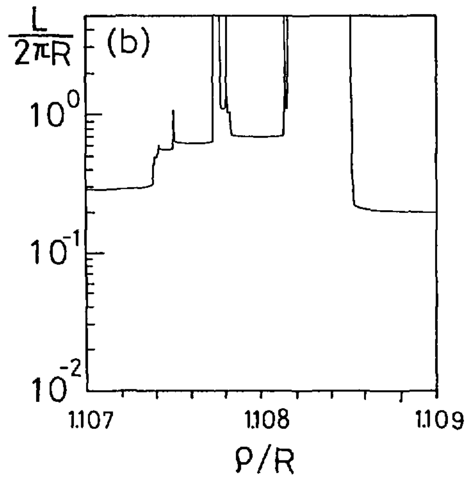
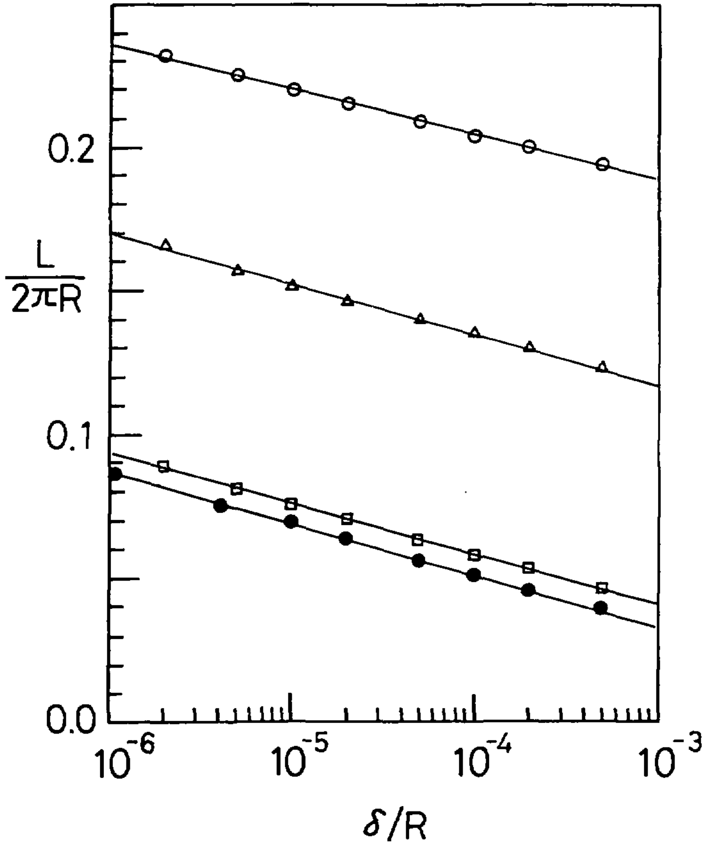

$\frac{L}{2\pi R}$
(b)
10^0
10^{-1}
10^{-2}
1.107
1.108
1.109
ρ/R

Fig. 2(b)



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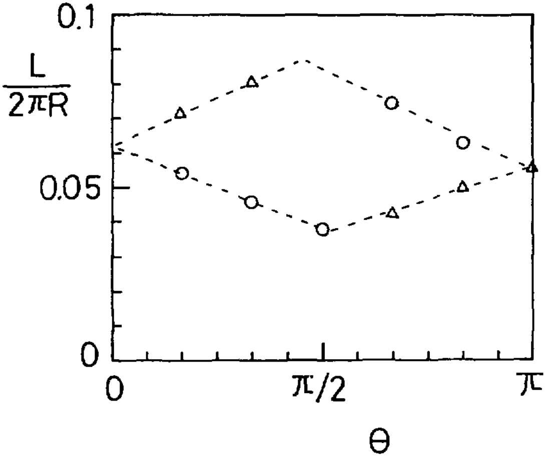

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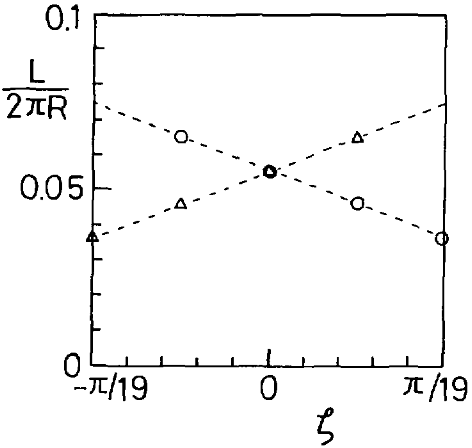

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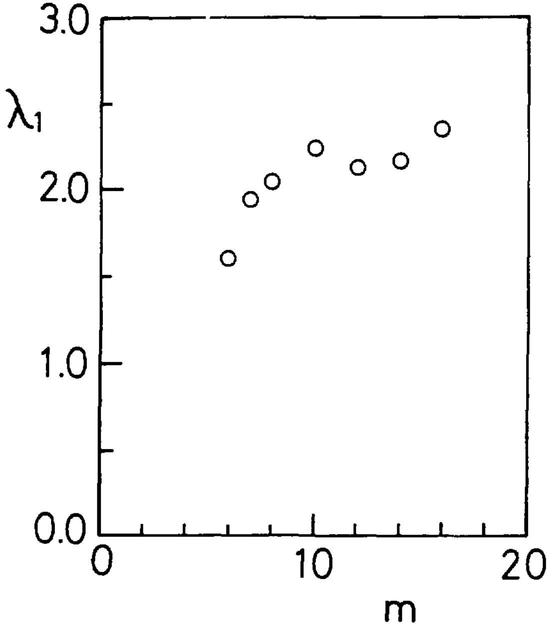

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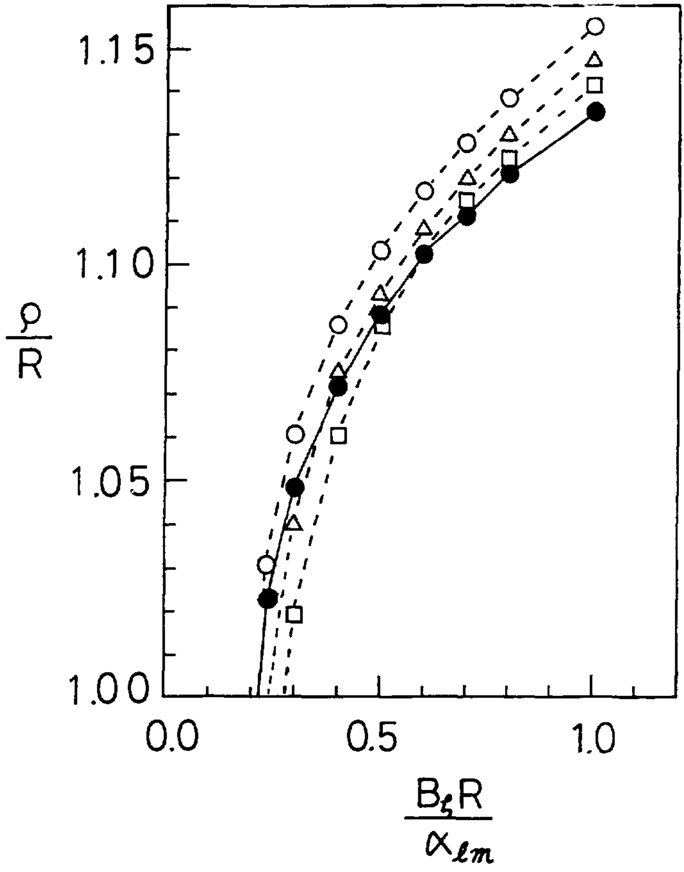

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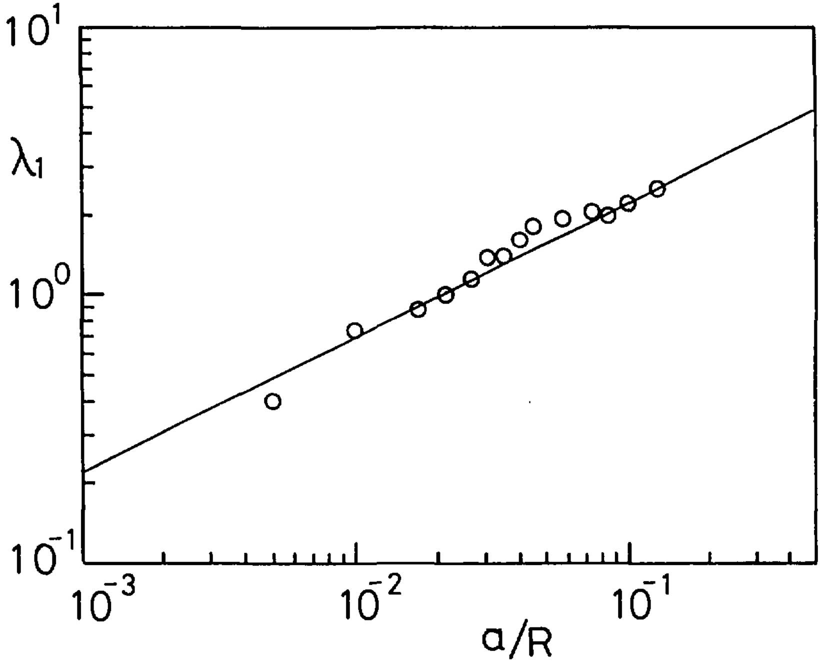
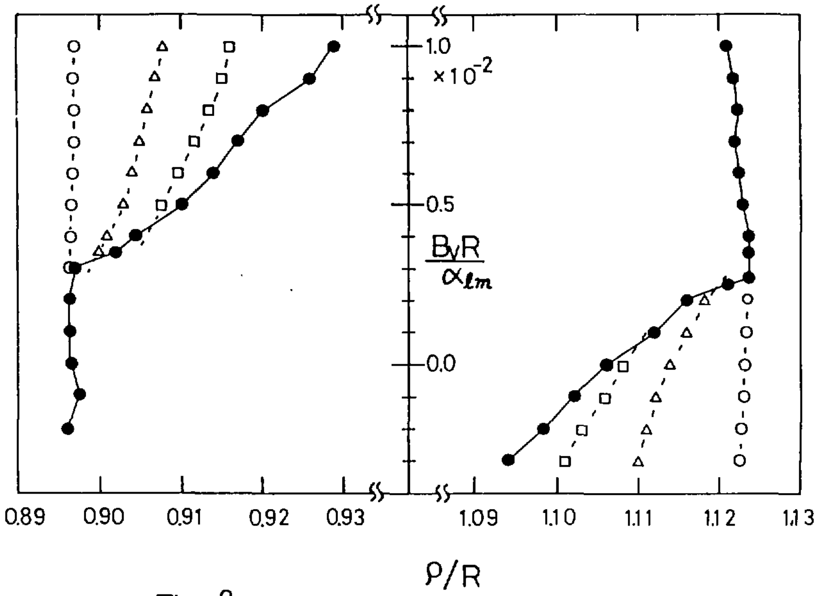


Fig. 7



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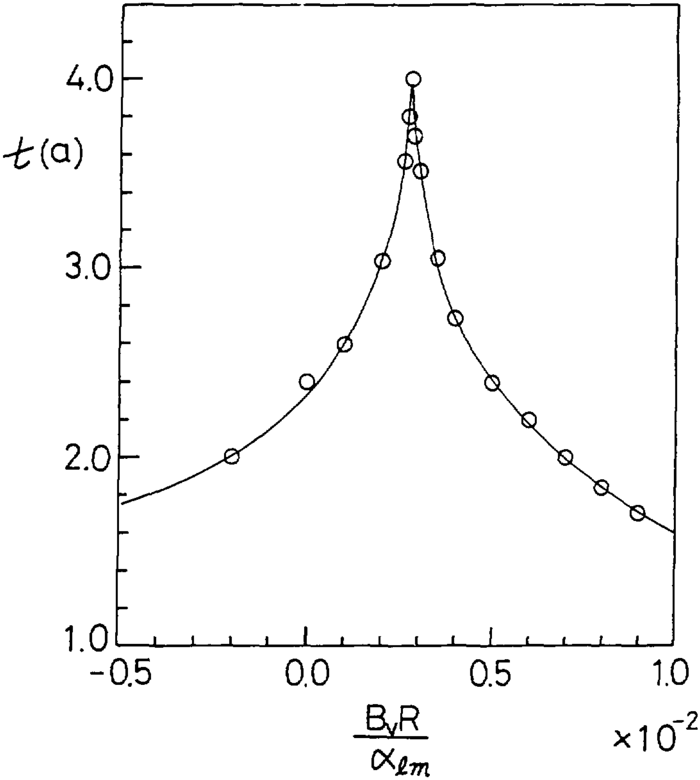

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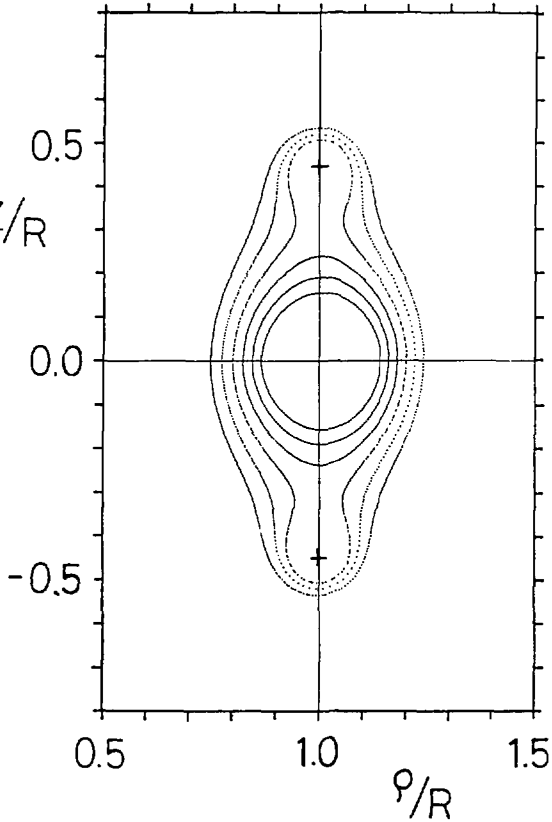
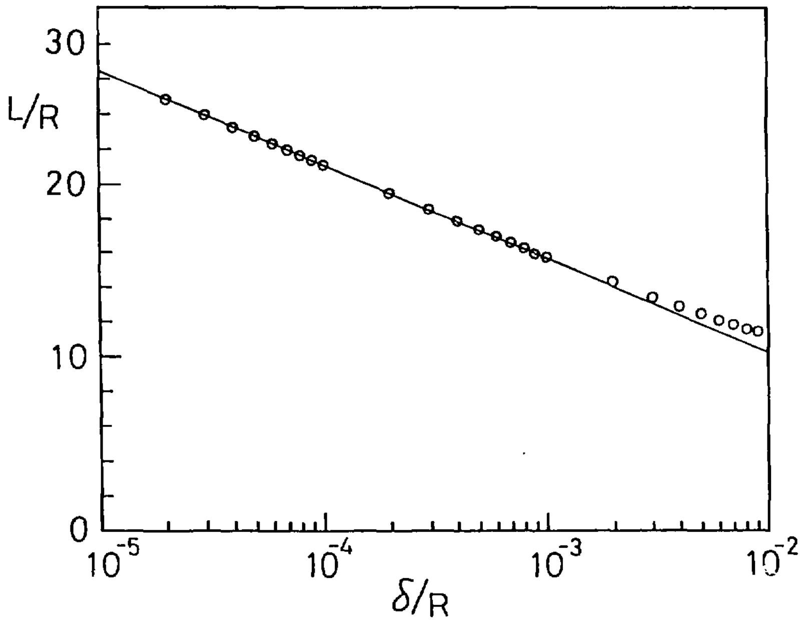


Fig.10



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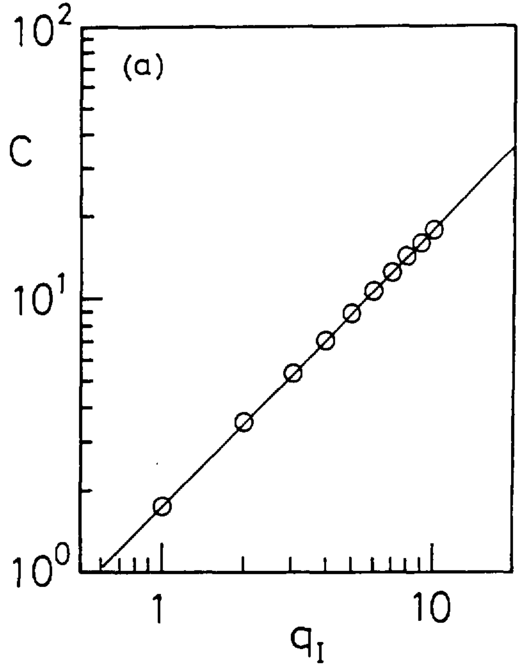

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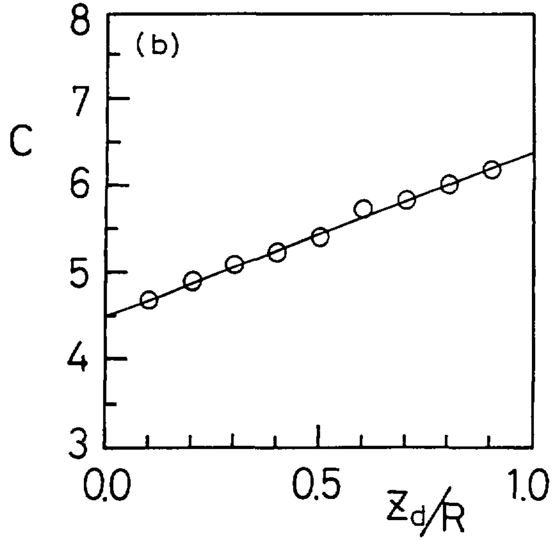

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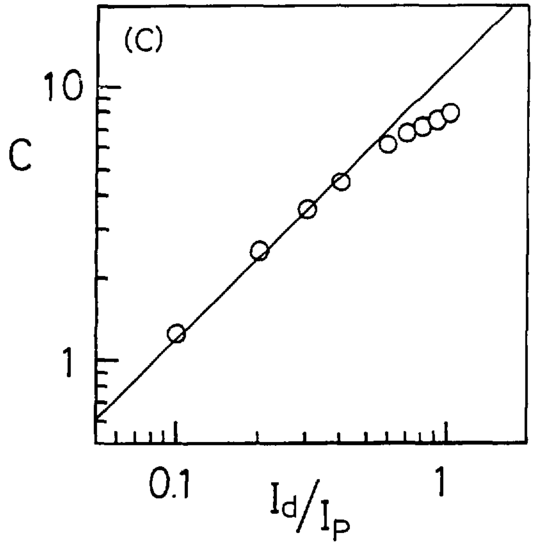

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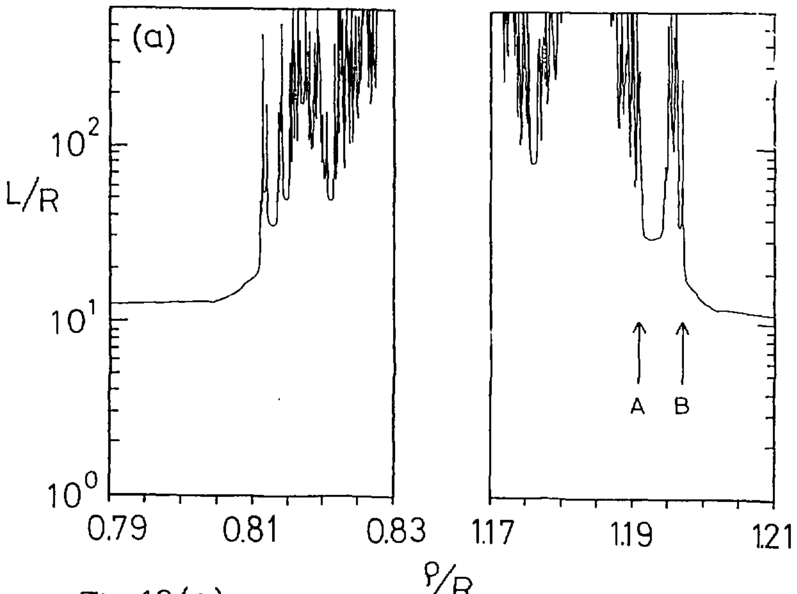

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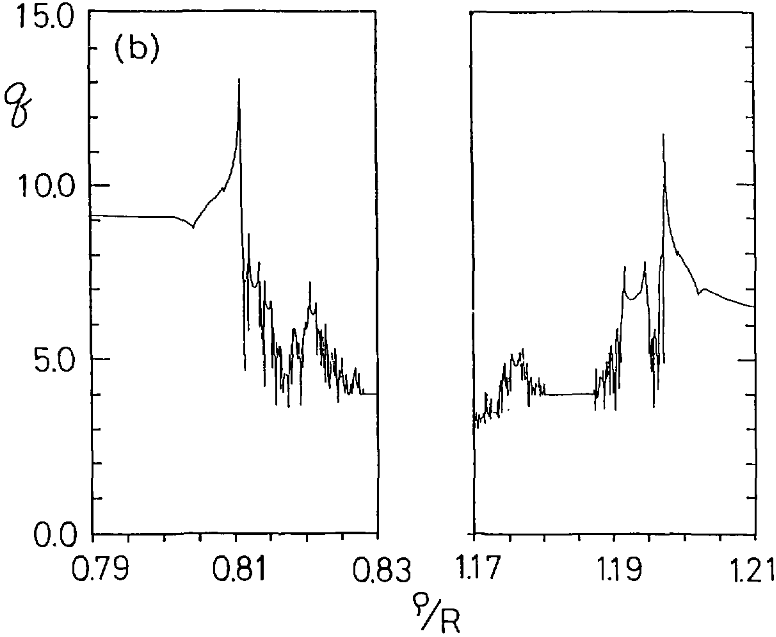

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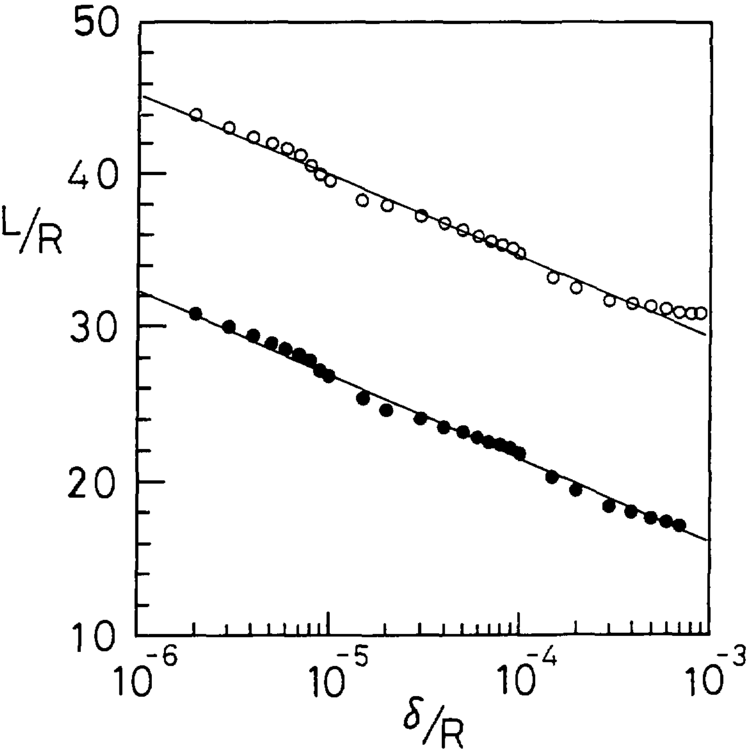

Fig. 14

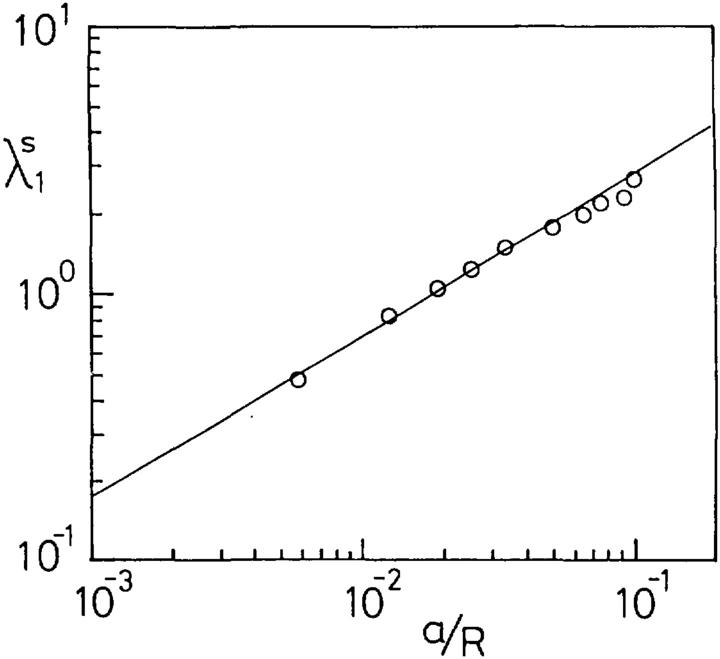

Fig. A1