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INTERMITTENCY IN MULTIPARTICLE PRODUCTION ANALYZED BY MEANS OF STOCHASTIC THEORIES*

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Abstract

Intermittency in multiparticle production is described by means of probability distributions derived from pure birth stochastic equations. The UA1, TASSO, NA22 and cosmic ray data are analyzed.

In the present contribution we report some new results of our studies of intermittency in multiparticle production at high energies by means of stochastic theories[1]. We investigate under which conditions a behaviour like[2]

$$\log\langle F_i\rangle \overset{\delta y\to 0}{\longrightarrow} f_i\cdot\ln\frac{1}{\delta y} \qquad (1)$$

can be obtained for the normalized factorial moments

$$\langle F_i\rangle = \frac{\langle n(n-1)\ldots(n-i+1)\rangle}{\langle n\rangle^i} \qquad (2)$$

in certain stochastic distributions. δy is the rapidity interval considered and f_i is the slope parameter. (For a recent review of intermittency in multiparticle production, see e.g. Ref. 3.) We thereby rely on the quantum statistical approach in describing hadronic multiplicity distributions[4-6]. It is well known that the negative binomial distribution (NB) and the pure birth distribution (PB) give good descriptions of the multiplicities in different y windows[7-10]. Furthermore, the solutions of the underlying stochastic differential equations show the property of self-similarity, the essential ingredient leading to intermittent behaviour[2,11].

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We start with the PB stochastic differential equation[1,9,10,12]

$$\partial P(n,t)/\partial t = \lambda(n-1)P(n-1;t) - \lambda n P(n;t) \tag{3}$$

with the initial condition at $t = 0$

$$P(n;t=0) = \langle m\rangle^n e^{-\langle m\rangle}/n!. \tag{4}$$

In this case the normalized factorial moments of the PB distribution read

$$\langle F_i\rangle = \Gamma(i)\xi^{1-i}L_{i-1}^{(1)}(-\xi) \tag{5}$$

where $L_n^{(1)}(x)$ is the associated Laguerre polynomial and

$$\xi(t) = \langle m\rangle(p+1)/p, \qquad p = e^{\lambda t} - 1. \tag{6}$$

This means that the $\langle F_i\rangle$ can be expressed completely in terms of the single variable ξ. For $i = 2$ we obtain

$$\langle F_2\rangle = \langle n(n-1)\rangle/\langle n\rangle^2 = 1 + 2/\xi. \tag{7}$$

Choosing as initial condition

$$P(n;t=0) = \delta_{nk} \tag{8}$$

one obtains the Furry distribution[9] as solution of eq. (3). In this case the normalized factorial moments are

$$\langle F_i\rangle = k\sum_{j=0}^{i}\begin{pmatrix} i \\ j \end{pmatrix}\frac{(k+i-j-1)!}{(k-j)!}p^{i-j}/\langle n\rangle^i. \tag{9}$$

The Furry distribution was also obtained in Ref. 12 for a certain class of branching models.

We shall also use the NB distribution[7–10] in our analysis of the δy dependence of the moments. It follows from a more complicated differential equation than eq. (3) which also contains migration terms. The normalized factorial moments are

$$\langle F_2\rangle = 1 + 1/k \tag{10}$$

$$\langle F_{i+1}\rangle = \langle F_i\rangle\cdot(1+i/k). \tag{11}$$

Here the $\langle F_i\rangle$ can also be expressed in terms of a single variable, k.

In order to compare the normalized factorial moments of the PB distribution, eq. (5), and of the NB distribution, eq. (11), with the data, we make the ansatz $\xi^{-1} = \alpha + \beta\log(1/\delta y)$, and $k^{-1} = \alpha' + \beta'\ln(1/\delta y)$, and determine the parameters α, β, α', β' using the data for $\langle F_2\rangle$ as input. In the following we present some of our numerical results of the comparison with the experimental data of Refs. 13–16. Further results are also in Ref. 1, and will be given elsewhere.

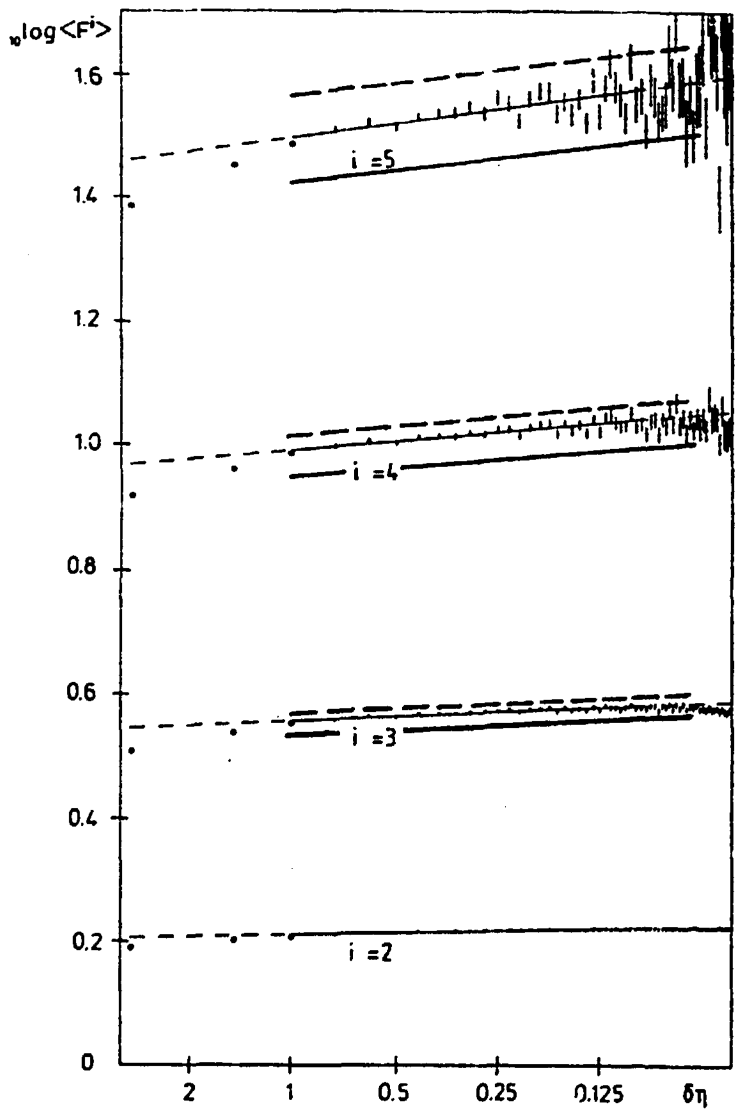

Fig. 1: Comparison of the UA1 $\bar{p}p$ data at $\sqrt{s} = 630$ GeV for $\log\langle F_i\rangle$ vs. $\log(1/\delta\eta)$ with the predictions of the PB distribution (full line) and the NB distribution (dashed line), eqs. (5) and (11), taking $\log\langle F_2\rangle$ as input

The UA1 data[13] for $\log\langle F_i\rangle$, $i = 2-5$, at $\sqrt{s} = 630$ GeV have very good statistics and show a significant intermittency signal. They are shown in Fig. 1 for $\log\langle F_i\rangle$ vs. $\delta\eta$, together with our predictions for the PB distribution and the NB distribution. Both distributions reproduce the slopes very well, whereas the intercepts of the higher $\langle F_i\rangle$ show some deviation from the data. The numerical results for the slopes f_i are given in Table 1.

In Table 1 we also compare the e^+e^- data of TASSO[14] and the cosmic ray data of Ref. 15, for the slopes f_i, with the predictions of the PB distribution and the NB distribution. For determining the slopes f_i of the TASSO data we used the rapidity intervals between 0.7 and 0.12, as was also done in Ref. 14. Furthermore, the normalization of the factorial moments was taken as $1/\langle n\rangle^i$, like in eq. (2), with $\langle n\rangle = 8.6$, instead of the normalization used in Ref. 14. Quite generally, multiplicity data in e^+e^- annihilation and in cosmic rays have the highest values for the slopes f_i[17]. As can be seen in Table 1, the experimental values of Refs. 14 and 15 are well reproduced by the PB and NB distributions.

Table 1 also shows that in the case of the NA22 data[16] neither the PB nor the NB distribution can reproduce the slope f_i in a satisfactory manner. A possible explanation could be that at lower energies such as in Ref. 15 a stochastic equation more complicated than eq. (3) has to be used[18].

Apart from the NA22 data, in most of the examples studied we found that the slopes f_i are better reproduced by the PB and NB distributions than the intercepts of $\log\langle F_i\rangle$. The capability of the NB distribution to reproduce intermittency has also been studied in Refs. 19–21. In the case of the Furry distribution, eq. (9), the normalized factorial moments do not depend on a single variable, but on $\langle n\rangle$ and the variable p (see eq. (6)) separately. Consequently, for performing a similar analysis of the $\log\langle F_i\rangle$ with the Furry distribution, additional information on the dependence of $\langle n\rangle$ on δy is necessary.

Finally, we want to comment on the problem of expressing all moments $\langle F_i\rangle$, $i > 2$, via a linked-pair ansatz for the normalized factorial cumulants, by the lowest moment $\langle F_2\rangle$ which itself is related to the two-particle correlation function. This was proposed in Refs. 22 and 23 and investigated in the case of the NB distribution in Ref. 21 (see also Ref. 24). The multiplicative constants a_i, $i = 3, 4, 5, \ldots$ introduced in Ref. 22 and obtained by a fit to the data on $\langle F_i\rangle$, $i = 3, 4, 5$, are completely determined in the PB distribution. They follow from eq. (5) as $a_3 = 1.2$, $a_4 = 1.4$, $a_5 = 1.7, \ldots$, to be compared with the values $a_3 = 1.3$, $a_4 = 1.6$, $a_5 = 2.8$ of Ref. 22 (for the NB distribution they were obtained in Ref. 21 as $a_3 = 1.4$, $a_4 = 1.8$, $a_5 = 2.2, \ldots$). On the other hand, we obtain for the Furry distribution from eq. (9) the normalized factorial cumulants: $k_2 = \rho$, $\rho = (p-1)/\langle n\rangle$, $k_3 = 2\rho^2 + 2\rho/\langle n\rangle + 2/\langle n\rangle^2$ etc. While k_2 is given by the single variable ρ, the higher normalized factorial cumulants k_i, $i > 3$, are determined by two variables, ρ and $\langle n\rangle$. The Furry distribution, therefore, is a counterexample to the linked pair approximation.

Table 1: Comparison of the data of Refs. 13–16 for the slope parameters f_i with our results from the PB and the NB distributions, eqs. (5) and (11)

UA1 $\bar{p}p$ at $\sqrt{s} = 630$ GeV[13]

f_i	Exp	PB	NB
$i = 2$	0.011 ± 0.001	input	input
3	0.025 ± 0.003	0.026	0.027
4	0.050 ± 0.005	0.042	0.047
5	0.077 ± 0.011	0.060	0.068

TASSO e^+e^- at $\langle\sqrt{s}\rangle = 35$ GeV[14]

f_i	Exp	PB	NB
$i = 2$	0.023 ± 0.003	input	input
3	0.080 ± 0.014	0.067	0.068
4	0.134 ± 0.052	0.124	0.130

Cosmic rays, hadrons[15]

f_i	Exp	PB	NB
2	0.22 ± 0.08	input	input
3	0.61 ± 0.16	0.49	0.51
4	1.04 ± 0.14	0.71	0.80
5	1.60 ± 0.24	1.08	1.20
6	2.03 ± 0.22	1.42	1.59

NA22 $(n_{ch} < 7 - 8)$ at $\sqrt{s} = 22$ GeV[16]

f_i	Exp	PB	NB
2	0.0127 ± 0.0008	input	input
3	0.0499 ± 0.0022	0.0312	0.0330
4	0.148 ± 0.007	0.0534	0.0583
5	0.328 ± 0.019	0.0783	0.0872

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