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FOKKER-PLANCK TRANSPORT IN SOLID STATE ACCELERATOR CONCEPTS

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ABSTRACT

Particle transport in a crystalline solid under channeling conditions is considered by means of a Fokker-Planck description. The model includes electron multiple scattering, radiation damping and an accelerating electric field. Analytic solutions have been obtained using a harmonic potential model to describe the channeling forces. These solutions will be described.

INTRODUCTION

The phenomenon of the channeling[1] of energetic charged particles along axes and planes of high symmetry in crystalline solids, has prompted several investigators[2–7] to suggest schemes for accelerating particles in which the lattice fields would play the part of the magnetic transport system in conventional accelerators. Different schemes would use different methods for obtaining the accelerating gradient. Recent work has suggested an optical field in the x-ray region,[2] akin to the microwave fields driving conventional accelerators, while another[3] would exploit the fields of collective modes driven by an external source similar to plasma based accelerators operating at lower density. The accelerating gradients can be considerable with 1 GeV/em suggested in the first case and fields as large as 100 GeV/cm in the second. Most of these researchers recognized that the multiple scattering would be considerable even though it is reduced by the channeling for positive particles. Some have estimated this, accounting for the (+) effect of the accelerating field on the emittance.[3,5] In this work, we have examined this problem by means of a Fokker-Planck transport model. Analytic solutions have been obtained for a harmonic channel potential. We will first define the model and then discuss one solution we have found.

FOKKER-PLANCK MODEL

We have adopted a one-dimensional Fokker-Planck treatment for the distribution function of the particles in the crystal accelerator. In the case of axial channeling, this describes the behavior of the projected distribution function in a radial plane, all of which are equivalent in an axisymmetric channel. While this is not necessarily so in general, it should be reasonable in the case of proper channeling. In any case, the qualitative features of the results might well be expected to persist even in more general situations. For planar channeling, the

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problem is one-dimensional a *priori*. Furthermore, we make a paraxial approximation $p_\perp/p_z \ll 1$, $p_z \simeq p \gg 1$ and retain the pitch angle only to lowest order: $\theta \simeq p_\perp/p$. Under these assumptions, the Fokker-Planck equation becomes

$$\begin{aligned} 0 &= \frac{1}{pc}\frac{\partial f}{\partial t} + \theta\frac{\partial f}{\partial x} + \frac{\partial f}{\partial z} + \tilde{E}\frac{\partial f}{\partial p} - \frac{\tilde{u}_x}{p}\frac{\partial f}{\partial \theta} - \beta\frac{\partial f}{\partial p} \\ &\quad - \beta_R\frac{\partial(\theta f)}{\partial \theta} - \frac{1}{2}\frac{\partial}{\partial \theta}\left(\frac{\langle \Delta\theta^2 \rangle}{\Delta t}\frac{\partial f}{\partial \theta}\right) . \end{aligned} \tag{1}$$

Momenta are expressed in units of the rest momentum of the projectile $m_I c$ and energy in units of the rest energy.

The first five terms arise from the convective derivative in phase space; the fourth and fifth terms due to the accelerating field (taken to be in the z-direction) and the channeling potential respectively. The next term accounts for energy losses such as bremsstrahlung or collisional ionization. The remaining terms are due to radiation damping arising from the emission of channeling radiation and multiple scattering.[8] Under channeling conditions for positive particles, the electrons in the medium dominate the multiple scattering. For our purposes, the situation in which the acceleration gradient exceeds the losses is of interest. A boundary value problem is appropriate and therefore we look for steady-state solutions, $\partial f/\partial t = 0$.

Analytic solutions can be readily obtained when $\tilde{u}$ is harmonic. More general cases could be treated by perturbation theory (weak anharmonicity) or numerical solutions based on the method of solution to now be briefly described. The procedure is to introduce the characteristics of the first order operator in Eq. (1). (The term $-\beta_R f$ can be handled trivially by introducing $\chi = e^{-\beta_R z} f$. This has no effect on the statistical averages.) The general solution of this coupled set of ODE's contains a set of integration constants. These, through the solution of the ODE's define a transformation of variables from $(z, \theta, x, p) \to (s, \eta, \zeta, \xi)$. Applying this to Eq. (1) casts it in a form to which a lemma of Chandrasekhar[9] applies. The Green's function solution of Eq. (1) can then be written down immediately. For a given distribution of particles at $z = 0$, the distribution at interior points $z > 0$ is given by a convolution. Moments over the distribution as functions of z can be obtained in a straightforward way.

STRONG ACCELERATION RADIATION-FREE SOLUTION

The case of strong acceleration (acceleration > losses) is of interest. For heavy particles ($m \geq m_p$), the radiation is negligible (Ultimately it would be very nice to get into the radiative regime for them, too.) The solution of the characteristic equations can be expressed in terms of Bessel functions. The calculation of the moments can be done analytically. First we observe that

$$\tilde{u} = \frac{kx^2}{2} = \frac{v_0 x^2}{2m_I c^2} \tag{2}$$

where v_0 is the channel well curvature. It is typically $\sim$ few $\times 10^{16}\,\text{ev/cm}^2$. Even for an accelerating gradient of 100 GeV/cm, $k/\alpha \gg 1$. The appropriate asymptotic expansions of the Bessel functions can be used. We find

$$\langle x^2 \rangle \simeq \frac{D}{\alpha k}\left[\left(\frac{\alpha z + p_0}{p_0}\right)^{1/2} - 1\right] \tag{3}$$

$$\langle \theta^2 \rangle \simeq \frac{D}{\alpha(\alpha z + p_0)}\left[\left(\frac{\alpha z + p_0}{p_0}\right)^{1/2} - 1\right] \tag{4}$$

$$\langle x\theta \rangle = 0 ,$$

where p_0 is the initial energy and $D \equiv \frac{p^2}{2}\left\langle \frac{\Delta\theta^2}{\Delta z}\right\rangle$.
Note that in the absence of acceleration, $\alpha = 0$, the result just reduces to a random walk. The (unnormalized rms emittance, ϵ, given by[10]

$$\epsilon = 4\left[\langle x^2\rangle\langle\theta^2\rangle - \langle x\theta\rangle^2\right]^{1/2} \quad \text{is then} \tag{5}$$

$$\epsilon \simeq \frac{4D}{\alpha\sqrt{k}}\frac{1}{(\alpha z + p_0)^{1/2}}\left[\left(\frac{\alpha z + p_0}{p_0}\right)^{1/2} - 1\right] . \tag{6}$$

In the limit $\alpha z \gg p_0$, this becomes $\epsilon \simeq 4D/\alpha\sqrt{kp_0}$, constant. This is consistent with the estimates in Ref. 3. Using the expression in Ref. 8 for the multiple scattering coefficient, we have

$$D = 2\pi\left(\frac{1}{137}\right)^4 a_0^2\, N\, Z_{\text{val}}\left(\frac{m_e}{m_I}\right)^2 L_R \tag{7}$$

where $L_R \simeq 10$, $a_0 =$ Bohr radius $= .53 \times 10^{-8}$cm, $N =$ electron number density $\approx 10^{22}/\text{cm}^3$ and $Z_{\text{val}} =$ no. of valence electrons/atom. For positrons, $D \simeq 50/\text{cm}$. Note that logarithmic energy dependencies in fundamental processes have been neglected throughout.

Recall that the solutions given here are for the Green's function solution. If a different initial distribution is specified, the moments are modified accordingly.

As a numerical example consider protons at an initial energy of 1 TeV, $p_0 = 1.1 \times 10^3$. With a channel potential of about 50 eV, which is rather typical.[1] $v_0 \simeq 2 \times 10^{16}\,\text{eV/cm}^2$ for typical channel radii of 4×10^{-8} cm. This gives the following parameters:

An accelerating gradient of 100 GeV/cm

$$\Rightarrow \alpha = 1.1 \times 10^2/\text{cm}$$

$$k = v_0/m_I c^2 \simeq 2.2 \times 10^7/\text{cm}^2$$

$$E_f = 100\,\text{TeV} \Rightarrow z = 10^3\,\text{cm}$$

Then Eq. (6) yields a final emittance value of

$$\epsilon \simeq 5 \times 10^{-12} \,\text{rad-cm} \,.$$

This is larger than the channel acceptance which would be obtained from the expressions in Ref. 3. Of course, when the emittance is comparable to the channel acceptance, the simple form adopted for the channel well is suspect, although it would seem that estimates in this case would be too optimistic. Nevertheless, the qualitative behavior is consistent with other work. It is clear that means to reduce the multiple scattering could be of some benefit to improve the error margin. Some possibilities for this will be discussed elsewhere.[10]

SUMMARY AND CONCLUSIONS

We have considered the transport of charged particles under channeling conditions in crystal accelerator schemes using a Fokker-Planck model. Analytic solutions have been obtained. These can be used to find the statistical properties of the particle beam. Solutions, given any specified distribution at the boundary, can be obtained by convolution. As an example, we have considered the emittance growth of a δ-function beam of protons as it is accelerated under channeling conditions. Extensions of the work to include the effect of radiation damping which is important for light particles (positrons and muons) and to channeling in bent crystals is in progress. The latter is important in the application to beam steering in accelerators.

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