IC/90/261
INTERNAL REPORT
(Limited Distribution)

International Atomic Energy Agency
and
United Nations Educational Scientific and Cultural Organization
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

TRANSIT TIME FOR RESONANT TUNNELING *

Gastón García-Calderón** and Alberto Rubio***
International Centre for Theoretical Physics, Trieste, Italy.

ABSTRACT

This work considers properties of the partial widths in one dimensional elastic resonant tunneling in order to propose a transit time $\tau_{tr} = \hbar/(\Gamma_n T^{res})$ where Γ_n is the elastic width and T^{res} the transmission coefficient at resonance energy. This time is interpreted as an average over the resonance energy width. It is shown that the tunneling current density integrated across a sharp resonance is inversely proportional to τ_{tr}. This transit time may be much larger than the values predicted by other definitions.

MIRAMARE – TRIESTE
September 1990

* Submitted for publication.

** Permanent address: Instituto de Fisica, Universidad Nacional Autónoma de Mexico, Apartado Postal 20-364, Mexico 01000 D.F., Mexico.

*** Permanent address: Escuela Superior de Ciencias, Universidad Autónoma de Baja California, Apartado Postal 1880, Ensenada, B.C., Mexico.

The last decade has witnessed the rapid development of novel semiconductor structures at nanometric scales which has given rise to a new generation of electronic devices[1]. Common features of many of these devices are their low dimensionality and the importance of quantum effects. In particular in multibarrier heterostructures, amongst others, electron transport takes place via resonant tunneling essentially as a one dimensional process[2,3].

The above developments have motivated the necessity for a more precise description of tunneling effects, particularly the question of the relevant time scales involved. The essential feature of the resonance situation is a complex constructive interference process. The physics is well known[4]. The simplest case corresponds to a situation where an electron of well defined energy E is incident upon a system formed by two consecutive barriers each of a height greater than E and a classically allowed region between them. At nearly all incident energies the electron is almost totally reflected. However it may be that for a small energy range of width Γ_n around an energy ϵ_n, the particle finds itself with a large probability to enter the system to be eventually transmitted or reflected. The case of multibarrier systems is similar to the above situation though the interference process is now more complex.

Regarding resonant tunneling times, one question that has been asked is, How long does it take for an electron to traverse the system?. This question is crucial regarding the ultimate speed of resonant tunneling devices. Away from resonance the subject of tunneling times has become controversial in recent years [5]. On the other hand at resonance energy the times provided by various approaches coincide for the case of full transmission ($T = 1$) and hence are non controversial[6-9]. The most common answer to the above question has been the transmission phase time[6].

However, as shown below, this time is not adequate in general as a transit time.

It is the purpose of this work to exploit the physical properties of the partial widths for elastic resonant tunneling processes[7] to provide an appropriate time scale for the transit of electrons through the structure. This leads to a transit time τ_{tr} for resonant tunneling in multibarrier systems. We show that the tunneling current density arising from a sharp isolated resonance is inversely proportional to τ_{tr}. Since the current involves an integral across the energies within the width Γ of the resonance, it follows that the transit time τ_{tr} refers to an average over such energy interval.

In the description of resonant processes it is well known the conceptual advantage of associating a complex energy, $E_n = \epsilon_n - i\Gamma_n/2$, to each resonance[10,11]. In particular this allows an analytic description of relevant quantities such as the transmission coefficient near resonance energy in a Breit-Wigner fashion[11,12]. However the eigenfunctions $u_n(x)$ corresponding to each of these complex energies possess interesting properties that have not been fully exploited in the context of one dimensional resonant tunneling. The eigenfunctions $u_n(x)$ are defined as the solutions of the Schrödinger equation,

$$\frac{d^2u_n(x)}{dx^2} + \left[k_n^2 - V(x)\right] u_n(x) = 0 \qquad (1)$$

with $k_n = a_n - ib_n$ and $U(x) = 2mV(x)/\hbar^2$. The complex eigenvalue is $E_n = \hbar^2k_n^2/2m = \epsilon_n - i\Gamma_n/2$, where ϵ_n represents the position of the resonance and Γ_n the corresponding width. $V(x)$ stands for an arbitrary potential profile of finite

length, i.e., $V(x) = 0$ for $x < 0$ and $x > L$, capable of sustaining resonances. The functions $u_n(x)$ satisfy outgoing boundary conditions at $x = 0$ and $x = L$, that is, $u_n'(0) = -ik_nu_n(0)$ and $u_n'(L) = ik_nu_n(L)$, where the prime denotes derivative with respect to x. The eigenfunctions $u_n(x)$ may also be seen as the residues at the complex poles $E_n = \hbar^2k_n^2/2m = \epsilon_n - i\Gamma_n/2$ of the full propagator of the system[7,12]. In general one may write the full expression for the resonant eigenfunction as $u_n(x,t) = u_n(x)exp[-i\epsilon_nt/\hbar]exp[-\Gamma_nt/2\hbar]$, which shows explicitly the intrinsic decaying nature of these states. Since we are considering an elastic process, the requirement of probability conservation,

$$d/dt(I_nexp[-\Gamma_nt/\hbar]) = J_n(0,t) - J_n(L,t) \qquad (2)$$

where $I_n = \int_0^L |u_n(x)|^2\,dx$ and $J_n(x,t)$ stands for the probability current, together with the above outgoing boundary conditions yields the total elastic width, $\Gamma_n = \Gamma_n^0 + \Gamma_n^L$, which defines the partial widths,

$$\Gamma_n^0 = \hbar(\hbar a_n/m)|u_n(0)|^2/I_n\,; \qquad \Gamma_n^L = \hbar(\hbar a_n/m)|u_n(L)|^2/I_n \qquad (3)$$

Eq.(3) has been obtained in an alternative form in a previous work [12]. Notice that the partial widths are independent of the normalization condition on the eigenfunctions $u_n(x)$. The quantities $\Gamma_n^0/\hbar$ and $\Gamma_n^L/\hbar$ represent respectively the probability per unit time of electronic decay through the extremities of the system at $x = 0$ and $x = L$. Γ_n^0 and Γ_n^L are the widths associated to the two channels involved in a one dimensional scattering process. They are proportional to the velocity of the electron times the probability to find the electron at the corresponding boundary divided by

the probability to find it within the internal region. One therefore sees, from Eq.(3), that the partial widths are quantities which depend coherently, through $u_n(x)$, on the system as a whole. The above quantities define the characteristic times[7],

$$\tau_n^0 = \frac{\hbar}{\Gamma_n^0}; \qquad \tau_n^L = \frac{\hbar}{\Gamma_n^L} \tag{4}$$

Obviously the characteristic times defined by Eq.(4) also depend coherently on the system as a whole. This is to be contrasted with treatments which assign time scales to different regions of the system ignoring the inherent coherence of the resonant process. In particular in the case of a double barrier structure, the above characteristic times should not be confused with the traversal times through the barriers which are calculated as if the barriers were independent of the rest of the system. One may use a semiclassical calculation to compare these times. If w denotes the width of the well, v_n the resonant speed inside the well, and T^0 and T^L the corresponding transmission coefficients through each barrier, the WKB approximation yields for the characteristic times [9], $\tau_n^0 = 2w/(v_n T^0)$ and $\tau_n^L = 2w/(v_n T^L)$, whereas for the traversal times gives $t_0 = d_0/v_n^0$ and $t_L = d_L/v_n^L$, where d_0 and d_L are the thicknesses of the barriers and v_n^0 and v_n^L the corresponding velocities. It follows then that the ratios of the characteristic times to the traversal times are exponentially large in the thicknesses of the respective barriers, namely,

$$\frac{\tau_n^0}{t_0} \approx \frac{w}{d_0} exp(2\alpha d_0); \qquad \frac{\tau_n^L}{t_L} \approx \frac{w}{d_L} exp(2\alpha d_L) \tag{5}$$

where $\alpha = [2m(V-\epsilon_n)/\hbar^2)]^{1/2}$. Eq.(5) shows that as the thicknesses increase the characteristic times will be in general much larger than the traversal times.

The partial widths can be used to describe relevant physical processes near resonance energy. For example it is well known that the transmission coefficient around a sharp isolated resonance i.e., $\epsilon_n \gg \Gamma_n$ and $|\epsilon_n - \epsilon_{n\pm1}| \gg \Gamma_n$, may be written in terms of the partial widths as [12,13],

$$T(E) = \frac{\Gamma_n^0 \Gamma_n^L}{(E-\epsilon_n)^2 + \Gamma_n^2/4} \tag{6}$$

The above equation shows in particular that unity transmission is achieved at resonance energy when the two partial widths are equal, i.e. $\Gamma_n^0 = \Gamma_n^L$. Reciprocately, when $\Gamma_n^0 \neq \Gamma_n^L$, the transmission is manifestly less than one.

The partial decay widths are also relevant for the discussion of resonant tunneling times. In fact the dwell times[7,8] t_{0L} and t_{L0}, the phase transmission time[6,14] t_θ, and the traversal Larmor time[9] τ_τ, may be written in terms of partial widths. In particular, for $T = 1$ all of them coincide at the same value $2\hbar/\Gamma_n$. However, as pointed out by Smith[15], since the energy of the resonance cannot be characterized by an exact value as it is not well defined within the interval given by the width Γ_n, one has to average these times across this interval with an appropriate weight[15]. Both the dwell time and the transmission phase time when averaged yield the lifetime, i.e.

$$\bar{t}_{0L} = \bar{t}_{L0} = \bar{t}_\theta = \frac{\hbar}{\Gamma_n} \tag{7}$$

As remarked by Smith, the collision experiments are an alternative way, to the decay experiments, of measuring the lifetime. It is interesting to note that the averaged

traversal Larmor time gives instead $\tau_T = 2\hbar/\pi\Gamma_n$. As is well known the dwell time is the average time that electrons spend within the system and since for $T = 1$ all particles are transmitted, this time yields the transit time through the system for this case. Clearly the phase time gives also the transit time for the same case.

In the general case of irregular asymmetric potential profiles where $\Gamma_n^0 \neq \Gamma_n^L$, and hence $T < 1$, the averaged dwell times become $\bar{t}_{0L} = 2\hbar\Gamma_n^0/\Gamma_n^2$ and $\bar{t}_{L0} = 2\hbar\Gamma_n^L/\Gamma_n^2$ which as is well known do not distinguish between channels and therefore are not adequate to define a transit time[5]. The expression for the averaged phase transmission remains as in Eq.(7) except that now $\Gamma_n^0 \neq \Gamma_n^L$, namely, $\bar{t}_\theta = \hbar/(\Gamma_n^0 + \Gamma_n^L)$. However in contrast to the situation described by Eq.(7), the fact that now the partial widths are different implies that the transmission phase time is not in general adequate as a transit time. This follows from the fact that the lifetime and hence the average transmission phase time are a measure of the decay out of the system through the two channels. The rate of decay will be dominated by the larger of the decay widths. Actually if the partial widths differ appreciably and we denote by Γ_n^{max} and Γ_n^{min} the larger and the smaller, respectively of Γ_n^0 and Γ_n^L then,

$$\bar{t}_\theta \approx \frac{\hbar}{\Gamma_n^{max}} \tag{8}$$

The fact that Eq.(8) does not depend on the decay through the other channel, i.e. Γ_n^{min}, could be interpreted as indicating that the corresponding time is negligible. However, such an interpretation is incorrect, as follows from the analysis leading to Eq.(5). The delay time given by the transmission phase time merely provides an alternative form of obtaining the lifetime. The traversal Larmor time behaves in a

similar way to the phase time except for a factor of $2/\pi$, i.e., $\tau_T \approx 2\hbar/\pi\Gamma_n^{max}$.

One should expect an adequate definition of a transit time to incorporate information on the time scales associated to electrons entering and leaving the system. Clearly the characteristic times provide the time scale for decay of the electron, respectively, through $x = 0$ and $x = L$. The time scale for capture may be obtained by reversing the direction of the electron velocities. For coherent processes it follows, from general analytical properties of the propagator of the problem, that for each resonant pole at $k_n = a_n - ib_n$ there corresponds a pole at $k_{-n} = -k_n^*$ with associated eigenfunction $u_n(x) = u_n^*(x)$ which describes an incoming electron. Indeed, the probabilty density along the internal region $0 < x < L$, increases with time i.e., $|u_n(x,t)|^2 = |u_n(x)|^2 exp[\Gamma_n t/\hbar]$ as expected for a capture process [16]. It is straightforward to see that the equation for probability conservation ,i.e. Eq.(2), now applied to $u_n(x,t)$ gives as before Γ_n as the sum of the partial widths given by Eq.(3). As a result it follows that the time scale for capture through each boundary is the same as that for decay. A similar statement has been made by Ricco and Azbel[4] and Tsuchiya *et. al.*[17]. Recent work by Guo *et. al.* [18] reports that build up times are different from decay times. Actually we do not find contradiction with our result since they consider wavepackets of energy width larger than the resonance width, as appropriate to study time evolution of decay, whereas our work involving plane waves corresponds to the opposite situation, namely, wave packets of energy width much smaller than the resonance width. It follows from the above considerations that in the scattering process , say from $x < 0$, of an electron near resonance energy, the time scale for entering the system will be τ_n^0 and the time scale for leaving the system through the opposite side will be τ_n^L, and accordingly

for an electron approaching the system from the right. Therefore we expect that the transit time will be of the order of the sum of τ_n^0 and τ_n^L. Since the notion of decay widths does not correspond to a specific resonance energy but applies equally well to all electrons whose energies are within the resonance width, the transit time must be interpreted as an average time. Thus it must, for the case $T = 1$, coincide with the value given by Eq.(7), i.e. the lifetime $\hbar/\Gamma_n$. Therefore a definition for the transit time consistent with the above condition is,

$$\tau_{tr} = \frac{1}{4}(\tau_n^0 + \tau_n^L) = \frac{\hbar}{\Gamma_n T^{res}} \tag{9}$$

In writing the expression on the far right hand side we have made use of Eq.(4) and of Eq.(6) for the transmission coefficient, at resonance energy, i.e., $T^{res} = T(\epsilon_n) = 4\Gamma_n^0\Gamma_n^L/\Gamma_n^2$. Eq.(9) implies that for diminishing transmission the transit time will increase beyond limit. This is to be contrasted with the transmission phase time which instead tends to a finite value, i.e., Eq.(8).

For the case of multibarrier resonant tunneling structures, the current density integrated over a sharp isolated resonance is inversely proportional to the transit time τ_{tr}. To see this we follow Tsu and Esaki[3]. In the low temperature limit and for a voltage V_{bias} larger than the Fermi energy E_F , i.e., $eV_{bias} > E_F$, the tunneling current density is given by,

$$J = \frac{em}{2\pi^2\hbar^3}\int_0^{E_F}(E_F - E)T(E)\,dE \tag{10}$$

Our previous discussion on the resonant formalism can be easily generalized to consider the effect of the applied voltage. The analytical expression for the transmission coefficient given by Eq.(6) remains valid and upon substitution into Eq.(10) yields for a sharp isolated resonance the expression,

$$J = \frac{em(E_F - \epsilon_n)}{4\pi\hbar^2}\frac{\Gamma_n T^{res}}{\hbar} = \frac{em(E_F - \epsilon_n)}{4\pi\hbar^2}\frac{1}{\tau_{tr}} \tag{11}$$

The fact that the above expression involving τ_{tr} results from an integration over energies across the resonance width is consistent with its interpretation as an average transit time. For an asymmetric structure $J \approx \Gamma_n^{min}$ and hence the time scale of the process will be $\hbar/\Gamma_n^{min}$, i.e. according to Eq.(9), the longest of τ_n^0 and τ_n^L. This is to be contrasted with the transmission phase time which instead goes like $\hbar/\Gamma_n^{max}$, i.e. Eq.(8). For the symmetric case $J \approx \Gamma_n$ and hence the corresponding time scale, as discussed earlier, will be given by the lifetime, which is consistent with a result by Guéret *et. al.*[19]. Eq.(11) establishes a relationship between our definition and a relevant measurable quantity.

In conclusion, this work is based on a framework that emphasizes the properties of the partial widths. This approach leads to a definition of transit time for arbitrary resonant tunneling structures. An important consequence is that the transit time may be much larger than previously expected on analysis based on other definitions. This may be of interest because in practical cases the potential profiles are in general asymmetrical and hence do not provide full transmission. It is worth mentioning that the resonance energies and decay widths may be readily calculated by a straightforward application of the transfer matrix technique[20].

ACKNOWLEDGMENTS

The authors would like to acknowledge partial support from DIGICSA-SEP, México and to thank Professor Abdus Salam, The International Atomic Energy Agency and UNESCO for hospitality at The International Centre for Theoretical Physics in Trieste.

REFERENCES

1. *The Physics and Fabrication of Microstructures and Microdevices*, edited by M.J. Kelly and C. Weissbuch (Springer-Verlag, Berlin, 1986) and references therein.
2. F. Capasso, K.Mohammed, and A.Y. Cho, IEEE J. Quantum Electron. **22**, 1853 (1986).
3. R. Tsu and L. Esaki, Appl. Phys. Lett. **22**, 562 (1973).
4. B. Ricco and M. Ya. Azbel, Phys. Rev. B **29**, 1970 (1984).
5. E. H. Hauge anf J. A. Stovneng, Rev. Mod. Phys. **61**, 917 (1989) and references therein.
6. S. Collins, D. Lowe and J. R. Barker, J. Phys. C **20**, 6233 (1988), *ibid* 6233.
7. G.García-Calderón and A. Rubio., Phys. Rev. B **36**, 4462 (1987).
8. B. Gu, L. Gu and M. He, J. Appl. Phys. **66**, 704 (1989).
9. M. Büttiker, in *Electronic Properties of Multilayers and Low Dimensional Semiconductor Structures*, edited by J. M. Chamberlain, L. Eaves and J. C. Portal (Plenum, New York, 1990).
10. G. Gamow, Z. Phys. **51**, 204 (1928).
11. L.D. Landau And E.M. Lifshitz, *Quantum Mechanics: Non-relativistic Theory* (Pergamon, Oxford, 1977), 3rd edition, Sec. 119.
12. G. García-Calderón, Solid State Commun. **62** ,441 (1987).
13. P.J. Price, Phys. Rev. B **38**, 1994 (1988); V. Kalmeyer R.B. Laughlin, Phys. Rev. B **35**, 9805 (1987).
14. G. García-Calderón and A. Rubio, Solid State Commun. **71**, 237 (1989).

15. F. Smith, Phys. Rev. **118**, 349 (1960).
16. V. De Alfaro and T. Regge, *Potential Scattering*, (North-Holland, Amsterdam and John Wiley and Sons, New York, 1965) p.67.
17. M. Tsuchiya, T. Matsusue and H. Sakaki, Phys. Rev. Lett. **59**, 2356 (1987).
18. H. Guo, K. Diff, G. Neofotistos and J. D. Gunton, Appl. Phys. Lett., **53**, 131 (1988).
19. P. Guéret, C. Rossel, E. Marclay and H. Meier, J. Appl. Phys. **66**, 278 (1989).
20. G. García-Calderón, A. Rubio and R. Romo, unpublished 1990.