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DEPENDENCE OF CIT PF COIL CURRENTS ON PROFILE AND SHAPE PARAMETERS USING THE CONTROL MATRIX*

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ABSTRACT

The plasma shaping flexibility of the Compact Ignition Tokamak (CIT) poloidal field (PF) coil set is demonstrated through MHD equilibrium calculations of optimal PF coil current distributions and their variation with poloidal beta, internal inductance, plasma 95% elongation, and 95% triangularity. Calculations of the magnetic stored energy are used to compare solutions associated with various plasma parameters. The Control Matrix (CM) equilibrium code[1], together with the nonlinear equation and numerical optimization software packages HYBRD[2], and VMCON[3], respectively, are used to find equilibrium coil current distributions for fixed divertor geometry, volt-seconds, and plasma profiles in order to isolate the dependence on individual parameters. A reference equilibrium and coil current distribution are chosen, and correction currents dI are determined using the CM equilibrium method to obtain other specified plasma shapes. The reference equilibrium is the $\kappa = 2$ divertor at beginning of flattop (BOFT) with a minimum stored energy solution for the coil current distribution. The pressure profile function is fixed.

The Control Matrix Equilibrium Code

A typical iterative algorithm for solving the separatrix limited free-boundary equilibrium problem $\Delta^* \psi = r^2 \nabla \bullet (r^{-2} \nabla \psi) = -\mu r J_\phi$, for the poloidal flux function ψ in a rectangular region Ω, consists of two major computational parts:

(1) For fixed plasma current density and coil current distributions, compute boundary values of ψ on $\partial\Omega$, solve for $\psi(r,z)$ in the interior of Ω, and locate the separatrix to evaluate ψ_x.

(2) Given ψ_x and $\psi(r,z)$ in Ω, compute the plasma current density distribution $J_\phi(r,z)$ using the assumed pressure and toroidal flux profile functions $P(\psi)$ and $F(\psi)$, respectively.

The CM method (Figure 1) consists of computing the shape control matrix **A** after step (1), using it to determine the vector of correction currents d**I** producing a desired change in the plasma shape, and recomputing step (1) with the coil current vector **I** = **I** + d**I**. The calculation in part (2) is then carried out with the corrected coil current distribution.

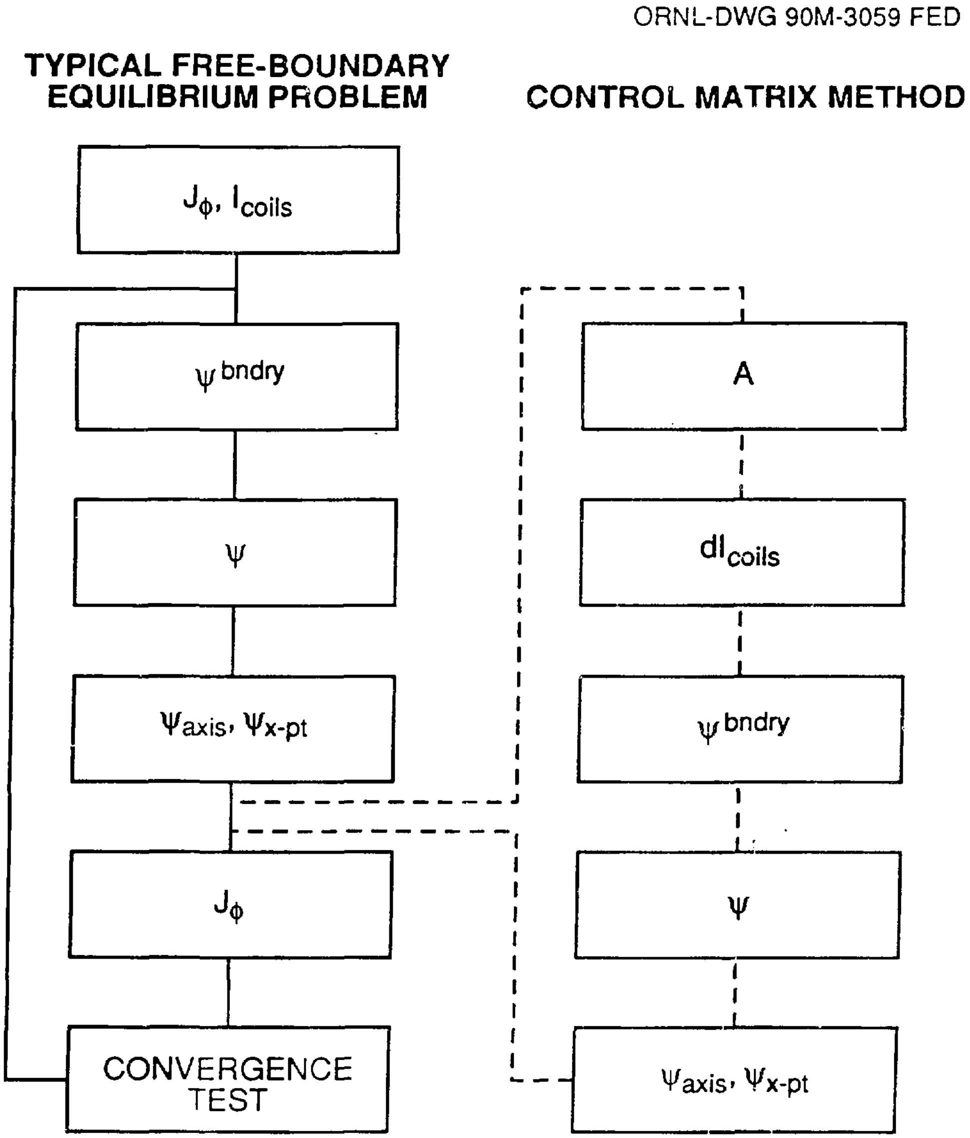

Figure 1. For fixed X-point equilibria, the shape control matrix **A** is computed and used to determine a correction $d\mathbf{I}_{\text{coils}}$ to the coil current distribution, prior to updating the plasma current distribution $\mathbf{J}_\phi$.

Profile Functions

$$J_\phi = r\, dP/d\psi + F(dF/d\psi)/(\mu r),$$

$$dP/dx = P_0\, [\exp(-Ax) - \exp(-A)]/[\exp(-A) - 1],$$

$$dF^2/dx = 2\mu R_o^2\, P_0\, (1/\beta_J - 1)[\exp(-Bx) - \exp(-B)] \\ /[\exp(-B) - 1],$$

where $x = (\psi - \psi_0)/(\psi_x - \psi_0)$.

P_0 is scaled so that the total plasma current is fixed, ie.

$$I_p = \iint J_\phi d\Omega.$$

Plasma equilibrium pressure and current density profiles are characterized by the poloidal beta,

$$\beta_p = 4 \int P\, dV/(\mu R_o I_p^2),$$

and the plasma internal inductance,

$$l_i/2 = \int B_p^2 dV/(\mu^2 R_o I_p^2),$$

respectively.

Profile Constraints and Minimum-Energy Solutions

The numerical software package HYBRD[2] is designed to solve n nonlinear equations in n variables. The CM equilibrium subroutine is called from HYBRD to compute values of free parameters β_J and B in the F-profile to constrain the profile parameters β_p and $l_i/2$.

In the CIT, seven independent coil sets control five plasma shape and flux parameters. It is possible to fix the values of two coil currents in the CM solution, i.e. reduce the size of the control matrix from 7x5 to 5x5. In this case the nonlinear optimization package VMCON[3] is used to find values of the two additional currents to minimize the stored energy in the coil system $W_{PF} = 1/2\mathbf{I}^T M \mathbf{I}$. This is referred to as a minimum energy equilibrium solution.

Reference Equilibrium

R_o = 2.138 m
a = 0.661 m
B_t = 11 T
I_p = -12.3 MA
κ = 2.0
δ = 0.25
β_p = 0.16
$l_i/2$ = 0.37
ϕ = 38 V-s

Pressure profile: A = -0.5

CIT PF System

Table 1. Poloidal field coil set based on GEM-29.

Coil	R_c (m)	Z_c (m)	ΔR (m)	ΔZ (m)
PF1	0.592	0.410	0.406	0.821
PF2	0.592	1.307	0.406	0.770
PF3	0.592	2.179	0.406	0.770
PF4	1.630	2.891	0.563	0.423
PF5	3.190	2.870	0.291	0.271
PF6	4.187	2.000	0.275	0.324
PF7	4.225	0.749	0.354	0.448

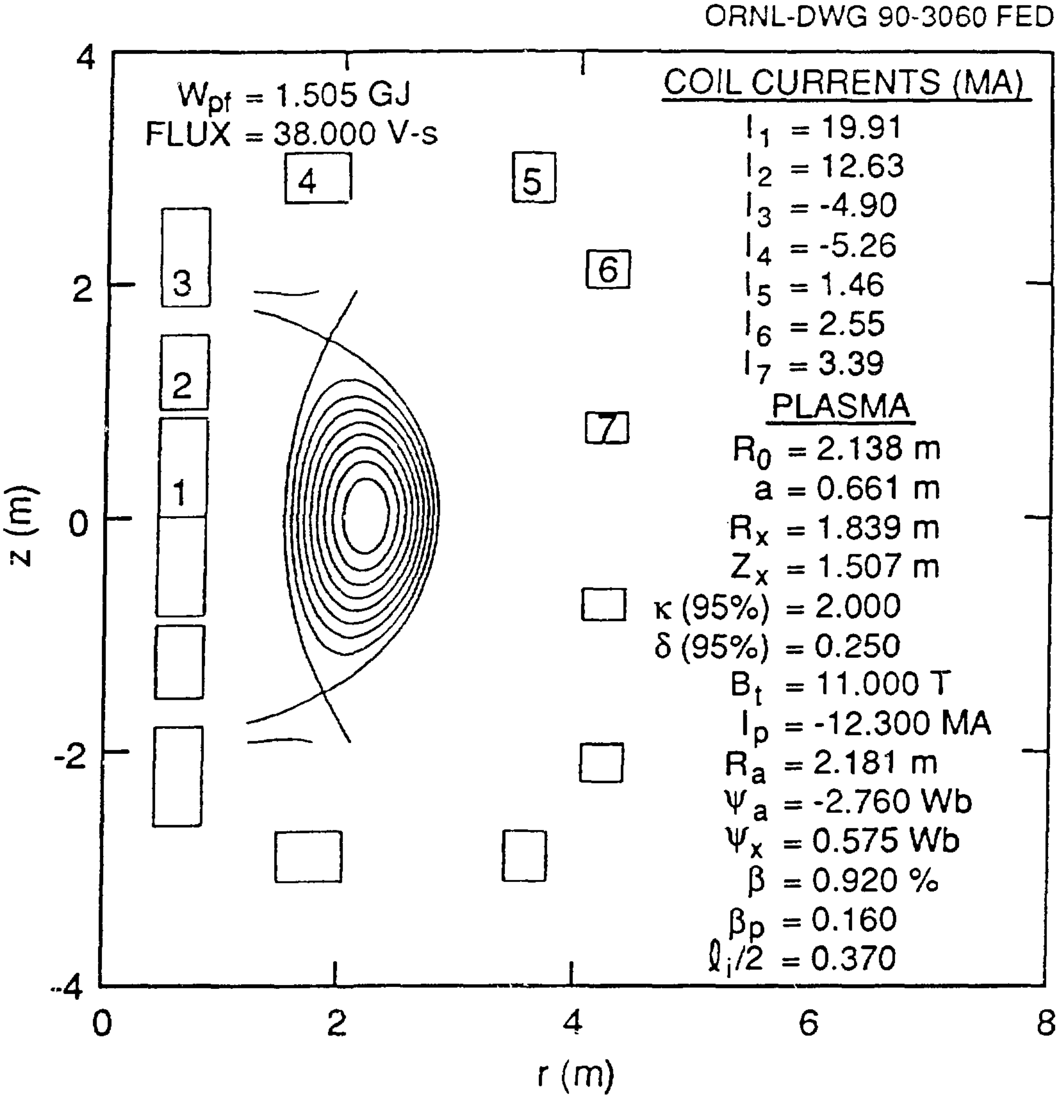

Figure 2. The reference CIT equilibrium solution is the $\kappa = 2$, diverted plasma at beginning of flattop.

Variation with Plasma Profile

Poloidal Beta

Table 2. PF coil current variation with poloidal beta.

β_p	0.08	0.16	0.24	0.32
Currrent (MA)				
PF1	20.406	19.900	19.414	18.915
PF2	12.651	12.660	12.624	12.638
PF3	-5.095	-4.914	-4.708	-4.530
PF4	-5.237	-5.273	-5.294	-5.333
PF5	1.633	1.468	1.285	1.121
PF6	2.553	2.550	2.544	2.541
PF7	3.229	3.383	3.543	3.695
W_{PF}(GJ)	1.512	1.506	1.501	1.499

Variation with Plasma Profile

Internal Inductance

Table 3. PF coil current variation with internal inductance.

$l_i/2$	0.27	0.37	0.47	0.52
Current (MA)				
PF1	15.121	19.900	23.347	24.607
PF2	25.410	12.660	3.185	-0.181
PF3	-12.144	-4.914	0.705	2.308
PF4	-9.895	-5.273	-2.200	-1.014
PF5	5.858	1.468	-1.899	-3.269
PF6	3.538	2.550	1.827	1.592
PF7	1.367	3.383	5.000	5.644
W_{PF}(GJ)	2.593	1.506	1.448	1.600

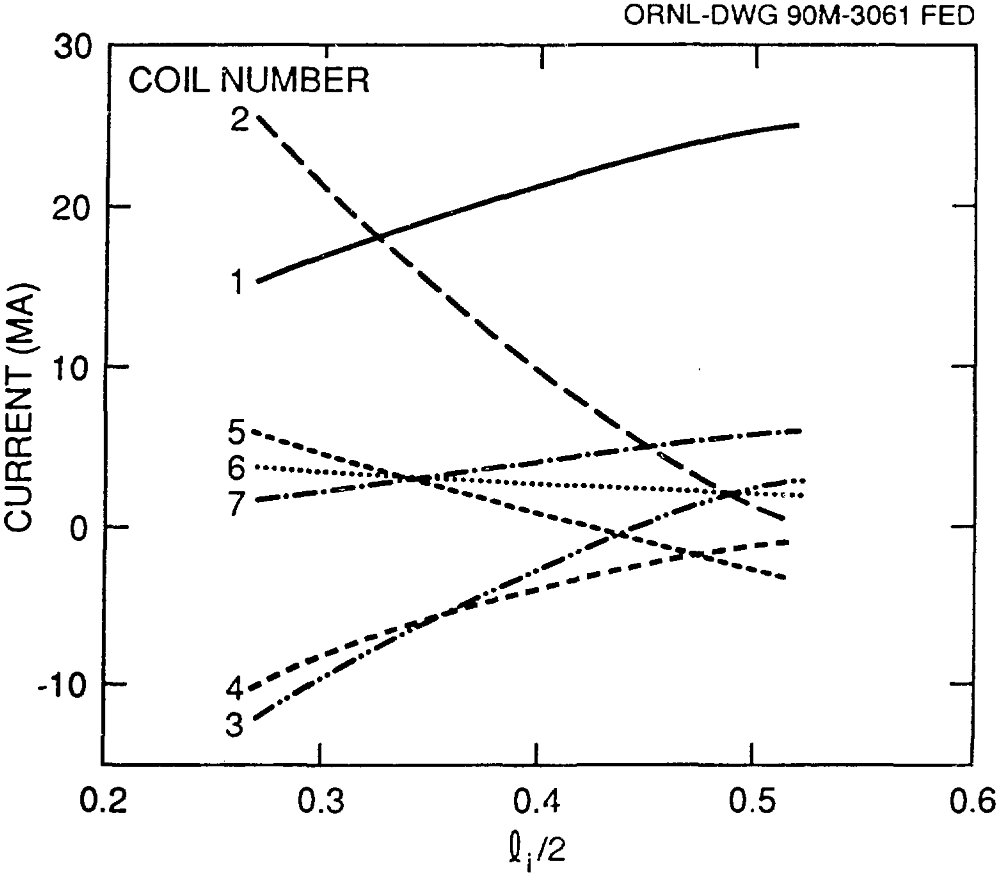

Figure 3. The dependence of coil currents PF1 - 7 on internal inductance for fixed plasma 95% shape parameters, poloidal beta, and flux-linkage (V-s).

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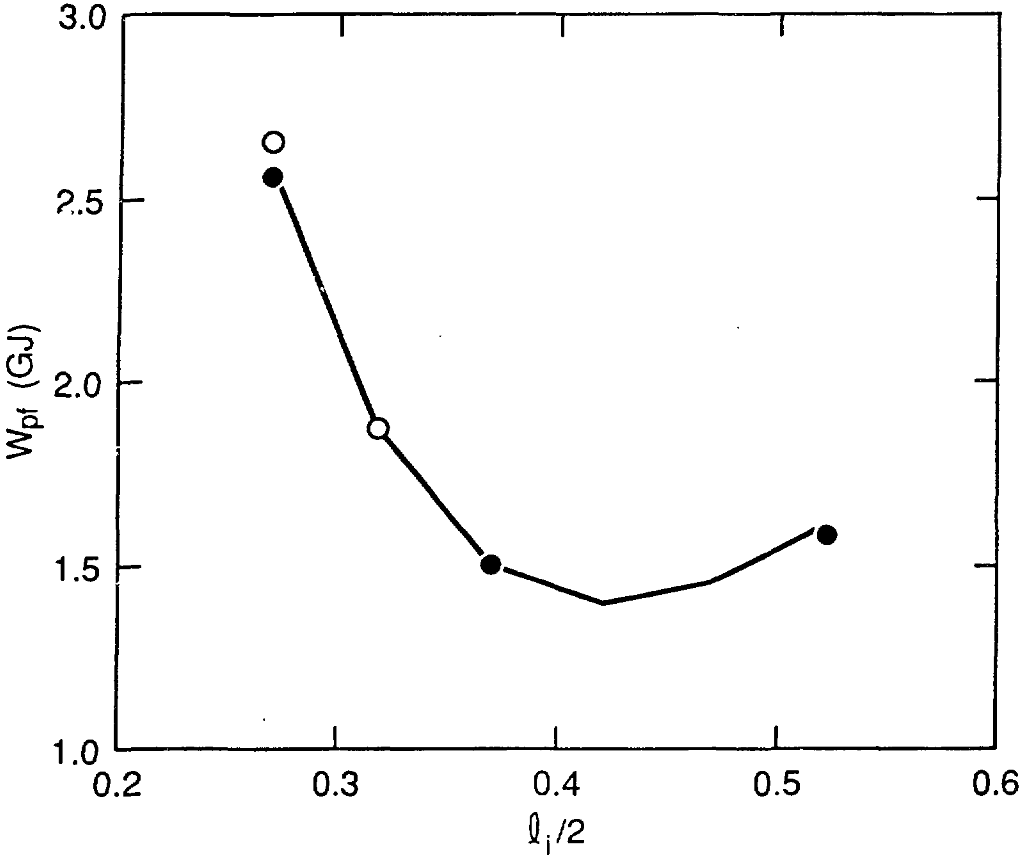

Figure 4. The dependence of PF coil stored energy on internal inductance (solid line). The dots are minimum energy solutions. The open circles are CM solutions with I_3 = -7.5MA (fixed).

Variation with Plasma Shape

Elongation

Table. 4. PF coil current variation with plasma elongation.

κ_{95}	1.80	1.90	2.00	2.10
Current (MA)				
PF1	16.939	18.694	19.900	20.938
PF2	22.597	16.623	12.660	9.415
PF3	-17.146	-9.832	-4.914	-0.907
PF4	-12.299	-8.104	-5.273	-3.316
PF5	6.924	3.681	1.468	-0.134
PF6	3.915	3.109	2.550	2.088
PF7	1.369	2.563	3.383	4.025
W_{PF}(GJ)	3.155	1.965	1.506	1.351

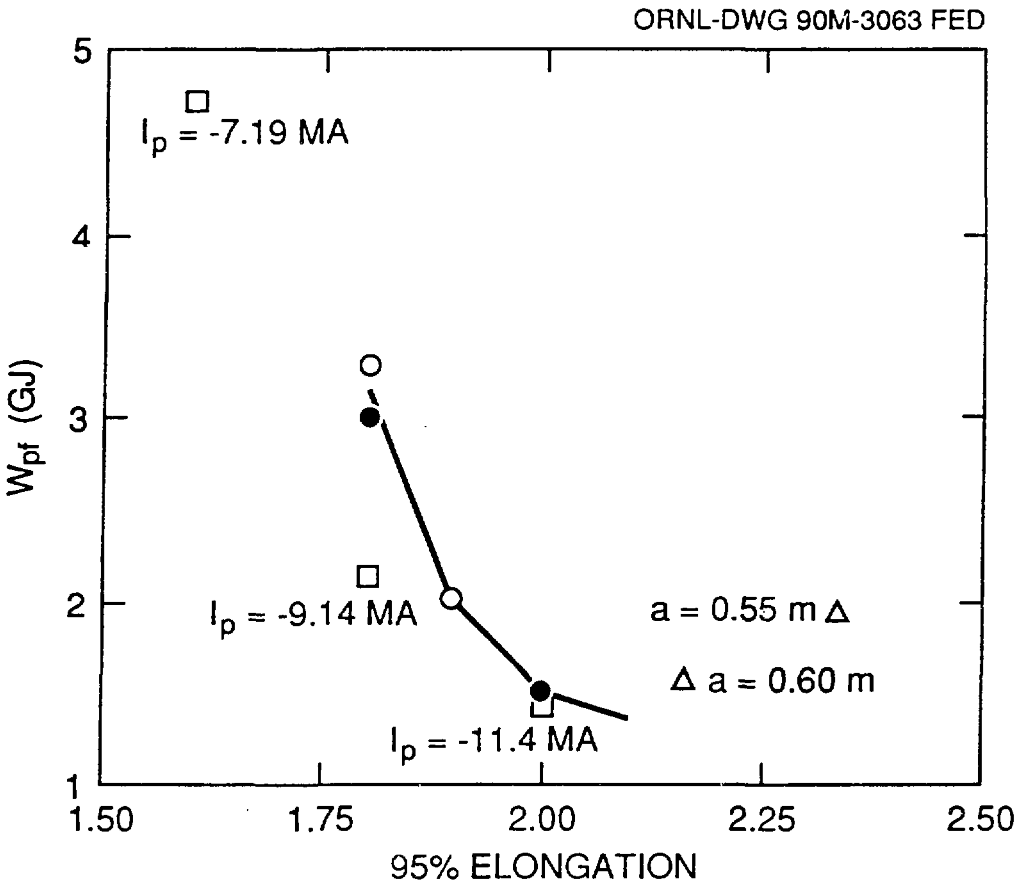

Figure 5. The dependence of PF coil stored energy on 95% elongation (solid line). The dots are minimum energy solutions. The open circles are CM solutions with I_3 = -7.5MA (fixed). The squares are CM solutions with fixed 95% safety factor (q = 3.2, variable I_p). The diamonds correspond to solutions where κ is increased by reducing the plasma minor radius.

Variation with Plasma Shape

Triangularity

Table. 5. PF coil current variation with plasma triangularity.

δ_{95}	0.20	0.25	0.30	0.35	0.40
Current (MA)					
PF1	18.997	19.900	20.774	21.873	22.904
PF2	14.728	12.660	10.588	7.842	5.333
PF3	-4.311	-4.914	-5.335	-5.360	-6.422
PF4	-5.656	-5.273	-4.774	-3.956	-2.609
PF5	1.005	1.468	1.780	1.844	1.852
PF6	2.519	2.550	2.546	2.483	2.353
PF7	3.610	3.383	3.223	3.171	3.116
W_{PF}(GJ)	1.544	1.506	1.470	1.416	1.364

Summary

The CM equilibrium code, together with numerical software for nonlinear equations and constrained optimization, are used to find equilibrium coil current distributions for fixed divertor geometry, volt-seconds, and plasma profiles in order to determine the variation with plasma profile and shape parameters. Two input toroidal flux function profile parameters in the equilibrium code are varied by HYBRD to constrain poloidal beta and internal inductance. Variation in coil currents with internal inductance is much stronger than the variation with poloidal beta, and for $\beta_p = 0.16$, shows a distinct minimum in stored energy in the interval $0.40 < l_i/2 < 0.45$. With a minimum energy reference equilibrium, CM solutions for variations in profile parameters are near minimum energy. For constant minor radius, shaping coil currents and stored energy rise in magnitude with decreasing elongation. For variable minor radius, stored energy rises as elongation increases.

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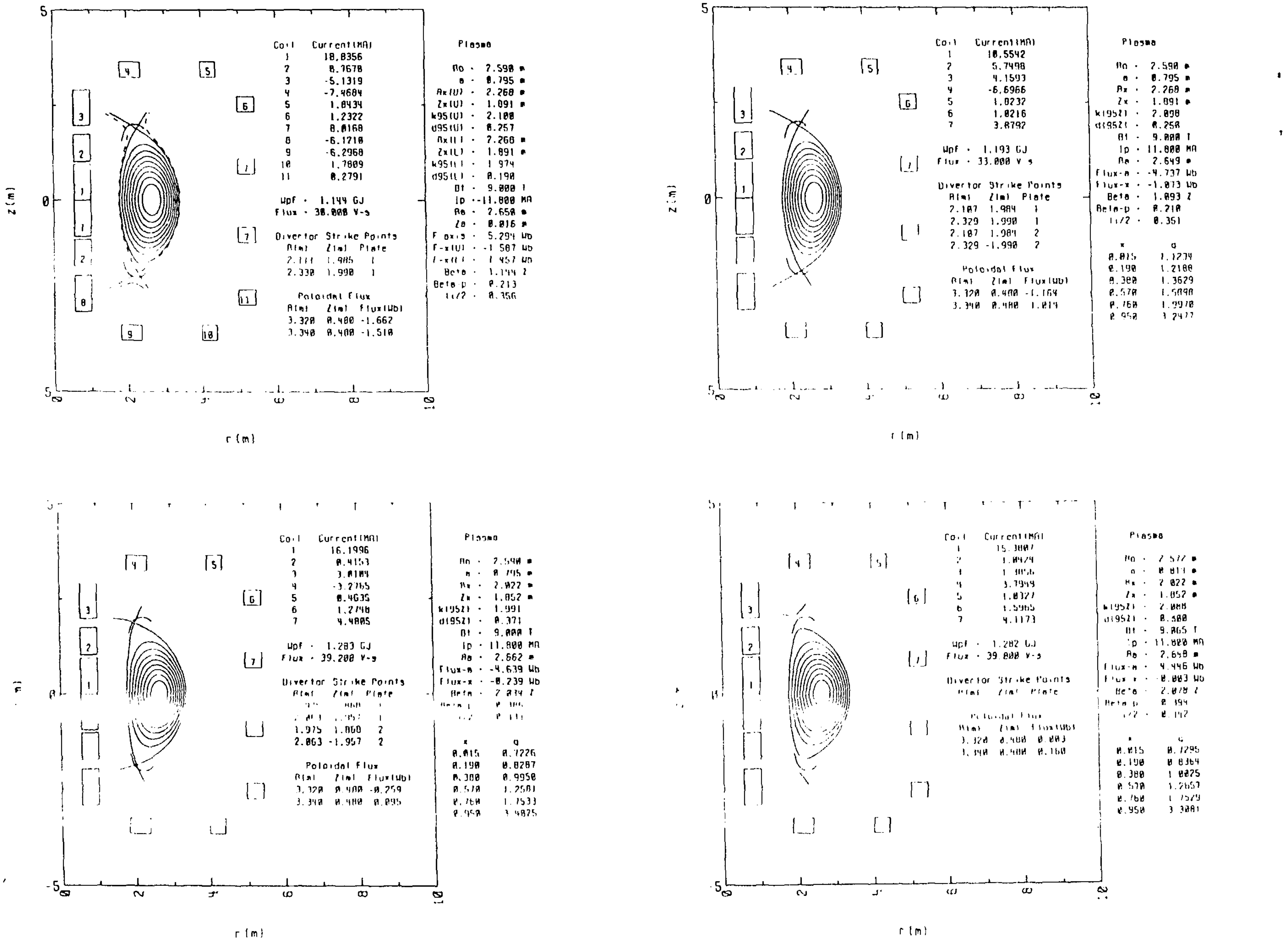
CIT Equilibrium Sequence - R_o = 2.59 m



```
Coil   Current(MA)
 1      10.0356
 2       0.7670
 3      -5.1319
 4      -7.4684
 5       1.0434
 6       1.2322
 7       0.0160
 8      -6.1210
 9      -6.2960
10       1.7009
11       0.2791

 Wpf = 1.144 GJ
Flux = 30.000 V-s

Divertor Strike Points
  R(m)   Z(m)  Plate
  2.111  1.985   1
  2.330  1.990   1

    Poloidal Flux
  R(m)   Z(m)  Flux(Wb)
  3.320  0.400 -1.662
  3.340  0.400 -1.510
```

```
      Plasma

     Ro =  2.590 m
      a =  0.795 m
  Rx(U) =  2.260 m
  Zx(U) =  1.091 m
 k95(U) =  2.100
 d95(U) =  0.257
  Rx(L) =  2.260 m
  Zx(L) =  1.091 m
 k95(L) =  1.974
 d95(L) =  0.190
     Bt =  9.000 T
     Ip = -11.000 MA
     Ra =  2.650 m
     Za =  0.016 m
 F axis =  5.294 Wb
 F-x(U) = -1.507 Wb
 F-x(L) =  1.457 Wb
   Beta =  1.144 %
 Beta-p =  0.213
   li/2 =  0.356
```

```
Coil   Current(MA)
 1      10.5542
 2       5.7490
 3       4.1593
 4      -6.6966
 5       1.0232
 6       1.0216
 7       3.0792

 Wpf = 1.193 GJ
Flux = 33.000 V s

Divertor Strike Points
  R(m)   Z(m)  Plate
  2.107  1.984   1
  2.329  1.990   1
  2.107  1.984   2
  2.329 -1.990   2

    Poloidal Flux
  R(m)   Z(m)  Flux(Wb)
  3.320  0.400 -1.164
  3.340  0.400  1.014
```

```
      Plasma

     Ro =  2.590 m
      a =  0.795 m
     Rx =  2.260 m
     Zx =  1.091 m
 k(95%) =  2.090
 d(95%) =  0.250
     Bt =  9.000 T
     Ip = 11.000 MA
     Ra =  2.649 m
 Flux-a = -4.737 Wb
 Flux-x = -1.073 Wb
   Beta =  1.093 %
 Beta-p =  0.210
   li/2 =  0.351

     x          q
   0.015     1.1274
   0.190     1.2100
   0.380     1.3629
   0.570     1.5090
   0.760     1.9970
   0.950     3.2477
```

```
Coil   Current(MA)
 1      16.1996
 2       0.4153
 3       3.0184
 4      -3.2765
 5       0.4635
 6       1.2740
 7       4.4005

 Wpf = 1.283 GJ
Flux = 39.200 V-s

Divertor Strike Points
  R(m)   Z(m)  Plate
  [illegible] [illegible] 1
  [illegible] 1.957   1
  1.975  1.060   2
  2.063 -1.957   2

    Poloidal Flux
  R(m)   Z(m)  Flux(Wb)
  3.320  0.400 -0.259
  3.340  0.400  0.095
```

```
      Plasma

     Ro =  2.590 m
      a =  0.795 m
     Rx =  2.022 m
     Zx =  1.052 m
 k(95%) =  1.991
 d(95%) =  0.371
     Bt =  9.000 T
     Ip = 11.000 MA
     Ra =  2.662 m
 Flux-a = -4.639 Wb
 Flux-x = -0.239 Wb
   Beta =  2.034 %
 Beta-p =  0.[illegible]
   li/2 =  0.[illegible]

     x          q
   0.015     0.7226
   0.190     0.8287
   0.380     0.9950
   0.570     1.2501
   0.760     1.7533
   0.950     3.4875
```

```
Coil   Current(MA)
 1      15.3007
 2       1.0479
 3       1.3056
 4       3.7949
 5       1.0327
 6       1.5965
 7       4.1173

 Wpf = 1.282 GJ
Flux = 39.000 V-s

Divertor Strike Points
  R(m)   Z(m)  Plate

    Poloidal Flux
  R(m)   Z(m)  Flux(Wb)
  3.320  0.400  0.003
  3.340  0.400  0.160
```

```
      Plasma

     Ro =  2.577 m
      a =  0.811 m
     Rx =  2.022 m
     Zx =  1.052 m
 k(95%) =  2.000
 d(95%) =  0.500
     Bt =  9.065 T
     Ip = 11.000 MA
     Ra =  2.648 m
 Flux-a =  4.446 Wb
 Flux-x = -0.003 Wb
   Beta =  2.078 %
 Beta-p =  0.394
   li/2 =  0.147

     x          q
   0.015     0.7295
   0.190     0.8364
   0.380     1.0075
   0.570     1.2657
   0.760     1.7579
   0.950     3.3081
```