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COHERENT CHARGE FLUCTUATIONS IN JOSEPHSON JUNCTIONS AND THE OSCILLATIONS OF THE EFFECTIVE CAPACITANCE *

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ABSTRACT

We predict novel voltage oscillations of the effective capacitance of small Josephson junctions. This macroscopic effect involves coherent charge fluctuations with charge $2e$, leading to a period of oscillations, $V_c = 2e/C$, where C is the junction capacitance. The amplitude of the effect decreases with temperature as $\exp(-\pi^2 T/\varepsilon_c)$, where $\varepsilon_c = (2e)^2/C$.

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In the past few years the investigation of macroscopic systems has become a subject of great interest. One particular realization of these effects is reflected in small Josephson junctions where much of the recent activity is connected with the existence of Bloch oscillations [1] (referred to as "secondary quantum phenomena" in Ref.2).

In this paper we point out a new type of oscillation in weak–link systems which have their origin in the fluctuations in the charge of a small capacitance junction.

To introduce the effect we argue by using the general mathematical structure of the well–known problem of a particle on a circle with quasiperiodic boundary conditions, (the so–called θ–vacuum problem [3]). The problem found a realization in the instanton Aharonov–Bohm effect in a charge–density–wave system [4], in which case the parameter θ is determined by the magnetic flux, Φ, penetrating through the ring. Unfortunately, the magnitude of the effect in that case outside the reach of present experimental capability, and we were led to consider another system namely the small Josephson junctions, as an experimentally accessable candidate.

It is well–known that weak–link superconducting devices are characterized by a single dynamical variable, φ, the relative phase between two bulk superconductors.

Since φ is an angle, the states of the system differing by $2\pi n$ (n – an integer) are physically equivalent. In finite systems these vacua are connected via tunnelling, resulting in an exponentially small lowering of the ground state energy. (This coherence effect is completely analogous to the more physically apparent coherence around the ring in a charge–density–wave system Aharonov–Bohm problem [4].) In what follows we show that the coherent fluctuations of phase φ lead, in the preence of a DC voltage to an oscillatory dependence of the dielectric characteristics of the junction on DC bias, V. Although we will be using the terminology of quantum mechanics it is amusing to note that Plank's constant disappears from all results suggesting that our effect may be derived from purely classical considerations [6].

Let us study the model of a distributed Josephson junction (with $L \gg \lambda_J$) biased by a DC voltage, V. At low temperature the electrodynamics of this system is described by the Lagrangian [5]

$$\mathcal{L} = N_o \left\{ \left(\frac{\partial \varphi}{\partial t} \right)^2 - c_o^2 \left(\nabla_\perp \varphi \right)^2 - \omega_J^2 (1 - \cos\varphi) \right\} \tag{1}$$

where $N_o = \hbar^2 C_s / 8e^2$, C_s is the capacitance per square, c_o is the Swihart velocity, ω_J, is the Josephson frequency, and $\lambda_J = c_0/\omega_J$ is the Josephson penetration depth.

In the presence of DC voltage, at sufficiently low temperatures (such that $T \ll \hbar I_c / 2e$ where I_c is the critical current) the phase variable can be written as $\varphi = \varphi_o + \chi$, where $\varphi_o = 2eVt/\hbar$ and χ is the fluctuating component. In the limit of interest, $eV \gg \hbar c_o / L$, and thus χ is a slowly varying function of x and t. With these qualifications, the path integral representation of the partition function (vacuum–to–vacuum amplitude) reduces after averaging over fast time scales associated with $T_V = \pi\hbar/eV$ to an integral over slowly varying trajectories, $\chi(\vec{x},t)$:

$$Z = \int D\chi \exp\left\{\frac{i}{\hbar}\int ds\, dt\, \mathcal{L}_{\text{eff}}(\chi)\right\} \tag{2}$$

where

$$\begin{aligned}\mathcal{L}_{\text{eff}} &= N_o\left\{\left(\frac{\partial\chi}{\partial t}\right)^2 + \frac{4\,eV}{\hbar}\,\frac{\partial\chi}{\partial t} - c_o^2\,(\nabla_\perp\chi)^2\right\} = \\ &= N_o\left\{\left(\frac{\partial\chi}{\partial t}\right)^2 - c_o^2\,(\nabla_\perp\chi)^2\right\} + \frac{\theta\hbar}{2\pi A}\,\frac{\partial\chi}{\partial t}\,. \end{aligned}\tag{3}$$

Here $\theta = 2\pi V/V_c, V_c = 2e/C$, C is the total capacitance and A is the area of the junction. The main feature of Eq.(3) is the appearance of the total derivative (the θ–term). This does not affect the equations of motion yet modifies the connection between the canonical momentum, $\pi_\chi = \delta\mathcal{L}/\delta\dot{\chi}$ and the velocity $\dot{\chi}$

$$\pi_\chi = \dot{\chi} + \frac{1}{2\pi A}\,\theta \tag{4}$$

thus leading to an observable dynamical effect.

It is easy to see that the main contribution of the θ–term to the free energy comes from homogeneous trajectories in space, $\chi(\vec{x},t) = \chi(t)$ [4]. Imposing the cyclic boundary conditions in imaginary time,

$$\chi(\tau+\beta) - \chi(\tau) = 2\pi n \tag{5}$$

(n is an integer and $\beta = T^{-1}$ is the inverse temperature) we obtain the following simple expression for the oscillatory part of the free energy $F = -T\,lnZ$ (for details see Ref.4)

$$\Delta\ F_\theta = -T\ \ell n\left[\frac{v_3\left(\frac{\theta}{2\pi},q\right)}{v_3(0,q)}\right]\ , \tag{6}$$

where $v_3(v,q)$ is the Jacobi function, and

$$q = \exp\left(-\pi^2\,\frac{T}{\varepsilon_c}\right), \qquad \varepsilon_c = \frac{(2e)^2}{2C}\,. \tag{7}$$

The asymptotic form of Eq.(6) at high ($T \gtrsim \varepsilon_c$) and low temperatures ($T \ll \varepsilon_c$) can be written as

$$\Delta\ F_\theta \simeq \begin{cases} 2T\exp\left(-\pi^2\,\frac{T}{\varepsilon_c}\right)\left[1-\cos\left(2\pi\,\frac{V}{V_c}\right)\right], & T \gtrsim \varepsilon_c \\ \varepsilon_c\left\{\left\{\frac{V}{V_c}\right\}\right\}^2, & T \ll \varepsilon_c \end{cases}\,. \tag{8}$$

Here $\{\{x\}\}$ denotes the fractional part of x proximate to the nearest integer.

In conclusion we briefly discuss the possibility of the experimental observation of the effect. It is most convenient to consider the effective capacitance, obtained from (8) through a second derivative with respect to voltage. For $T \gtrsim \varepsilon_c$ we obtain

$$C_{\text{eff}}(V) = C\left\{1 + 4\pi^2 \frac{T}{\varepsilon_c} e^{-\pi^2 T/\varepsilon_c} \cos\left(2\pi \frac{V}{V_c}\right)\right\} . \qquad (9)$$

The typical parameter values for point junctions (for which our considerations also hold) are $A = 10^{-10}\, cm^2$ and $V_C \sim \mu V$. Given these values we expect that the dependence given in (9) can be observed in parametric resonance experiments. In particular, the condition $\omega\,\tau \ll 1$ with $\tau = RC$ and $\hbar\omega = eV_C$ implies $R \gg \hbar/e^2$; finally, together with the previously mentioned condition, $hI_c/2e \gg T$, this leads to the inequality, $T \ll T_c\, h/e^2 R$ (where T_c is the critical temperature of the bulk superconductor), a condition easily satisfied in typical low resistance contacts in the temperature range, $T \sim 1 + 10\ K$.

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