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ABSTRACT

The concept of "nucleon in nuclei" has often been referred to in recent literature. What it is used for is rarely precised however. In this paper, it is shown (or reminded) that the "nucleon in nuclei" is a model dependent object. As an illustration, it is shown that nuclear matter in its ground state may be described to a good approximation, if not exactly, by an independent particle state and that the on-shell G-matrix used in calculating its binding energy gets its effective character from that of those particles. The expression of these particles in terms of free nucleon operators is given.

The nuclear shell model, which has partly been inspired by the atomic one, supposes an interaction between one nucleon and the other ones which is relatively weak with respect to the nucleon-nucleon interaction itself. This apparent contradiction has been resolved by earlier works on nuclear matter[1-4] but, due to its theoretical character, the explanation and some of its consequences are still largely unknown. In particular, it seems currently believed among those who are not aware about these theoretical subtleties that nucleons in a Woods-Saxon well, or submitted to 2-body effective forces (like a Skyrme force for instance) are identical to free ones as it would be for an electron in the Coulomb field produced by other charged particles in the atom. The essential difference between nuclear and atomic shell models, which resides in the dressed character of the nucleon in the first case is largely ignored. It is possible that, for a part, the above situation has its origin in Hartree-Fock approaches themselves, whose successes have supported the idea of a complete analogy of the nuclear shell model with the atomic one, the effective character of the interaction in the first case being forgotten.

There are in the past or recent literature several examples where the "nucleon in nuclei" is identified to the free nucleon without any founded justification. Predictions of charge form factors of nuclei from Hartree-Fock approaches with effective forces generally employ the free nucleon charge form factor for the "nucleon in nuclei" without any word of caution. In the same approaches, Coulomb energies are calculated assuming that the Coulomb interaction of "nucleons in nuclei" is the same as for free nucleons. In an other field, the isoscalar nucleon structure function was first measured in iron, this nucleon being considered as a target of free nucleons moving independently. In the 3 above examples, the independent nucleons the nucleus is supposed to be made of are considered as free nucleons. It is also because nucleons in nuclei were assumed to be identical to the free ones that it could be asserted that calculations in nuclei were missing Pauli contributions at the quark level. Finally, although the example is somewhat different, it is because the "nucleon in nuclei" is generally considered to be uniquely defined that the origin of the quenching of Gamov-Teller transitions was so strongly debated. It is our belief that the two current explanations, excitation of the Δ resonance and tensor correlations, may well coexist in some energy range if one accepts the idea that they essentially refer to different degrees of freedom, closer to quasi-particles in the first case, to free particles in the second one.

In our mind, the above situation has partly its origin in the fact that we are usually dealing with amplitudes, and not with the degrees of freedom they are referring to. To some extent, this is understandable since wave-functions are not observables and the degrees of freedom used for its modelization, just as the choice of some basis, can be chosen at will [5]. Moreover, in the case of nuclear matter, there is a one to one correspondance which is rather misleading. As a result, it might be thought that the nucleon in a nuclear environnement is the same as the free one with, at best, different properties. The idea that different properties may be connected with a modification of the nucleon in the nuclear medium, which would have its origin in the dressing of nucleons by eliminated degrees of freedom is not currently considered. In this paper, we want to present our viewpoint on the example of nuclear matter and explicitly show that : i) nuclear matter can be described to a good approximation, if not exactly, by an independent particle state, ii) the current on-shell G matrix used in calculating its binding energy does refer to these particles and not to free nucleons as sometimes thought. In this order, we designed an approach to nuclear matter which puts the emphasis on the degrees of freedom and is different from other approaches, as far as we know.

Our starting point is the two-body interaction :

$$H(a) = \Sigma\, a^{+}_{i} \frac{p^{2}_{i}}{2M} a_{i} + \frac{1}{2} \sum_{ijk\ell} a^{+}_{i}\, a^{+}_{j} < i\, j \mid V \mid k\ell > a_{k}\, a_{\ell} \tag{1}$$

where a_i and a^{+}_{i} are destruction and creation operators of a nucleon with momentum p_i.They are considered as structureless objects, although they represent nucleons dressed by the mesons whose elimination as dynamical degrees of freedom has given rise to the interaction V. Quite generally, the groundstate of nuclear matter can be written as a sum of products of creation operators acting on the vacuum $\mid 0 >$:

$$\mid \phi> = \sum_{C} x_{C}(\prod_{i\in C} a^{+}_{i}) \mid 0 > \tag{2}$$

where C denotes some configuration and x_C its relative amplitude.

The question we are interested in concerns the possibility to rewrite the state$\mid \phi>$ given by (2) as a unique product of creation operators, namely :

$$\mid \phi> = (\prod_{i\in C_0} A^{+}_{i}) \mid 0 > \tag{3}$$

In this expression, C_0 refers to the configuration where the individual states with a momentum lower than the Fermi momentum are occupied, while the operators A^{+}_{i} (together with A_j) obey the usual anticommutation relations of fermions, but refer to objects we will call particles in the following.

Part of the answer to our question is provided by Goldstone[3] who showed that the correlated nucleon state, $\mid \phi>$, given by (2) can be written as the product of an unitary operator acting on the ground state of non correlated nucleons :

$$| \phi > = U \, | \phi_0 > \tag{4}$$

with

$$| \phi_0 > = (\prod_{i \in C_0} a^+_i) \, | 0 > \tag{5}$$

Inserting between the a^+_i in (5) the unit operator written as $1 = U^+ U$ then provides us with the desired result by making the following identification :

$$A^+_i = U \, a^+_i \, U^+ \tag{6}$$

and by taking into account the fact that the operator U acting on the vacuum $| 0 >$ can only give a phase which can be absorbed into its definition. It is important to notice that the above result only holds for the ground state of nuclear matter which is generally supposed to be unique. It does not apply in presence of degenerate states.

In the following, we detail the construction of the operator U, which can be written as exp (-S(A)) where S(A) is an anti-hermitic operator. Inverting (6), we have :

$$\begin{aligned} a^+ &= e^{S(A)} A^+ e^{-S(A)} \\ a &= e^{S(A)} A \, e^{-S(A)} \end{aligned} \tag{7}$$

After making the substitution (7) in (1), we then get an hamiltonian :

$$H(a) = H(S,A) = e^{S(A)} H(A) \; e^{-S(A)} \tag{8}$$

which in terms of particles can a priori excite from the ground state given by (3) components with 2 particles - 2 holes, 3 particles-3 holes, ... Components with 1 particle-1 hole are excluded by momentum conservation. In order that the state (3) be an eigenstate, it is necessary to choose the operator S(A), undetermined until here, in such a way that terms in (3) which can give rise to the above components cancel.

It is easy to see that the absence of 2p-2h will be assured at the lowest order by taking :

$$S(A) = S_2 (A) = \frac{1}{2} (A^+_{\nu_1} A^+_{\nu_2} A_{\rho_2} A_{\rho_1} - A^+_{\rho_1} A^+_{\rho_2} A_{\nu_2} A_{\nu_1}) \, \alpha_{\rho_1 \rho_2 \nu_2 \nu_1} \tag{9}$$

where the indices v and ρ respectively refer to particles below and above the Fermi level. Retaining only the dominant terms, the coefficients $\alpha_{\rho_1 \rho_2 v_2 v_1}$ will verify the following equations :

$$\left(\frac{P^2{}_{v_1}+P^2{}_{v_2}-P^2{}_{\rho_1}-P^2{}_{\rho_2}}{2M}+V_{v_1}+V_{v_2}-V_{\rho_1}-V_{\rho_2}\right)\ \alpha_{\rho_1\rho_2 v_2 v_1}$$

$$=<\rho_1\,\rho_2|\,V|\,v_2\,v_1>+\sum_{\rho'_1\rho'_2}<\rho_1\rho_2|V|\,\rho'_2\,\rho'_1>\alpha_{\rho_1\rho_2 v_2 v_1}+\ldots,$$

with $V_v = \sum_{v'} < vv'\,|V|\,v'v - vv' >$

$$V_\rho = \sum_{v'} < v'\rho|V|\rho v' - v'\rho> \qquad (10)$$

It can be checked that the above equation is just the one determining the on-shell G-matrix (to be identified to the first member).

To calculate the groundstate energy, terms involving the expansion of H(S,A) up to the second order in S have to be considered. Once this is done, the part of the hamiltonian which acts on the ground state given by (3) can be written :

$$H_{g.s.} = \sum_{v} A^+{}_v \frac{P_v{}^2}{2M} A_v + \frac{1}{2}\sum_{\substack{v_1 v_2\\ v'_1 v'_2}} A^+{}_{v_1} A^+{}_{v_2} <v_1\,v_2\,|G^o|\,v'_2 v'_1> A_{v'_2}\,A_{v'_1} \qquad (11)$$

with

$$< v_1 v_2\,|G^o|\,v'_2 v'_1> = < v_1 v_2\,|V|v'_2 v'_1> + \sum_{\rho'_1\rho'_2} <v_1 v_2\,|V|\rho'_2\rho'_1> \alpha_{\rho'_1\rho'_2 v'_2 v'_1} \qquad (12)$$

The expression achieves our demonstration since it clearly shows that the G matrix, which is often used as a model potential, does refer to objects which are different from free nucleons. More exactly, the modification of the potential V by the last term in (12), which is essential to get attraction, simultaneously appears with the dressing of a nucleon into a particle given by (6). For completeness, we give the expression for the particle creation operator in terms of creation and annihilation operators for nucleons :

$$A^{+}_{\nu} = a^{+}_{\nu} + \sum_{\rho_1 \rho_2 \nu'} a^{+}_{\rho_1} a^{+}_{\rho_2} a_{\nu'} (\alpha_{\rho_1 \rho_2 \nu' \nu} - \alpha_{\rho_1 \rho_2 \nu \nu'}) + \ldots \qquad (13)$$

As an approach to nuclear matter, the present one could be improved. On the other hand, it is not without relation with the e^S approach of Coester[6], but contrarily to what is done here, his e^S is not unitary. The unitary character of our e^S allows for a close similarity with Hartree Fock approaches, but with effective forces.

The results shown in this paper do not strictly apply to finite nuclei. We however believe that they may be of some validity for them in the same measure as Hartree-Fock approaches with effective forces are.

In this paper, we have shown (or reminded) that nuclear matter in its ground state, which can be considered as a state of correlated nucleons, could also be considered, at an other extreme, as a state of independent particles, quite similarly as for the Landau theory. In between, there is a continuum of descriptions corresponding to different unitary transformations performed on the nucleon. Obviously in such cases, it would not be any more possible to write the wave function in the form of an independent particle state. In the usual conception of a nucleus made of structureless objects, the above transformation is a pure mathematical one. However, as reminded at the beginning, nucleons have a meson cloud and to some extent the transformation just corresponds to perform a modification of the meson cloud of the nucleon. It can also be considered as a change in the basis used to describe the physics. The independent particle picture used together with effective forces might then be seen as a system of nucleons whose constituents, especially its meson cloud, would continuously be modified with the nuclear environnement, while keeping their identity.

One of the consequences of the above results is that the same physics can be pictured in quite different ways, depending on the choice of the degrees of freedom used to describe the nuclear system under consideration. In particular, employing an independent particle picture to describe the ground state of some system does not necessarily appear as an approximation as often thought, but rather as a less detailed description. When making comparison with experiment, in particular in the field of electron scattering which partly motivated the present work, it is therefore important to precise which modelization of the nucleus is used since each of them can allow for different interpretations. Referring to the example treated here, it may be said that what is interpreted as an effect of correlations by some people may well be interpreted by other ones as a modification in the nuclear medium of the intrinsic nucleon properties.

While we were thinking that the question raised in the title has a clear and unambiguous answer, at least in nuclear matter, we were surprised that many colleagues had quite different opinions about it. We are tempted to believe that some clarification is necessary and hope that the present paper will help in this aim. We are very grateful to all of them for having accepted to give their viewpoint. By advance, we acknowledge further comments that the present paper may raise.

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