RADIAL NODALIZATION EFFECTS ON
BWR STABILITY CALCULATIONS

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ABSTRACT

Computer simulations have shown that stability calculations in boiling water reactors (BWRs) are very sensitive to a number of input parameters and modeling assumptions. In particular, the number of thermohydraulic regions (i.e., channels) used in the calculation can affect the results of decay ratio calculations by as much as 30%. This paper presents the background theory behind the observed effects of radial nodalization in BWR stability calculations. The theory of how a radial power distribution can be simulated in time or frequency domain codes by using "representative" thermohydraulic regions is developed. The approximations involved in this method of solution are reviewed, and some examples of the effect of radial nodalization are presented based on LAPUR code solutions.

RESUME

Il a été démontré par le moyen d'ordinateur que la stabilité des réacteurs du type "BWR" est très sensible à un certain nombre de paramètres et de considérations de modèle choisie. En particulier, le nombre de régions thermohydraulique utilisée dans les calculs peut affecter les résultats de "decay ratio" aussi loin que 30%. Cette publication présente la théorie derrière l'effet observé d'une nodalisation radiale utilisée pour le calcul de la stabilité des réacteurs du type "BWR". La théorie basé sur la distribution radiale d'énergie simulé par les programmes utilisant l'espace temp ou l'espace fréquence est ceci par le billet d'une régionale thermohydraulique représentation, a été développé. Les approximations utilisées par cette méthode sont revues et quelques exemples de la nodalisation radiale sont présentées basé sur les résultats du programme LAPUR.
INTRODUCTION

This paper presents the background theory behind the observed effects of radial nodalization in BWR stability calculations. The theory of how a radial power distribution can be simulated in time or frequency domain codes by using "representative" thermohydraulic regions is developed. The approximations involved in this method of solution are reviewed, and some examples of the effect of radial nodalization are presented based on LAPUR [1,2] code solutions.

SPACE-DEPENDENT REACTIVITY FEEDBACK

The reactivity feedback, \( \Delta \rho \), due to a void perturbation, \( \Delta \alpha \), can be estimated in cylindrical coordinates form from the following expression:

\[
\Delta \rho(t) = \frac{1}{V} \int_0^H \left( \int_0^R \Phi^*(r,z) \Phi(r,z) \frac{d\rho}{d\alpha}(r,z,t) \Delta \alpha(r,z,t) 2\pi r dr \right) dz
\]

where \( \Phi \) and \( \Phi^* \) are the normalized neutron flux and its adjoint, respectively, \( \frac{d\rho}{d\alpha} \) is the local density reactivity coefficient, and \( V \) is the core volume.

To solve this equation in a digital computer code, the integrals in Eq. (1) are typically approximated by a summatory over a series of nodes. In this way, Eq. (1) becomes

\[
\Delta \rho(t) = \sum_{i=1}^{NR} \frac{N_{ir}}{N} \times P_{ir}^2 \times \Delta \rho_{ir}(t)
\]

where \( \Delta \rho_{ir}(t) \) is the time dependent reactivity feedback from region \( ir \), which is given by Eq. (3)

\[
\Delta \rho_{ir}(t) = \frac{1}{NZ} \sum_{iz=1}^{NZ} \varphi_{ir,iz}^2 \left( \frac{d\rho}{d\alpha}(t) \right)_{ir,iz} \Delta \alpha_{ir,iz}(t)
\]

In Eqs (2) and (3), \( NR \) is the number of representative thermohydraulic "regions" (also called thermohydraulic channels), and \( NZ \) is the number of axial nodes in each channel, \( N_{ir} \) is the number of bundles in region \( ir \), and \( N \) is the total number of bundles in the whole core. The parameters with an \( ir \) subscript represent region-averaged parameters, the \( iz \) subscript represent the axial node position; \( P_{ir} \) is the relative power of a bundle in region \( ir \), \( \frac{d\rho}{d\alpha}_{ir,iz} \) is the density reactivity coefficient of region \( ir \) as a function of axial node \( iz \), \( \Delta \alpha_{ir,iz} \) is the time dependent void perturbation in region \( ir \), and \( \varphi_{ir,iz} \) is the normalized axial flux (also called axial power shape) at axial node \( iz \) in region \( ir \). In Eq. (2), the adjoint flux has been assumed equal to the forward flux, which is an exact solution under one-energy-group diffusion theory.
Thus, the numerical solution of Eq (2) is only an approximation to the real reactivity feedback given by Eq. (1). Equation (2) reverts to Eq. (1) only if \( NR \) equals the total number of bundles in the core, and if \( NZ \) is large enough. The smaller the number of regions, the poorer the approximation. In this approximation, the radial dependence in Eq. (1) has been substituted by a series of representative channel calculations for each region. Typically, calculations are performed with a number of regions varying from 1 to 20; few codes are capable of modelling all the bundles in a core. Even for these codes, the calculations with such a large number of thermohydraulic regions are cost prohibitive.

The approach of Eq. (2) is essentially exact if the number of thermohydraulic regions, \( NR \), equals the number of bundles in the core; however, cost constraints typically result in a small number of regions being modeled. This modelling of the radial power distribution affects the reactivity-feedback calculations through two main effects: (1) The nonlinear weighing of the reactivity contribution from each region due to the \( P^2 \) weighing of the void fraction perturbation, and (2) the void dependence of the void reactivity coefficient.

Basically, the error involved in the above approximations can be expressed by the inequality expressed in Eq. (4), which says that the sum of the product is not equal to the product of the sum unless the operands are constant.

\[
\sum_{ir=1}^{NR} \frac{N_{ir}}{N} \left( P_{ir}^2 \times \Delta \rho_{ir}(t) \right) \neq \overline{P^2} \times \overline{\Delta \rho(t)}
\]

\[
\overline{P^2} = \sum_{ir=1}^{NR} \frac{N_{ir}}{N} P_{ir}^2, \quad \overline{\Delta \rho(t)} = \sum_{ir=1}^{NR} \frac{N_{ir}}{N} \Delta \rho_{ir}(t)
\]

The channel density feedback, \( \Delta \rho_{ir}(t) \), is a strong function of the channel power, \( P_{ir} \), and, thus, the inequality in Eq. (4) holds. \( \Delta \rho_{ir}(t) \) depends on \( P_{ir} \) through two effects: (1) directly through the generation of larger void perturbations, and (2) indirectly, through the effect of the density reactivity coefficient.

The direct effect is due to the fact that neutron flux tends to oscillate as a percentage of the initial steady state power. In other words, if the reactor power oscillates by 10%, all the channel powers oscillate by 10%. This results in a larger absolute power oscillation in the high power channels, which causes increased void fraction oscillations and, thus, larger reactivity feedback from the high power channels than from the low power channels. The indirect effect is due to the fact that the density reactivity coefficient is a strong function of the local void fraction, as seen in

![Figure 1. Typical density reactivity coefficient as a function of local density](image-url)
Fig. 1. The larger the void fraction (i.e., smaller density), the larger the magnitude of the reactivity coefficient. Thus, high power channels have larger reactivity feedback coefficients than low power channels.

Typical fuel loadings result in what is known as a checker board arrangement, where fresh fuel is loaded in one third of the core and is surrounded by two thirds of old irradiated fuel. This loading results in a third of the bundles having high power (for example, 140% of average power) and the remaining two thirds having low power (approximately 80%). If only one representative region or channel (i.e., \( NR = 1 \)) is used to model this type of core, reactivity feedback can be seriously underestimated. For example, in a checker board pattern like the one described above, a simple calculation indicates that the reactivity feedback with a single average channel is underestimated by 25%. This is equivalent to reducing the density reactivity coefficient by the same factor, which has dramatic effects on the estimated stability of the core.

Therefore, in order to model accurately the reactivity feedback, a sufficient number of thermohydraulic regions need to be represented in the model to describe the radial power distribution accurately.

**SPACE-DEPENDENT INLET FLOW FEEDBACK**

In the previous section we have discussed the effect the radial power distribution has on the reactivity feedback. It is well known that BWR dynamics relevant to density-wave instabilities have two main feedback paths: (1) the reactivity feedback and (2) the core inlet flow feedback. The reason we have discussed the reactivity feedback first is simply because it is easier to understand, not because of its importance. Indeed, the inlet flow feedback, which constitutes part of the thermodynamics of the channel, is probably more affected by the radial power distribution than the reactivity feedback.

A typical BWR core is composed of a large number of independent bundles that are connected hydraulically at both ends by the upper and lower plena. Because of these large plena, the pressure drop across all bundles must be maintained equal among all channels, although not necessarily constant in time, so that low power channels must have the same pressure drop as high power channels. Thus, to maintain this boundary condition, the inlet flow in the high power channels must oscillate significantly more than in the low power channels. This effect is caused by the fact that, as described above, the void oscillations are larger in the high power channels. This has a very significant effect on channel pressure drop; thus, the inlet flow in the high power channels has to compensate more for the increased flow oscillations, resulting in a more unstable configuration.

The effect of the radial power distribution can be observed in Table I, that shows the decay ratio calculated by the LAPUR code for a core configuration formed by two thermohydraulic regions of an equal number of bundles. We observe that as the radial power distribution becomes more skewed (i.e., power is taken from Region 1 to region 2), the reactor becomes more unstable. The conditions for Table I are 1500 MWth power and 27 Mlb/h flow, with a sinusoidal axial power shape.
An interesting result that highlights the effect of radial power distributions on inlet flow feedback can be observed in Table II. From this table, we can conclude that a 1500 MWth reactor with a skewed radial power shape is more unstable than a reactor condition with a higher power. For example, if the power is raised to 2100 MWth (approximately 140% of 1500 MWth) but only one thermohydraulic region is used, the calculated decay ratio is only 0.68, which is larger than the one-region decay ratio at 1500 MWth (0.54), but smaller than the two region (60%, 140%) reactor with only 1500 MWth that results in a decay ratio of 0.98 (see Table I). This is a counter intuitive result. It says that if we take a reactor that operates at 2100 MWth and reduce the power of half of the bundles (maintaining the rest of the bundles at constant power) so that the average power is now 1500 MWth, the resulting lower-power configuration is more unstable than the original high power configuration.

The above effect can be explained by the effect the low power channels have on the inlet flow feedback of the high power channels. In the low power channels, the void fraction does not oscillate as much as in the high power channels following a reactivity perturbation. The large void fraction oscillation in the high power channels would tend to produce large pressure drop oscillations; however, the pressure drop across the high power channels must equal that of the low power channels. To accomplish this, the high-power channels inlet flow must have greater oscillation amplitude, resulting in an increased flow feedback that destabilizes these channels and the whole core.

### Table I. Effect of checker board pattern loading on core-wide decay ratio

<table>
<thead>
<tr>
<th>Relative Power</th>
<th>Decay Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Region 1</td>
<td>Region 2</td>
</tr>
<tr>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>80%</td>
<td>120%</td>
</tr>
<tr>
<td>60%</td>
<td>140%</td>
</tr>
<tr>
<td>40%</td>
<td>160%</td>
</tr>
</tbody>
</table>

### Table II. Sensitivity of calculated core-wide decay ratio to recirculation loop gain for different radial power distributions

<table>
<thead>
<tr>
<th>Relative Power</th>
<th>Decay Ratio Calculated With Recirculation Loop Gain Multiplier</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.</td>
</tr>
<tr>
<td>Region 1</td>
<td>Region 2</td>
</tr>
<tr>
<td>60%</td>
<td>140%</td>
</tr>
<tr>
<td>140%</td>
<td>140%</td>
</tr>
</tbody>
</table>
The above theory is confirmed by the data in Table II, that shows the decay ratio as function of the gain of the recirculation loop pressure-drop-to-flow transfer. This gain may be increased physically by reducing the friction in the recirculation loop. If this loop has no friction, then it has an infinite gain and the pressure drop is maintained constant (and equal to the density plus pump heads) regardless of the amount of core inlet flow required. A large gain, then, essentially decouples the core channels from each other by forcing a constant pressure drop boundary condition. A small gain forces a constant core inlet flow and the dynamic flow distribution between channels has a dominant effect. This effect is seen in Table II. For high gain values, the channels are uncoupled and the one-region core with the highest power is more unstable. For low gain values, the two-region core (i.e. 60% 140% powers) is more unstable than the one-region core, because the low power channels act as a source of flow to the high power channels by oscillating its inlet flow out-of-phase.

In physical terms we can explain this process as follows: when a perturbation of power causes an increase of voids in the high power channels, the low power channels increase their flow to attempt to have the same pressure drop as the high power channels. This reduces the flow available for the high power channels and, consequently, reinforces the original oscillation (i.e., more voids are produced). Thus this configuration is more unstable. For the case in which the recirculation loop has low friction (i.e., high gain), the increase in pressure drop in the high-power channels is compensated by an increase in total flow, most of which is redirected through the low-power channels.

The above effect can be observed in Figs 2 and 3. These figures show the gain and phase of the LAPUR calculated transfer function from total power to individual channel inlet flow in power to individual channel inlet flow

![Figure 2](image1.png)  
**Figure 2.** Gain of transfer function from total power to individual channel inlet flow

![Figure 3](image2.png)  
**Figure 3.** Phase of transfer function from total power to individual channel inlet flow
normalized units. As can be observed, the low power channel has a smaller gain, which indicates that the flow oscillations in this type of channel are smaller than in the high power channel for the same total power perturbation. These results were expected from the above discussion, and tend to confirm that the low power channels act as a destabilizer for the high power channels by forcing a smaller oscillation in the core pressure drop. The phases of these transfer functions (Fig 3), as expected, are out of phase (i.e., 180° apart), which indicates that the increased flow through the high power channels is somehow compensated by the decreased flow through the low power channel. An interpretation of this effect is that the low power channel act as a source of flow oscillations for the high power channel, a fact that tends to destabilize the whole core. It has to be noted that both of these transfer functions are calculated by LAPUR for the core-wide in-phase mode of power oscillation, which (as seen in Figs 2 and 3) does not preclude some form of out-of-phase radial flow oscillations.

What we have seen in this section is that flow redistribution among bundles of different power is a significant effect that must be modeled accurately to estimate the stability of a particular reactor configuration. This flow redistribution is only possible if the radial power distribution has been modeled accurately enough to allow for different pressure drop responses following a power perturbation. The recirculation loop dynamics also plays an extremely important role in allowing this flow redistribution to occur. Proper modeling is, thus, required for the recirculation loop that includes the upper and lower plena, the separators and driers, the downcommer and the recirculation pumps.

HOT CHANNEL DESTABILIZATION

We have seen in the previous two sections the destabilizing effect of skewed radial power distributions by two mechanisms: the nonlinear power-square weighing of the reactivity feedback, and the inlet flow redistribution. A third mechanism is the hot channel destabilization. The flow dynamics in high power channels is, in general, more unstable than in low power channels. This is due to the fact that high power channels produce more voids and, consequently, have a larger two-phase pressure drop component that tends to make the channel flow oscillate more. When one combines this effect with the power-square weighing of the reactivity feedback, it is observed that the increased instability of the high power channels is not compensated by the decreased instability of the low power channels. For instance, if a core is composed of half the channels with 60% power and the other half with 140%, the high power channels weight when computing the reactivity feedback is 196% (i.e., $1.4^2$), while the weight of the low power channels is only 36% (i.e, $0.6^2$). Thus, the total core stability is heavily affected by the stability of the high power channels, while the low power channels have a relatively small effect. Thus, from this point of view, a skewed radial power distribution is a destabilizing effect, because the core stability is controlled in a large way by the hot channel power and not by the average power.
THE CLOSED-LOOP SYSTEM SOLUTION

We have discussed in the above sections some of the pieces of the whole system as they affect the solution. In reality, the radial power distribution affects the final result (i.e., the reactor stability) through all the mechanisms described above together, and probably some others too. The only reasonable way to study the effect on the whole system is to perform a series of whole-reactor calculations for expected operating conditions and to study the effect of different assumptions on the results. To this end we have taken a relatively extreme (but probable) radial power distribution and performed a series of decay ratio calculations using different radial nodalization schemes with the LAPUR code.

Figure 4 presents the result of these analyses. LAPUR can model a maximum of seven thermohydraulic regions; thus, the seven-region calculation is used as a reference radial power distribution model. This model corresponds to the first case in Fig. 4, where we see that there are 83 bundles with relative power $P_1 = 1.47$, 87 bundles with $P_2 = 1.40$, and so on. The last 92 bundles correspond to the periphery region which has a high inlet flow restriction and low power, $P_7 = 0.36$. As stated before, this radial power distribution is one of the worst expected during normal reactor operation.

The results of our analysis show that, if all the bundles are grouped into a single region (Case 7 in Fig. 4), the calculated decay ratio is 0.86, which compares with a 1.11 value for the reference case. Thus, using a reduced number of regions may result in calculated decay ratios that are 30% lower than more accurate calculations with detailed radial power distributions. The last two cases in Fig. 4 shows that, if code limitations permit only a small number of regions, it is better to model the high power channels with more detail than the low power channels. Thus, it is recommended that the maximum number of regions allowed by the particular code be used for stability calculations, with a minimum of 5 or 6 regions.

<table>
<thead>
<tr>
<th>In Friction</th>
<th>low</th>
<th>low</th>
<th>low</th>
<th>low</th>
<th>low</th>
<th>low</th>
<th>high</th>
<th>Decay Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>N Chann</td>
<td>83</td>
<td>87</td>
<td>100</td>
<td>110</td>
<td>122</td>
<td>170</td>
<td>92</td>
<td></td>
</tr>
<tr>
<td>Power</td>
<td>1.47</td>
<td>1.40</td>
<td>1.22</td>
<td>1.11</td>
<td>0.99</td>
<td>0.71</td>
<td>0.36</td>
<td>1.11</td>
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<tr>
<td>Power</td>
<td>1.47</td>
<td>1.40</td>
<td>1.22</td>
<td>1.05</td>
<td>0.91</td>
<td>0.71</td>
<td>0.36</td>
<td>1.06</td>
</tr>
<tr>
<td>Power</td>
<td>1.47</td>
<td>1.40</td>
<td>1.22</td>
<td></td>
<td>0.97</td>
<td>0.71</td>
<td>0.36</td>
<td>1.01</td>
</tr>
<tr>
<td>Power</td>
<td>1.47</td>
<td>1.40</td>
<td>0.97</td>
<td></td>
<td></td>
<td></td>
<td>0.36</td>
<td>1.00</td>
</tr>
<tr>
<td>Power</td>
<td>1.09</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.36</td>
<td>0.91</td>
</tr>
<tr>
<td>Power</td>
<td>1.00</td>
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<td></td>
<td></td>
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<td></td>
<td></td>
<td>0.86</td>
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<tr>
<td>Power</td>
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<td></td>
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<td></td>
<td></td>
<td>0.36</td>
<td>1.08</td>
</tr>
<tr>
<td>Power</td>
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<td>1.22</td>
<td></td>
<td>0.88</td>
<td></td>
<td>1.17</td>
<td></td>
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</table>

Figure 4. LAPUR calculated decay ratios for different radial power distribution models
ACKNOWLEDGEMENTS

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