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A LANGEVIN SIMULATION OF THE GROSS-NEVEU SPECTRUM*

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We study the order parameter of chiral symmetry, and fermion and boson masses in the Gross-Neveu model as a function of the flavour number N and of the Langevin time step ϵ, in the scaling region. The 1/N dependence of the ϵ=0 value of the order parameter is in excellent agreement with an analytical calculation up to second order. Care is taken of the important two fermion contribution in the bosonic correlation functions. Mass ratios are found to be ϵ dependent, but their ϵ=0 extrapolation is compatible with the analytic expectation.

1. INTRODUCTION

The Gross-Neveu model [1,2] (2 d model whith n_f copies of interacting fermions without gauge fields) is far simpler than QCD . Thus it allows the study of algorithms used in numerical simulation of QCD including the fermionic determinant. For this reason our simulation does not take advantage of d=2 peculiar properties but use standard Langevin technics with noise estimator of the fermionic determinant. Comparison of numerical results with analytical calculations (mainly from saddle point method , thanks to asymptotic freedom) gives a deep control on our simulation.

An interesting property of the Gross-Neveu model is its spectrum which satisfies [2,3]

$$m_n = 2m \sin \frac{\pi n}{2(n_f - 1)}$$

where m_n is the mass of a fermion (boson) for n odd (even), m being the kink mass. From that formula one expects the first boson mass being lower than the two fermion cut and thus we are faced to the very interesting problem of the separation of bound state and continuum contributions. Once the masses are determined, one can study the dependence of mass ratio on the discretised Langevin time step.

As in QCD correlations can be built out of the square of the fermionic propagator, $G(0,x)^2$. Here one can also build a correlation function which is related to the connected part of $G(0,0)G(x,x)$ (without full inversion of the propagator) and compare the masses of the two flavour singlet bosons.

2. THE SIMULATION

We simulate a lattice version of the model with Susskind fermions [4]. Once the Grassmann variables have been integrated out one has an effective action in terms of σ fields conjugate to $\overline{\chi}\chi$

$$S_{eff} = N\left(\sum_x \frac{\sigma_x^2}{2\lambda} - Tr\ ln(D+\Sigma)\right)$$

where D is the Susskind differential operator and

$$\Sigma = \frac{1}{4}\delta_{xy}(\sigma_x + \sigma_{x-\hat{1}} + \sigma_{x-\hat{2}} + \sigma_{x-\hat{1}-\hat{2}})$$

with N twice the continuum flavour number n_f.

This action allows analytical calculations with the saddle point method [5,6], in particular for the order parameter of the spontaneously broken chiral symmetry which can be taken as $< \sigma >$. It determines also our numerical simulation, the σ field configurations being generated with the Langevin

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algorithm

$$\sigma_x^{n+1} = \sigma_x^n - \epsilon N\left(\frac{\sigma_x^n}{\lambda} - \frac{1}{2}\eta^b(D+\Sigma)^{-1}\frac{\delta\Sigma}{\delta\sigma_x^n}\eta^b\right) + \sqrt{\epsilon}\eta_x^a$$

where ϵ is the Langevin time step size and η^a and η^b are orthonormalized (to 2) gaussian white noises. The total number of iterations n_{max} (after thermalization) varies from 100 000 to 1 600 000 according to the parameter (N,λ,ϵ) values; it was 800 000 for correlation function calculations. We have used 20x20 lattices at N=60,24,12 and 6 and a size 40x40 at N=2 to eliminate any finite size effect. At each iteration we kept mean values on the lattice. Correlations in the Langevin time of these quantities gave control on the errors along with a binning procedure and indicated if ϵ was small enough.

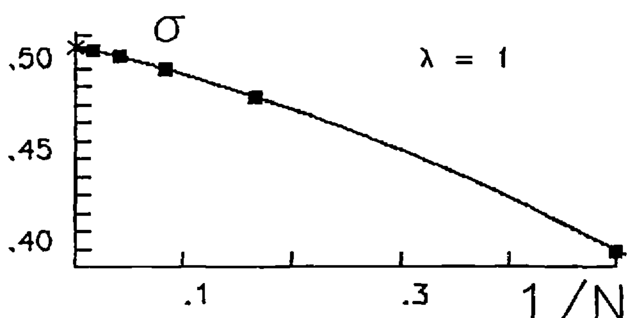

Fig. 1

Fig.1 presents data for $<\sigma>$ extrapolated to ϵ=0, as a function of 1/N. The curve results from a quadratic fit, which compares as follows to a 1-loop calculation

$$\sigma^{fit}(N) = .5116(2) - .143(7)\frac{1}{N} - .16(4)\frac{1}{N^2}$$

$$\sigma^{th}(N) = .5114 - .1473\frac{1}{N} + O(\frac{1}{N^2})$$

Encouraged by this excellent agreement we next consider the following correlation functions

$$C_F(t) = <\frac{1}{L_2}\sum_{x_2}(D+\Sigma)^{-1}_{0x}>$$

$$C_\sigma^\pm(t) = <\frac{1}{L_2}\sum_{x_2}(\pm)^{x_2}\sigma(0)\sigma(x)>$$

$$C_B^\pm(t) = <\frac{1}{L_2}\sum_{x_2}(\pm)^{x_2}[(D+\Sigma)^{-1}_{0x}]^2>$$

It is convenient to replace C_σ^+ by

$$C_\sigma^0(t) = C_\sigma^+(t) - \frac{\lambda}{N}\delta_{t0}$$

$$+\frac{\lambda^2}{4N}(-)^t[2C_B^-(t) - C_B^-(t+1) - C_B^-(t-1)]$$

because one can show that

$$C_\sigma^0(t) \simeq \lambda^2\sum_{x_2}<(D+\Sigma)^{-1}_{00}(D+\Sigma)^{-1}_{xx}>_c$$

whereas

$$C_\sigma^-(t) = \frac{\lambda}{N}\delta_{t0}$$

3. THE SPECTRUM

The spectrum is obtained from the analysis of C_F,C_B^+,$C_B^-$and C_σ^0 whose t-dependence at N=2,λ=.8 and ϵ=.01 are shown in Fig.2 respectively in (a),(b),(c) and (d) (in fact (c) corresponds to $|C_B^-|$, with triangles instead of boxes for negative values)

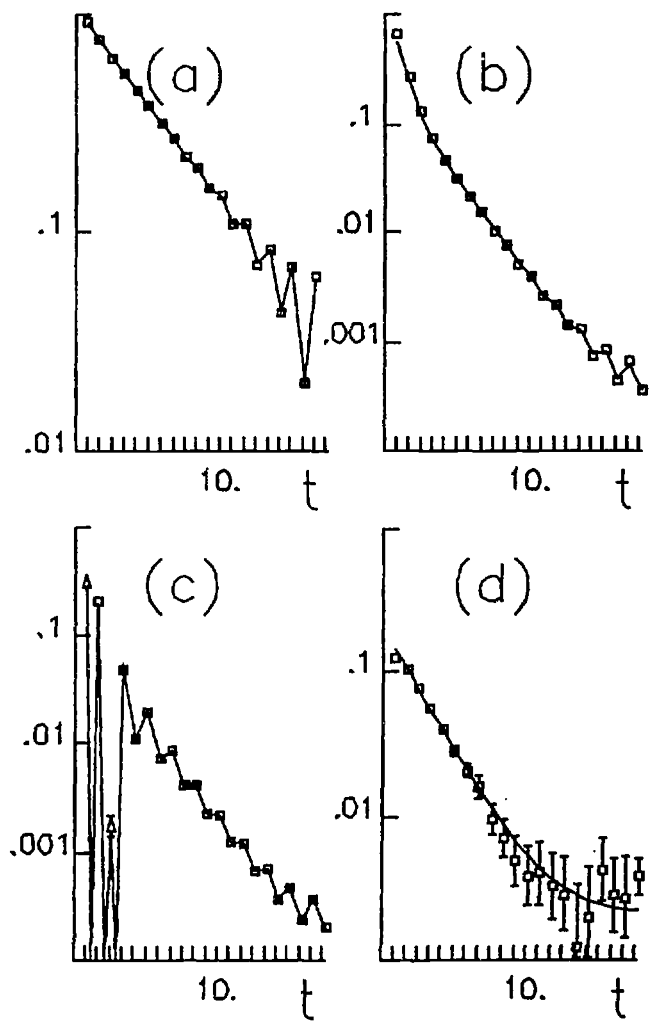

Fig. 2

The fit shown includes standard fermion and boson pole contributions, but also that of the fermion-antifermion continuum [6] to $C_B^{\pm}$. No agreement can be found without both pole *and* cut contributions.

In the $N = \infty$ limit one expects free fermion with mass $m_f = \sinh^{-1}(\sigma)$ and residue R=1. Fig.3 shows $sinh(m_f)/\sigma$ and R_f at $\lambda = 1$ as a function of ϵN, for N=60 (full triangles), N=24 (full boxes), N=6 (empty triangles) and N=2 (empty boxes). Both quantities actually get closer and closer to 1 as 1/N and ϵ decrease. Note that the ratio $\sinh(m_f)/\sigma$ remains ϵ dependent, with however a weaker variation than m_f or σ.

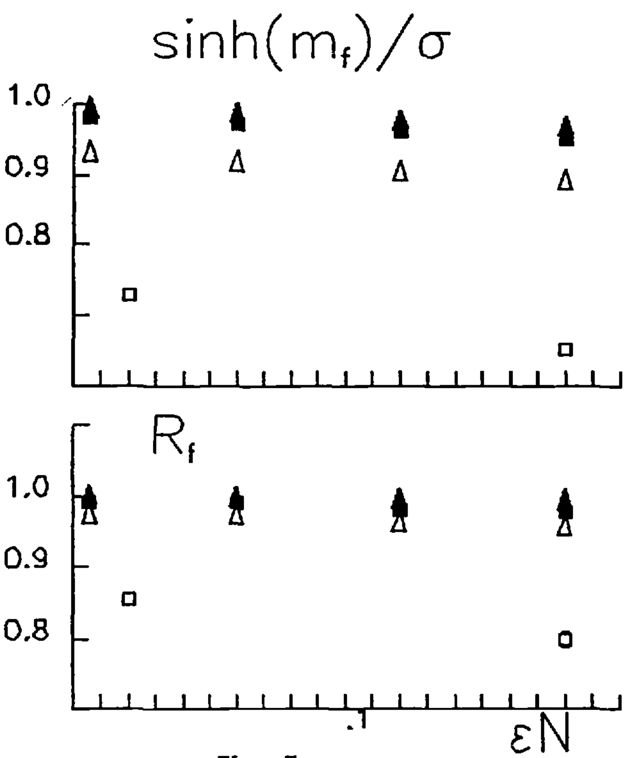

Fig. 3

The analysis of $C_B^+(t)$ clearly shows a "π" bound state ($J^{PC} = 0^{-+}$) without any oscillating contribution from $J^{PC} = 0^{+-}$:the oscillations observed in (b) of Fig.2 are all given by the cut contribution. The π residue is weakly ϵ dependent: it decreases with N increasing in agreement with a pure fermion-antifermion contribution at $N = \infty$. The corresponding masses are compared to $2m_f$ in Fig.4 where horizontal lines indicate the analytical result

$$\frac{2m_f}{m_b} = 2\frac{\sin(\pi/2(2N-1))}{\sin(\pi/(2N-1))}$$

At N=60,24 and 6 there is agreement for all ϵ values. As the cut contribution is dominating, such an agreement may appear as not very significant and related to corrections to cut contribution. We have tried to avoid such a confusion but some uncertainty should be included in a systematic error. Nevertheless the results at N=2 are very significant because the larger t extent and the mass ratio substantially different from 1 allow a better cut/pole separation. We found a clear ϵ dependence, but the agreement with the analytical result is quite good at ϵ=0. This means that the Langevin time discretization generates new interactions.

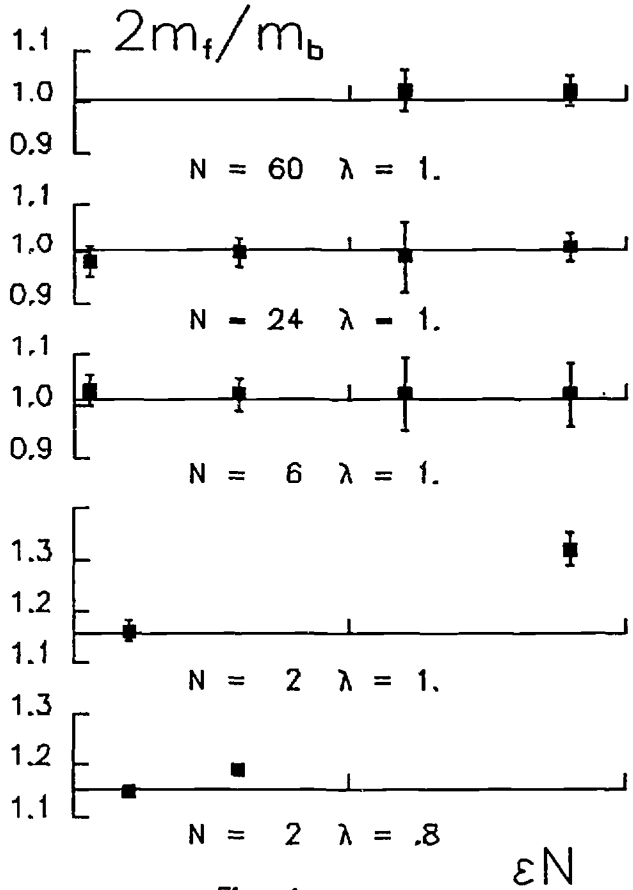

Fig. 4

In the analysis of $C_B^-(t)$ we have found signals for two bound states, with $J^{PC} = 0^{-+}$ and 0^{++}. However the residues of these states appear to be much smaller than in C_B^+ by a factor of 4-6.

So the need for a precise determination of the cut contribution, here assumed to be that of *free* fermions, is still more important than for the C_B^+ case. This sensitivity to the cut parametrization is probably the reason why our results for the ratio

π'/π of the 0^{-+} masses in C_B^- and C_B^+ (Table 1, statistical error only) are significantly different from 1.

	$\lambda = 1.$	$\lambda = 1.$	$\lambda = .8$	$\lambda = .8$
ϵ	.01	.09	.01	.03
π'/π	.94 (3)	.89 (5)	.78 (5)	.74 (2)

Table 1: Ratio of 0^{-+} masses in C_B^- and C_B^+.

In the same way we can compare the 0^{++} mass in C_B^- with the one obtained in C_σ^0. Their ratios are given in Table 2 without errors. Indeed for the correlation C_σ^0 the signal/noise ratio is too small in individual bins to allow for an error estimate. The discrepancy at $\lambda = 1$ presumably comes from an ill-determined value. On the contrary the result at $\lambda = 0.8$ agrees with symmetry restoration.

	$\lambda = 1.$	$\lambda = 1.$	$\lambda = .8$	$\lambda = .8$
ϵ	.01	.09	.01	.03
σ'/σ	.7	.8	1.0	1.1

Table 2: Ratio of 0^{++} masses in C_B^- and C_σ^0.

As a check of our statistical sample we have analyzed $C_\sigma^-(t)$, which should be $\lambda/N\delta_{t0}$ at ϵ=0. At $t \neq 0$, we actually find a result compatible with 0 within errors. The quantity $NC_\sigma^-/\lambda - 1$ at t=0 is shown in Table 3 : the data are clearly compatible with a vanishing extrapolation at $\epsilon = 0$. Such an impressive agreement makes us confident in our $\sigma\sigma$ correlation despite the statistical noise, and it means that we were able to evaluate $< (D+m)^{-1}_{00}(D+m)^{-1}_{xx} >_\epsilon$ without huge matrix inversion.

4. CONCLUSION

We have shown that a high statistics Langevin simulation of a particular asymptotically free theory allows for a good insight into the spectrum.

0	.18	.06	.02	.018	.006
1	.16(2)	.051(2)		.012(4)	.003(6)
2				.013(4)	.005(2)
3	.098(2)		.008(2)		
4		.037(3)	.012(3)		

Table 3: $C_\sigma^-(0)N/\lambda - 1$ as function of ϵN (line 0) for [λ,N] = [.7,60] [.7,6] [1.,2] and [.8,2] (lines 1,2,3 and 4)

Althought the separation of bound state and cut contribution was difficult, especially when the cut position is close to the pole, we give evidence for several bound states of the model. In the best case (π state), where the cut contribution is not overwhelming, we obtained an excellent agreement with the analytical expectation (Fig. 4). However this result can be reached only after extrapolation at zero Langevin time step.

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