CONF IAEA-CN--53/D-4-13

DE91 002632

INTERNATIONAL ATOMIC ENERGY AGENCY

THIRTEENTH INTERNATIONAL CONFERENCE ON
PLASMA PHYSICS AND CONTROLLED NUCLEAR FUSION RESEARCH

Washington, DC, United States of America, 1–6 October 1990

IAEA-CN-53/

D-4-13

TOKAMAK FLUIDLIKE EQUATIONS, WITH APPLICATIONS TO TURBULENCE AND TRANSPORT IN H MODE DISCHARGES*

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* This research was supported by the United States Department of Energy under Grant No DE-FG03-88ER-53275 (University of California at San Diego), Grant No. DE-FG02-86ER53218 (University of Wisconsin, Madison), and Contract No. DE-AC05-84OR21400 (Oak Ridge National Laboratory and Martin Marietta Energy Systems, Inc.).

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Abstract

Significant progress has been made in developing tokamak fluidlike equations which are valid in all collisionality regimes in toroidal devices, and their applications to turbulence and transport in tokamaks. The areas highlighted in this paper include: (1) the rigorous derivation of tokamak fluidlike equations via a generalized Chapman-Enskog procedure in various collisionality regimes and on various time scales; (2) their application to collisionless and collisional drift wave models in a sheared slab geometry; (3) applications to neoclassical drift wave turbulence; i.e. neoclassical ion-temperature-gradient-driven turbulence and neoclassical electron-drift-wave turbulence; (4) applications to neoclassical bootstrap-current-driven turbulence; (5) numerical simulation of nonlinear bootstrap-current-driven turbulence and tearing mode turbulence; (6) transport in Hot-Ion H mode discharges.

1 INTRODUCTION

Neoclassical MHD theory was first set forth four years ago at the Kyoto IAEA meeting [1]. Considerable progress was reported two years ago at the Nice IAEA meeting [2]. Since then the theory has been developed in two parallel approaches: One is to develop neoclassical MHD equations (here called "tokamak fluidlike equations" to include kinetic effects) rigorously based on a generalized Chapman-Enskog procedure. Using this procedure, closure relations are being developed for various time scales, mode structures and collisionality regimes, including kinetic effects. The other is to refine the theory of neoclassical bootstrap-current-driven turbulence, develop the theory of neoclassical drift wave turbulence and compare these results with experimental data. The remainder of this brief paper highlights recent developments and progress in tokamak fluidlike theory and its applications since last Nice IAEA meeting.

2 HYBRID FLUID/KINETIC MOMENT DESCRIPTIONS OF TOKAMAK PLASMAS:

Conventional fluid moment descriptions (resistive MHD, Braginskii equations [3], etc.) which are often used to explore macroscopic behavior, microscopic fluidlike instabilies, and transport properties of tokamak plasma, do not contain a number of important kinetic effects such as Landau damping, magnetic particle trapping, etc. On the other hand, conventional kinetic studies do not include a number of important fluid effects such as neoclassical polarization, poloidal flow damping, bootstrap current, etc. that emerge from fluid moment descriptions. In order to bridge the gap and to facilitate a comprehensive theory of tokamak plasmas that includes both types of effects, we have developed an exact generalized Chapman-Enskog approach for determining the non-Maxwellian part of the distribution function that is needed for calculating the fluid moment closure relations. The needed closure relations are obtained by calculating the viscous stress tensors $\mathbf{\Pi}$ and Θ from solutions of the kinetic equation for the distribution function.

The basic procedure is to divide the total distribution function f into a heat-flow-shifted Maxwellian and the remaining kinetic part of the distribution function, which must be determined from kinetic theory, depending on geometry, collisionality regime, time scale, mode structure, etc. By taking account of exact fluid moment equations for $\partial n/\partial t, \partial T/\partial t, \partial \mathbf{V}/\partial t$ and $\partial \mathbf{q}/\partial t$, we obtain the recasted kinetic equation for the kinetic part of distribution function F [4,5]:

$$\frac{dF}{dt} - C^l(F) = -f_m\left[\frac{2}{3p}\mathbf{\Pi} : \nabla \mathbf{V} L_1^{(1/2)} + \mathbf{v}' \cdot \mathbf{U} + \left(\mathbf{v}'\mathbf{v}' - \frac{v'^2}{3}\mathbf{I}\right) : \mathbf{W}\right], \quad (1)$$

where

$$\begin{aligned} f &\equiv f_m\left[1 + \frac{2\mathbf{v}'}{v_t^2}\cdot\left(-\frac{2\mathbf{q}}{5p}\right)L_1^{(3/2)}\right] + F, \\ \mathbf{U} &= -\frac{1}{p}\nabla\cdot\mathbf{\Pi}L_0^{(3/2)} - \frac{2}{5p}\nabla\cdot\Theta L_1^{(3/2)}, \\ \mathbf{W} &= \frac{m}{T}\nabla\mathbf{V}L_0^{(3/2)} - \frac{2m}{5pT}\nabla\mathbf{q}L_1^{(3/2)}. \end{aligned}$$

Here, higher order terms (in $\rho_\theta/L_\perp$) in RHS have been neglected for simplicity. Although Eq. (1) looks more complicated than the kinetic equation for the total distribution function, it has several advantages. It preserves exact conservation properties of $n, T, \mathbf{V}, \mathbf{q}$ on various time scales. Hence, it can be used to treat plasma instabilities with $\omega \sim \nu$, including both collisionless and collisional kinetic effects such as magnetic pumping, Landau damping, etc. It rigorously conserves fluid moments $(n, T, \mathbf{V}, \mathbf{q})$, which the kinetic approach sometimes fails to do. Also, it explicitly includes the irreversible, dissipative processes that are needed for net transport. Once we solve Eq. (1) for F and then viscous stress tensors $\mathbf{\Pi}$ and Θ, we can substitute these into moment equations to determine transport fluxes. In a strongly magnetized plasma, Eq. (1) can be gyroaveraged in a standard way to obtain the drift kinetic equation for $\bar{F}$. The resultant equation has been solved in equilibrium in toroidal geometry in references [6,7]. For $\partial \mathbf{q}/\partial t = 0, \partial \Pi/\partial t = 0$ and $\lambda_{mfp}\mathbf{b}\cdot\nabla \ln B \ll 1$, this formalism exactly reproduces the Braginskii's equations [3]. For $\omega \sim k_{\parallel}v_t \sim \nu$, we can obtain the collisionless and collisional Landau damping contribution to the viscous stress tensors [5]. Approximate dynamic viscosity coefficients in banana regime have been derived in ref. [8]. More accurate dynamic viscosity coefficients in all collisionality regimes are under development [9]. Also, procedures and closure relations are being developed for studying trapped-particle instabilities [10]. Treating trapped particles as seperate species, a reduced set of fluidlike equations in the long mean-free-path regime has been proposed [11] to exhibit the coupling of electrostatic trapped-particle modes with resistive ballooning modes in tokamak plasmas.

3 SHEARED SLAB ELECTRON DRIFT WAVES

The generalized Chapman-Enskog procedure has been applied to linearized drift type microinstability problems. Neglecting the $\partial \mathbf{q}/\partial t$ equation and assuming $\nabla T = 0$, Eq. (1) has been solved in a sheared slab geometry for a Krook-like collisional model. The linearized closure relations which include full Landau damping and some collisional effects can be written as [5]

$$\mathbf{b}\cdot\mathbf{k}\cdot\tilde{\Pi}_{\parallel} = \frac{ZZ''}{D} nmv_t k_{\parallel}\tilde{V}_{\parallel} + \frac{2ZZ' - Z''}{D} nk_{\parallel}\tilde{T} \qquad (2)$$

$$\mathbf{k}\cdot\tilde{\mathbf{q}} = \frac{2ZZ' - Z''}{D} pv_t k_{\parallel}\tilde{V}_{\parallel} + \frac{3Z''' + 12(Z')^2}{8D} nv_t k_{\parallel}\tilde{T} \qquad (3)$$

where $D \equiv -2ZZ' - Z''/2$, and Z is the plasma dispersion function with argument $\zeta \equiv (\omega + i\nu_{eff})/k_{\parallel}v_t$. It is easy to show that Eqs. (2) and (3) are consistent with the usual drift kinetic results. Our results show that Landau damping comes into the fluid equations through both the viscous stress and heat flux contributions, instead of just through the heat flux [12]. Various methods (asymptotic expansions and multipole curve fitting) can be used to simplify the above equations.

4 NEOCLASSICAL DRIFT WAVE TURBULENCE

The theory of collisionless fluid ion temperature-gradient-driven turbulence has been extended to the banana-plateau collisionality regime [13]. Neoclassical ion nonlinear fluid evolution equations for flute-type modes are developed and utilized to study ion temperature-gradient-driven modes in the banana-plateau regime. Neoclassical effects modify negative compressibility η_i-modes by: introducing parallel viscous damping which makes the long wave length parallel ion flow response dissipative rather than inertial. and enhancing the linear [14] and nonlinear polarization drifts by a factor of B_t^2/B_p^2. As a result of these modifications, growth rates become dissipative rather than sonic [i.e., $\gamma \sim k_\parallel^2 c_s^2(\eta_i - 2/3)/\mu_i$] and radial mode widths are broadened $[\Delta_x \sim \rho_s(B_t/B_p)(1+\eta_i)^{1/2}]$. Neoclassical η_i-modes are fundamentally three-dimensional excitations. Thus, spectral transfer to small scale dissipation occurs, resulting in saturation. Renormalized turbulence theory is used to calculate the ion thermal diffusivity χ_i at saturation. For low k_θ $(\Delta\omega_{k_\theta} < \mu_i)$;

$$\chi_i \sim \left(\frac{3\pi}{4}\right)^2 \frac{c_s^2}{\mu_i}\rho_s^2 \frac{\overline{(k_\theta \rho_s)}^2}{L_s^2}\frac{B_t^4}{B_p^4}(1+\eta_i)^2(\eta_i - 2/3) \quad \text{for } \eta_i > \eta_{i\text{th}} = 2/3,$$

where $\overline{k_\theta}$ is restricted to $\overline{k_\theta \rho_s} < \mu_i(L_s/c_s)(B_p/B_t)(\eta_i - 2/3)^{-1/2}(1+\eta_i)^{-1/2}$. For moderate k_θ $(\Delta\omega_{k_\theta} > \mu_i)$;

$$\chi_i \sim \mu_i \rho_s^2 \frac{B_t^2}{B_p^2}(1+\eta_i).$$

In both cases, a strong favorable dependence on B_p (and hence I_p) is exhibited. Furthermore, the χ_i for the long wavelength mode exhibits favorable density scaling ($\chi_i \sim 1/\mu \sim 1/n$). Both of these results are in agreement with experimental findings.

Neoclassical ∇T_i-driven turbulence is a natural candidate for modelling of strong, moderate collisionality turbulence in L-mode edge plasmas and the L $\rightarrow$ H transition. This claim is motivated by the predominantly electrostatic character of such turbulence, the robust character of neoclassical ∇T_i-driven modes, and the observation that such modes typically have low threshold (i.e. $\eta_i \cong 2/3$). Moreover, the steepening of the edge density gradient which accompanies the L $\rightarrow$ H transition naturally quenches the turbulence, consistent with the notion of an edge transport barrier in H-mode. The quenching of neoclassical ∇T_i-driven turbulence causes simultaneous reduction in $\chi_i, \chi_\phi, \chi_e$ and D, consistent with experimental findings.

However the conventional ∇T_i-driven mode theories have difficulties in explaining plasma current scaling and are valid for the rather collisionless core region. These difficulties are resolved by the neoclassical ∇T_i-driven turbulence theory. Ongoing work in this area is concerned with exploring the effects of sheared rotation on the linear stability and nonlinear dynamics of neoclassical ∇T_i-driven turbulence.

The neoclassical polarization drift also changes the mode structure of the standard shear damped electron drift wave [15] from an outgoing mode to a localized one, and introduces an explicit B_p-dependence into the mode width through the neoclassical polarization, Hence, explicit favorable plasma current scaling will appear in neoclassical electron drift wave induced transport, as well [13]. Further progress in this topic awaits the development of a dynamic viscous damping coefficient valid for $\omega \sim \mu_i$ in low collisionality regimes.

5 NEOCLASSICAL BOOTSTRAP-CURRENT-DRIVEN TURBULENCE

The nonlinear evolution and saturation of neoclassical bootstrap-current-driven turbulence (NBCDT), evolving from linear bootstrap current driven instabilities [16] described by the neoclassical MHD equations has been studied [17]. For high-T_i discharges (such as DIII-D Hot-Ion H mode), the decorrelation rate $\Delta\omega_{\mathbf{k}}$ usually exceeds the neoclassical viscous damping frequency, so the enhancement factor in the neoclassical polarization is reduced to $(B_t^2/B_p^2)(\mu_i/\Delta\omega_{\mathbf{k}})$. The calculation of the electron heat transport resulting from stochastic magnetic fields driven by NBCDT is revisited. Taking account of high frequency modification, we obtain [19]

$$\chi_e \cong 0.09|v_\parallel|L_s\left(\frac{\epsilon}{q}\beta_p\right)\left(\frac{\mu_e}{\mu_e+\nu_e}\right)^{3/2}\frac{1}{k_\theta L_p}S_M^{-1/2}(\mu_i\tau_A)^{1/2}\Lambda^* \qquad (4)$$

$$\frac{<\tilde{B}_r>}{B} \cong 0.2\left(\frac{\epsilon}{q}\beta_p\right)\left(\frac{\mu_e}{\mu_e+\nu_e}\right)^{11/8}\left(\frac{L_s^3}{k_\theta L_p^4}\right)^{1/4}S_M^{-3/8}(\mu_i\tau_A)^{3/8}\Lambda^{-7/12}\Lambda^*,$$

where

$$\Lambda^* \equiv \Lambda^{7/3}\left(1-\frac{1}{\Lambda}\right)\left(1-\frac{\sqrt{2}}{\Lambda+\Lambda^3}\right)^{-1/2},$$

and other notations are explained in Ref. [17]. The magnetic fluctuation levels and associated electron thermal conduction are enhanced by increasing

β_p and a steep pressure gradient, but are suppressed by strong shear. While resistive MHD turbulence models [18] are relevant primarily at the edge of the plasma, the region of NBCDT applicability extends over a wide zone between the center and the edge of the plasma. Also, since NBCDT is aggravated as the pressure is increased by additional heating, NBCDT is of particular relevance to regimes of moderate to high plasma β_p, such as DIII-D Hot-Ion H-mode and TFTR Supershots.

6 SIMULATION OF BOOTSTRAP-CURRENT-DRIVEN TURBULENCE

Numerical calculations of NBCDT are of importance for: (a) identification of the turbulent saturation mechanism and as a test of the analytic theory, (b) to obtain the k_θ spectrum, which has not yet been analytically calculated, and (c) generating detailed 3-D isolated magnetic field structures for use in studies of electron thermal conductivity enhancement by parallel losses along ergodic field lines. The numerical model consists of a neoclassical Ohm's law, neoclassical parallel and perpendicular momentum balance equations, and a continuity equation. This results in four coupled time evolution equations for the poloidal flux function, the fluid vorticity, the density and the parallel ion flow velocity. If the assumption of rapid ion flow damping ($\mu_i \gg \gamma$) is invoked then the system can be reduced to three equations (the $V_{\parallel i}$ equation can be eliminated).

The linear numerical results have been compared to the analytical predictions. Results indicate generally good agreement. The only deviation occurs at high mode numbers where the numerical growth rates are somewhat lower than the analytical ones, due to the breakdown of the rapid parallel ion flow damping assumption used in the analysis. In the nonlinear regime, the numerical model has been investigated using both 3-field and 4-field versions. Although systematic comparisons have not yet been made between the two calculations, there do not appear to be any significant qualitative differences, in the parameter regimes which have been considered. In Figure 1 the typical time evolution for the 3-field model is shown with the fluctuating potential, density, and radial and poloidal components of magnetic field plotted at a fixed radial point ($r/a = 0.6$), demonstrating the achievement of saturation. This calculation was run with 111 modes and 200 radial grid points. The spectrum includes resonant modes for the rational surfaces over a radial range from $r/a = 0.39$ to 0.97. A comparison of the saturated radial and poloidal magnetic field fluctuation levels is shown in Fig. 2 where the analytical predictions [17] have been evaluated locally using the profile values and the saturated k_θ spectrum of the numerical calculation. As may be seen, there is semi-quantitative agreement between the nonlinear numerical results and analytical theory in the outer half of the plasma. Similar agreement results when the fluctuating densities and potentials are studied.

7 APPLICATIONS TO H MODE DISCHARGES

As an application of NBCDT model, consider core transport in Hot-Ion H-mode discharges. Such discharges have flat density profiles and are characterized by $\chi_e > \chi_i \sim \chi_\phi$. They thus present a challenge to conventional drift wave turbulence theory. However, the moderately high values of β attained in Hot-Ion H mode, along with the disparity between χ_e and χ_i, suggests that magnetic turbulence may control transport in such plasmas. We have undertaken detailed comparisons of the NBCDT model electron thermal diffusivity (χ_e) with the effective one obtained by power-balance calculations using profiles from DIII-D Hot-Ion H-mode plasmas [20]. Figure 3 shows the comparison of the theoretical prediction with the experimental results for a $B_t = 2T, I_p = 1.0MA, P = 8.8MW$ Hot Ion H-mode discharge. The agreement in the confinement zone is clearly good, both in regards to profile and magnitude of χ_e. Since $\nabla n \rightarrow 0$ in such plasmas, diamagnetic corrections to the neoclassical fluid turbulence theory are unimportant. The favorable results obtained in this comparison underscore the need for supporting fluctuation and runaway electron confinement studies, and suggest that NBCDT and other electromagnetic dissipative fluid turbulence models offer considerable promise, particularly in high-β regimes, and thus should be considered serious candidates.

8 SUMMARY

Tokamak fluidlike equations including kinetic effects (Landau damping, particle trapping, etc.) and geometrical effects (magnetic pumping, neoclassical polarization, bootstrap current, etc.) have been developed rigorously via a generalized Chapman-Enskog approach. These equations are applied to neoclassical drift wave turbulence and neoclassical bootstrap-current-driven turbulence models. The resultant transport fluxes and fluctuation levels seem to agree reasonably well with tokamak experimental results from DIII-D. Further developments of rigorous derivation of tokamak fluidlike equations and detailed comparison of neoclassical MHD turbulence predictions with experimental results are promising and should be encouraged.

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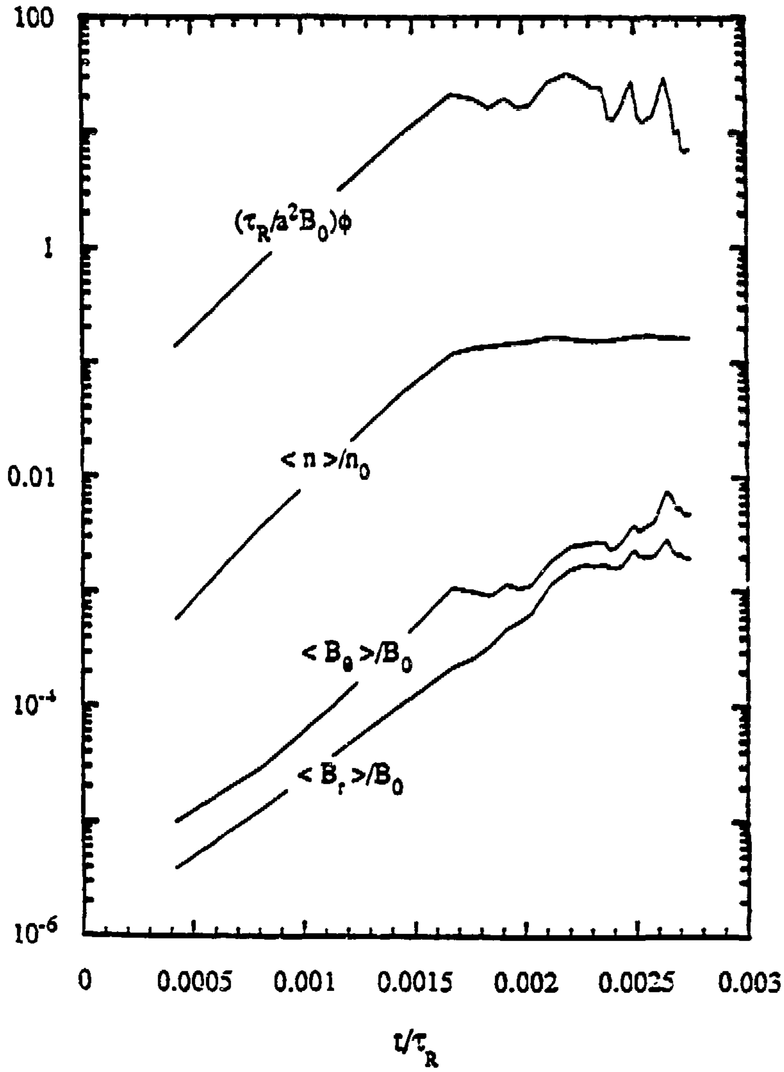

Figure 1: Typical time evolution of fluctuating quantities in 3-field model

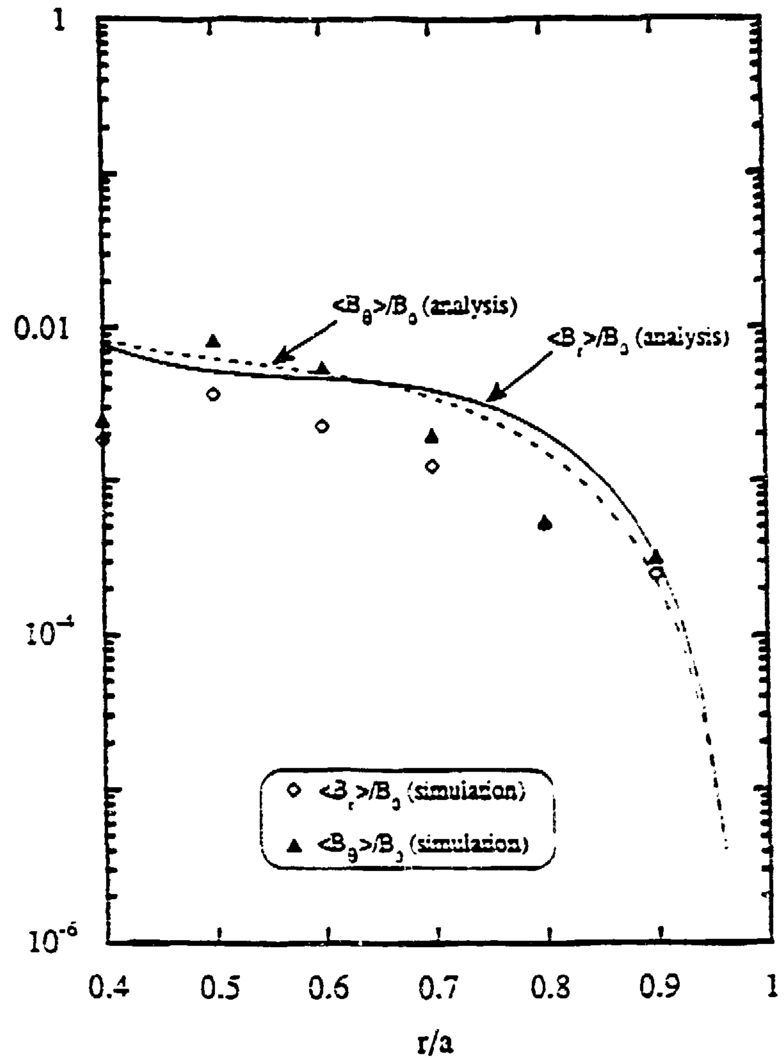

Figure 2: A comparison of the saturated radial and poloidal magnetic fields with analytical predictions

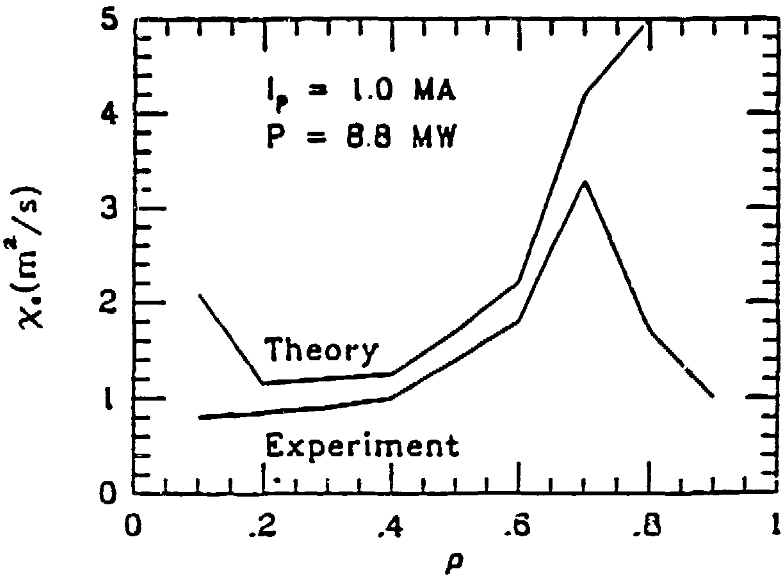

Figure 3: Comparison between χ_e from NBCDT [Eq.(4)] and effective χ_e from a DIII-D Hot Ion H-mode experiment [20].