SHADOWING EFFECTS IN NUCLEAR PARTON DISTRIBUTIONS FOR SMALL VALUES OF $x$.

RAPORT No 1457/PH

Jan Kwiecinski

Department of Theoretical Physics, H. Niewodniczanski Institute of Nuclear Physics, ul. Radzikowskiego 152, 31-342 KRAKOW, Poland.

Abstract.

The shadowing corrections to gluon and quark distributions in nuclei in the region of small values of $x$ are discussed. They are related to parton distributions in a Pomeron which are in principle measurable in hard diffractive processes on the nuclear target. Multiple scattering corrections to shadowing are considered in a model dependent way. The perturbative QCD evolution of shadowing is also taken into account. Various possibilities of the partonic content of a Pomeron are considered. It is shown in particular that the conventional parametrisations of parton distributions in a Pomeron which are based on the assumption that it consists mostly of gluons imply substantial nuclear shadowing in gluon distributions in heavy nuclei. Possible phenomenological implications of shadowing corrections in nuclear parton distributions for various semi-hard processes with nuclear targets are briefly discussed.
1. INTRODUCTION.

The term shadowing is usually used to describe the situation when the cross-section for some process on a nuclear target normalised per nucleon is smaller than the cross-section for this process on a free nucleon.

The recent EMC data [1-2] on deep inelastic muon scattering on nuclear targets which showed depletion of the nuclear structure functions per nucleon with respect to the structure function of a free nucleon for small values of the Bjorken variable $x$ and for moderately large values of $Q^2 = -q^2$ where $q$ is the four momentum transfer between the scattered muons have revived interest in nuclear shadowing of virtual photons [3-7]. This depletion persists to be present for relatively large values of $Q^2$ (i.e. for $Q^2 > 4 \text{GeV}^2$ or so) [2] suggesting that shadowing may exist also in the Bjorken scaling region. This implies that the partonic mechanism of shadowing may be present besides the more conventional mechanism based on the vector meson dominance model which corresponds to nuclear absorption of virtual vector mesons coupled to virtual photons [3-11]. The latter leads to vanishing shadowing corrections for large $Q^2$.

The partonic mechanism of nuclear shadowing of virtual photons reflects shadowing effects in quark (and antiquark) distribution functions in nuclei. Similar effects are of course expected to be present also in gluon distributions and they may manifest themselves in (semi) hard nuclear interactions in which gluons are expected to participate. These are for instance the gluon minijet production processes in heavy ion collisions at high
energies [12], the $J/\Psi$ production processes in hadron-nucleus or nucleus-nucleus collisions which are expected to be dominated by the gluon-gluon fusion mechanism [13] etc. The shadowing effects in gluon distributions should reflect themselves through the QCD evolution also in the quark (and antiquark) distributions and so in the deep inelastic lepton-nucleus scattering as well.

Possible origin of shadowing in parton distributions can be understood within a simple space time picture of the interaction of the virtual probe with the hadronic target [8,10,11]. On the basis of the uncertainty principle one finds that for $x<1/(2mR_A)$ where $R_A$ is the nuclear radius and $m$ the nucleon mass the partons can occupy distances comparable to the nuclear radius and so the partons from different nucleons can overlap spatially and interact. This interaction of partons from different nucleons leads to reduction of effective number of partons in a nucleus. This is the basic mechanism of shadowing effects in parton distributions in perturbative QCD [14,15] which in the region of very small values of $x$ can be enhanced by parton overcrowding in a nucleon implied by perturbative QCD [15]. In the region $1/(2mR_A)<x<1/mr$ where $r$ is the average distance between nucleons in a nucleus (i.e. $r \approx 1$ fm or so) one expects the shadowing to reduce gradually and for $x>1/mr>0.1$ the shadowing is expected to become negligible.

The shadowing corrections in parton distributions should satisfy the Altarelli-Parisi evolution equations implied by perturbative QCD [17]. The parton recombination mechanism leads to modification of those equations [15]. Solution of the evolution
equations requires the input shadowing corrections to parton distributions at some scale $Q^2=Q_0^2$. Their understanding is important since magnitude of shadowing corrections is essentially controlled by the magnitude of the input \([10,11,15]\). Possible origin of the input shadowing corrections within perturbative QCD was discussed in \([30]\).

In the discussion of nuclear effects in which several nucleons participate like the nuclear shadowing effects in parton distributions it is useful to be able to express those effects in terms of quantities which are in principle measurable in the processes on a nucleon target. In the discussion of multiple scattering of hadrons on nuclear targets an appropriate extension of the Regge theory has proved to be particularly useful in this respect \([18,19]\). In the case when two nucleons participate in the interaction it directly links the screening corrections to the hadron nucleus cross-section with the inclusive diffractive hadron nucleon cross-section. Multiple interaction of the diffractively produced system within a nucleus requires some model however.

The purpose of this paper is to explore the Regge theory of inelastic screening suitably adapted to semihard interactions at small $x$. This will make it possible to link the shadowing corrections in parton distributions in a nucleus with the hard diffractive processes on a nucleon. We shall predominantly discuss shadowing in the region of moderately small values of $x \geq 10^{-2}$ and for moderately large values of $Q^2 \approx 10 (\text{GeV}/c)^2$ or so which may be relevant phenomenologically. In this region the QCD effects of
parton overcrowding are unimportant. The content of our paper is as follows. In the next Section we establish the quantitative relation between the shadowing corrections to nuclear parton distributions and the parton distributions in a Pomeron which are in principle measurable in hard diffractive processes on a nucleon target. We also introduce in a model dependent way the multiple scattering corrections. In Sec. 3 we recall the perturbative QCD corrections to shadowing which affect its $Q^2$ evolution. The Section 4 is devoted to estimate of shadowing corrections mainly in gluon distributions. One of our aims will be to estimate the effect of the potentially large gluonic content of a Pomeron on shadowing. Using the standard gluon distributions in a Pomeron (22-24) we find substantial (i.e. of the order of 30%-50% or so) shadowing corrections to gluon distributions in heavy nuclei even for moderately small values of $x \approx 10^{-2}$. The QCD evolution of shadowing is also taken into account. We also estimate the contribution of the parton recombination mechanism of shadowing corrections to gluon distributions implied by perturbative QCD and find that it is relatively small. Finally in Section 5 a brief summary of our results is given.

2. Partonic content of a Pomeron and the shadowing corrections in nuclear parton distributions.

Let us consider the interaction of some external probe of large virtuality $Q^2 = -q^2$ with a nuclear target (see Fig.1). This probe may be either the virtual photon which can couple to quarks and antiquarks or some other probe which may couple to gluons.
The Bjorken scaling variable $x$ is defined as:

$$x = \frac{Q^2}{2p_T^2}$$

where $A$ is the atomic weight of the nucleus. The total cross section for this process can be related through the optical theorem to the imaginary part of the forward scattering amplitude of a probe on a nucleus. One can represent this amplitude in a form of multiple scattering series (see Fig. 2):

$$T_A(x, Q^2) = \sum_n T_A^{(n)}(x, Q^2)$$

and the total cross section corresponds to a sum of imaginary parts of the corresponding terms of this series. The first term corresponds to the impulse approximation and in this approximation the total cross-section normalised per nucleon equals to the cross-section on a free nucleon. Any deviations from this relation come from the contributions in which at least two nucleons participate in the interaction.

For large $Q^2$ the probe is expected to interact with partons in a nucleus and the multiple scattering corrections to the total cross-section can be retranslated on the corresponding corrections to parton distributions in a nucleus i.e.:

$$p_A(x, Q^2) = p_N(x, Q^2) + \Delta p_A(x, Q^2)$$

$$\Delta p_A(x, Q^2) = \sum_{n>2} \Delta p_A^{(n)}(x, Q^2)$$

where $p_A(x, Q^2)$ denotes the parton distribution function in a
nuclei per nucleon and $p_n(x,Q^2)$ is the parton distribution function of a nucleon (in general $p_n(x,Q^2)=([A-Z]p_n(x,Q^2) + Zp_p(x,Q^2))/A$ where $p_n(x,Q^2)$ and $p_p(x,Q^2)$ are the parton distributions in a neutron and proton respectively). In the high energy limit (i.e. for small values of $x$) one expects the multiple Pomeron exchange to dominate (see Fig. 3). The double interaction diagram of Fig. 3 relates the corresponding corrections $\Delta x^{(2)}(x,Q^2)$ to a parton content of a Pomeron. The latter is described in terms of the parton distribution functions in a Pomeron which are in principle measurable in hard diffractive processes on a nucleon target. Thus the quark (and antiquark) distribution functions in a Pomeron control the diffractive deep inelastic lepton - nucleon scattering i.e. the process $\mu^- + N \rightarrow \mu^- + N + X$ with large rapidity gap between the nucleon in a final state and the hadrons produced in a system $X$ [21,22]. The corresponding differential diffractive structure function $d^2F_2^{\text{dif}}/dx dt$ is related in the following way to the Pomeron structure function $F_2^p(x',Q^2,t)$ (see Fig. 4):

$$\frac{d^2F_2^{\text{dif}}}{dx dt} = \frac{1}{16\pi} f^2(t) \xi^2 - 2 \zeta \xi F_2^p(x',Q^2,t)$$

(2.4)

where

$$t = s^2$$

$$\xi = \frac{Q^2}{pq}$$

$$x' = \frac{Q^2}{s^2}$$

(2.5)
The function $a_p(t)$ is the Pomeron trajectory and $(xt)$ is the Pomeron coupling to a nucleon. In the region of large $Q^2$ the parton model should be applicable to a virtual photon-Pomeron scattering. This model relates the structure function $F_2^p(x',Q^2;t)$ to quark and antiquark distribution functions in a Pomeron:

$$F_2^p(x',Q^2;t) = x' E_l^2 \left[ q^p(x',Q^2;t) + q^{\bar{p}}(x',Q^2;t) \right]$$  \hspace{1cm} (2.6)

The gluon distribution functions in a Pomeron can be measured in hard processes like large $p_t$ jet production or heavy quark production etc within the diffractively produced system in hadronic collisions [33-35] (see Fig.5).

Adapting the Regge theory of inelastic screening in hadron-nucleus collisions [18] to a scattering of the probe on a nucleus one gets the following relation between the shadowing corrections $d\sigma^{'2}_A(x,Q^2)$ and the parton distributions $p^p(x',Q^2)$ in a Pomeron where $p^p(x',Q^2)=p^p(x',Q^2;t=0)$ and they denote either the quark, antiquark or gluon distribution functions:

$$\Delta \sigma_A^{1/2}(x,Q^2) = \frac{1}{2} \sum_x \int d^2 bdz_{l} dz_{z} \frac{1}{E(z_{1}-z_{2})} n_{A}(b,z) n_{A}(b,z)$$

$$\cos[mg(z_{z}-z_{1})] \xi_{1/2}^{1/1} \xi_{1/2}^{1/0} \bar{p}^{p}(x/Q^2)$$

\hspace{1cm} (2.7)

where $m$ is the nucleon mass and $n_{A}(b,z)$ is the nucleon number density in a nucleus. Strictly speaking this formula is exact only for purely imaginary amplitude corresponding to Pomeron exchange i.e. for $\alpha_p(0)=1$ and in general there are additional contributions.
containing the real part of the Pomeron exchange amplitude. Derivation of the formula (2.7) is straightforward. The inelastic screening contribution to the total cross-section for the interaction of a probe with the nucleus corresponding to double interaction diagram of Fig. 3 contains the probe-Pomeron total cross-section in the integral (2.7) in place of the parton distribution function in a Pomeron multiplied by $x/Q$ \cite{18,19}. In the parton model the cross-section equals to a convolution of the probe-parton cross-section and the parton distribution. Undoing this convolution one obtains the formula (2.7) containing only parton distributions. The negative sign of the shadowing corrections reflects the fact that the Pomeron exchange is described by the imaginary amplitude. It should be noticed that the $x'$ scaling of the parton distribution in a Pomeron ($x'=x/Q$) implies the non-vanishing shadowing corrections in the region of large $Q^2$. The $Q^2$ evolution of shadowing is linked with the QCD evolution of parton distribution in a Pomeron.

The double scattering diagram neglects however the interaction of the diffractively produced system within the nucleus. This approximation may be inadequate both for the very small values of $x<1/2m_R$ where $R_A$ is the nuclear radius as well as in the region of moderately small values of $x$ $1/2m_R<x<1/m_r$ where $r$ is the average distance between nucleons in a nucleus. In the former region the double scattering approximation overestimates the shadowing giving $\Delta p_\alpha^{(2)}(x,Q^2) \sim A^{1/3}$ while one expects that the leading behaviour of $\Delta p_\alpha(x,Q^2)$ should be independent of $A$ with corrections behaving as $A^{-1/3}$ \cite{18}. In the
latter region this approximation tends to underestimate the shadowing. The simplest way to account for the reabsorption of the diffractively produced system within the nucleus is to assume that the total cross-section $\sigma$ for the interaction of this system with nucleons is independent of its mass and that only diagonal transitions are retained (see Fig. 6). The shadowing corrections in parton distributions are then given by the following formula:

$$\Delta x_{p_A}(x, Q^2) = \frac{1}{2A} \int \frac{dz}{x} \int d^2 \theta d^2 z \delta(z - z_1) n_A(b, z_1) n_A(b, z_2) \cos(m\vec{z}_1 - \vec{z}_2) \beta(x, Q^2) \exp\left[ \frac{-\sigma}{2} \int \frac{dz'}{z'} n_A(b, z') \right]$$

(2.8)

For simplicity we shall use the same parameter $\sigma$ in the case of quark and gluon distributions. This approximation which is in the spirit of the aligned jet model of nuclear shadowing [10, 11, 26] may be reasonable in the region of moderately small values of $x$ ($x \lesssim 10^{-2}$ or so) which we shall be mostly interested in. It may however be inadequate for very small values of $x$ where one has to take into account various "enhanced" diagrams with several triple Pomeron vertices etc. which go beyond the simple approximation of diagonal transitions.

3. Parametrisations of parton distributions in a Pomeron and perturbative QCD corrections to shadowing.
The shadowing corrections $\Delta q_A^{(i)}(x, Q^2)$ and $\Delta g_A(x, Q^2)$ to quark and gluon distributions in a nucleus satisfy the ordinary Altarelli-Parisi evolution equations which follow from perturbative QCD [17]:

$$Q^2 \frac{d\Delta q_A^{(i)}(x, Q^2)}{dQ^2} =$$

$$\frac{\alpha_s(Q^2)}{2\pi} \int_1^{x'} \frac{dx'}{x'} \left[ P_{qq}(x/x') \Delta q_A^{(i)}(x', Q^2) + P_{qg}(x/x') \Delta g_A(x', Q^2) \right]$$

$$Q^2 \frac{d\Delta g_A(x, Q^2)}{dQ^2} =$$

$$\frac{\alpha_s(Q^2)}{2\pi} \int_1^{x'} \frac{dx'}{x'} \left[ 2P_{gg}(x/x') \Delta g_A(x', Q^2) + P_{qg}(x/x') \Delta g_A(x', Q^2) \right]$$

(3.1)

where the functions $P_{ab}(z)$ are the Altarelli-Parisi splitting functions and the function $\alpha_s(Q^2)$ is the running QCD coupling

$$\alpha_s(Q^2) = \frac{12\pi}{(n_f - 2) \ln(Q^2/\Lambda^2)}$$

(3.2)

where $n_f$ is the number of flavours. We also have $\frac{\Delta q_A^{(i)}(x, Q^2) - \Delta q_A^{(i)}(x, Q^2)}{x}$ where $\Delta q_A^{(i)}(x, Q^2)$ are the shadowing corrections to antiquark distributions. The index $i$ numerates the quark flavours. The input shadowing corrections at some scale
are related to the parton distributions in a Pomeron at this scale (see the formulas (2.8) or (2.7) of the preceding Section). Possible parametrisations of the quark and gluon distributions in a Pomeron were discussed in [21-25]. The standard assumption is that the Pomeron consists almost entirely of gluons and the following two parametrisations of gluon distribution functions were considered [22-25]:

(i) the "soft" gluon distribution:

\[ g_{\text{soft}}^P(x, Q^2) = 8(1-x)^5/x \tag{3.3} \]

(ii) the "hard" gluon distribution:

\[ g_{\text{hard}}^P(x, Q^2) = 8(1-x)^3 \tag{3.4} \]

In both cases the gluon distributions are contrived to saturate the momentum sum rule i.e.:

\[ \int_0^1 dx\, xg^P(x, Q^2) = 1 \tag{3.5} \]

One expects however on the basis of the factorisation of the Pomeron couplings that the quark distribution functions should be non-zero (albeit small) in the small x limit at least [4]. Also, on the basis of the Pomeron-photon analogy, one expects that the quark box diagram should contribute to the quark distribution functions in a Pomeron [20,21]. We shall therefore parametrise the input quark distribution functions in a Pomeron as below:
\[ q_p^{(u)}(x,Q_0^2) = (a^{(u)} + b^{(u)} x) (1-x)/x \]
\[ q_p^{(d)}(x,Q_0^2) = q_p^{(u)}(x,Q_0^2) \]

(3.6)

where the index \( i \) numerates the quark flavours and the constants \( a^{(u)} \) and \( b^{(u)} \) can be estimated using the factorisation of the Pomeron couplings [4] and on the basis of the Pomeron -photon analogy [20,21] respectively. The details of this estimate will be given in the next Section. The input gluon distributions in a Pomeron will then be parametrised either in a form of the soft distribution:

\[ g_p^{\text{soft}}(x,Q_0^2) = a_p^g (1-x)^{5/3}/x \]

(3.7)

or in a form of the hard distribution:

\[ g_p^{\text{hard}}(x,Q_0^2) = a_p^g (1-x)^{5/3} \]

(3.8)

with the constant \( a_p^g \) determined from the momentum sum rule i.e.:

\[ a_p^g = 5(1 - \frac{1}{2} (a^{(u)} + b^{(u)})/3) \]

(3.9)

Perturbative QCD gives also the additional contribution to shadowing which comes from the recombination of partons from different nucleons. This recombination mechanism modifies the evolution equations introducing nonlinear terms in those equations [15]. Retaining only that term which corresponds to the gluon recombination from two nucleons one gets the following
expression for the shadowing corrections to gluon distributions in a nucleus:

\[
\Delta x g_{acb}^{\text{nc}}(x,Q^2) = \frac{n}{2A} \int_{\xi}^{1} \frac{dz}{z} \int d^2k \left[ \frac{3\alpha_s(k^2)}{k^2} \right] \int d^2bdz \theta(z-z_1) n_A(b,z) n_A(b,z_2) 
\]

\[
cos[m_2(z-z_1)] (z g_N(\xi,k^2))^2 dx/\xi, k^2, Q^2)
\]

(3.10)

where \( g_N(\xi,k^2) \) is the gluon distribution function in a nucleon and the distribution function \( DX/\xi,k^2,Q^2 \) satisfies the same evolution equation as the gluon distribution in a gluon but with the boundary condition \( DX/\xi,k^2,k^2=1 \). In the next Section we shall present the numerical estimate of the shadowing corrections to quark and gluon distributions in nuclei.

4. Estimate of the shadowing corrections to gluon and quark distributions in nuclei.

Estimate of the shadowing corrections requires a model for the nuclear number density \( n_A(b,z) \). For simplicity we choose the uniform density model i.e.

\[
n_A(b,z) = \rho_0 \Theta[R_A - (b^2 + z^2)^{1/2}]
\]

(4.1)

where for the nuclear radius \( R_A \) we put

\[
R_A = r_0 A^{1/3}
\]

(4.2)

with \( r_0 = 1.25 \) fm. The normalisation condition for the nuclear
number density \( n_A(b,z) \)

\[
\int d^2bdz \ n_A(b,z) = A
\]  

(4.3)

fixes the normalisation constant \( \rho_0 \) in the formula (4.1)

\[
\rho_0 = \frac{3}{4\pi r_0^2}
\]  

(4.4)

Rewriting the formula (2.8) for the shadowing corrections in a form of the convolution of the parton distribution in a Pomeron and the nuclear "form-factor" \( H_A(\xi;\sigma) \):

\[
\Delta p_\rho(x,Q^2) = \int dx_1 \frac{1}{2} x_1^2 A_0(0) H_A(\xi;\sigma) \frac{d^2 P(x_1,Q^2)}{d^2 x_1} H_A(\xi;\sigma)
\]

(4.5)

one obtains the following expression for the "form-factor" \( H_A(\xi;\sigma) \) in the uniform density approximation:

\[
H_A(\xi;\sigma) = -\rho_0^2 \delta^2(0) \left[ 2^s x^s \frac{\lambda}{\xi^2 + m^2 \xi^2} - \frac{1}{2^s} \frac{1}{A} e^{-1/2} \frac{\xi^2}{\lambda^2 + m^2 \xi^2} \right] + \frac{1}{4^s} \exp \Delta(\xi - \lambda^2 - \lambda^2) \frac{1}{(\lambda - \xi)^4}
\]

(4.6)

where

\[
\lambda = -\frac{\rho_0 \sigma}{2}
\]  

(4.7)
The formulas (4.8) and (4.6) display explicitly the A dependence of shadowing corrections.

We have calculated the shadowing corrections for $Q^2=10$ GeV$^2$ evolving from the input shadowing corrections at $Q^2=2$ GeV$^2$. The latter were calculated from (4.5) and (4.6) using the gluon and quark distributions in a Pomeron defined by the formulas (3.5)–(3.9) of the preceding Section. The Pomeron intercept was set equal to 1 while the parameter $\alpha(0)$ was put equal to 10 GeV$^{-1}$ (this corresponds to the contribution of the Pomeron to the total pp cross section equal approximately 40 mb). The parameters $a_u$ and $b_u$ defining the input quark distribution functions in a Pomeron [see the formula (3.6) of the preceding Section] were assumed to be flavour independent i.e. we put $a_u=a$ and $b_u=b$ and we took into account three quark flavours. The parameter $a$ corresponds to the triple Pomeron contribution to the Pomeron structure function and can be estimated from factorisation of the Pomeron couplings [41] (see Fig. 7a):

$$a = \frac{4\pi}{3\beta^2(0)} \left[ \frac{d^2\sigma_{pp+px}}{dt} \right]_{t=0} \left. \frac{F_z(a)}{\sigma_{pp}} \right|_{t=0}$$

(4.8)

In this formula the quantity $\left[ \frac{d^2\sigma_{pp+px}}{dt} \right]_{t=0}$ is the diffractive inclusive pp+px cross-section at $t=0$, $F_z(a)$ is that part of the nucleon structure function $F_z$ in deep inelastic muon nucleon scattering which corresponds to the quark sea contribution in the small $x$ limit (i.e. $F_z^{(q)} = \text{const}$) and $\sigma_{pp}$ is that part of the total cross section which corresponds to the Pomeron
contribution. We set \( \frac{d^2 \sigma_{\text{pp}+pX}}{dx_1 dx_2} \bigg|_{t=0} = 3.54 \text{ mbGeV}^{-2} \) [23], \( F_2 = 0.3 \) and \( a^p_{\text{pp}} = 40 \text{ mb} \) respectively. The parameter \( b \) corresponding to the quark box diagram contribution of Fig. 7b was estimated to be equal to 0.065 [21]. The parton distributions in a free nucleon were obtained from the evolution equations assuming the CDHS input parametrisation of those distributions at \( Q_0^2 = 5 \text{ GeV}^2 \) [27]. The parameter \( \Lambda \) defining the QCD running coupling was put equal to 200 MeV. The calculations were performed for two values of the parameter \( \alpha \), i.e. for \( \alpha = 0 \) and for \( \alpha = 20 \text{ mb} \).

In Fig. 8 we plot the quantity \( R_g \):

\[
R_g = 1 + \frac{\Delta g(x,Q^2)}{g_N(x,Q^2)}
\]

for the iron nucleus for two values of \( \alpha \) (\( \alpha = 0 \) and \( \alpha = 20 \text{ mb} \)) and for two parametrisations of the input gluon distributions in a Pomeron (3.7) and (3.8). In Fig. 9 we plot the same quantity for uranium for \( \alpha = 20 \text{ mb} \) and for two input gluon distributions. We find that the shadowing corrections are substantial even in the region \( x \approx 0.01 \). For \( x \approx 0.01 \) they can reduce gluon distribution in a nucleus almost by a factor equal to two. Relatively large value of the shadowing corrections are closely connected with the large amount of gluons in a Pomeron. By "large amount of gluons in a Pomeron" we understand the conjecture that (1) the parton distributions in a Pomeron can be normalised using the conventional momentum sum rules (2) those momentum sum rules are at some moderately large scale \( Q_0^2 \) (in our case \( Q_0^2 = 2 \text{ GeV}^2 \)).
approximately saturated by gluons with only small admixture of quark and antiquark contributions. We illustrate this point in Fig. 10 where we plot the shadowing corrections to gluon distributions in iron \(-\Delta g(x,Q^2)\) corresponding to the soft and hard input gluon distributions in a Pomeron and to the input quark distributions alone in a Pomeron at the scale \(Q_0^2 = 2\text{GeV}^2\) respectively. In the latter case the shadowing corrections to gluon distributions are entirely generated through the QCD evolution from the shadowing corrections to the quark distributions. It can also be seen from Figures 8-10 that the adopted approximation concerning the multiple rescattering within the nucleus may be unreliable for very small values of \(x > 10^{-3}\) where it may lead to negative gluon distributions in the nucleus. This is particularly evident for the case of soft input gluon distributions in a Pomeron. In the very small \(x\) region \(x \ll 10^{-2}\) one should take into account the multiple interaction of Pomeron in multiple nuclear rescattering. The soft distribution may be itself unreliable since its small \(x\)' limit is by an order of magnitude larger than the estimate obtained from the factorization of the Pomeron couplings [24]. The soft gluon distribution in Pomeron seems to be however favored by the experimental data on jet production in the diffractively produced system [25].

The \(x\) dependence of shadowing in the region of moderately small values of \(x > 10^{-2}\) or so is mainly controlled by the nuclear "form-factor" \(H_A(\xi; c)\) which cuts-off the region of large \(\xi\) in the convolution integral which is dominated by the region \(\xi \gg x\). For
this reason the hard gluon distributions in a Pomeron give bigger shadowing corrections in this region of $x$ than the soft distributions. On the other hand in the region of very small values of $x \times 10^{-2}$ or so the effect of the nuclear form-factor is less important for the $x$ dependence of shadowing than the fact that the soft gluon distributions in a Pomeron give logarithmically divergent shadowing corrections in the limit $x \to 0$ while the hard distributions give finite result. Those qualitative results are essentially unchanged by the QCD evolution and so the main feature of shadowing corrections depends in a crucial way upon the input.

We have also estimated the perturbative QCD contribution to shadowing corrections to gluon distributions corresponding to parton recombination [see the formula (3.10) of the preceding Section] in the approximation of neglecting the QCD evolution in the gluon distribution $g_N(\xi,k^2)$ in a nucleon and in the distribution $g(x',k',Q^2)$. The gluon distribution in a nucleon in the integral (3.10) was parametrised as $g_N(\xi,k^2)=3(1-\xi)^5$. The corresponding shadowing corrections were calculated for two values of the lower cut-off parameter $Q_0^2$ in the integral (3.10) i.e. for $Q_0^2=2$ GeV$^2$ and for $Q_0^2=5$ GeV$^2$. The results are presented in Fig.11 and compared with the shadowing corrections corresponding to the hard input distributions in a Pomeron and to $\sigma=20$mb. We see that QCD corrections to shadowing are very sensitive to the lower cut-off $Q_0^2$ as expected are in general much smaller than the shadowing calculated from (2.9). For $Q_0^2=5$ GeV$^2$ which seems to be more appropriate [15] the QCD recombination contribution to
shadowing does not exceed few percent of the shadowing corrections calculated from large gluon distributions in a Pomeron.

We have also calculated the shadowing corrections to quark distributions and estimated their effect on the nuclear structure function $F_{2A} (x, Q^2)$ per nucleon which is related to the shadowing corrections to nuclear quark and antiquark distributions as below:

$$F_{2A} (x, Q^2) = F_{2} (x, Q^2) + x E_{q} \{ \Delta q_{A}^{(1)} (x, Q^2) + \Delta \bar{q}_{A}^{(1)} (x, Q^2) \}$$

where $F_{2} (x, Q^2)$ is the structure function of the free nucleon. We also considered shadowing corrections in the structure function $F_{2A} (x, Q^2)$ which might come from the nuclear rescattering of low mass vector mesons which couple to virtual photons in addition to the shadowing corrections in quark and antiquark distributions i.e.:

$$F_{2A} (x, Q^2) = F_{2A}^{\text{part}} (x, Q^2) + \frac{Q^2}{4 \pi A} \sum_{\nu} \frac{m_{\nu}^{4} \Delta \sigma_{\nu A} (x, Q^2)}{\gamma_{\nu}^{2} (m_{\nu}^2 + Q^2)^2}$$

where $F_{2A}^{\text{part}} (x, Q^2)$ is given by the formula (4.10), $m_{\nu}$ denotes the mass of the vector meson $\nu$, the constants $\gamma_{\nu}$ correspond to the coupling of the photons to vector mesons and can be obtained from their leptonic widths. In the sum we have restricted ourselves to the $\rho, \omega$ and $\phi$ vector meson contributions. The cross-sections $\Delta \sigma_{\nu A} (x, Q^2)$ correspond to the nuclear rescattering of vector mesons:
\[ \Delta \sigma_{V_A}(x,Q^2) = -\frac{\alpha_S}{\pi} \int d^2dz_1 dz_2 \Theta(z_2-z_1) n_{V_A}(b,z_1)n_{V_A}(b,z_2) \]
\[ \cos(m^2/(2Q^2))(z_2-z_1) \exp \left[ -\frac{\alpha_S}{\pi} \int dz' n_{V_A}(b,z') \right] \]

(4.12)

where the cross-sections \( \sigma_{Vn} \) are the total vector meson-nucleon cross-sections and we set \( \sigma_{pn} = 24 \text{ mb} \) and \( \sigma_{nn} = 14 \text{ mb} \). The results are presented in Figs. 12 and 13 for the iron and tin nuclei respectively. In both figures we plot the quantity \( R_{Z} = F_{Z}(x,Q^2)/F_{Z}(x,Q^2) \) for \( Q^2 = 10 \text{ GeV}^2 \) for the iron nucleus and for \( Q^2 = 7 \text{ GeV}^2 \) for the tin nucleus. The latter value of \( Q^2 \) is approximately equal to the average value of \( Q^2 \) corresponding to the experimental points [21]. We plotted only those experimental points which correspond to shadowing region. The function \( R_{Z} \) containing the effects of rescattering of virtual vector mesons is represented in Figs. 12 and 13 by the solid line while that corresponding only to the shadowing corrections in quark and antiquark distributions is represented by the dashed line respectively. It should be noticed that the shadowing corrections coming from the rescattering of vector mesons are still non-negligible even for \( Q^2 = 10 \text{ GeV}^2 \). It has to be observed that the shadowing predicted by the model seems to be smaller than implied by the data. Both the QCD evolution which enhance the quark content of a Pomeron and the effects of nuclear rescattering (\( \alpha_S \approx 0 \)) tend to enhance the shadowing corrections for \( x > 0.03 \) or so in comparison with estimates done in [4] where those effects were
neglected. The cut-off induced by the nuclear "form-factor" $H_A(x;0)$ however supresses those effects in the region of moderately large $x$ (i.e. for $x > 0.1$). One has to be also aware of the fact that the simple minded addition of two separate contributions to shadowing as in the formula (4.10) may contain double counting. Possible way to avoid this double counting is described in \cite{4}.

5. Summary and conclusions.

In this paper we have elaborated consequences of mutual relation between the parton content of a Pomeron and shadowing corrections to parton (i.e. quark and gluon) distributions in nuclei. We were mostly concerned in the region of moderately small values of $x$ ($10^{-2} \leq x \leq 10^{-1}$ or so) which may be relevant phenomenologically. We have shown in particular that the potentially large amount of gluons in a Pomeron can lead to a substantial nuclear shadowing in nuclear distributions. By "large amount of gluons" we understand the conjecture that (1) the parton distributions in a Pomeron can be normalised using the conventional momentum sum rules (2) those momentum sum rules are at some moderately large scale $Q^2_o$ (we have set $Q^2_o = 2 \text{GeV}^2$) approximately saturated by gluons with only small admixture of quark and antiquark contributions. Perturbative QCD evolution changes those proportions and the large gluon distribution in a Pomeron reflects itself through the QCD evolution in shadowing corrections in quark and antiquark distributions in nuclei enhancing the shadowing effects also in the deep inelastic lepton...
nucleus scattering. The shadowing corrections to gluon distributions were found to be of the order of 50% of the gluon distributions of a free nucleon for \( x \approx 10^{-2} \) or so and to decrease gradually with increasing \( x \). The shadowing is negligible at \( x \approx 0.1 \). The shadowing corrections reduce the magnitude of parton distributions in nuclei in comparison with the parton distributions of free nucleons and as mentioned above this reduction can be as large as a factor equal to two for \( x \approx 0.01 \). This should imply significant suppression of cross-sections of the semihard processes with nuclei which are sensitive to gluon distributions in the small \( x \) region like the gluon minijet production in heavy ion collisions at very high energies. Unlike the absorptive corrections in the initial and final states which cancel in the inclusive cross sections thanks to the Adler rules [28] the shadowing corrections to gluon distributions affect these cross-sections. This comes from the fact that the inclusive jet production cross section is still expected to be given by the standard QCD improved parton model formula but with the gluon distributions containing shadowing corrections. Significant shadowing corrections imply in this case that the average multiplicity of minijets in heavy ion collisions should be much smaller than the multiplicities which were estimated using the unshadowed gluon distributions [12]. Shadowing effects in gluon distributions may also manifest themselves in \( J/\psi \) production in hadron-nucleus or nucleus-nucleus collisions where the dominant production mechanism is presumably that of gluon-gluon fusion. In this case however, additional and important source of suppression
can come from the nuclear reabsorption of the J/ψ particle itself [13]. This mechanism is also required by the experimental data which show significant suppression in the region corresponding to $x < 0.1$ [29] where the shadowing effects in parton distributions are negligible. The shadowing corrections to gluon distributions may also affect the prompt photon production [31] in hadron-nucleus or nucleus-nucleus collisions where the important production mechanism is the reaction $qg \to j + q$ on the partonic level. Since, however, the shadowing corrections are expected to be small, it only for the very small values of $x < 0.05$ or so that they are still unimportant for presently available energies. They may play some role however for the RHIC energy range.

Acknowledgments.

I would like to thank Patrick Aurenche, Leonard Bosted, Evgeni Levin, Jianwei Oiu, Mark Strikman and the members of the muon experiment E-565 at the Fermi National Accelerator Laboratory for several illuminating discussions. I am very much indebted to Patrick Aurenche for providing his program computing the prompt photon production cross-sections and for assistance in the numerical computations concerning the estimate of shadowing effects in this process. I am also grateful to the Theoretical Physics Department of the Fermi National Accelerator Laboratory where this work was completed for its hospitality.
References.

(19) J. Kwieciński, L. Leśniak and K. Zalewski, Nucl. Phys., B79
(27) CERN, CDHS; H. Abramowicz et al., Z. Phys. C17 (1983) 283;
E. Eichten, I. Hinchliffe, K. Lane and C. Quigg, Rev. Mod.
(28) A. Abramovsky, V.N. Gribov and O.V. Kancheli, Yad. Fiz. 18
(1973) 598.
2121.
(30) F.E. Close, J. Qiu and R.G. Roberts, "QCD parton
recombination and applications to nuclear structure
functions", ANL-HEP-PR-89-22.
Figure captions.

Fig. 1. Kinematics of the probe-nucleus forward scattering.

Fig. 2. Multiple scattering expansion for the probe-nucleus scattering amplitude. The dashed and solid lines correspond to the probe and nucleons in a nucleus respectively.

Fig. 3. Multiple Pomeron exchange contributions to probe-nucleus scattering. The zigzag lines correspond to Pomerons and the solid lines in the upper part of the diagram correspond to partons which couple to a probe. The solid lines in the lower part of the diagram represent nucleons in a nucleus.

Fig. 4. Deep inelastic diffractive scattering and the Pomeron structure function. The wavy and zigzag lines correspond to the virtual photon and to a Pomeron respectively.

Fig. 5. Hard gluon-gluon interaction within the diffractively produced system. The zigzag and wavy lines represent Pomerons and gluons respectively.

Fig. 6. Multiple scattering of the diffractively produced system within the nucleus.

Fig. 7. (a) The triple Pomeron contribution to the Pomeron structure function. (b) The quark box diagram contribution to the Pomeron structure function. The zigzag and wavy lines represent Pomeron and the virtual photon respectively.

Fig. 8. The quantity $R_g$ defined by the formula (4.9) at $Q^2=10\text{GeV}^2$ for iron for various values of the parameter $\sigma$ and for...
different input gluon distributions in a Pomeron at the
scale $Q_0^2=2\text{GeV}^2$. (......) $\sigma=0$ and soft input gluon
distribution; (-.-.-.) $\sigma=20\text{mb}$ and soft input gluon
distribution; (- - - - -) $\sigma=0$ and hard input gluon
distribution; (-----) $\sigma=20\text{mb}$ and hard input gluon
distribution.

Fig. 9. The same as above for uranium. (- - - - -) $\sigma=20\text{mb}$ and soft
input gluon distribution; (-----) $\sigma=20\text{mb}$ and hard input
gluon distribution.

Fig. 10. The shadowing corrections to gluon distributions in iron
nucleus at $Q^2=10\text{ GeV}^2$ \([\Delta G(x,Q^2)\equiv \Delta g_A(x,Q^2)]\) for different
input parton distributions in a Pomeron at the scale $Q_0^2=2\text{GeV}^2$. (- - -) input soft gluon distributions and
quark distributions; (-----) input hard gluon
distributions and quark distributions; (..........) input
quark distributions alone.

Fig. 11. The shadowing corrections to gluon distributions at
$Q^2=10\text{GeV}^2$ in iron \([\Delta G(x,Q^2)\equiv \Delta g_A(x,Q^2)]\) compared with the
gluon recombination contribution. (-----) shadowing
corrections corresponding to the input hard gluon
distribution in a Pomeron and to $\sigma=20\text{mb}$; (-.-.-.) the
gluon recombination contribution corresponding to the
lower cut-off in the integral (3.10) equal to $2\text{GeV}^2$;
(- - - - -) the gluon recombination contribution corresponding to the lower cut-off in the integral (3.10)
equal to $5\text{GeV}^2$.

Fig. 12. The quantity $R_2=\frac{F_2}{Z_A Z_2}$ at $Q^2=10\text{ GeV}^2$ for iron.
(- - - -) partonic mechanism of shadowing alone;
(-----) partonic mechanism of shadowing and the nuclear rescattering of low mass vector mesons.

Fig. 13 The same as above for the tin nucleus and for $Q^2=7\text{GeV}^2$.
The experimental data are from [2].
Fig. 4

\[ F_2^P(x_1, Q^2, t) \]
Fig. 5
$Q^2 = 10 \text{ GeV}^2$

$A = 56$
$Q^2 = 10 \text{ GeV}^2$

$A = 56$
$Q^2 = 7 \text{GeV}^2$

$A = 120 \pm$