COMMISSARIAT A L'ENERGIE ATOMIQUE

CENTRE D'ETUDES NUCLEAIRES DE SACLAY
Service de Documentation
F91191 GIF SUR YVETTE CEDEX

FR 9100419

CEA-CONF --10246

L2

UNIFIED CHIRAL MODELS OF MESONS AND BARYONS

MENDEZ-GALAIN R.- RIPKA G.
CEA Centre d'Etudes Nucleaires de Saclay, 91 - Gif-sur-Yvette (FR).
Service de Physique Theorique

Communication présentée à : Conference on Hadron Physics with Multi-GeV Electrons

Les Houches (FR)
6-15 Feb 1990

UNIFIED CHIRAL MODELS OF MESONS AND BARYONS.

by

Ramon Mendez Galain and Georges Ripka

SERVICE DE PHYSIQUE THEORIQUE
Centre d'Etudes Nucleaires de Saclay
F-91190 Gif-sur-Yvette Cedex, France

ABSTRACT: Unified chiral models of mesons and baryons are presented. Emphasis is placed on the underlying quark structure of hadrons including the Skyrmion. The Nambu Jona-Lasinio model with vector mesons is discussed.

Lectures delivered at the Les Houches school on "Hadron Physics with Multi-GeV Electrons", February 1990.

Saclay SPhT/90-54

1. Introduction.

In these lectures we discuss unified theories of mesons and baryons. Mesons are low energy vibrations of the vacuum. They provide us with information about the spontaneously broken chiral symmetry and the long range order of the physical vacuum. On the other hand, baryons are excited stationary states of the vacuum. They can be described either as solitons of a meson lagrangian or as bound states of quarks.

A theory of low energy hadron physics must describe the vacuum as a system with spontaneously broken chiral symmetry in its ground state. As early as 1960, Skyrme proposed such a unified theory [21]. It was written in terms of a pion field. In his model baryons emerge as topological solitons which have acquired the name of Skyrmions. They are described in sections 2-4. Later, quarks were discovered and QCD became the theory of strong interactions. Low energy QCD is non-perturbative and can only be handled numerically in lattice calculations. The latter provide us with scarce information concerning hadron structure and dense hadronic matter. This explains why quark models of hadrons have proliferated [48].

The most successful quark models have been bag models [54], constituent quark models [54], color dielectric models [55] and the Nambu Jona-Lasinio model [33,34,35] which is discussed in these lectures (sections 6-8). In the first three models, confinement is mimicked by introducing an effective potential. The Nambu Jona-Lasinio model lacks confinement but it provides a mechanism for the spontaneous breakdown of chiral symmetry. It describes quarks with a 4-fermion zero-range effective interaction which preserves chiral symmetry. Mesons are described as $q\bar{q}$ excitations of the vacuum. They are low-amplitude vibrations which are solutions of the Bethe-Salpeter equation. Baryons are described as bound (but not confined) states of quarks, also referred to as non-topological solitons.

In these lectures we discuss both Skyrme lagrangians (which involve only chiral fields) and chiral quark lagrangians. We stress that most of the essential ingredients of the Skyrme model (the baryonic current, the Wess-Zumino term and the 1/2 integer spin quantization) are reflecting the underlying quark structure of the soliton. They can be obtained in terms of gradient expansions of a chiral quark lagragian [18-20].

This set of lectures is the third of a series devoted to chiral models of hadrons [34]. They complete a discussion of the Nambu Jona-Lasinio model by introducing vector mesons.

2. From Weinberg to Skyrme.

In the early 1960's low energy pion physics was described by the Weinberg action [1]:

$$I = \frac{f_\pi^2}{4} \int d_4x \ \mathrm{tr}(\partial_\mu U)(\partial^\mu U^\dagger) \qquad U \equiv e^{i\vec{\theta}.\vec{\tau}} \tag{2.1}$$

which is invariant under the global chiral rotations:

$$U \Rightarrow A^{-1} U A^{-1} \qquad A \equiv e^{-i\vec{\alpha}.\vec{\tau}/2} \tag{2.2}$$

The vacuum can be chosen to be the translationally invariant stationary point $\theta_a(x) = 0$ where $a=1,2,3$ is the isospin index. Any chiral rotation of the vacuum yields an equivalent degenerate vacuum. This degeneracy is responsible for the appearance of zero mass Goldstone bosons which are identified to the pions. To see this we expand the action in powers of the field θ:

$$I^{(2)} = \frac{f_\pi^2}{2} \int d_4x \ (\partial_\mu \theta_a)\left(\partial^\mu \theta_a\right) = \frac{1}{2} \int d_4x \ (\partial_\mu \pi_a)\left(\partial^\mu \pi_a\right) \tag{2.3}$$

We obtain an action of zero mass particles described by the pion field $\vec{\pi}(x) \equiv f_\pi \vec{\theta}(x)$. The axial current $j^a_{\mu A}(x)$ is the Noether current associated to the chiral rotation (2.2). We can quantize the pion field and calculate the matrix element of the axial current between the vacuum and a one-pion state. The result is:

$$\left\langle 0 \left| j^a_{\mu A}(x) \right| \pi_b, \vec{q} \right\rangle = -iq_\mu \ \delta_{ab} \ \frac{e^{iqx}}{\sqrt{2\omega_q \Omega}} \ f_\pi \tag{2.4}$$

where $qx \equiv \vec{q}.\vec{r} - \omega_q t$ and $\omega_q = q$ since the pions are massless. We recognize in (2.4) the definition of the pion decay constant f_π which is measured to be $f_\pi = 93$ MeV. The Weinberg action (2.1) is trivially modified to include a chiral-symmetry breaking term which gives the pion its 138 MeV mass. When it is expanded in powers of θ, it is quite successful in describing low-energy properties of pions.

The Weinberg action (2.1) is often referred to as the non-linear σ-model. It can be viewed as a limit of the linear σ-model [2], defined by the action:

$$\begin{aligned} I &= \int d_4x \ \left(\frac{1}{2}(\partial_\mu \sigma)(\partial^\mu \sigma) + \frac{1}{2}(\partial_\mu \pi_a)\left(\partial^\mu \pi_a\right) - \frac{\kappa^2}{8}\left(\sigma^2 + \pi_a^2 - f_\pi^2\right)^2\right) \\ &= \int d_4x \ \left(\frac{1}{2}(\partial_\mu \Phi)(\partial^\mu \Phi) - \frac{\kappa^2}{8}\left(\Phi^2 - f_\pi^2\right)^2 + \frac{\Phi^2}{4} \ \mathrm{tr} \ \frac{1}{2}(\partial_\mu U)(\partial^\mu U^\dagger)\right) \end{aligned} \tag{2.5}$$

where we used the linear representation of the chiral field:

$$\Phi U \equiv \sigma + i\vec{\pi}.\vec{\tau} \qquad \sigma = \Phi\cos\theta \qquad \vec{\pi} = \Phi \frac{\vec{\theta}}{\theta} \sin\theta \tag{2.6}$$

(The linear representation (2.6) with real fields σ and $\vec{\pi}$ is only valid when the $\vec{\tau}$ matrices are the generators of the SU(2) group. Linear representations, valid for SU(3) and higher order groups have been developped by Kuznetzov [3].)

The linear σ-model describes a world in which the vacuum is the translationally invariant stationary point $\sigma = f_\pi$, $\vec{\pi} = 0$. Low amplitude vibrations of the vacuum are composed of zero-mass pions and σ mesons of mass $m_\sigma = \kappa f_\pi$. In the limit $\kappa \Rightarrow \infty$ of infinite sigma-meson mass, the σ and π fields are constrained to the chiral circle, meaning that $\sigma^2 + \vec{\pi}^2 = f_\pi^2$ at every point in space and time, so that the linear σ-model (2.5) becomes equivalent to the non-linear model (2.1). Many calculations of the non-linear model are simpler in terms of the four fields σ and $\vec{\pi}$ of the linear model.

As early as 1960 Skyrme [4,21] proposed to describe baryons in terms of solitons obtained from a chirally invariant lagrangian. He was however unable to obtain a stable soliton with the Weinberg action (2.1) for reasons described below.

A soliton is a localized (therefore not translationally invariant) stationary point of the action. When we speak of a particular "point" of the action, we mean a point in the infinite dimensional field space which is defined by a set of values of the fields $\theta_a(x)$ at all space-time points $x \equiv (\vec{r},t)$. Solitons are, like the vacuum, time-independent stationary points of the action. They differ from the vacuum in that they are localized in space. (Instantons are stationary points of the action which are localized both in space and time.) Solitons are to be distinguished from mesons which are low-amplitude vibrations of the vacuum. Whether they actually are low-amplitude vibrations or not can be checked *a posteriori* by computing the quadratic fluctuations of the the field strength although nobody seems to take the trouble to do so. Solitons can, of course, also have vibrations. They represent excited states of baryons as well as strange baryons [5].

It is not possible to construct stable solitons from either the linear or non-linear σ-model because of the so-called Derrick instability. Consider the linear σ-model (2.5). Since a soliton has time-independent fields, it can be calculated by minimizing its classical energy:

$$E = \int d_3 r \; \frac{1}{2}(\nabla_i \sigma)^2 + \frac{1}{2}(\nabla_i \pi_a)^2 + \frac{\kappa^2}{8}\left(\sigma^2 + \pi_a^2 - f_\pi^2\right)^2 \qquad (2.7)$$

The energy E(R) calculated with the fields $\sigma(\vec{r}/R)$ and $\pi_a(\vec{r}/R)$ has the form $E(R) = BR + CR^3$ where B and C are the coefficients defined below in Eq.(2.10). We see that the soliton size shrinks to zero because the minimum energy occurs at R=0.

Skyrme suggested to stabilize the soliton by adding a 4-derivative term to the lagrangian. Before we discuss this, let us discuss two recent works which show how quantum fluctuations can stabilize the soliton [6,7]. Consider, for example, the quantum fluctuations of the size parameter R. Assume that the fields are functions of the variable $\vec{x} = \vec{r}/R(t)$ and that the only time dependence of the fields is due to the time dependence of the size parameter R:

$$\sigma(\vec{r},t) = \sigma(\vec{r}/R(t)) \qquad \pi_a(\vec{r},t) = \pi_a(\vec{r}/R(t)) \tag{2.8}$$

Substituting these forms into the action (2.5), we get:

$$I = \int dt \left(\frac{AR}{2}\dot{R}^2 - BR - CR^3\right) = \int dt \left(\frac{2A}{9}\dot{Y}^2 - BY^{2/3} - CY^2\right) \tag{2.9}$$

where $Y \equiv R^{3/2}$ and:

$$A \equiv \int d_3x \left(x_i \frac{\partial\sigma}{\partial x_i}\right)^2 + \left(x_i \frac{\partial\pi_a}{\partial x_i}\right)^2, \quad B \equiv \frac{1}{2}\int d_3x \left(\frac{\partial\sigma}{\partial x_i}\right)^2 + \left(\frac{\partial\pi_a}{\partial x_i}\right)^2, \quad C \equiv \int d_3x \frac{\kappa^2}{8}\left(\sigma^2 + \pi_a^2 - f_\pi^2\right)^2 \tag{2.10}$$

The Y variable can be quantized and the calculation of radial vibrations reduces to a simple one-dimensional Schrödinger equation. The mass parameter for the radial motion is $m = 4A/9$ and the hamiltonian is $h = P^2/2m + BY^{2/3} + CY^2$. Semi-classically, the momentum is about $P = 1/Y$ and the energy is approximately $E(Y) = 1/(2mY^2) + BY^{2/3} + CY^2$. Assume that $C = 0$. The value Y_0 which makes the energy stationary is $Y_0 = (3/2mB)^{3/8}$. This is an equilibrium radius $R_0 = \left(\frac{27}{8AB}\right)^{1/4}$. When the coefficients A and B are calculated with a hedgehog field, defined below in Eq.(3.1) and an exponential profile, a more careful calculation [6] yields a nucleon mass of 1140 MeV compared to its observed mass of 989 MeV. Moreover, the two nucleon excited states N(1440) and N(1710), the first of which is the Roper resonance, are obtained at 1470 and 1740 MeV respectively [6]. Such reasonable agreement with a simple and parameter-free calculation shows to what extent the justification of Skyrme's 4-derivative term, discussed below as a stabilizing agent, is arbitrary. The soliton can also be stabilized taking into account its rotations [7] and, possibly, any other of its quantized vibrational modes.

Skyrme suggested to stabilize the soliton by adding to the action (2.1) a term which is quartic in the derivatives of the fields. Skyrme's action is:

$$I = \int d_4x \; \mathrm{tr}\left(-\frac{f_\pi^2}{4} L_\mu L^\mu + \frac{1}{16e^2}[L_\mu, L_\nu][L^\mu, L^\nu]\right) \tag{2.11}$$

where $L_\mu \equiv U^\dagger(\partial_\mu U)$ and e is an extra parameter of the theory. The classical energy of a system described by this action is:

$$E = \int d_3 r \left(\frac{f_\pi^2}{2} \varphi_i . \varphi_i + \frac{1}{4e^2} \left((\varphi_i . \varphi_i)^2 - (\varphi_i . \varphi_j)(\varphi_i . \varphi_j) \right) \right) > 0 \quad (2.12)$$

where we used the notation:

$$U \equiv \varphi_0 + \vec{\varphi}.\vec{\tau} \qquad \varphi_0^2 + \vec{\varphi}^2 = 1 \qquad \varphi_i \equiv (\partial_i \varphi) \qquad (2.13)$$

The scaling argument given above yields a classical energy of the form $E(R) = A'R + B'/R$ which has an equilibrium radius $R_0 = \sqrt{B'/A'}$. This feature, together with its property of being only quadratic in the time derivatives, was Skyrme's admitted motivation for proposing his action. More profound motivations must have acted however since he was also a mathematician and, as such, he was seduced by the rich mathematical properties of his action [15] . Some of these are discussed in section 3. Others still being discovered and applied today, a quarter of a century later [17].

Physics with the Skyrme action, to which vector mesons are added, is a well trodden field which has reached high-tech sophistication. Three Physics Reports have been devoted to the subject during the last three years [8] and nothing will be gained by trying to summarize them. In these lectures we will stress those features of the Skyrme model which strongly suggest the underlying quark structure of the Skyrmion. We shall see that many complications of the Skyrme model can be avoided by expliciting the quark degrees of freedom.

3. From Skyrmions to quarks.

Skyrme invented the "hedgehog" shaped chiral field which minimizes the energy (2.12):

$$\theta_a(\vec{r}) = \hat{r}_a \theta(r) \qquad \hat{r} \equiv \vec{r}/r \qquad U(\vec{r}) = e^{i\hat{r}.\vec{\tau}\,\theta(r)} \qquad (3.1)$$

The originality of the hedgehog shape is that it couples the isospin and 3-space directions, both denoted by the index a in Eq.(3.1). The coupling between the isospin and 3-space orientations can be understood as a broken symmetry which occurs because of the classical treatment of the fields. The Skyrme action is invariant with respect to both isospin and 3-space rotations whereas the hedgehog field U, which depends on the scalar product $\vec{r}.\vec{\tau}$, is only invariant with respect to the joint rotations. We shall see in section 4 that this broken symmetry gives rise to collective rotations. It is well to remember that translational symmetry is also broken by the field which describes the soliton. Otherwise it would not be localized in space. This broken symmetry gives rise to centre of mass corrections which will be discussed in section 6.

For a baryon number B = 1 (see below) all chiral models favor energetically the hedgehog "spherical" solution (3.1). We shall see in section 4 that such a solution has the angular momentum and isospin coupled to zero and that it gives rise to a rotational band composed of states with J = T [12]. Applied to the nucleon system, the first two members of the rotational band are the nucleon (J=T=1/2) and the delta (J=T=3/2). Whether the rotational band continues to higher values or is cut off at the delta (as the naïve quark model suggests), is discussed in section 5. Since the delta is strongly excited when a nucleon absorbs a pion, Skyrme's hedgehog is well suited to nucleon spectroscopy. It also helps the interpretation of π-N scattering [14].

One can express the energy (2.12) of the Skyrmion in terms of the "profile function" θ(r) defined in Eq.(3.1):

$$E = \frac{f_\pi}{e} 4\pi \int_0^\infty x^2 dx \left(\frac{1}{2}\left(\left(\frac{d\theta}{dx}\right)^2 + \frac{2\sin^2\theta}{x^2} \right) + \frac{\sin^2\theta}{2x^2}\left(\frac{\sin^2\theta}{x^2} + 2\left(\frac{d\theta}{dx}\right)^2 \right) \right) \quad (3.2)$$

The energy is to be minimized with respect to variations of the profile function θ(x) subject to the two boundary conditions:

$$\theta(x) \underset{x \Rightarrow 0}{\sim} n\pi \quad (n \text{ integer}), \qquad \theta(x) \underset{x \Rightarrow \infty}{\sim} 0 \quad (3.3)$$

The boundary condition at the origin is required to keep the energy (3.2) finite. The boundary condition at infinity states that the vacuum prevails outside the soliton.

The integer n turns out to be the baryon number. Indeed Skyrme defined the current:

$$B^\mu \equiv \frac{tr}{24\pi^2} e^{\mu\alpha\beta\gamma} L_\alpha L_\beta L_\gamma = \frac{1}{12\pi^2} \epsilon_{\mu\alpha\beta\gamma}\epsilon^{abcd}\varphi_a\varphi_b^\alpha\varphi_c^\beta\varphi_d^\gamma \quad (3.4)$$

where φ is defined in (2.13) and where $\varphi_b^\mu \equiv \left(\partial^\mu\varphi_b\right)$. The current B^μ is conserved: $\partial_\mu B^\mu = 0$. This property follows from the definition (3.4) of B^μ. It is independent of the form of the lagrangian. Skyrme identified the conserved current B^μ to the baryonic current not only because of its conservation, but also because he found that $\int d_3 r\, B^{\mu=0}(\vec{r}) = n$, where n is an integer. He could thus identify $B^{\mu=0}(\vec{r})$ to the baryon density and n to the baryon number.

When the chiral field has the hedgehog shape (3.1), the baryon density is:

$$B^{\mu=0}(\vec{r}) = -\frac{1}{2\pi^2 r^2}\sin^2\theta\left(\frac{d\theta}{dr}\right) \qquad \int d_3 r\, B^{\mu=0}(\vec{r}) = \frac{1}{\pi}\left|\theta - \frac{\sin 2\theta}{2}\right|_0^\infty = n \quad (3.5)$$

To obtain this result we used the boundary conditions (3.3).

This result is not limited to chiral fields with a hedgehog shape. From the outset Skyrme constructed $B^{\mu=0}(\vec{r})$ to be the Jacobian of the mapping $U(\vec{r})$ of compactified 3-space S_3 onto SU(2). Such mappings can be classified according to a winding number which is equal to $\int d_3r\, B^{\mu=0}(\vec{r})$ and which takes only integer values [21].

Let us pause for a moment to assess what has been achieved. We have obtained a baryon number which is quantized to integer values using only classical fields. Usually, particle number is quantized when the fields are postulated to obey quantum mechanical commutation (or anti-commutation) rules. Here, somehow, topology has replaced quantization. This does not suprise us so much today because such a phenomenon also occurs in the study of monopoles for example. We have avoided introducing a fermion field for the description of the baryon. Skyrme did not like fermion fields. He thought that they should not be introduced into a theory because they have no classical analog in Hilbert space [15]. So he was very pleased to manage without them. He also later showed that his soliton could obey the Dirac equation [16].

We will betray Skyrme in these lectures in the sense that we will introduce explicitely fermions (in the form of quarks) into the theory. We will see that, provided the gradients of the fields are not too strong, quarks can produce the baryonic current (3.4) introduced by Skyrme. They also yield, under the same conditions, the Wess-Zumino term [24] which Witten found neccessary to add to Skyrme's lagrangian [22] . Indeed Witten remarked that the Skyrme action, just like the Weinberg action (2.1), is invariant under the transformation $U \Rightarrow U^{-1}$ which is equivalent to changing the sign of the pion field: $\vec{\pi} \Rightarrow -\vec{\pi}$. This is not however an invariance of the chiral quark models discussed in sections 5 and 6, nor is it an invariance of QCD. This spurious invariance of the Skyrme action forbids for example processes such as $K^+K^- \Rightarrow \pi^+\pi^-\pi^0$ which are allowed by QCD (such processes can only be described when the Skyrme lagrangian is extended to flavor SU(3)). Witten showed that the lowest order symmetry breaking term which could be added to the Skyrme action is the Wess-Zumino term which he showed to be proportional to the number of colors:

$$I_{W.Z.} = \frac{N_c}{240\pi^2} \int d_4x\, \epsilon^{\mu\nu\alpha\beta}\, \mathrm{tr}\, \vec{\theta}.\vec{\tau}\left(\partial_\mu\vec{\theta}.\vec{\tau}\right)\left(\partial_\nu\vec{\theta}.\vec{\tau}\right)\left(\partial_\alpha\vec{\theta}.\vec{\tau}\right)\left(\partial_\beta\vec{\theta}.\vec{\tau}\right) \quad (3.6)$$

This is only the generic form of the Wess-Zumino term. It vanishes when $\vec{\tau}$ are the SU(2) generators. However its gauged form does not vanish and it yields Skyrme's baryonic curent.

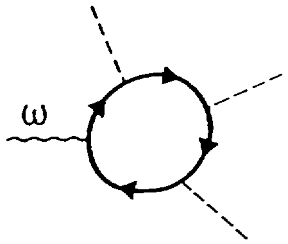
The chiral quark models, described in sections 5 and 6 do not require the Wess-Zumino term because they include it. It can be shown that the Wess-Zumino term is generated by a quark loop in a gradient expansion of the chiral field [19].

There are two confronting approaches to low energy hadronic physics. One is the quark representation adopted in these lectures. It claims that Skyrmion physics, with its quantized baryon number and with its added Wess-Zumino term, is simply begging for quarks and that, once this is realized and the quark degrees of freedom explicitely introduced, the theory becomes richer and simplified both conceptually and in practice. However, the reader should bear in mind the opposite approach which we refer to as the Skyrme picture. It does not introduce quarks and it describes low energy physics in terms of local meson lagrangians involving lowest order gradients of the fields [25].

The relation between Skyrme's baryonic current (3.4) and quarks was explicited by Goldstone and Wilczek [18] in another context. They considered a system of fermions (quarks in our context) interacting with an external chiral field U. The lagrangian which describes this system is:

$$\mathcal{L} = \bar{\psi}\left(i\partial_\mu\gamma^\mu - MU\right)\psi \qquad U \equiv e^{i\gamma_5\vec{\vartheta}.\vec{\tau}} \tag{3.7}$$

where M is an arbitrary mass. The γ_5 appearing in U is required to make the action (3.7) chirally invariant. We want to calculate the current $\langle\bar{\psi}\gamma^\mu\psi\rangle$ carried by the quarks in the ground state of the system described by the lagrangian (3.7). To do this we introduce a small source term $\omega_\mu\gamma^\mu$ into the lagrangian which becomes $\mathcal{L} = \bar{\psi}\left(i\partial_\mu\gamma^\mu - MU - \omega_\mu\gamma^\mu\right)\psi$. We split the lagrangian into an unperturbed part with U=1, ω=0 and a perturbation proportional to (U-1) and ω. The Feynman graphs are composed of a single fermion loop with vertices representing either the perturbation M(U-1) or $\omega_\mu\gamma^\mu$. For the purpose of calculating the current $\langle\bar{\psi}\gamma^\mu\psi\rangle$ we only need the graphs with one vertex $\omega_\mu\gamma^\mu$. The lowest order non-vanishing graph is third order in M(U-1):

$\Gamma =$ 

$$= \mathrm{Tr}\ \omega_\mu\gamma^\mu \frac{1}{i\partial_\mu\gamma^\mu - M} M(U-1) \frac{1}{i\partial_\mu\gamma^\mu - M} M(U-1) \frac{1}{i\partial_\mu\gamma^\mu - M} M(U-1) \frac{1}{i\partial_\mu\gamma^\mu - M} \tag{3.8}$$

It is finite and requires no renormalization. The current carried by the quarks in the ground state is $\left\langle\bar{\psi}(x)\gamma_\mu\psi(x)\right\rangle = \left.\frac{\delta\Gamma}{\delta\omega_\mu(x)}\right|_{\omega=0}$. An explicit calculation of the graph (3...) yields the curent:

$$\left\langle\bar{\psi}(x)\gamma_\mu\psi(x)\right\rangle = \frac{\nu}{48\pi^2}\,\epsilon_{\mu\alpha\beta\gamma}\epsilon^{abcd}\varphi_a\varphi_b^\alpha\varphi_c^\beta\varphi_d^\gamma \tag{3.9}$$

where φ is defined in (2.13), ν is the spin, isospin and color degeneracy of the quarks ($\nu = 4\,N_c = 12$ for u and d quarks). The baryonic current of the quarks is $B_\mu(x) = (1/N_c)\left\langle\bar{\psi}(x)\gamma_\mu\psi(x)\right\rangle$. It is independent of the mass M appearing in the lagrangian (3.7) and it is identical to Skyrme's baryonic current (3.4).

We see that Skyrme's current (3.4) may be thought of as the baryonic current of quarks which are coupled to the chiral field U. It is only identical to the quark current in lowest order perturbation theory.

To see this more explicitely, let us calculate exactly the quark orbits in the presence of a chiral field with the hedgehog shape (3.1). This calculation is required for all chiral soliton models based on quarks. When working with time-independent fields it is simpler to use the hamiltonian:

$$H = \int d_3 r\; \psi^\dagger\left(\frac{\alpha.\nabla}{i} + \beta MU\right)\psi \equiv \int d_3 r\; \psi^\dagger h\, \psi \qquad (3.10)$$

instead of the action (3.7).

The eigenstates of the hamiltonian H are Slater determinants composed of quark orbits which are the eigenstates of the Dirac hamiltonian:

$$h = \frac{\vec{\alpha}.\vec{\nabla}}{i} + \beta M\left(\cos\theta(r) + i\gamma_5\hat{r}.\vec{\tau}\sin\theta(r)\right) \qquad (3.11)$$

The mass M may be eliminated from the Dirac hamiltonian by working with the dimensionless length $\vec{x} = M\vec{r}$, in which case we have:

$$\frac{h}{M} = \frac{\vec{\alpha}.\vec{\nabla}_x}{i} + \beta\left(\cos\theta + i\gamma_5\hat{x}.\vec{\tau}\sin\theta\right) \qquad (3.12)$$

The quark orbits are the normalized eigenstates of h:

$$h|\lambda\rangle = \epsilon_\lambda|\lambda\rangle \qquad\qquad \langle\lambda|\lambda\rangle = 1 \qquad (3.13)$$

and they can be calculated numerically [23] . The Dirac hamiltonian commutes with the parity operator. It is invariant with respect to joint rotations in 3-space and isospin space. It therefore also commutes with the "grand spin" operator $\vec{K} = \vec{j} + \vec{t}$ where $\vec{j}$ and $\vec{t}$ are the angular momentum and isospin operators. The orbits can be labelled by their parity and grand spin K. For example, a 0^+ orbit has grand-spin K=0 and positive parity. The spectrum obtained with an exponential profile $\theta(r) = \pi\exp(-r/R) = \pi\exp(-x/RM)$ is plotted on Fig.1. The parameter R measures the size of the region of space where the chiral field differs from its vacuum value. For very small sizes, the chiral field does not act at all and the quark orbits are those of free Dirac particles. As the size parameter is increased, one positive energy 0^+ orbit lowers its energy and joins the negative energy orbits. The number of negative energy orbits is increased by one unit. The number of quarks which can fill a 0^+ orbit is N_c. Thus the state, obtained by filling all the orbits up to and including this 0^+ orbit, has baryonic number B=1 relative to the vacuum. Fig.1 also shows the spectrum of the orbits in the case where the field has winding number n=2. The profile function is now $\theta(r) = 2\pi\exp(-r/R)$. We see that two K=0 orbits of opposite parities come down and join the negative energy orbits. Each K=0 orbit can be filled with N_c quarks so that the state constructed by filling all the orbits up to and including the two K=0 orbits has baryon number B=2. Needless to say, this continues for any n.

Consider next the baryon density. It may be calculated from the quark orbits (3.14):

$$\langle\bar{\psi}\gamma^{\mu=0}\psi\rangle = \langle\psi^\dagger\psi\rangle = \sum_{e_\lambda \in D}\langle\lambda|\vec{r}X\vec{r}|\lambda\rangle - \sum_{e_k \in D_0}\langle k|\vec{r}X\vec{r}|k\rangle + \sum_{v}\langle v|\vec{r}X\vec{r}|v\rangle \tag{3.14}$$

The first term is the contribution of the Dirac sea orbits. The Dirac sea orbits may be identified by continuity starting from MR=0 (see Fig.1). The second term subtracts the contribution of the unperturbed Dirac sea which represents the vacuum. The last term is the contribution of the quarks which are added to the Dirac sea and which are in valence orbits. Figure 2 shows the integrated baryonic density (3.14) and they compare it to Skyrme's density (3.4) calculated with the same profile function. The densities shown in Fig.2 were calculated with a chiral field with winding number n=1. All the negative energy orbits together with the valence 0^+ orbit (Fig.1) were filled. We see that for a large enough size RM the exact density (3.14) agrees well with Skyrme's density (3.4) whereas for a small size the agreement is not so good.

We can now interpret the perturbative calculation (3.8) of the quark current. A careful analysis reveals that the perturbation series is an expansion in the gradients of the chiral field U. The result (3.9) is the lowest order contribution. Higher order contributions involve sixth and higher order derivativers of the chiral field. They depend on the mass parameter M and the expansion is in powers of 1/RM [19].

The ground state of the Hamiltonian (3.10) is the state in which all the negative energy orbits are filled and this is the state which is described by the perturbation theory. The perturbation theory is valid for large values of RM. Fig.1 shows that for large RM the ground state of H contains the "valence" 0^+ orbit when the winding number of the field is n=1, the two K=0 orbits when n=2, etc.

Fig.2 illustrates an instance in which the Skyrme picture is a low gradient approximation of the quark picture. When solitons are calculated in quark models (sections 5 and 6), one can check whether the soliton size allows a gradient expansion or not. The gradient expansion seems to break down at values of RM $\simeq$ 2 at which the 0^+

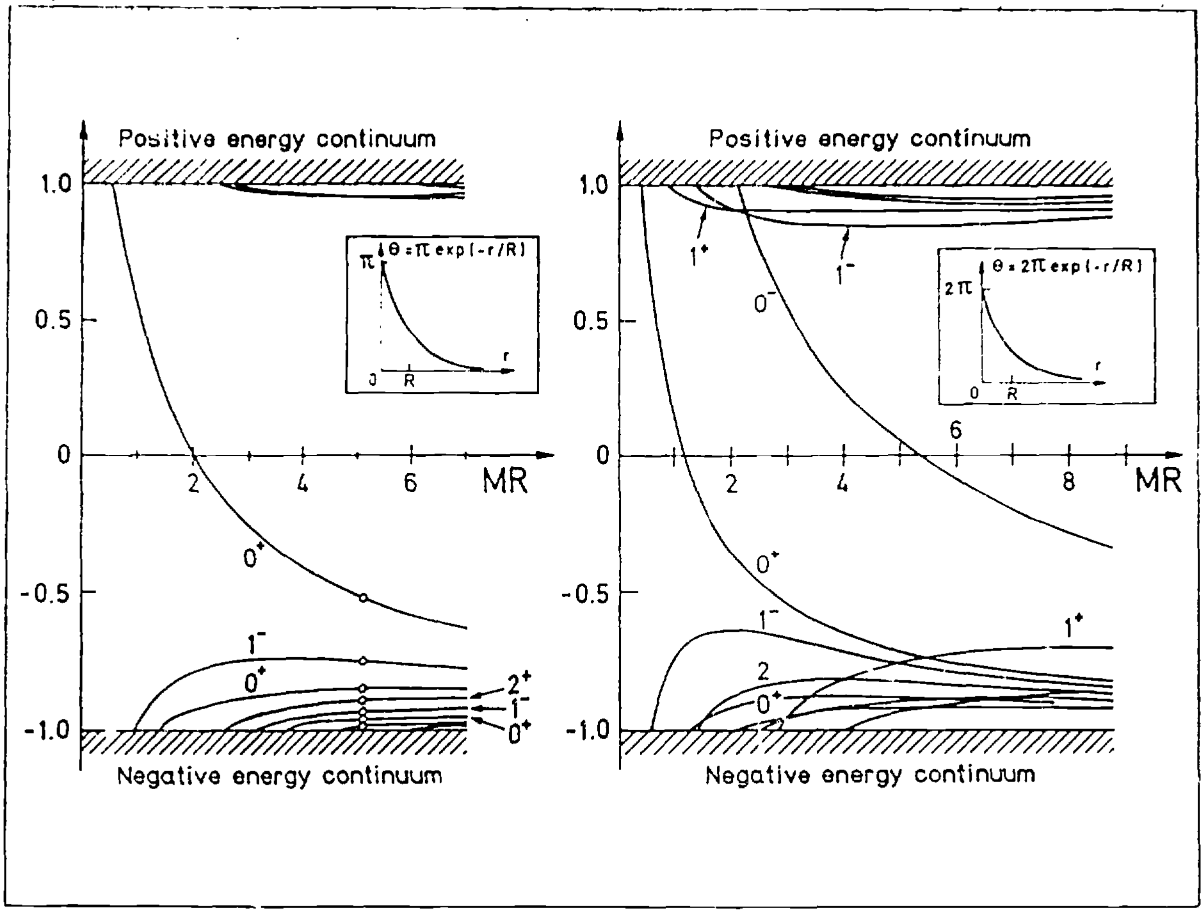


Figure 1: spectrum of the Dirac hamiltionian (3.11) for a hedgehog field with winding number n=1 (left figure) and n=2 (right figure) The profile function (3.1) of exponential form is displayed in the window. The energies of the orbits are expressed in units of the mass M. They are plotted in terms of the dimensionless mass parameter MR. The orbits are labelled by their grand spin and parity.

orbit becomes positive. The gradient expansion is thus valid for sizes $R > 2/M$. The larger M is the closer will the baryon density be to Skyrme's density. Typical values of M which occur in chiral solitons range from 300 MeV to 800 MeV (see sections 5 and 6). The quark radius of the proton is expected to be about 0.5 fm = $(0.4\ \text{GeV})^{-1}$ so that the gradient expansion often fails for the chiral fields used to describe the nucleon. This conclusion is however model dependent because the value of M is model dependent. The validity of the gradient expansion is discussed in Ref.[20].

In general, Skyrme picture (including the Wess-Zumino term) can be viewed as the lowest order expansion of a quark representation in the limit of large RM, that is, in a gradient expansion of the chiral field. The quark picture differs from Skyrme's in several other aspects. In quark models such as the Nambu Jona-Lasinio model (section 6) the boundary condition (3.3) is dynamic in the sense that it

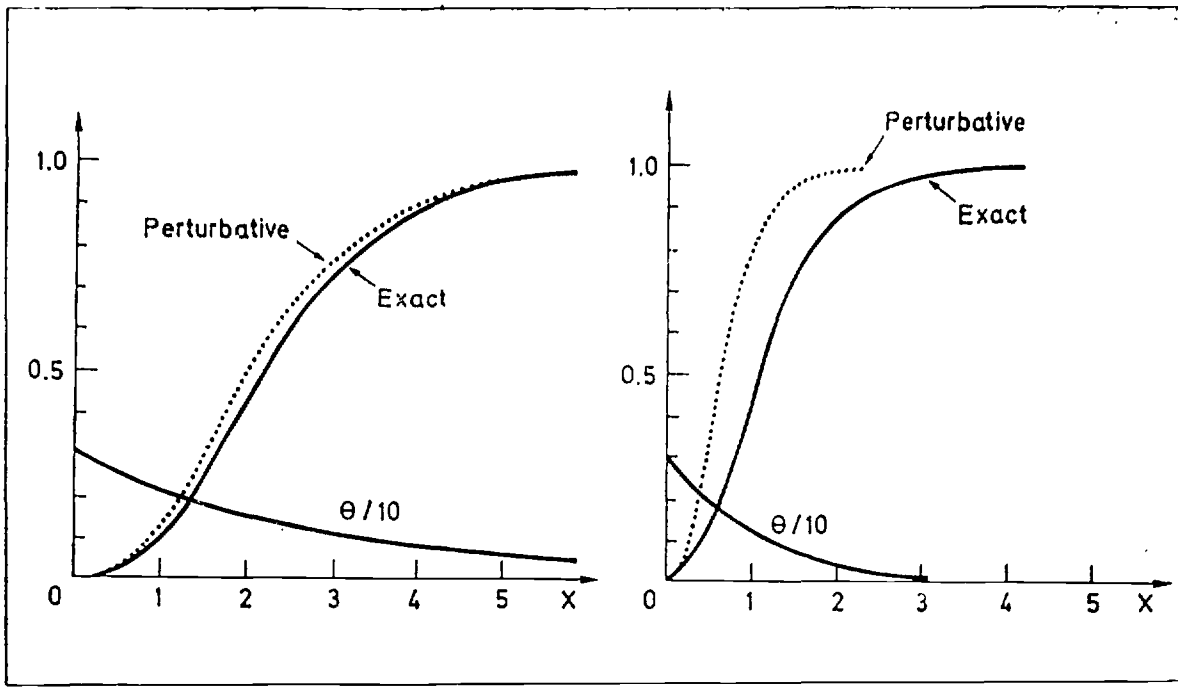
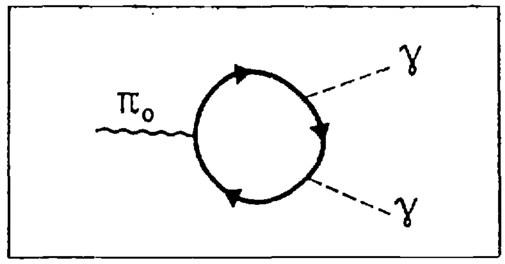


Figure 2: the integrated baryon density $\int_0^{Mr} 4\pi x^2 \langle \psi^\dagger(x)\psi(x) \rangle dx$ of quarks in a hedgehog shaped chiral field with winding number n=1, displayed in the window of the upper Fig.1. The left curve is for a size parameter MR=3 and the right curve for a smaller size parameter MR=1. The full curve labelled "exact" is calculated using Eq.(3.14). The dashed curve labelled "perturbative" is the lowest order perturbative calculation (3.9) which is identical to Skyrme's density (3.4).

minimizes the energy. But, in contrast to the Skyrme model, the energy remains finite if the chiral angle at the origin differs from a multiple of π. One can go continuously from a boundary condition with n=1 at the origin to a boundary condition with n=2. The topological constraints have lost much of their relevance to the dynamics of the quark system. In particular the topological stability of the Skyrmion is lost. This is why solitons calculated in quark models are said to be non-topological.

It is amusing to speculate what might have happened if Skyrme had been taken seriously in 1960. Some bright young physicist, called Goldwitt or Goldwilk, might have discovered that Skyrme's baryonic current could be derived by introducing fermions which interact with Skyrme's chiral field U. This might have triggered the idea that the nucleon is composed of these fermions. There would have to be an odd number of these fermions in order to give the nucleon half-integer spin. He might have tried the idea that the nucleon is composed of not one but of three fermions and he would have guessed their charge by getting the proton and neutron charge right. He might even have gone on to compute the π^0 electromagnetic decay rate assuming that the process was mediated by his fermions:



(3.15)

His calculation would have revealed that, in order to fit experiment, the fermions would require an additional internal quantum number equal to 3. He might have discovered quarks and color before Gell-Mann...

Chiral quark models will be discussed in sections 5 and 6. Before, we pause briefly for a study of the rotations of the Skyrmion.

4. Rotations of the Skyrmion.

An isospin rotation of the chiral field may be expressed as the following transformation of the matrix U:

$$U \Rightarrow AUA^{-1} \qquad U^{\dagger} \Rightarrow AU^{\dagger}A^{-1} \qquad A \equiv e^{-i\vec{\alpha}.\vec{\tau}/2} \tag{4.1}$$

The isospin rotation is parametrized by the three real variables $\vec{\alpha}$. Instead, we could have parametrized the rotation in terms of three real Euler angles α,β and γ by writing:

$$A = e^{-i\alpha\tau_3/2}e^{-i\beta\tau_2/2}e^{-i\gamma\tau_3/2} \tag{4.2}$$

If we use the linear representation (2.13) of the matrix U, we find that an infinitesimal isospin rotation of the form (4.1) transforms the fields as $\varphi_0 \Rightarrow \varphi_0$, $\vec{\varphi} \Rightarrow \vec{\varphi} + (\vec{\alpha}\times\vec{\varphi})$ so that φ_0 (the σ-meson field) is isospin invariant and $\vec{\varphi}$ (the pion field) is an isovector.

We study rotations in the same way as the radial vibrations discussed in section 2. We only sketch the procedure here. We assume that the angles which define the matrix A are time dependent and that the only time dependence of the fields is due to the rotation [26]:

$$U(\vec{r},t) = A(t)U_0(\vec{r})A^{-1}(t) \qquad \partial_t U = A\left[A^{-1}\dot{A},U_0\right]A^{-1} \tag{4.3}$$

The field U_0 may be viewed as the chiral field which is stationary (time independent) in the rotating frame. The angles defining the rotation matrix A are collective coordinates which are time but not space dependent.

The rotational frequency $\vec{\Omega}$ is defined by the expression:

$$A^{-1}\dot{A} = -i\,\frac{\vec{\Omega}.\vec{\tau}}{2} \qquad (4.4)$$

Using the linear representation (2.13) of U, we find that:

$$(\partial_t U) \equiv (\partial_t \varphi_0) + i\vec{\tau}.\left(\partial_t \vec{\varphi}\right) - i\vec{\tau}.A(\vec{\Omega}x\vec{\varphi})A^{-1} \qquad (4.5)$$

If we sustitute the field (4.3) into the Skyrme action (2.11), we find:

$$I = \int dt \left(-E + \int d_3 r \left(\frac{f_\pi^2}{2} (\vec{\Omega}x\vec{\varphi})^2 \right.\right.$$

$$\left.\left. \frac{f_\pi^2}{2}(\vec{\Omega}x\vec{\varphi})^2 + \frac{1}{2e^2}(\vec{\Omega}x\vec{\varphi})^2 \left((\partial_i \varphi_0)^2 + \left(\partial_i \vec{\varphi}\right)^2\right) - \frac{1}{2e^2}\left(((\vec{\Omega}x\vec{\varphi}) . \left(\partial_i \vec{\varphi}\right)\right)^2 \right)\right) \qquad (4.6)$$

where E is the static energy (2.12).

This is a quadratic form in the angular velocity $\vec{\Omega}$ which simplifies if one assumes the hedgehog shape (3.1) for the chiral field. It reduces then to the expression:

$$I = \int dt \left(E - \frac{1}{2} \mathcal{J} \vec{\Omega}^2 \right) \equiv \int dt\ L \qquad (4.7)$$

where $\mathcal{J}$ is a moment of inertia equal to:

$$\mathcal{J} = \frac{2}{3e^3 f_\pi} \int 4\pi x^2 dx\ \sin^2\theta \left(1 + \frac{\sin^2\theta}{x^2} + \left(\frac{d\theta}{dx}\right)^2 \right) \qquad (4.8)$$

It can be calculated in terms of the profile function θ. It should be stressed however that the action (4.7) is only valid for low angular velocity $\vec{\Omega}$ because it assumes that the chiral field keeps its hedgehog shape (3.1) in the rotating frame. At large angular velocity the terms proportional to $\vec{\Omega}x\vec{\varphi}$ will tend to distort the chiral field from its hedgehog shape. Whether the distortions are large or small, when the system acquires an isospin 3/2 as in the Δ for example, is discussed at the end of section 5.

The action (4.7) describes a rotor with moment of inertia $\mathcal{J}$. The rotation may be quantized by choosing, for example, the Euler angles (4.2) as dynamical variables. The canonical quantization of the Euler angles leads to a hamiltonian for the collective motion of the form $H = \frac{\mathcal{L}^2}{2\mathcal{J}} + E$ where $\mathcal{L}_a$ (a=1,2,3) are the body-fixed isospin operators. The eigenfunctions of the rotating soliton are then rotation matrices $D^I_{MK}(\alpha,\beta,\gamma)$ [27]. The energy of the soliton in this state is $E_T = E + \frac{T(T+1)}{2\mathcal{J}}$.

A more careful analysis would have lead us to consider separately rotations in 3-space and in isospin space. The analysis would have revealed that, due to the invariance of the hedgehog shaped U-field with respect to joint rotations in isospin and 3-space, only states with I=T are generated by the collective rotation. The nucleon and the Δ are obvious candidates for such a rotational band.

There is nothing in the Skyrme lagrangian to tell us what values the isospin T can take. It can be integer of half-integer. To discover that a Skyrmion with winding number n=1 is a fermion with odd integer spin, we must have recourse to the Wess-Zumino term [22]. In the quark models described in sections 5 and 6, there is no such ambiguity since the nucleon is composed of $N_c=3$ quarks.

In section 5 we shall see that in chiral quark models the Δ is not composed of three quarks with aligned spin and isospin because about 75% of the spin is carried by the chiral field. We shall also discuss there the question of whether the rotational band cuts off at J=T=3/2, as constituent quark models suggest, or if it continues to higher values, as Skyrme's model suggests.

5. Chiral solitons as bound states of quarks.

The perturbative calculation (3.8) of Skyrme's baryonic current and the derivation of the Wess-Zumino term from a quark loop [19] strongly suggest that Skyrme lagrangians describe systems of quarks interacting with a chiral field. In this section we introduce the quark dynamics explicitely.

The simplest thing one can do is to add quarks to the linear (or non-linear) σ-model (2.5). The action of the system is then:

$$I = \int d_4x \, \bar{\psi}\left(i\partial_\mu\gamma^\mu - g\left(\sigma + i\gamma_5\vec{\pi}.\vec{\tau}\right)\right)\psi$$

$$+ \int d_4x \left(\frac{1}{2}(\partial_\mu\sigma)(\partial^\mu\sigma) + \frac{1}{2}(\partial_\mu\pi_a)\left(\partial^\mu\pi_a\right) - \frac{\kappa^2}{8}\left(\sigma^2 + \pi_a^2 - f_\pi^2\right)^2\right) \quad (5.1)$$

The first term describes quarks represented by the spinor ψ and interacting with the chiral field $(\sigma,\vec{\pi})$. If one sets the pion field to zero, the action (5.1) reduces to the soliton model of Friedberg and Lee [28] . It therefore extends this model to the pion degree of freedom and chiral symmetry. In the physical vacuum, the classical values of the fields are $\sigma = f_\pi$ and $\vec{\pi} = 0$ and the quarks acquire a mass gf_π which we call the "constituent quark" mass.

The first calculations of a bound state were performed using only valence quark orbits and neglecting the Dirac sea orbits [29,30]. In this approximation, the the energy of the soliton is:

$$E = N_c e_\lambda + \int d_3 r \left(\frac{1}{2}(\nabla_i \sigma)^2 + \frac{1}{2}(\nabla_i \pi_a)^2 + \frac{\kappa^2}{8}\left(\sigma^2 + \pi_a^2 - f_\pi^2\right)^2\right) \qquad (5.2)$$

where e_λ is the energy of the valence quark orbit obtained by solving the Dirac equation:

$$\left(\frac{\alpha.\nabla}{i} + g\beta\left(\sigma + i\beta\gamma_5 \vec{\pi}.\vec{\tau}\right)\right)|\lambda\rangle = e_\lambda |\lambda\rangle \qquad \langle\lambda|\lambda\rangle = 1 \qquad (5.3)$$

The valence orbit is occupied by N_c quarks.

The fields σ and $\vec{\pi}$ are obtained by minimizing the energy (5.3). A hedgehog field with winding number n = 1 is found to minimize the energy of a system of N_c quarks. The state is bound (and therefore localized) if its energy is lower than $N_c g f_\pi$ otherwise it could decay into N_c free quarks in the vacuum. Such a state is a non-topological soliton.

The spectrum displayed on Fig.1 suggests that if the chiral field is given a hedgehog shape (3.1), the quarks which fill the valence 0^+ orbit can considerably lower their mass and the soliton energy (5.2). A bound state may thus be formed when N_c quarks fill the 0^+ orbit provided that the kinetic energy of the chiral field (the gradient terms) is not too large. When the size parameter R increases, the energy of the 0^+ orbit decreases while the kinetic energy of the chiral field grows linearly with R. Thus there should be an equilibrium radius for the bound state of quarks. Calculations show that a stable bound state of quarks is indeed formed by a hedgehog chiral field, provided the coupling constant g is strong enough. A strong coupling constant implies a large constituent quark mass mass $g f_\pi$ and it ensures that the 0^+ orbit is strongly bound. This can been seen from the scaling property (3.12). Not suprisingly, the soliton calculated this way has properties quite similar to the Skyrmion.

The main result of these calculations is illustrated on Fig.3 which displays the effective potential of the σ-model, that is, the energy per unit volume of the vacuum, as a function of the σ and π fields. The energy has the well known "mexican hat" or "wine bottle" shape. The vacuum is the point $\sigma = f_\pi$, $\pi = 0$ marked V on Fig.3. Imagine that you enter the soliton from the outside and that you measure σ and π on the way. In the Friedberg-Lee model, you start at the vacuum point V and, as you enter the soliton, you climb up to the point B where the constituent quark mass is zero and where chiral symmetry is restored. Indeed, in that model, the pion degree of freedom is absent and there is no other way for the quarks to reduce their mass. The Friedberg-Lee model was designed to generate dynamically the bag model. The quarks form a bound state by reducing their mass inside the soliton at the expense, not only of kinetic energy of the σ-field, but also of volume energy, expressed by the last term of (5.2). The energy per unit volume at the point $\sigma = \pi = 0$ is in fact just the MIT bag constant.

Now suppose you enter a chiral soliton calculated with a hedgehog field of winding number n=1. You start from the same point $\sigma = f_\pi$, $\pi = 0$ marked V on Fig.3 and, as you enter, instead of climbing up to the point B, you follow the chiral circle along the rim of the mexican hat until you reach the point S, as may be checked from Eqs.(3.3). This is energetically favored for two reasons. First the quarks reduce more their mass in a hedgehog field than in a σ field because, in the absence of a pion field, a valence orbit has always positive energy. Second, all the volume energy is saved because along the chiral circle $\sigma^2 + \pi^2 - f_\pi^2 = 0$. All chiral models, including the Nambu Jona-Lasinio model described below, favor energetically the path which follows the chiral circle rather than climbing up to the point B. The chiral models differ from bag models in the way they bind quarks so as to form a soliton. At present there is no evidence coming either from experiment or from QCD lattice calculations which might indicate what the inside of the nucleon is actually like. In the absence of better evidence we prefer the energetically favored chiral models.

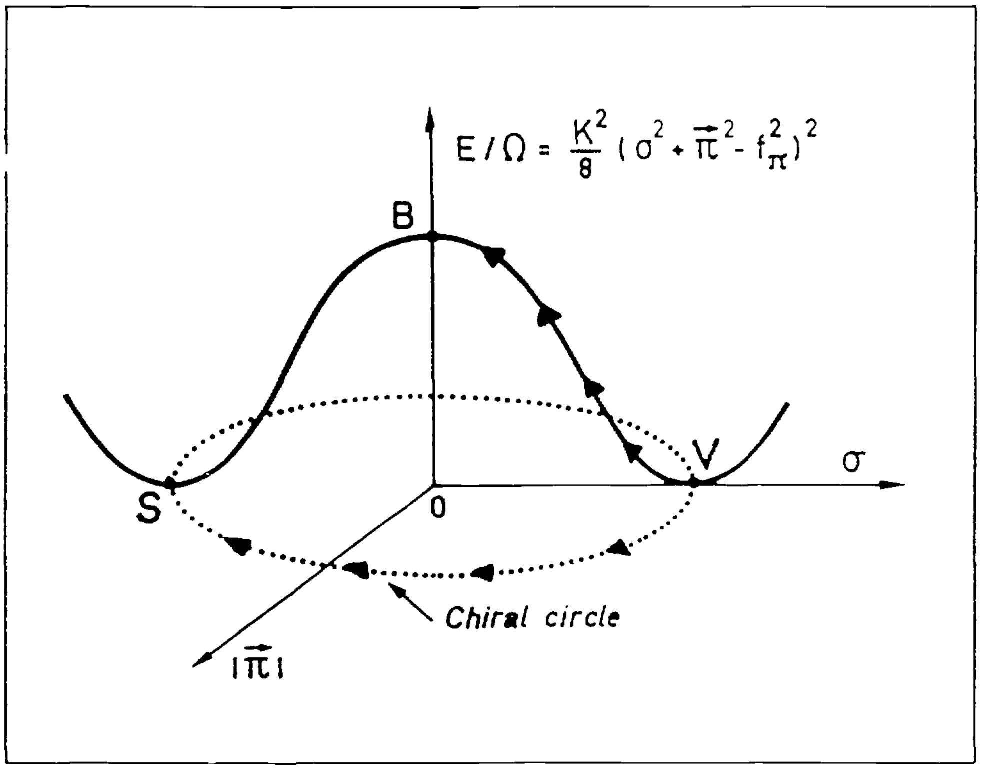


Figure 3: The energy per unit volume of the vacuum in the σ-model is plotted as a function of the σ and π classical fields. The vacuum is represented by the point V ($\sigma = f_\pi$, $\pi = 0$). The center of a Friedberg Lee soliton (or of a bag) is at the point B ($\sigma = \pi = 0$) where chiral symmetry is restored and where the energy per unit volume is the MIT bag constant B. The center of a chiral soliton (and of a Skyrmion) is the point S where the angle θ is rotated by π relative to the vacuum.

Later calculations [23,31] included the effects of Dirac sea quarks. The σ-model, being renormalisable, is ideally suited to the purpose. However, it was soon realized [32] that in spite of the renormalisability of the σ-model, the Dirac sea introduces a new instability in the form of Landau ghosts. This is due to high gradients of the fields. For this reason, more recent work has concentrated on Nambu Jona-Lasinio type models which are not renormalizable but which are free from Landau ghost instabilities.

The quark model discussed in this section shares with the Skyrmion an interesting scaling property which tells us how the soliton is modified when it is immersed in a nuclear medium. The energy (5.2) may be expressed in terms of the following dimensionless variables:

$$\varphi_0 \equiv \frac{\sigma}{f_\pi} \qquad \vec{\varphi} \equiv \frac{\vec{\pi}}{f_\pi} \qquad \vec{x} \equiv g f_\pi \vec{r} \qquad \epsilon_\lambda \equiv \frac{e_\lambda}{g f_\pi} \tag{5.4}$$

It becomes:

$$\frac{E}{N_c g f_\pi} = \epsilon_\lambda + \frac{1}{N_c g^2} \int d_3x \left(\frac{1}{2}\left(\frac{\partial \varphi_0}{\partial x_i}\right)^2 + \frac{1}{2}\left(\frac{\partial \varphi_a}{\partial x_i}\right)^2 + \frac{\kappa^2}{g^2}\left(\varphi_0^2 + \varphi_a^2 - 1\right)^2 \right) \tag{5.5}$$

To obtain this result we used the scaling property (3.15) of the Dirac equation. Far outside the soliton, the σ field acquires its vacuum value f_π:

$$\lim_{r \Rightarrow \infty} \sigma = f_\pi \tag{5.6}$$

However, in a nuclear medium of finite density f_π is reduced [34,36,39] so that at infinity the σ-field acquires a smaller value. Since the right hand side of Eq.(5.5) is independent of f_π, we deduce that the soliton energy decreases (and the soliton size increases) by the same fraction as the asymptotic value of σ [48]. In Walecka's theory of nuclear matter saturation for example, the σ-field in nuclear matter at normal density is reduced by a factor of $M^*/M \simeq 0.6$. If this was true, then the Skyrmion and the quark model discussed in this section would swell by a factor of 1.6! A more recent estimate [53] is $M^*/M \simeq 0.95$. This would imply a small increase of about 5% of the nucleon size in a nucleus. In the Nambu Jona-Lasinio model (section 6) the scaling property is more complicated.

Another application of the quark model is the study of non-linear distortions of the rotating solitons [50]. In section 4 we saw that the rotating Skyrmion could be studied by seeking a stationary configuration of the chiral field in a rotating frame. The same method can be applied to the quark model. The only difference is the appearance of an additional coriolis term of the form $-\vec{\Omega}.\vec{T}$ in the Dirac equation (5.3) which tries to align the quark isospins along the rotation axis. As a result the hedgehog shape of the chiral field is distorted. We refer the reader to Ref. [50] for details. At zero rotational frequency, the soliton has a hedgehog shape. As the system rotates, the hedgehog shape is distorted. However the spin and isospin of the soliton remain equal at all rotational frequencies. It is found that, at a finite rotational frequency, the quarks align their spins to form a $J=T=N_c/2=3/2$ state (see Fig.4). However, contrary to what constituent quark models predict, this state is not the Δ. The Δ turns out to be a more slowly rotating $J=T=3/2$ state in which about 1/4 of the spin is carried by the quarks and 3/4 by the pion field. Recent experiments might confirm this [51] . The angular velocity of the soliton in this $J=T=3/2$ state is small enough to give some justification to a perturbative treatment of the rotation as done in section 4. When studying rotating solitons, one has to take into account the radiation of pions because the Δ mass is well above the pion emission threshold. When this is done, the calculated rate of pion emission from the $J=T=3/2$ state compares well with the observed 115 MeV width of the Δ (see Fig.4).

When the rotational frequency attains a critical value, the quarks align their spin so as to form a $J=T=3/2$ state. But before this, the spin of the rotating soliton barely exceeds 5/2 (the maximum attained spin depends on the size of the soliton). Unfortunately we cannot observe a $T=5/2$ state in π-nucleon scattering so that we don't know if such a state exists.

The $J=T=3/2$ state consisting of aligned quark spins cannot be studied perturbatively. It is most likely an excited state of the Δ. It is not easy to study this state in a Skyrme model because, when the quarks align, the pion field has only one non-vanishing component along the rotation axis. Eq.(3.4) shows that such a field would produce zero baryonic current. For such a state, higher order contributions to the baryonic current would have to be introduced into the Skyrme model...

6. The Nambu Jona-Lasinio model.

The Nambu Jona-Lasinio model [33,34,35] describes a system of quarks with a chirally symmetric 4-fermion local interaction. The hamiltonian of the system is:

$$H = \int d_3 r \left(\psi^{\dagger} \frac{\alpha . \nabla}{i} \psi - \frac{1}{2a^2} \left((\bar{\psi}\psi)^2 - \left(\bar{\psi}\gamma_5 \vec{\tau}\psi\right)^2 \right) \right) \qquad (6.1)$$

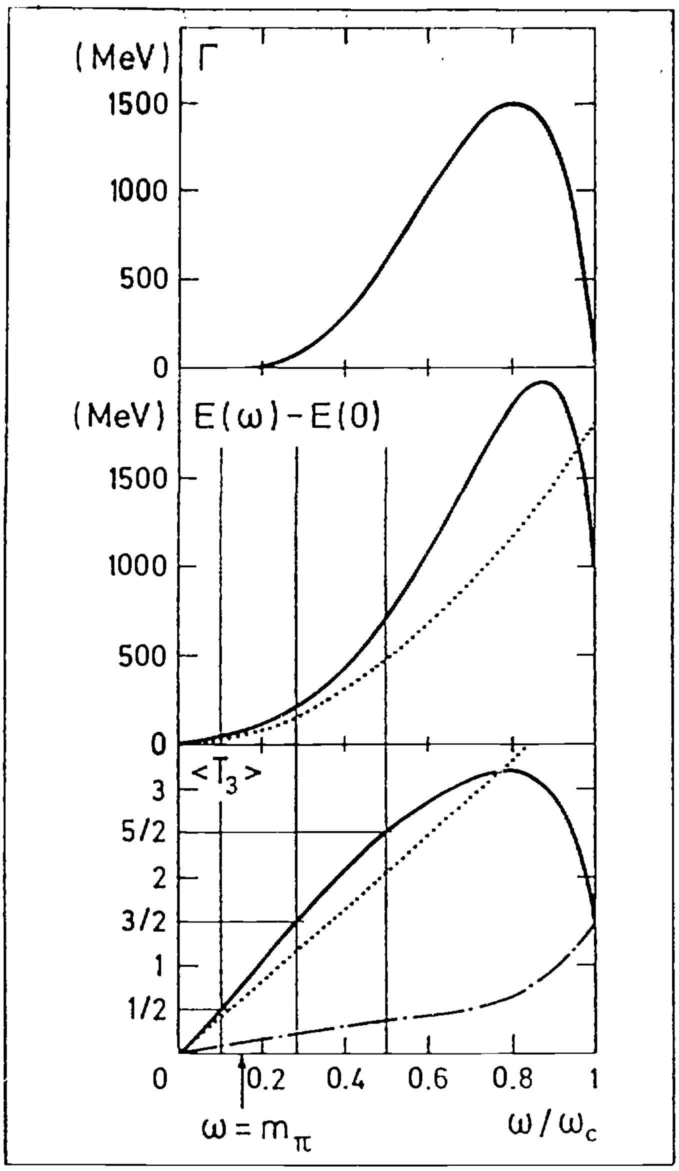


Figure 4: Properties of a rotating chiral soliton plotted againt the ratio of the rotational frequency ω to the critical frequency ω_c at which the quarks align their isospin. The full line of the lower figure shows the isospin $\langle T_3 \rangle$ acquired by the rotating soliton. The spin and isospin of the soliton are equal and opposite at all frequencies. The dot-dashed line shows the quark contribution to the spin. At low rotational frequencies, the quarks contribute about 25% of the spin, the rest being due to the pion field. At the critical frequency ω_c all the spin is carried by the aligned quarks and none by the pion field. The dashed curve shows the spin $\mathfrak{J}\omega$ where $\mathfrak{J}$ is the moment of inertia calculated perturbatively at low angular frequency. The full line of the middle figure shows the energy of the rotating system. The dashed line is the perturbative energy $\mathfrak{J}\omega^2/2$. The upper figure shows the pion emission rate (in MeV) of the rotating soliton. It vanishes when the rotational frequency is smaller than the pion mass m_π. The vertical lines indicate the frequencies at which the isospin is equal to 1/2, 3/2 and 5/2 respectively. The isospin does not exceed a value of 5/2.

This hamiltonian has an equivalent form:

$$H = \int d_3 r \left(\psi^{+}\left(\frac{\alpha.\nabla}{i} + \beta\Phi U\right)\psi + \frac{a^2\Phi^2}{2} \right) \qquad U = e^{i\gamma_5\vec{\theta}.\vec{\tau}} \tag{6.2}$$

where Φ is a real scalar field and U a chiral field. Notice that, in contrast to the σ-model (5.1), no kinetic energy of the chiral field appears in the hamiltonian (6.2). The chiral field acts as a constraint. Indeed, the equivalence of the forms (6.1) and (6.2) can be derived using the linear representation $\Phi U = S + i\gamma_5\vec{P}.\vec{\tau}$ and by requiring the eigenvalues of H to be stationary with respect to variations of the fields S and $\vec{P}$. The Hartree approximation applied to the form (6.1) is equivalent to the assumption that the field ΦU is a classical field. We will work in this approximation. In section 7 we discuss an extension of the model which includes vector fields.

The Nambu Jona-Lasinio model may be viewed as a generalization to relativistic systems of the Landau theory of fermi liquids. The aim is to see how far the low-energy properties of hadronic matter can be described with zero-range effective interactions. The Nambu Jona-Lasinio model has the advantage of introducing a minimum number of parameters and this makes its limits of applicability appear quite clearly.

The hamiltonian (6.2) produces a spontaneous chiral symmetry breakdown. This is easily seen by considering a translationally invariant system in which the chiral field is independent of $\vec{r}$. A Weinberg rotation $\psi \Rightarrow q = U^{1/2}\psi$ can then eliminate the U field from the hamiltonian which becomes:

$$H = \int d_3 r \; q^{+}\left(\frac{\alpha.\nabla}{i} + \beta\Phi\right)q + \frac{a^2\Phi^2\Omega}{2} \tag{6.3}$$

where $\Omega \equiv \int d_3 r$ is the volume of the system. The hamiltonian (6.3) describes a system of free Dirac particles of mass Φ. The ground state is a Dirac sea with an energy per unit volume equal to:

$$\frac{E}{\Omega} = -\frac{\nu}{\Omega}\sum_k \sqrt{k^2 + \Phi^2} + \frac{a^2\Phi^2}{2} \tag{6.4}$$

In the physical vacuum, the field Φ acquires the value Φ_0 which makes the energy stationary:

$$\left.\frac{\partial E}{\partial \Phi}\right|_{\Phi=\Phi_0} = 0 \qquad a^2 = \frac{\nu}{\Omega}\sum_k \frac{1}{\sqrt{k^2 + \Phi_0^2}} \tag{6.5}$$

where ν is the quark degeneracy. Equation (6.5) is usually referred to as the "gap equation" because $2\Phi_0$ is the mass gap separating the filled Dirac sea orbits from the empty positive energy orbits (see

Fig.5). We can redefine the energy (6.4) by subtracting the minimum energy $E(\Phi\to\Phi_0)$. We also use the gap equation (6.5) to eliminate the constant a^2 in favor of Φ_0. The energy per unit volume of the vacuum is then:

$$\frac{E}{\Omega} = -\frac{\nu}{\Omega}\sum_k \left(\sqrt{k^2+\Phi^2} - \sqrt{k^2+\Phi_0^2} - \frac{\Phi^2-\Phi_0^2}{2\sqrt{k^2+\Phi_0^2}}\right) \qquad (6.6)$$

When plotted against Φ, the energy (6.6) has the form displayed on Fig.3. The minimum energy occurs at a finite value of $\Phi = \Phi_0$ and in the vacuum the quarks acquire a mass Φ_0 which we call the "constituent quark" mass. Chiral symmetry is thus broken in the physical vacuum, as is observed. Note that Φ_0 is not not equal to f_π as Fig.3 might suggest. Indeed, in the linear representation (6.2) of the chiral field, S and $\vec{P}$ are proportional (but not equal) to the sigma and pion fields of the σ-model because they get renormalized. This is shown in section 7.

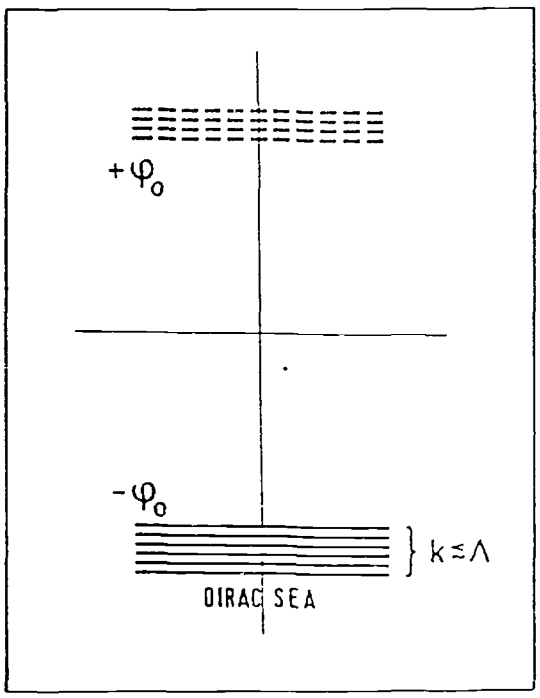


Figure 5: The spectrum of the vacuum quark orbits in the Nambu Jona-Lasinio model.

The vibrations of the vacuum consist of zero mass pions (Goldstone bosons due to the spontaneously broken chiral symmetry) and σ mesons of mass $2\Phi_0$ [33,34,35,36] . We would like the Nambu Jona-Lasinio to provide us with a unified description of the vacuum, mesons and baryons. For this reason we want to fit the pion decay constant f_π. It can be shown to be given by the equation [36]:

$$f_\pi^2 = \Phi_0^2 \frac{\nu}{(2\pi)^4} \int d_4k \frac{1}{\left(k^2 + \Phi_0^2\right)^2} \tag{6.7}$$

The model so far appears to have only one parameter a^2. However the energy (6.6) and the pion decay constant (6.7) diverge logarithmically at large quark momenta k.

The regularization of the divergence is external to the model and requires justification. The Leningrad group for example derives a Nambu Jona-Lasinio type effective theory by assuming that the QCD vacuum is an instanton liquid [37]. In their approach, the field Φ turns out to be a momentum dependent quark mass $\Phi(k)$ which falls off sufficiently rapidly beyond $k \simeq \Lambda$ where Λ is a calculated cut-off. This ensures the convergence of the theory. In most other applications of Nambu Jona-Lasinio theory, a phenomenological cut-off is introduced, much like the cut-off introduced in BCS theory of superconductors [38]. Whether it is justified or not, the cut-off is an essential parameter of the theory. Because of its low value (see below) all physical quantities depend strongly on its value.

The expression (6.7) for the pion decay constant relates the constituent quark mass Φ_0 and the cut-off Λ. Fig.6 shows this relation in the case where a proper-time regularisation is used:

$$f_\pi^2 = \Phi_0^2 \frac{\nu}{(2\pi)^4} \int d_4k \int_{1/\Lambda^2}^{\infty} sds\ e^{-s\left(k^2 + \Phi_0^2\right)} = \frac{\nu\Phi_0^2}{16\pi^2} \int_1^{\infty} \frac{dy}{y}\ e^{-\frac{\Phi_0^2}{\Lambda^2}} \tag{6.8}$$

One can compute the π^0 electromagnetic decay width from the process (3.15). It is found [41] that the observed value is only obtained when the ratio Λ/Φ_0 of the cut-off to the constituent quark mass exceeds a value of the order of 4. This only occurs on the lower branch of Fig.6 where the constituent quark mass is less than 300 MeV. This is a serious shortcoming of the model in the form (6.1). Indeed such a low constituent quark mass would imply a mass gap smaller than $2\Phi_0 \simeq 600$ MeV . Any meson calculated as a $q\bar{q}$ excitation with a mass higher than the mass gap would not be bound: it would decay into an unphysical state composed of a free quark and anti-quark pair. Thus constituent quark masses lower than 300 MeV would exclude bound ω and ρ mesons, for example, because they have a masses of about 770 MeV. We shall see below that, with such low constituent quark masses, no bound state of three quarks occurs so that the model would also be unable to form baryons as bound states of quarks. In section 7 we shall see that the introduction of vector mesons into the hamiltonian significantly changes the relation between the cut-off and the constituent quark mass (the dotted line of Fig.6).

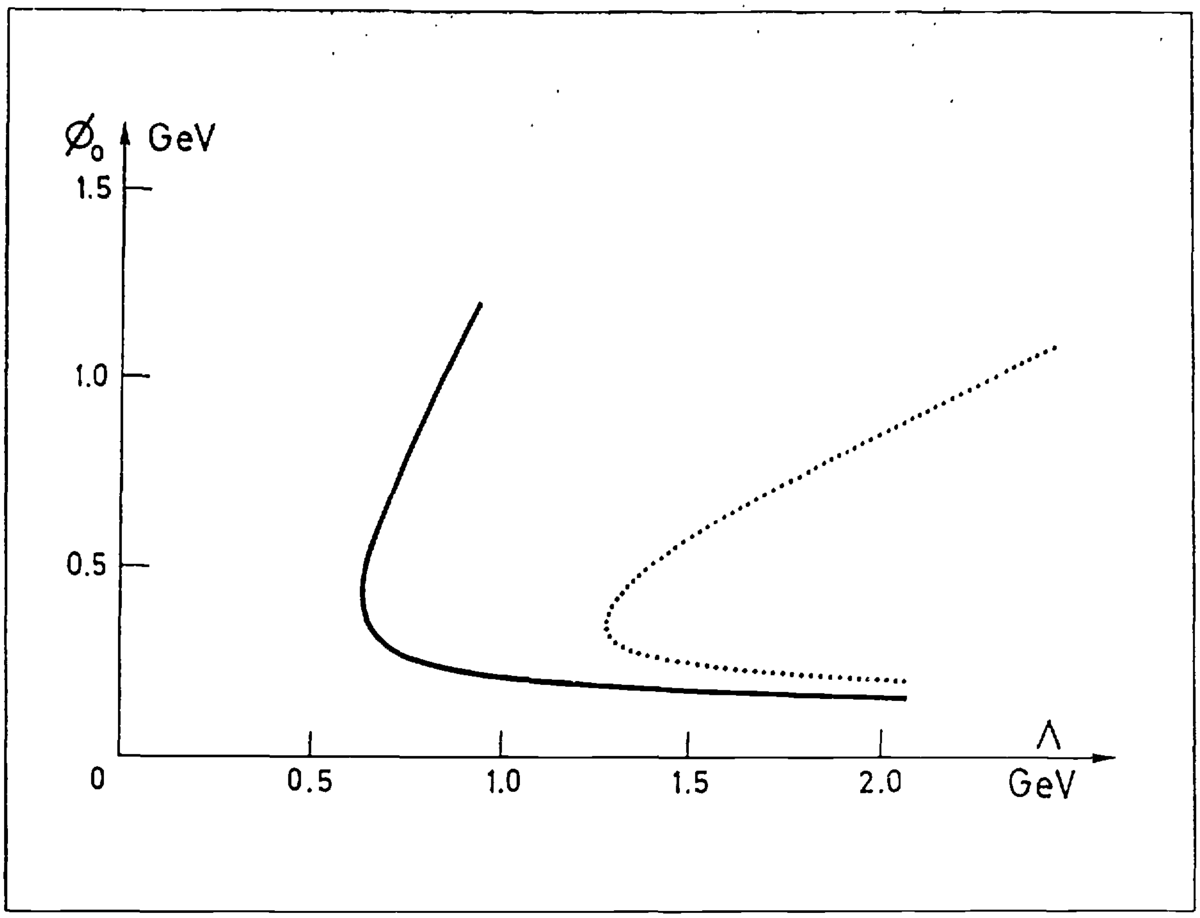


Figure 6: The relation between the cut-off Λ and the quark constituent mass φ_0 obtained by fitting the pion decay constant f_π. The full curve is obtained from Eq.(6.8) in a Nambu Jona-Lasinio model involving only chiral fields. The dotted line is obtained from (9.1) when the π and A_1 fields mix.

Several applications [35,39] insist on fitting also the quark condensate vacuum expectation value $\langle \bar{\psi}\psi \rangle \simeq -(250\ \text{MeV})^3$. However, this is a quadratically divergent quantity which is very sensitive to the way in which the quark-loop momentum is cut off. It is always possible to fit it, together with f_π, by using as two-parameter cut-off profile [40,42].

Solitons are constructed in the Nambu Jona-Lasinio model (6.2) in much the same way as in the σ-model as described in section 5. The quarks are assumed to occupy orbits which are solutions of the Dirac equation (5.3) with σ and $\vec{\pi}$ replaced by S and $\vec{P}$. The energy of the soliton is then:

$$E = \sum_{\lambda_{occ}} e_\lambda + \nu \sum_k \left(\sqrt{k^2 + \Phi_0^2} + \frac{\langle k | \Phi^2 - \Phi_0^2 | k \rangle}{2\sqrt{k^2 + \Phi_0^2}} \right) \tag{6.9}$$

The fields are calculated by minimizing the energy (6.9). In the case of translationally invariant fields, the expression (6.9) reduces to the expression (6.6). The first sum in (6.9) includes the Dirac sea orbits as well as the occupied valence orbits. A baryon is formed by a hedgehog field with winding number n=1. In this case the valence orbit is the prominent 0^+ orbit shown on Fig.1.

Several groups have calculated baryons with this model [42,43,44,45] . The results depend strongly on the constituent quark mass Φ_0 as shown on Fig.7. No bound state occurs when the constituent quark mass Φ_0 is less than about 350 MeV. For masses in the range 350 MeV < Φ_0 < 450 MeV a quasi-bound state is formed, which is a local minimum with an energy higher than $N_c\Phi_0$ so that it is not a bound state. A bound state is only formed with constituent quark masses Φ_0 > 450 MeV . The increase of binding with Φ_0 may be understood by inspecting Fig.6. We see that for high constituent quark masses the ratio Λ/Φ_0 decreases. The repulsion due to the Dirac sea therefore diminishes relative to the binding of the valence orbit. However we face then again the problem that a small ratio Λ/Φ_0 yields a wrong π^0 electro-magnetic decay width [41]. One should also mention that the formation of a bound soliton depends on how the quark loop momenta are cut off. The behavior displayed on Fig.6 is obtained by a proper-time

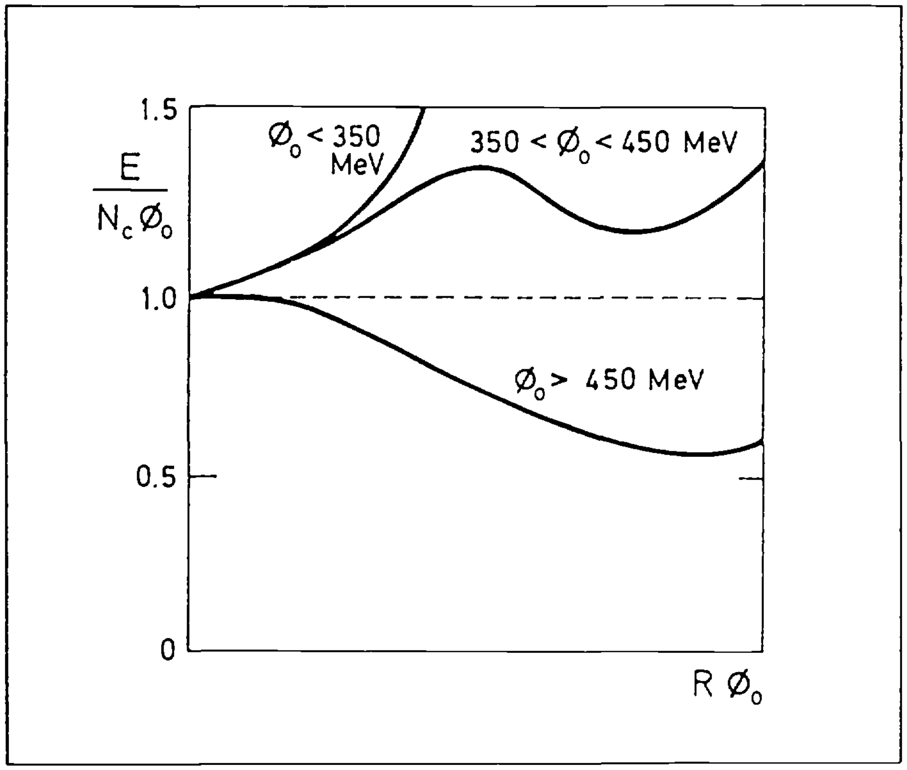


Figure 7: The behavior of the energy (6.9) of a soliton in terms of the size parameter R in the Nambu Jona-Lasinio model with a proper time regularization. The energy is expressed in units of $N_c\Phi_0$ so that a bound state occurs when the energy is smaller than 1 (dashed line). The hedgehog field has winding number n=1. The three curves indicate the energy obtained with different values of the constituent quark mass.

regularisation of the energy (6.9). A sharp 3-momentum cut-off does not produce a bound state [46]. Thus the calculation of chiral solitons with the Nambu Jona-Lasinio model reduced to chiral fields is still plagued with problems.

Of course several corrections to the Hartree approximation may come to the rescue. They are all $1/N_c$ effects which may however be important in the $N_c \Rightarrow 3$ limit. A typical effect is the centre of mass motion of the quarks which could lower the energy by about 350 MeV. This crude estimate is based on the oscillator model. A bound state of N_c quarks may be compared with a system of N_c particles in a harmonic oscillator potential. The energy of such a system is $N_c \hbar\omega/2$. It is known [47] that the centre of mass of such a system has a kinetic energy equal to $3\hbar\omega/4$ where ω is the oscillator frequency. The value of $\hbar\omega$ can be estimated from the nucleon spectrum which shows a group of negative parity states at about 500 MeV above the ground state. Thus $\hbar\omega \simeq 500$ MeV for the oscillator which binds the quarks and the centre of mass energy is about 0.75x500 MeV = 375 MeV. This is a spurious energy which should be subtracted from the calculated soliton energy (6.9) or (5.2).

7. Vector fields.

In this section we extend the Nambu Jona-Lasinio model to vector fields. Vector fields have been extensively studied in Skyrmion models [8]. In the Nambu Jona-Lasinio model vector fields have been more applied to mesons [35] than to baryons.

Mesons are time dependent vibrations of the vacuum. They are more easily calculated from the lagrangian than from the hamiltonian. The action of the Nambu Jona-Lasinio model (6.2) is:

$$I = \int d_4 x \left(\bar{\psi}\left(i\partial_\mu \gamma^\mu - S_0 - i\gamma_5 \vec{P}.\vec{\tau}\right)\psi - \frac{a^2}{2}\left(S_0^2 + \vec{P}^2\right)\right) \qquad (7.1)$$

where we have written the chiral field in the form $\Phi U \equiv S_0 + i\gamma_5 \vec{P}.\vec{\tau}$ for reasons which will appear shortly. We wish to extend this action so as to include vector fields.

We begin by coupling a family of scalar and vector fields to the quarks. Extra mass terms will be added later. Consider the action:

$$I = \int d_4 x \left(\bar{\psi}\left(i\partial_\mu \gamma^\mu - S - iP\gamma_5 - V_\mu \gamma^\mu - A_\mu \gamma^\mu \gamma_5\right)\psi - \frac{a^2}{2}\left(S_0^2 + \vec{P}^2\right)\right) \quad (7.2)$$

S,P,V and A denote respectively scalar, pseudo-scalar, vector and axial fields. Each one of these fields has an isoscalar and an isovector part. So all together eight fields are coupled to the quarks in expression (7.2). They can be explicited as follows:

$$S \equiv S^0 + \vec{S}.\vec{\tau} \qquad P \equiv P^0 + \vec{P}.\vec{\tau} \qquad (7.3a)$$

$$V_\mu \equiv V^0_\mu + \vec{V}_\mu.\vec{\tau} \qquad A_\mu \equiv A^0_\mu + \vec{A}_\mu.\vec{\tau} \qquad (7.3b)$$

We want the action (7.2) to be invariant under the four strong-interaction symmetries, namely isospin, parity, charge conjugation and G-parity. These imply specific transformation properties and quantum numbers of the fields. They are listed in the following two tables which also specify the observed mesons which can be associated to the fields.

field	couples to	isospin I	G-parity $G=e^{i\pi I_2}C$	spin J	parity P	charge conjugation C	$I^G(J^{PC})$	meson
S^0	$\bar{\psi}\psi$	0	+	0	+	+	$0^+(0^{++})$	σ
$\vec{S}$	$\bar{\psi}\vec{\tau}\psi$	1	-	0	+	+	$1^-(0^{++})$	$\vec{a}_0$ ex $\vec{\delta}$ 983 MeV
P^0	$\bar{\psi}\gamma_5\psi$	0	+	0	-	+	$0^+(0^{-+})$	η 548.8 MeV 957.5 MeV
$\vec{P}$	$\bar{\psi}\gamma_5\vec{\tau}\psi$	1	-	0	-	+	$1^-(0^{-+})$	$\vec{\pi}$ 138 MeV

(7.4a)

field	couples to	isospin I	G-parity $G=e^{i\pi I_2}C$	spin J	parity P	charge conjugation C	$I^G(J^{PC})$	meson
V^0_μ	$\bar{\psi}\gamma_\mu\psi$	0	-	1	-	-	$0^-(1^{--})$	ω 783 MeV
$\vec{V}_\mu$	$\bar{\psi}\gamma_\mu\vec{\tau}\psi$	1	+	1	-	-	$1^+(1^{--})$	$\vec{\rho}$ 770 MeV
A^0_μ	$\bar{\psi}\gamma_\mu\gamma_5\psi$	0	+	1	+	+	$0^+(1^{++})$	f_1 ex D 1283 MeV
$\vec{A}_\mu$	$\bar{\psi}\gamma_\mu\gamma_5\vec{\tau}\psi$	1	-	1	+	+	$1^-(1^{++})$	$\vec{a}_1$ ex $\vec{A}_1$ 1260 MeV

(7.4b)

When we associate fields to observed mesons, we must keep in mind that the η meson is a mixture of u,d and s quarks so that it does not make sense to calculate it with only u and d quarks. It is only listed for completeness. We shall see in section 8 that the $\vec{a}_1$ turns out to be a linear combination of the $\vec{P}$ and $\vec{A}$ fields and the f_1 a linear combination of the P^0 and A^0 fields.

We have assumed minimal coupling of the vector mesons to the quarks. For this reason, the quark action (7.2) is gauge invariant. It is invariant with respect to four *local* flavor transformations U(1), $U_5(1)$, $SU_V(2)$ and $SU_A(2)$. The quark spinors transform as follows:

	U(1)	$U_5(1)$	$SU_V(2)$	$SU_5(2)$
$\psi \Rightarrow$	$e^{-i\alpha}\psi$	$e^{-i\alpha\gamma_5}\psi$	$e^{-i\vec{\alpha}.\vec{\tau}/2}\psi$	$e^{-i\gamma_5\vec{\alpha}.\vec{\tau}/2}\psi$
$\bar{\psi} \Rightarrow$	$\bar{\psi}e^{i\alpha}$	$\bar{\psi}e^{-i\alpha\gamma_5}$	$\bar{\psi}e^{i\vec{\alpha}.\vec{\tau}/2}$	$\bar{\psi}e^{-i\gamma_5\vec{\alpha}.\vec{\tau}/2}$

(7.5a)

For infinitesimal transformations, the scalar and vector fields transform as follows:

	U(1)	$U_5(1)$	$SU_V(2)$	$SU_5(2)$
$S^0 \Rightarrow$	S^0	$S^0 - 2\alpha P^0$	S^0	$S^0 - \vec{\alpha}.\vec{P}$
$\vec{S} \Rightarrow$	$\vec{S}$	$\vec{S} - 2\alpha\vec{P}$	$\vec{S} + \vec{\alpha}\times\vec{S}$	$\vec{S} - \vec{\alpha}P^0$
$P^0 \Rightarrow$	P^0	$P^0 + 2\alpha S^0$	P^0	$P^0 + \vec{\alpha}.\vec{S}$
$\vec{P} \Rightarrow$	$\vec{P}$	$\vec{P} + 2\alpha\vec{S}$	$\vec{P} + \vec{\alpha}\times\vec{P}$	$\vec{P} + \vec{\alpha}S^0$

(7.5b)

	U(1)	$U_5(1)$	$SU_V(2)$	$SU_5(2)$
$V^0_\mu \Rightarrow$	$V^0_\mu + (\partial_\mu\alpha)$	V^0_μ	V^0_μ	V^0_μ
$\vec{V}_\mu \Rightarrow$	$\vec{V}_\mu$	$\vec{V}_\mu$	$\vec{V}_\mu + \vec{\alpha}\times\vec{V}_\mu - \frac{1}{2}(\partial_\mu\vec{\alpha})$	$\vec{V}_\mu + \vec{\alpha}\times\vec{A}_\mu$
$A^0_\mu \Rightarrow$	A^0_μ	$A^0_\mu + (\partial_\mu\alpha)$	A^0_μ	A^0_μ
$\vec{A}_\mu \Rightarrow$	$\vec{A}_\mu$	$\vec{A}_\mu$	$\vec{A}_\mu + \vec{\alpha}\times\vec{A}_\mu$	$\vec{A}_\mu + \vec{\alpha}\times\vec{V}_\mu + \frac{1}{2}(\partial_\mu\vec{\alpha})$

(7.5c)

The four flavor symmetries are observed to be global rather than local symmetries of strong interactions. This allows us to add vector field mass terms which preserve global but not local flavor symmetries. The most general action which includes mass terms and which preserves the global flavor symmetries is:

$$I = \int d_4x \; \bar{\psi}\left(i\partial_\mu\gamma^\mu - S - iP\gamma_5 - V_\mu\gamma^\mu - A_\mu\gamma^\mu\gamma_5\right)\psi$$

$$- \frac{a^2}{2}\left(S_0^2 + \vec{P}^2\right) - \frac{a'^2}{2}\left(\vec{S}^2 + P_0^2\right)$$

$$- \frac{b^2}{2}\left(\vec{V}_\mu.\vec{V}_\mu + \vec{A}_\mu.\vec{A}_\mu\right) - \frac{c^2}{2}\,V^0_\mu V^0_\mu - \frac{d^2}{2}\,A^0_\mu A^0_\mu \qquad (7.6)$$

The constants a,a',b,c and d can be determined from the observed meson masses. To do this we must study the dynamics of the vector fields.

8. Dynamics of vector fields.

Mesons are more easily studied by eliminating the quark fields from the action (7.6) which may be replaced by the equivalent action:

$$I = \int d_4x\ i\ \mathrm{tr}\ \ln\left(i\partial_\mu\gamma^\mu - S - iP\gamma_5 - V_\mu\gamma^\mu - A_\mu\gamma^\mu\gamma_5\right)$$

$$- \frac{a^2}{2}\left(S_0^2 + \vec{P}^2\right) - \frac{a'^2}{2}\left(\vec{S}^2 + P_0^2\right) - \frac{b^2}{2}\left(\vec{V}_\mu.\vec{V}_\mu + \vec{A}_\mu.\vec{A}_\mu\right) - \frac{c^2}{2} V_\mu^0 V_\mu^0 - \frac{d^2}{2} A_\mu^0 A_\mu^0 \tag{8.1}$$

This is easily proved by either integrating out the quark fields from the Feynman path integral of by quantizing the quark fields and eliminating them using their equation of motion.

By setting to zero the derivatives of the action (8.1) with respect to the fields, we can show that a translationally invariant stationary point, which can descibe the physical vacuum, is:

$$S_0 = \Phi_0, \quad \vec{S} = 0, \quad P_0 = \vec{P} = 0, \quad V_0^\mu = \vec{V}^\mu = 0, \quad A_0^\mu = \vec{A}^\mu = 0 \tag{8.2}$$

where Φ_0 is a "constituent quark" mass given by the equation:

$$a^2 = -i\frac{2\nu}{(2\pi)^4}\int d_4k\,\frac{1}{-k^\mu k_\mu + \Phi_0^2} = \frac{\nu}{(2\pi)^3}\sum_{\vec{k}}\frac{1}{\sqrt{\vec{k}^2 + \Phi_0^2}} \tag{8.3}$$

This is the same equation as the gap equation (6.5) so that chiral symmetry is broken in the physical vacuum the same way as before.

To calculate the mesons, we need to express the action in terms of the fluctuating parts of the fields, that is, in terms of the difference between the fields and their vacuum values (8.2). We set:

$$S \equiv \tilde{S} + \Phi_0 \qquad \tilde{S} \equiv \tilde{S}_0 + \vec{S}.\vec{\tau} \tag{8.4}$$

We subtract from the action (8.1) its value at the stationary point (8.2) and we use the gap equation (8.3) to eliminate the constant a^2. A straightforward calculation leads to the action:

$$I = i\mathrm{Tr}\ln\left(1 - \frac{1}{i\partial_\mu\gamma^\mu - \Phi_0}\left(\tilde{S} + iP\gamma_5 + V_\mu\gamma^\mu + A_\mu\gamma^\mu\gamma_5\right)\right) + \frac{1}{2}\mathrm{Tr}\,\frac{1}{\partial_\mu\partial^\mu + \Phi_0^2}\left(S_0^2 + \vec{P}^2 - \Phi_0^2\right)$$

$$- \frac{a'^2}{2}\left(\vec{S}^2 + P_0^2\right) - \frac{b^2}{2}\left(\vec{V}_\mu.\vec{V}_\mu + \vec{A}_\mu.\vec{A}_\mu\right) - \frac{c^2}{2} V_\mu^0 V_\mu^0 - \frac{d^2}{2} A_\mu^0 A_\mu^0 \tag{8.5}$$

where, we used the notation $\mathrm{Tr}\ A \equiv \int d_4x\ \mathrm{tr}\ A$. The mesons are (hopefully) low-amplitude vibrations of the vacuum. To calculate them it is sufficient to expand the log term in (8.5) to second order. But before we begin, we discuss a well known problem associated with the regularization of the log term.

The log term, which we will refer to as the quark-loop term, is gauge invariant. It is invariant with respect to the four local flavor transformations (7.5). If one attempts to regularize the quark term by cutting off the momenta at some value $k \simeq \Lambda$, its gauge invariance is lost. If one insists on preserving its gauge invariance, then one may attempt a Pauli-Villars regularization. This however introduces mass dependent counterterms which break chiral invariance. A popular way out of this difficulty has been the use of the gauge invariant proper-time regularization invented by Schwinger [52,35] who used it for electro-dynamics. In this method one separates the quark loop term I_q into real and imaginary parts by writing:

$$I_q \equiv \mathrm{Tr}\ln D - \mathrm{Tr}\ln D_0 = \frac{1}{2}\mathrm{Tr}\ln DD^{\dagger} - \frac{1}{2}\mathrm{Tr}\ln D_0 D_0^{\dagger} + \frac{1}{2}\left(\mathrm{Tr}\ln D - \mathrm{Tr}\ln D^{\dagger}\right) \qquad (8.6)$$

where $D \equiv i\partial_\mu\gamma^\mu - S - iP\gamma_5 - V_\mu\gamma^\mu - A_\mu\gamma^\mu\gamma_5$ and D_0 the corresponding unperturbed term.

The real part, the first two terms of (8.6), consists of two positive definite terms and it is regularized thus:

$$\frac{1}{2}\,\mathrm{Tr}\,\ln DD^{\dagger} - \frac{1}{2}\,\mathrm{Tr}\,\ln_0 D_0^{\dagger} = \mathrm{Tr}\int_{1/\Lambda^2}^{\infty}\frac{ds}{s}\left(e^{-sD_0D_0^2} - e^{-sDD^{\dagger}}\right) \qquad (8.7)$$

The integral over the "proper time" s is made convergent by cutting off the low values at $s = 1/\Lambda^2$ where Λ is a cut-off with a dimension of mass. This method yields a gauge and chiral invariant real part. It also leads to elegant forms when it is used in the euclidean metric and when the $\mu=0$ components of the vector fields are continued to imaginary values in order to make the Dirac operators D hermitian. Finally, it removes the quadratic dependence of the quark loop on the cut-off leaving only a logarithmic dependence. The third term of (8.6), the imaginary part, does not contribute to the second order expansion of the action (8.5). It does however contribute to higher order processes such as $\omega \Rightarrow \pi\pi\pi$. These do not diverge and they are usually calculated without regularization.

This way of proceeding is pleasant to the theorist because it adapts the physics to the mathematics. Whether nature has it that way is another question. It does not matter what regularization method you use when you have a renormalisable theory and if you define properly the renormalisation conditions. But here we are dealing with an effective theory and the regularization is a physical effect which has its roots outside the model. We do not know for sure what physics the regularization is mocking up. Results turn out to be strongly dependent on the cut-off and in such a situation it is not sufficient to say that the Nambu Jona-Lasinio model is valid at such and such a scale, which is then vaguely related to the QCD scale.

We do not even know whether there is any point in preserving the gauge invariance of the quark loop term. We require the observed global chiral invariance but not flavor gauge symmetry because it is not observed. The quadratic divergences which are introduced in a non-gauge invariant regularization (such as a momentum cut-off) cause no trouble and they can easily be absorbed into the constants a,a',b,... of the mass terms (see tables (8.15)). For this reason, we prefer to calculate the mesons in a formalism which allows one to try out any regularization method. In phenomenology it is always instructive to know what works best. The reasons why are sometimes understood later.

We proceed with the expansion of the action (8.5). The log term is expanded up to second order, the Dirac and Pauli matrices are traced out and plane waves are used to evaluate the remaining traces in configuration space. This is a somewhat lengthy job which requires some book-keeping but it is otherwise straightforward. It is simplified by the fact that most of the fields decouple and yield independent second-order contributions to the action.

The scalar fields S_0 and $\vec{S}$ decouple and give independent contributions of the form:

$$I = \frac{1}{2(2\pi)^4} \int d_4 q \ \tilde{\varphi}(q)\tilde{\varphi}(-q) z(q) \left(q^\mu q_\mu - m^2(q)\right) \tag{8.8}$$

where $\tilde{\varphi}(q) \equiv \int d_4 x \ e^{i q_\mu x^\mu} \tilde{\varphi}(x)$. The propagator of the field φ can be read off the form (8.8). It is diagonal in momentum space and equal to:

$$K(q) = \frac{1}{z(q)\left(q^\mu q_\mu - m^2(q)\right)} \tag{8.9}$$

$m^2(q)$ is the meson mass operator (or self-energy) and the corresponding on-shell meson is identified by the pole:

$$q^\mu q_\mu - m^2(q) = 0 \tag{8.10}$$

The vector fields V_μ^0 and $\vec{V}_\mu$ decouple and give independent contributions of the form:

$$I = \frac{1}{2(2\pi)^4} \int d_4 q \ V_\mu(q) V_\nu(-q) z(q) \left(\left(\delta^{\mu\nu} q_\alpha q^\alpha - q^\mu q^\nu\right) - \delta^{\mu\nu} m^2(q)\right) \tag{8.11}$$

Note that $V_\mu(q)V_\nu(-q)\left(\delta^{\mu\nu}q_\alpha q^\alpha - q^\mu q^\nu\right) = \frac{1}{2} F_{\mu\nu}(q)F^{\mu\nu}(-q)$ where $F_{\mu\nu} \equiv (\partial_\mu V_\nu) - (\partial_\nu V_\mu)$.

The vector field propagator $K_{\mu\nu}(q)$ can be read off the form (8.11). It is diagonal in momentum space and equal to:

$$K_{\mu\nu}(q) = \frac{1}{z(q)} \frac{1}{q^\alpha q_\alpha - m^2(q)} \left(\delta_{\mu\nu} + \frac{q_\mu q_\nu}{m^2(q)}\right) \qquad (8.12)$$

Again, $m^2(q)$ is the meson mass operator (or self-energy) and the corresponding on-shell meson is identified by the pole (8.10).

The second order expansion of the action (8.5) has terms in which the A and P fields mix. The corresponding quadratic form can be diagonalized by defining new fields B which are linear combinations of the A and P fields:

$$B^0_\mu(q) = A^0_\mu(q) + \alpha(q) \frac{iq_\mu}{2\Phi_0} P^0(q) \qquad \vec{B}_\mu(q) = \vec{A}_\mu(q) + \beta(q) \frac{iq_\mu}{2\Phi_0} \vec{P}(q) \qquad (8.13)$$

The coefficients are chosen such that the mixed terms involving the B and P fields vanish. They turn out to be:

$$\alpha(q) = \alpha(-q) = \frac{4\Phi_0^2 \nu f(q)}{d^2 + \nu T(q) + 4\Phi_0^2 \nu f(q)}, \qquad \beta(q) = \beta(-q) = \frac{4\Phi_0^2 \nu f(q)}{b^2 + \nu T(q) + 4\Phi_0^2 \nu f(q)} \qquad (8.14)$$

where f(q) and T(q) are functions given in the appendix.

The following tables give the meson mass operators.

meson	field	z(q)	$m^2(q)$
σ	$\tilde{S}_0$	$\nu f(q)$	$4\Phi_0^2$
$\vec{a}_0$	$\vec{S}$	$\nu f(q)$	$4\Phi_0^2 + \frac{a'^2 - a^2}{\nu f(q)}$
$\eta^\dagger$	P_0	$\nu f(q) \frac{d^2 + \nu T(q)}{d^2 + \nu T(q) + 4\Phi_0^2 \nu f(q)}$	$\frac{a'^2 - a^2}{\nu f(q)} \frac{d^2 + \nu T(q) + 4\Phi_0^2 \nu f(q)}{d^2 + \nu T(q)}$
$\vec{\pi}$	$\vec{P}$	$\nu f(q) \frac{b^2 + \nu T(q)}{b^2 + \nu T(q) + 4\Phi_0^2 \nu f(q)}$	0

(8.15a)

† The η has strongly admixed strange quark components. It is only listed for the sake of completeness.)

meson	field	z(q)	$m^2(q)$
ω	V^0_μ	$\nu S(q)$	$\dfrac{\nu T(q) + c^2}{\nu S(q)}$
$\vec{\rho}$	$\vec{V}_\mu$	$\nu S(q)$	$\dfrac{\nu T(q) + b^2}{\nu S(q)}$
f_1	B^0_μ	$\nu S(q)$	$\dfrac{d^2 + \nu T(q) + 4\Phi_0^2 \nu f(q)}{\nu S(q)}$
$\vec{a}_1$	$\vec{B}_\mu$	$\nu S(q)$	$\dfrac{b^2 + \nu T(q) + 4\Phi_0^2 \nu f(q)}{\nu S(q)}$

(8.15b)

The functions f(q), S(q) and T(q) are given in the appendix. They can be regularized in various ways. If a gauge invariant regularisation is used, the quadratically divergent function T(q) vanishes.

Table (7.4b) shows that the vector meson pairs $(\omega,\vec{\rho})$ and $(f_1,\vec{a}_1)$ are almost degenerate. From table (8.15b) we see that this implies an equality of the three meson mass parameters: b=c=d. It means that the vector mesons have global $U_V(2)$ and $U_A(2)$ symmetry rather than SU(2) symmetry. The finite mass of the η shows that U(2) symmetry does not hold for the scalar mesons, probably because of the U(1) anomaly of QCD.

We end this section by playing a game. Imagine that we switch on the various mass terms one after the other to see what they do. To simplify the game we assume the functions f(q), S(q) and T(q) are regularized in a gauge invariant way and that the meson masses are evaluated at q=0. We start with the lagrangian (7.2) in order to have at least spontaneously broken chiral symmetry. The meson spectrum can be deduced from the tables (8.15). It is summarized in the table below:

U(1)	$U_5(1)$	$(SU2)_V$	$(SU2)_A$	lagrangian	spectrum
local	local	local	local	$\bar{\psi}(i\partial - S - iP\gamma_5 - V - A\gamma_5)\psi$ $- \frac{a^2}{2}(S_a^2 + P_a^2)$	$\sqrt{6}\,\phi_0$ —— a_1, f_1 $2\phi_0$ —— σ, a_0 0 —— ρ, ω

(8.16)

What happened? The lagrangian (8.16) has the four *local* flavor symmetries $U(1), U_5(1)$, $(SU2)_V$ and $(SU2)_A$. It is gauge invariant and chiral symmetry is spontaneously broken. We are facing the Higgs phenomenon. The σ and $\vec{a}_0$ mesons acquire a mass equal to $2\Phi_0$. The $\vec{\rho}$ and ω are massless gauge bosons. (This will only happen if a gauge invariant regularization is used.) The $\vec{\pi}$ and η are eaten up by the gauge fields $\vec{a}_1$ and f_1 which acquire a mass $\sqrt{6}\,\Phi_0$.

The spectrum (8.16) probably didn't please the Lord. So we add vector meson mass terms. The spectrum obtained is shown in the following table:

U(1)	$U_5(1)$	$(SU2)_V$	$(SU2)_A$	lagrangian	spectrum
global	global	global	global	$\bar{\psi}(i\partial - S - iP\gamma_5 - V - A\gamma_5)\psi$ $-\frac{a^2}{2}\left(S_a^2 + P_a^2\right)$ $-\frac{b^2}{2}\left(V_a^2 + A_a^2\right)$	—— a_1, f_1 $2\emptyset_0$ —— σ, a_0 —— ρ, ω 0 —— π, η

(8.17)

We have set $b^2 = c^2 = d^2$ to make the (ρ,ω) and (a_1, f_1) pairs degenerate, since this seems to have pleased the Lord. Gauge symmetry is now broken by the vector field mass terms. The local flavor symmetries are reduced to global symmetries. Four massless Golstone bosons appear: the $\vec{\pi}$ and the η.

The spectrum (8.17) still didn't please Him so we finally break $U_5(1)$ symmetry in order to give mass to the η. This symmetry breaking is related to the so-called U(1) anomaly in QCD. It is achieved by using two scalar meson mass terms which are proportional to a^2 and a'^2. The spectrum is now:

U(1)	$U_5(1)$	$(SU2)_V$	$(SU2)_A$	lagrangian	spectrum
global	BROKEN	global	global	$\bar{\psi}(i\partial - S - iP\gamma_5 - V - A\gamma_5)\psi$ $-\frac{a^2}{2}\left(S_0^2 + \vec{P}^2\right) - \frac{a'^2}{2}\left(\vec{S}^2 + P_0^2\right)$ $-\frac{b^2}{2}\left(V_a^2 + A_a^2\right)$	—— a_1, f_1 —— a_0 $2\emptyset_0$ —— σ —— ω, ρ —— η 0 —— π

(8.18)

The broken $U_5(1)$ symmetry gives mass to the η while maintaining the pions as massless Goldstone bosons. The ρ and ω now have masses smaller than the f_1, a_1 and a_0. Note that the particular form used for the symmetry breaking term, proportional to $a^2 - a'^2$, is probably only valid in a second order expansion of the fields.

9. Conclusion.

Although the meson spectrum (8.18) is in reasonable agreement with the observed meson masses, we must recall that it was calculated with q=0 mass operators. Had we not been playing a game, we would have attempted to calculate on-shell meson masses by solving Eq.(8.10). We would then have faced the problem that the a_0, a_1 and f_1 meson masses are higher than the σ mass which is equal to the $q\bar{q}$ threshold $2\Phi_0$. Such mesons would not be bound states and the functions f(q) and S(q) would develop singularities. In section 6 we encountered a similar problem when we tried to calculate the nucleon as a bound state of quarks.

The introduction of vector mesons does however improve the situation to some extent because it improves the ratio Λ/Φ_0 between the cut-off and the constituant quark mass. Let us compute the pion decay constant. In the presence of vector mesons, the expression (6.7) is modified because the $\vec{P}$ and $\vec{A}_\mu$ fields mix. The pion decay constant may be calculated by first computing the axial current which is the Noether current associated to the $SU_A(2)$ transformation. The pion field is then quantized and one computes the matrix element of the axial current between the vacuum and a one pion state. The result has the form (2.4) with a pion decay constant equal to:

$$f_\pi = \frac{\nu f(0)\Phi_0}{\sqrt{z_\pi}} \, \frac{b^2}{b^2 + \nu T(0) + 4\Phi_0^2 \nu f(0)} \tag{9.1}$$

where z_π is the function z(q) listed for the pion in table (8.15) and evaluated at q=0. The functions f(q) and T(q) are given in the appendix. In the absence of the vector field $\vec{A}_\mu$ the expression (9.1) reduces to the expression (6.7). If we evaluate the ρ and a_1 meson masses at q=0 and if we use gauge invariant regularisation which makes T(q)=0, we find the relations:

$$m_{a_1}^2 = m_\rho^2 + 6\Phi_0^2 \qquad\qquad \frac{b^2}{b^2 + \nu T(0) + 4\Phi_0^2 \nu f(0)} = \frac{m_\rho^2}{m_{a_1}^2} \tag{9.2}$$

The relation between the constituent quark mass Φ_0 and the cut-off Λ, obtained from Eqs.(9.1) and (9.2), is shown on Fig.6 (dashed line). If we combine the dashed curve with the observed masses of the ρ and the a_1, given in table (7.4b), we obtain a constituent quark mass $\Phi_0 \simeq 407$ MeV and a cut-off $\Lambda \simeq 1.35$ GeV . For this value of the quark mass we obtain a ratio $\Lambda/\Phi_0 \simeq 3.3$. This is an improvement since without vector mesons (the full line of Fig.6) the ratio would have been $\Lambda/\Phi_0 \simeq 1.5$.

The nice feature of the Nambu Jona-Lasinio model is its scarcity of parameters. If we are willing to assume degenerate f_1(1283 MeV) and a_1(1260 MeV) mesons, as well as degenerate ρ(770 MeV) and ω(783 MeV) mesons, then the lagrangian has only four parameters, namely a^2, a'^2, b^2 and the cut-off Λ. By fitting f_π the number of parameters is reduced to three. All the π,ω and ρ properties together with the N, Δ and of other nucleon excited states should be accounted for with these three parameters. So far baryon properties have not been calculated in the Nambu Jona-Lasinio model including vector mesons. Much work remains to be done and new ideas, such as a possible di-quark structure of the nucleon [52], remain to be tested.

Appendix: the functions f,S and T.

The functions f(q), S(q) and T(q) are defined as follows:

$$f(q) \equiv \frac{1}{(2\pi)^4}\int d_4k \frac{1}{\left(\left(k-\frac{q}{2}\right)^2+\varphi^2\right)\left(\left(k+\frac{q}{2}\right)^2+\varphi^2\right)} = f(-q)$$

$$= \frac{\pi^2}{(2\pi)^4}\int_{-1}^{1}\frac{du}{2}\int_{1/\Lambda^2}^{\infty}\frac{ds}{s}\, e^{-s\left(\varphi^2+\frac{q^2}{4}(1-u^2)\right)}$$

$$S(q) = \int_{-1}^{1}\frac{du}{2}\frac{1}{(2\pi)^4}\int d_4k \frac{1-u^2}{\left(k^2+\varphi^2+\frac{q^2}{4}(1-u^2)\right)^2} = S(-q)$$

$$= \frac{\pi^2}{(2\pi)^4}\int_{-1}^{1}\frac{du}{2}\int_{1/\Lambda^2}^{\infty}\frac{ds}{s}\, e^{-s\left(\varphi^2+\frac{q^2}{4}(1-u^2)\right)}(1-u^2)$$

$$T(q) = \int_{-1}^{1}\frac{du}{2}\frac{1}{(2\pi)^4}\int d_4k \left(\frac{k^2+q^2u^2}{\left(k^2+\varphi^2+\frac{q^2}{4}(1-u^2)\right)^2} - \frac{2}{k^2+\varphi^2}\right)$$

The functions can be regularized either by cutting off the momenta k (in a sharp or smooth way) or, as indicated, by cutting off the low values of the proper time s. The latter method is gauge invariant. The function T(q) vanishes when it is regularized this way.

REFERENCES

[1]. S. Weinberg, Physica 96A (1979) 327.
See also the discussion in the book by H.Georgi, "Weak interactions and modern particle theory", Benjamin (1984).

[2]. M.Gell-Mann and M.Lévy, Nuovo Cim. 16 (1960) 705.
B.W.Lee, "Chiral dynamics", Gordon and Breach, 1972.

[3]. D.Kusnezov and A.Bulgac, "Euler Relations for SU(N)", Michigan State University preprint.

[4]. T.H.R. Skyrme, Nucl.Phys. 31 (1962) 556

[5]. C.G.Callan, K.Hornbostel and I.Klebanov, Phys.Lett. B202 (1988) 269;
N.N.Scoccola, D.P.Min, H.Nadeau and M.Rho, Nucl.Phys. in press;
J.P.Blaizot, M.Rho and N.N.Scoccola, Phys.Lett. B207 (1988) 27.

[6]. P.Jain, J.Schechter and R.Sorkin, Phys.Rev. D39 (1989) 998;

[7]. J.A.Mignaco and S.Wulck, Phys.Rev.Lett. 62 (1989) 1449.

[8]. I.Zahed and G.E.Brown, Phys.Rep. 142 (1986) 1; Ulf.-G.Meissner, Phys.Rep. 161 (1988) 213; B.Schwesinger, H.Weigel, G.Holzwarth and A.Hayashi, Phys.Rep. 173 (1989) 173.

[9]. H.Forkel, A.D.Jackson, M.Rho, C.Weiss, A.W.Wirzba and H.Bang, "Chiral symmetry restoration and the Skyrme model", Stony-Brook preprint (1989).

[10]. For a recent review of this topic see G.Ripka, "Nambu Jona-Lasinio models applied to dense hadronic matter", lectures delivered at the XII Workshop on Nuclear Physics, Iguazu Falls, Argentina, August (1989), to be published by World Scientific.

[11]. B.D.Serot and J.D.Walecka, Advances in Phys. 16 (1986), Plenum Press.

[12]. A.Bohr and B.Mottelson, Nuclear Structure, Vol.2,page 21 (Benjamin) 1975.

[13]. J.P.Blaizot and G.Ripka, Phys.Rev. D38 (1988) 1556.

[14]. M.P.Mattis and M.Mukerjee, Phys.Rev.Lett. 61 (1989) 1344; see also the Phys.Rep. of Schwesinger et al. in Ref.[8].

[15]. T.H.R.Skyrme, "The origins of the Skyrmion", Internat.Journ.Mod.Phys. A3 (1988) 2745.

[16]. T.H.R.Skyrme, Journ.Math.Phys. 12 (1970) 1735.

[17]. R.Mackenzie and F.Wilczek, Int.Journ.Mod.Phys. A3 (1988) 2817;
N.S.Manton, Phys.Lett. B181 (1986) 137;Comm.Math.Phys. 111 (1987) 469;Phys.Lett. B198 (1987) 226;M.F.Atiyah and N.S.Manton, Phys.Lett. B222 (1989) 438; A.D.Jackson, Progr.Part.Nucl.Phys. 20 (1988) 65;
T.M.Samols, "Skyrmions on S^3 from instantons",Cambridge University preprint (1989).

[18]. J.Golstone and F.Wilczek, Phys.Rev.Lett. 47 (1984) 284.

[19]. I.A.R.Aitchison and C.W.Fraser, Phys.Rev. D31 1985) 2605;
I.A.R.Aitchison, C.M.Fraser and P.J.Miron, Phys.Rev. D33 (1985) 1984;

[20]. Elaine Tudor, University of Cambridge Thesis, 1985;

I.J.R.Aitchison, C.M.Fraser, E.Tudor and J.Zuk, "Failure of the derivative expansion for studying the stability of the baryon as a chiral soliton", University of Oxford preprint, 1985.

[21]. T.H.R.Skyrme, Proc.Roy.Soc.(London) A260 (1961) 127.

[22]. E.Witten, Nucl.Phys.B223 (1983) 422; Nucl.Phys. B223 (1983) 433.

[23] S.Kahana and G.Ripka, Nucl.Phys.A429 (1984) 462;

[24]. J.Wess and B.Zumino, Phys.Lett. 37B (1971) 95.

[25]. For a recent review of modern applications of the Skyrme picture see M.Rho, "Skyrmions revisited", Saclay preprint SPhT/89/155 to be published in Brief Reviews in Modern Physics Letters A.

[26]. G.S.Adkins, C.R.Nappi and E.Witten, Nucl.Phys. B228 (1983) 552.

[27]. A.R.Edmonds, "Angular momentum in quantum mechanics", Princeton University Press (1957).

[28]. R.Friedberg and T.D.Lee, Phys.Rev. D15 (1977) 1694; D16 (977) 1096.
R.Goldflam and L.Wilets, Phys.Rev. D25 (1982) 1951.

[29]. S.Kahana, G.Ripka and V.Soni, Nucl.Phys. A415 (1984) 351.
M.C.Birse and M.K.Banerjee, Phys.Lett. 136B (1984) 284.

[30]. K.Goecke, J.N.Urbano, M.Fiolhais and M.Harvey, Phys.Lett. 164B (1985) 249;
M.Fiolhais, F.Goecke, F.Grümmer and J.N.Urbano, Nucl.Phys. A481 (1988) 727.

[31]. S.Kahana, R.Perry and G.Ripka, Phys.Lett. B163 (1985) 37;
B.Moussalam, "Solitons in a σ-model with fermionic determinant", Orsay preprint IPNO/TH 89-25 (1989).

[32]. V.Soni, Phys.Lett. B183 (1987) 91; Phys.Lett. B195 (1987) 569;
G.Ripka and S.Kahana, Phys.Rev. D36 (1987) 1233.

[33]. Y.Nambu and Jona-Lasinio, Phys.Rev. 122 (1961) 354.

[34]. One of us (G.R.) has reviewed applications of the Nambu Jona-Lasinio model in G.Ripka, M.Jaminon and P.STassart, "Introduction to Nambu Jona-Lasinio models applied to hadronic matter", lectures delivered at the school on Hadrons and Hadronic Matter in Cargèse, August 1989; G.Ripka, "Nambu Jona-Lasinio models applied to dense hadronic matter", lectures delivered at the XII Workshop on Nuclear Physics, Iguazu Falls, Argentina, sptember 1989.

[35]. D.Ebert and H.Reinhardt, Nucl.Phys. B271 (1986) 188.
M.Wakamatsu and W.Weise, Zeitschr.Physik A331 (1988) 331.

[36]. M.Jaminon, G.Ripka and P.Stassart, Nucl.Phys. A504 (1989) 733;

[37]. D.I.Diakonov and V.Yu.Petrov, Nucl.Phys. B245 (1984) 293;
Nucl.Phys. B272 (1986) 457; Sov.Phys. JETP 62 (1985) 204.

[38]. T.R.Schrieffer, "The theory of superconductivity", Benjamin (1965).

[39]. V.Bernard, Ulf G.Meissner and I.Zahed, Phys.Rev.Lett. 59 (1987);V.Bernard and Ulf G.Meissner, Nucl.phys. A489 (1988) 647;Phys.Rev. B308 (1988) 753.

[40]. M.Jaminon, G.Ripka and P.Stassart, Phys.Lett. B227 (1989) 191.

[41]. A.H.Blin, B.Hiller and M.Schaden, Zeits.Phys. A331 (1988) 75.
[42]. D.I.Diakonov, V.Yu.Petrov and M.Praszalowicz, Nucl.Phys. B323 (1989) 53.
M.Praszalowicz, Nuovo Cim. A102 (1989) 39.
[43]. D.I.Diakonov, V.Yu.Petrov and P.V.Pobylitsa, Nucl.Phys. B306 (1988) 809;
D.I.Diakonov and V.Yu.Petrov, JETP Lett. 43 (1986) 75.
[44]. H.Reinhardt and R.Wunsch, Phys.Lett. B215 (1988) 577.
Th.Meissner, F.Grümmer and K.Goecke, Phys.Lett. B227 (1989)296;
Th.Meissner, E.Ruiz Arriola and K.Goecke, to be published in Zeit.Phys. (1990).
R.Alkofer, "The soliton of the Nambu Jona-LAsinio model", Saclay preprint S.Ph.T/89-181 (1989).
[45]. Th.Meissner, E.Ruiz Arriola, F.Grümmer, H.Mavromatis and K.Goecke, Phys.Lett. B214 (1988) 312.
[46]. S.Kahana and G.Ripka, unpublished.
[47]. J.P.Blaizot and G.Ripka, "Quantum theory of finite systems", MIT Press (1986).
[48]. G.Ripka, "Can we control the proliferation of nucleon models?", in "Windsurfing the Fermi Sea", vol.2,Ed.Kuo and Speth, North Holland (1987).
[49]. B.D.Serot and J.D.Walecka, Advances in Physics 16 (1986), Plenum Press.
[50]. J.P.Blaizot and G.Ripka, Phys.Rev.D38 (1988) 1566.
[51]. E.Leader and M.Anselmino, Zeit.Phys. C41 (1988) 239.
[52]. R.D.Ball, "The Good, the Bad and the Baryon", lecture delivered at the Lisbon meeting on "Hadrons and their interactions" (1989).
[53]. L.Frankfurt and M.Strickman, Phys.Rep. 160 (1988) 313.
[54]. See the lectures of V.Vento in this school.
[55]. See the lectures of H.Pirner in this school.