CTH-IEFT/PP-1990-18 . ISSN

CURRENT GENERATION BY ALPHA PARTICLES INTERACTING WITH LOWER HYBRID WAVES IN TOKAMAKS

V.S. Belikov, Ya.I. Kolesnichenko, M. Lisak and D. Anderson

INSTITUTIONEN FÖR ELEKTROMAGNETISK FÄLTTEORI

CHALMERS TEKNISKA HÖGSKOLA

CTH-IEFT/PP-1990-18

CURRENT GENERATION BY ALPHA PARTICLES INTERACTING WITH LOWER HYBRID WAVES IN TOKAMAKS

V.S. Belikov*, Ya.I. Kolesnichenko*, M. Lisak and D. Anderson

Institute for Electromagnetic Field Theory
and Plasma Physics
Chalmers University of Technology
Göteborg, Sweden

*Institute for Nuclear Research
Ukrainian SSR Academy of Sciences
Kiev, USSR

The problem of the influence of fusion generated alpha particles on lower-hybrid-wave current drive is examined. Analysis is based on a new equation for the LH-wave-fast ion interaction which is derived by taking into consideration the non-zero value of the longitudinal wave number. The steady-state velocity distribution function for high energy alpha particles is found. The alpha current driven by LH-waves as well as the RF-power absorbed by alpha particles are calculated.

1. Introduction

Lower Hybrid (LH) waves are expected to be one of the most efficient means of noninductive current drive in a steady state tokamak reactor, see cf [1,2]. It is also a well-established fact that lower hybrid waves can efficiently interact with ions in toroidal plasmas, cf [3-5]. The subsequent absorption results in a population of fast ions with mainly perpendicular energy, which in turn collisionally heats the bulk plasma. In particular, it has been shown that fusion generated alpha particles may, due to their very high birth energy, absorb a large fraction of the lower hybrid wave power to a large extent, thus reducing the current drive efficiency, [6-9]. However, absorption of RF-power is not the only effect of alpha particles on the current drive. Since the interaction between alpha particles and LH-waves gives rise to an anisotropy in the velocity distribution of alpha particles, the alphas may also generate an additional current. Depending on the direction of this current, the resulting total plasma current may either increase or decrease and both local and global effects on the LH current drive can be expected.

The primary local effect of the alpha current is associated with a changing of the radial profile of the LH-driven current. The alpha generated current, $J_\alpha(r)$, gives rise to a net current density, $J_{\alpha e}(r)$, according to, cf [2],

$$J_{\alpha e}(r) = J_\alpha(r)\left[1 - \frac{Z_\alpha \Phi(\varepsilon)}{Z_{eff}}\right] \tag{1}$$

where the second term in the square bracket results from the frictional force of electrons against alpha particles (Ohkawa current); the function $\Phi(\varepsilon)$ describes the toroidal effects on electrons excited by alpha particles: $\Phi(\varepsilon=0)=1$ and $\Phi(\varepsilon \to 1)=0$; $\varepsilon = r/R$; R is the major radius of the torus, Z_α is the charge number of the alpha particle, and Z_{eff} is the effective charge number of the plasma. Since the total driven current is $J(r) = J_e(r) + J_{\alpha e}(r)$, where $J_e(r)$ is the electron current driven by LH-waves, the local influence of the alpha current on the total current depends on the relative orientation of the currents $J_{\alpha e}(r)$ and $J_e(r)$. If $J_{\alpha e}(r)$ and $J_e(r)$ are in the same direction, then the alpha particles will increase the total driven current.

This can take place in the central plasma region with $Z_{eff} \sim 1$ if the LH-waves can penetrate into this region. However, in the case, when the LH-waves are restricted to the plasma periphery, the currents $J_{\alpha e}(r)$ and $J_e(r)$ have the same direction if the condition $Z_\alpha \Phi(\varepsilon))/Z_{eff} > 1$ is satisfied. Considering, for example, LH-current drive in an ITER-like plasma at $(r/a)_m > 0.5$, we take $a = 2.4$ m, $R = 5.5$ m, $\varepsilon_m = 0.22$ and find, [2], $\Phi(\varepsilon_m) \sim 0.3$. Hence, independently of Z_{eff} the above condition is not fulfilled and the alpha generated current will decrease the total driven current at the plasma periphery.

In order to understand the global effects associated with the alpha particle current, we represent the global efficiency, η, of non-inductive current drive in a plasma as

$$\frac{1}{\eta} = \sum_i \frac{\alpha_i}{\eta_i} \tag{2}$$

where the summation index "i" is over all applied current drive methods, e.g. LH-waves, NBI, etc., $\alpha_i = I_i/I$ and $\eta_i = I_i/P_i$. Here, I is the total driven current, and I_i and P_i are the fractions of current and absorbed power, respectively, corresponding to method "i". We see that if the α_i's are of the same order, then the global efficiency is defined by the method with the lowest efficiency.

Assuming now that $\eta \sim \eta_{LH}$ and representing η_{LH} as $\eta_{LH}^{-1} = (P_e+P_\alpha)_{LH}/I$, where $(P_e)_{LH}$ and $(P_\alpha)_{LH}$ are the fractions of power absorbed by electrons and alpha particles in connection with the LH current drive, it easily appears that the alpha particles significantly affect the total current drive efficiency if $(P_\alpha)_{LH} \sim (P_e)_{LH}$.

In the context of the preceding discussion it seems important to investigate the features of current generation by alpha particles interacting with LH-waves. The present paper presents the results of analytical approach to this problem.

The alpha generated current cannot be determined within the framework of the quasi-linear models used in Refs. [8, 9] where quasi-linear absorption of the LH-wave power by alpha particles has been studied. These models have been based on the theory of Karney, [11, 12], which describes ion acceleration by LH-waves having the parallel wave number $k_{\parallel} = 0$. According to previous theories, two acceleration mechanisms are possible. In the presence of a single plan wave, the ion motion becomes stochastic when the electric field exceeds a certain thresholds value. Alternatively, if a broad spectrum of waves is present, quasi-linear damping can take place. The second situation, i.e. the interaction of ions with wave packets, is likely to occur in practice and it does not require any threshold for the field intensity. Karney, [10], has shown that the two mechanisms lead to essentially the same velocity diffusion coefficient for the ions.

The appropriate diffusion coefficient can be obtained from quasi-linear theory, [13], by replacing a summation over the cyclotron harmonics by an integration. Formally, this can be performed if the resonance broadening being responsible for destroying the phase coherence is modelled by replacing the resonance condition $\delta(\omega - l\omega_{c\alpha})$ by resonance functions having finite width, [12, 13]. Here, δ is the Dirac delta function, ω is the wave frequency, $\omega_{c\alpha}$ is the cyclotron frequency of fast ions, and l is an integer. The last procedure seems, however, not to be consistent with the assumption $k_{\parallel} = 0$ which besides is never valid in reality. Only because of $k_{\parallel} = 0$ the Cerenkov resonances $\omega - l\omega_{c\alpha} - k_{\parallel}v_{\parallel} = 0$ with different values of l are overlapped providing that the width of the wave packet, $\Delta k_{\parallel}$, satisfies the condition

$$\frac{\Delta k_{\parallel}}{k_{\parallel}} > \frac{\omega_{c\alpha}}{N_{\parallel}\omega}\frac{c}{v_{\parallel}} \tag{3}$$

where $v_{\parallel}$ is the ion velocity along the magnetic field and $N_{\parallel} = ck_{\parallel}/\omega$ is the parallel refractive index of the injected LH -waves. For alpha particles interacting with LH-waves we can assume $v_{\parallel} \approx 10^9\ \mathrm{cm{\cdot}s^{-1}}$, $\omega_{c\alpha}/\omega \approx 10^{-2}$, $N_{\parallel} = 2$, which gives $\Delta k_{\parallel}/k > 10^{-1}$. In the following analysis it will be assumed that the condition (3) is fulfilled.

Because of the small value of $k_{\parallel}$ for the LH-current drive, ($k_{\parallel}/k_{\perp} \geq \sqrt{m_e/m_i}$), the quasi-linear distortion of the alpha particle distribution function will mainly take place in the perpendicular direction in velocity-space. This makes it possible to use the $k_{\parallel} = 0$ approximation when estimating the RF-power absorbed by alpha particles. However, to determine the alpha generated current one must take into account the finite $k_{\parallel}$-effects. The reason is that these effects give rise to the parallel distortion of the alpha distribution which in turn provides current generation.

The plane of the paper is as follows. The quasi-linear Fokker-Planck equation describing the effect of LH-waves on the alpha particle distribution function is formulated in Section 2. In Section 3, steady-state solutions of the quasi-linear equation are obtained for proper boundary conditions in velocity-space. The alpha generated current and the RF-power absorbed by alpha particles are determined in Section 4. Finally, Section 5 summarizes the results and presents conclusions.

2. The quasi-linear Fokker-Planck equation

Our present analysis will concern a situation where thermonuclear reactions in a tokamak reactor produce 3.5 MeV alpha particles with an almost mono-energetic and isotropic velocity distribution. During the slowing-down via Coulomb collisions the alpha particle distribution function remains isotropic, and there is no net current. Anisotropy develops, however, as a consequence of the interaction between externally excited LH-waves and energetic alpha particles with velocity $v \geq v_c$, where v_c is the critical velocity at which electrons and background ions contribute equally to the slowing-down.

We shall assume an ideal confinement of alpha particles. The collisional pitch angle scattering and energy diffusion of alpha particles are negligible in the velocity range under consideration. The toroidal effects as well as the energy spread of the alpha source will also be neglected.

Under the above assumptions, the behaviour of the velocity distribution function, f_α, of alpha particles interacting with LH-waves can be described by the quasi-linear Fokker-Planck equation in the form:

$$\frac{\partial f_\alpha}{\partial t} = C^{COLL}(f_\alpha) + C^{QL}(f_\alpha) + S \tag{4}$$

Here, the collision term, $C^{COLL}(f_\alpha)$, describes the slowing-down of alphas by electrons and background ions, i.e.

$$C^{COLL}(f_\alpha) = \frac{1}{\tau_s}\left(\frac{\partial}{\partial v_\parallel} v_\parallel + \frac{1}{v_\perp}\frac{\partial}{\partial v_\perp} v_\perp^2\right)\left(1 + \frac{v_c^3}{v^3}\right) f_\alpha \tag{5}$$

where the slowing-down time, τ_s, is

$$\tau_s = \frac{3\, m_\alpha\, m_e\, v_e^3}{4\sqrt{2\pi}\, e_\alpha^2\, e^2\, n_e\, \Lambda} \tag{6}$$

and

$$v_c^3 = \frac{3\sqrt{\pi}}{4}\,\frac{m_e}{n_e}\, v_e^3 \sum_i \frac{Z_i^2 n_i}{m_i} \tag{7}$$

v_e is the thermal velocity of electrons, Λ is the Coulomb logarithm, and the other notations are standard.

The quasi-linear diffusion term, $C^{QL}(f_\alpha)$, can be written as, cf [11],

$$C^{QL}(f_\alpha) = \sum_k \left(k_\parallel \frac{\partial}{\partial v_\parallel} + \frac{\omega - k_\parallel v_\parallel}{v_\perp}\frac{\partial}{\partial v_\perp}\right) D_k^{QL}\left(k_\parallel \frac{\partial f_\alpha}{\partial v_\parallel} + \frac{\omega - k_\parallel v_\parallel}{v_\perp}\frac{\partial f_\alpha}{\partial v_\perp}\right) \tag{8}$$

where the diffusion coefficient, D_k^{QL}, is defined by

$$D_k^{QL} = \frac{\pi e_\alpha^2}{m_\alpha^2} \sum_{l=-\infty}^{+\infty} \frac{|E_k|^2}{k^2} J_l^2(\xi)\, \delta(\omega - l\omega_{c\alpha} - k_\parallel v_\parallel) \tag{9}$$

Here, E_k is the amplitude of the electric wave packet field, $k_\perp$ is related to $k_\parallel$ via the dispersion relation for the LH-waves, $J_l(\zeta)$ is an ordinary Bessel function of integral order with argument $\zeta = k_\perp v_\perp/\omega_{c\alpha}$. Note that in deriving (9), E_k is taken to be infinitesimal and a spectrum of waves is assumed, [11].

Finally, we take the alpha particle source, S, in the form

$$S = \frac{S_\alpha}{4\pi v_\alpha^2}\,\delta(v - v_\alpha) \tag{10}$$

where $S_\alpha = n_D n_T \langle \sigma v \rangle$ is the rate of the thermonuclear reactions, and $v_\alpha = 1.3 \times 10^9\ \mathrm{cm \cdot s^{-1}}$ is the velocity of the generated alpha particles.

Let us now discuss the quasi-linear term given by eqs. (8) and (9) in more detail. In order to perform the summation over $k_\parallel$ we represent the field spectrum as

$$|E_k(k_\parallel)|^2 \simeq \frac{\Delta k_\parallel E_0^2}{\pi\left[(k_\parallel - k_{\parallel 0})^2 + (\Delta k_\parallel)^2\right]} \tag{11}$$

where $E_0^2 = \int_{-\infty}^{+\infty} dk_\parallel |E_k|^2$, and approximate the operator $C^{QL}(f_\alpha)$ by

$$C^{QL} \simeq \hat{\Pi}\left(\sum_k D_k^{QL}\,\hat{\Pi} f_\alpha\right) \tag{12}$$

where $\hat{\Pi} = \left(k_{\parallel 0}\frac{\partial}{\partial v_\parallel} + \frac{\omega - k_{\parallel 0} v_\parallel}{v_\perp}\frac{\partial}{\partial v_\perp}\right)$. Using now the delta function in eq. (9), we obtain

$$\begin{aligned} D^{QL} = \sum_k D_k^{QL} &= \frac{e_\alpha^2}{m_\alpha^2}\sum_{l=-\infty}^{+\infty}\frac{E_0^2}{k^2 v_\parallel} J_l^2(\zeta)\,\frac{\Delta k_\parallel}{\left[\left(\frac{\omega - l\omega_{c\alpha}}{v_\parallel} - k_{\parallel 0}\right)^2 + (\Delta k_\parallel)^2\right]} = \\ &= \frac{e_\alpha^2}{2m_\alpha^2}\frac{E_0^2}{k^2}\,\mathrm{Re}\int_{-\infty}^{+\infty} d\tau \exp\left[i\tau(\omega - k_{\parallel 0} v_\parallel + i\Delta k_\parallel v_\parallel)\right]\sum_{l=-\infty}^{+\infty} J_l^2(\zeta)\exp(-il\omega_{c\alpha}\tau) \end{aligned} \tag{13}$$

Since

$$\sum_{l=-\infty}^{+\infty} J_l^2(\xi) \exp(-l\omega_{c\alpha}\tau) = J_0\left[2\xi \sin\left(\frac{\omega_{c\alpha}\tau}{2}\right)\right] \tag{14}$$

the expression (13) becomes

$$D^{QL} = \frac{e_\alpha^2}{2m_\alpha^2} Re \int_{-\infty}^{+\infty} d\tau \; J_0\left[2\xi \sin\left(\frac{\omega_{c\alpha}\tau}{2}\right)\right] \exp\left[i(\omega - k_{\parallel 0}v_\parallel)\tau - \Delta k_\parallel v_\parallel \tau\right] \tag{15}$$

If now $\Delta k_\parallel v_\parallel > \omega_{c\alpha}$, see eq. (3), we can assume that the main contribution to the integral in (15) comes from the interval $\tau << 2/\omega_{c\alpha}$. The last condition means that the ions spend a small fraction of a cyclotron period in the wave field. Approximating then $\sin(\omega_{c\alpha}\tau/2)$ by $\omega_{c\alpha}\tau/2$ in the argument of the Bessel function, we obtain

$$D^{QL} \simeq \begin{cases} \dfrac{e_\alpha^2 E_0^2}{m_\alpha^2 k^2}\left[k_\perp^2 v_\perp^2 - (\omega - k_{\parallel 0} v_\parallel)^2\right]^{-1/2} & ; \quad v_\perp > v_{\perp r} \\ 0 & ; \quad v_\perp < v_{\perp r} \end{cases} \tag{16}$$

where $v_{\perp r} = |\omega - k_{\parallel 0}v_\parallel|/k_\perp$. Note that the same result can be obtained if we first replace the summation over the cyclotron harmonics by an integration and then use the asymtotic expansion of the Bessel functions for large $l = (\omega - k_\parallel v_\parallel)/\omega_{c\alpha} >> 1$ together with the approximation $|E_k|^2 \equiv E_0^2\,\delta(k_\parallel - k_{\parallel 0})$.

3. The alpha particle distribution function

We restrict ourselves to considering solutions of eq. (4) under steady-state conditions which are appropriate for times longer than the slowing-down time, cf [8]. It is then convenient to divide velocity space $(v_\perp, v_\parallel)$ into three regions as shown in Fig. 1. We see that the resonance line $v_\perp = k_\parallel/k_\perp\,|v_{\parallel p} - v_\parallel|$ where $v_{\parallel p} = \omega/k_\parallel$, separates the wave-particle interaction region 3, where $D^{QL} = 0$, from regions 1 and 2 where the quasi-linear diffusion is absent, $D^{QL} = 0$. Consequently, we solve eq. (4) separately in each region and then find the total solution by applying proper matching conditions.

A. Solutions in regions 1 and 2 ($v_\perp < |\omega - k_\parallel v_\parallel / k_\perp|$; $D^{QL} = 0$)

In the absence of the quasi-linear diffusion, the fusion produced alpha particles only experience collisional slowing-down. Since the characteristics of the operator C^{COLL} are $\chi = v_\parallel / v$ = const, it is natural to use the variables (v, χ) instead of $(v_\perp, v_\parallel)$. In these variables eq. (4) in steady-state reduces in regions 1 and 2 to

$$\frac{1}{\tau_s v^2} \frac{\partial}{\partial v}\left[\left(v^3 + v_c^3\right) f_\alpha\right] + \frac{S_\alpha}{4\pi v_\alpha^2} \delta(v - v_\alpha) = 0 \tag{17}$$

We now note that the points of intersection between $v = v_\alpha$ and the boundary line $v_\perp = k_\parallel |v_{\parallel p} - v_\parallel| / k_\perp$ are determined by

$$\chi_{1,2} = \frac{k_\parallel}{k} \frac{v_p}{v_\alpha} \mp \frac{k_\perp}{k}\left(1 - \frac{v_p^2}{v_\alpha^2}\right)^{1/2} \tag{18}$$

where $v_p = \omega / k$.

In region 1 ($\chi < \chi_1, \chi > \chi_2$), eq. (17) yields the well-known solution

$$f_\alpha^{(1)}(v) = \frac{S_\alpha \tau_s}{4\pi (v^3 + v_c^3)} \eta(v_\alpha - v) \tag{19}$$

where $\eta(x)$ is the Heaviside step function.

In region 2 ($\chi_1 < \chi < \chi_2$), the general solution of eq. (17) is given by

$$f_\alpha^{(2)}(v, \chi) = \frac{C(\chi)}{v^3 + v_c^3} \tag{20}$$

The function $C(\chi)$ is determined by matching the solutions in regions 2 and 3 along the boundary $v_\perp = k_\parallel |v_{\parallel p} - v_\parallel| / k_\perp$:

$$f_\alpha^{(2)}(v_r(\chi), \chi) = f_\alpha^{(3)}(v_r(\chi), \chi) \tag{21}$$

where

$$V_r(\chi) = \frac{V_{\parallel p}}{\chi + \frac{k_\perp}{k_\parallel}(1-\chi^2)^{1/2}} \tag{22}$$

Hence, we obtain

$$C(\chi) = \left[V_r^3(\chi) + V_c^3\right] f_\alpha^{(3)}(V_r(\chi), \chi). \tag{23}$$

B. Solution in region 3 ($v_\perp > |\omega - k_\parallel v_\parallel|/k_\perp$; $D^{QL} \neq 0$)

It follows from eq. (12) that the characteristics of the quasi-linear operator are defined by $v_\perp^2 + (v_\parallel - v_{\parallel p})^2 = \text{const.}$ Therefore we introduce a new orthogonal curvilinear coordinate system according to

$$w = v_\perp^2 + (v_\parallel - v_{\parallel p})^2$$

$$z = \frac{v_\perp}{v_{\parallel p} - v_\parallel} \tag{24}$$

In the variables (w, z), the expressions for $C^{COLL}(f_\alpha)$ and $C^{QL}(f_\alpha)$ take the form

$$C^{COLL}(f_\alpha) = \frac{1}{\tau_s}\left\{\frac{2}{w^{1/2}}\frac{\partial}{\partial w}\left(w^{3/2} - \frac{V_{\parallel p}\, w}{\sqrt{1+z^2}}\right) + \right.$$

$$\left. + \frac{V_{\parallel p}}{w^{1/2}}\frac{(1+z^2)^{3/2}}{z}\frac{\partial}{\partial z}\frac{z^2}{(1+z^2)}\right\}\left(1 + \frac{V_c^3}{V^3}\right) f_\alpha \tag{25}$$

and

$$C^{QL}(f_\alpha) = \frac{k_\parallel^2}{w}\frac{(1+z^2)^{3/2}}{z}\frac{\partial}{\partial z} D^{QL}(w,z)\frac{(1+z^2)^{3/2}}{z}\frac{\partial f_\alpha}{\partial z} \tag{26}$$

where

$$D^{QL}(w,z) = \frac{e_\alpha^2 E_0^2}{m_\alpha^2 k^2 k_\perp}\left[\frac{1+z^2}{w\,(z^2 - z_r^2)}\right]^{1/2} \tag{27}$$

Here, $v^2 = w^2 + v_{\parallel p}^2 - 2v_{\parallel p}\sqrt{w(1+z^2)}$ and $z_T = k_\parallel/k_\perp$. To find the solution of eq. (4) in region 3 we neglect the slowing-down across w. This approximation is plausible since, for $\xi = \tau_s/\tau_{QL} >> 1$, where

$$\xi \equiv \frac{\pi}{2} \frac{\tau_s e_\alpha^2 \omega^2 E_o^2}{m_\alpha^2 k^2 k_\perp V_\alpha^5} \tag{28}$$

most of the tail particles have small pitch angles, $\chi = \chi_1 v_\alpha/v_{max}$. Then, the steady-state equation determining $f_\alpha^{(3)}$ can be written as

$$\frac{\partial}{\partial z}\left[\frac{z^2}{(1+z^2)}\left(1+\frac{V_c^3}{V^3}\right)f_\alpha^{(3)}\right] + \frac{\partial}{\partial z}\left[\frac{2\xi}{\pi}\frac{V_\alpha^5}{V_{\parallel p}^3 W}\frac{(1+z^2)^2}{z\sqrt{z^2-z_T^2}}\frac{\partial f_\alpha^{(3)}}{\partial z}\right] +$$

$$+ \frac{S_\alpha \tau_s W^{1/2}}{4\pi V_\alpha^2 V_{\parallel p}} \frac{z}{(1+z^2)^{3/2}} \delta(v - v_\alpha) = 0 \tag{29}$$

The wave particle interaction region is now defined by $w_1 \leq w \leq w_2$ and $z \geq z_T$, where

$$W_{1,2} \equiv W(V = V_\alpha, \chi = \chi_{1,2}) = V_\alpha^2 + \frac{k_\perp^2 - k_\parallel^2}{k^2} V_{\parallel p}^2 \mp$$

$$\mp 2\frac{k_\perp}{k} V_{\parallel p} \sqrt{V_\alpha^2 - V_p^2} \tag{30}$$

Note that the interaction occurs only for $v_p < v_\alpha$.

We are looking for a continuous solution of eq. (29) satisfying the following boundary conditions:

$$\lim_{z\to\infty} f_\alpha^{(3)}(z, W) = 0 \tag{31}$$

$$\lim_{z\to\infty} \frac{\partial f_\alpha^{(3)}(z, W)}{\partial z} = 0 \tag{32}$$

and

(33)

After performing $\lim_{\kappa \to 0} \int_{z_r-\kappa}^{z_r+\kappa} d\kappa$ of eq. (29) one can easily verify that the last condition is satisfied. Then, eq. (29) can be integrated once to give

$$\frac{z^2}{(1+z^2)}\left(1+\frac{v_c^3}{v^3}\right) f_\alpha^{(3)}(w,z) - \frac{z_r^2}{(1+z_r^2)}\left(1+\frac{v_c^3}{v_r^3}\right) f_\alpha^{(3)}(w,z_r) +$$

$$+ \frac{2\xi}{\pi} \frac{v_\alpha^5}{v_{\parallel p}^3 w} \frac{(1+z^2)^2}{z\sqrt{z^2-z_r^2}} \frac{\partial f_\alpha^{(3)}(w,z)}{\partial z} + g(w)\eta[z - z_\alpha(w)] = 0 \tag{34}$$

where

$$g(w) = \frac{S_\alpha \tau_s w^{1/2}}{4\pi v_\alpha^2 v_{\parallel p}} \left.\frac{z}{(1+z^2)^{3/2}|\partial v/\partial z|}\right|_{z=z_\alpha(w)} = \frac{S_\alpha \tau_s}{4\pi v_\alpha v_{\parallel p}^2} \tag{35}$$

and

$$z_\alpha(w) = \left[\frac{4 w v_{\parallel p}^2}{(w + v_{\parallel p}^2 - v_\alpha^2)^2} - 1\right]^{1/2} \tag{36}$$

is the value of z at $v = v_\alpha$. Applying the boundary conditions (31) and (32) to eq. (34) yields

$$f_\alpha^{(3)}(w, z_r) = \frac{k^2}{k_\parallel^2} \frac{g(w)}{(1 + v_c^3/v_r^3)} \tag{37}$$

and we find that the solution of eq. (34) is given by

$$f_\alpha^{(3)}(w,z) = g\, e^{-\lambda(w,z)}\left\{\frac{k^2}{k_\parallel^2(1+v_c^3/v_r^3)} + \int_{z_r}^{z} dz'\varphi(w,z')\, e^{\lambda(w,z')}\eta[z_\alpha(w)-z']\right. \tag{38}$$

where

$$\lambda(w,z) = \frac{\pi}{2\xi} \frac{V_{\parallel p}^3 w}{V_\alpha^5} \int_{z_r}^{z} dz' \frac{z'\sqrt{z'^2 - z_r^2}}{(1+z'^2)^3}\left(1 + \frac{V_c^3}{V'^3}\right) \tag{39}$$

and

$$\varphi(w,z) = \frac{\pi g}{2\xi} \frac{V_{\parallel p}^3 w}{V_\alpha^5} \frac{z\sqrt{z^2 - z_r^2}}{(1+z^2)^2} \tag{40}$$

Finally, we return to the matching condition (23) which determines the solution in region 2. Using eqs. (23) and (38) we obtain

$$C(\chi) = g\frac{k^2}{k_\parallel^2} v_r^3(\chi) = \frac{S_\alpha \tau_s v_p k^3}{4\pi v_\alpha (k_\parallel \chi + k_\perp\sqrt{1-\chi^2})^3} \tag{41}$$

Consequently, eq. (20) can be written as

$$f_\alpha^{(2)}(v,\chi) = \frac{S_\alpha \tau_s v_p k^3}{4\pi v_\alpha (k_\parallel \chi + k_\perp\sqrt{1-\chi^2})^3 (v^3 + v_c^3)} \tag{42}$$

In summary, the expressions (19), (42) and (38) determine the steady-state alpha particle distribution function in the presence of LH-waves. The obtained result represents a generalization of the alpha particle distribution function considered in Ref. [9]. The new feature is the inclusion of the finite $k_\parallel$-effects which gives rise to the parallel quasi-linear distribution of the distribution function.

4. The alpha particle generated current

Using now the alpha particle distribution function found in the previous section, we shall give an analytical estimate of the alpha particle generated current defined by

$$J_\alpha = e_\alpha \int d\vec{v}\, v_\parallel f_\alpha = J_\alpha^{(1)} + J_\alpha^{(2)} + J^{(3)} \tag{43}$$

where $J_\alpha^{(i)}$ $(i = 1,2,3)$ are the partial currents generated in regions 1, 2 and 3, respectively. The calculations will be performed to leading order in the small parameter $k_{\parallel}/k$. We will also neglect corrections of $0(v_c^3/v_\alpha^3)$

Making use of eqs. (19) and (42) when calculating the currents $J_\alpha^{(1)}$ and $J_\alpha^{(2)}$, respectively, we obtain

$$J_\alpha^{(1)} = 2\pi e_\alpha \left\{ \int_{-1}^{x_1} dx \int_0^{v_\alpha} v_{\parallel} f_\alpha^{(1)} v^2 dv + \int_{x_2}^{1} dx \int_0^{v_\alpha} v_{\parallel} f_\alpha^{(1)} v^2 dv \right\} \simeq$$

$$\simeq -\frac{1}{2} e_\alpha S_\alpha \tau_s v_p \frac{k_{\parallel}}{k} \left(1 - \frac{v_p^2}{v_\alpha^2}\right)^{1/2} \tag{44}$$

and

$$J_\alpha^{(2)} = 2\pi e_\alpha \int_{x_1}^{x_2} dx \int_0^{v_r(x)} v_{\parallel} f_\alpha^{(2)} v^2 dv \simeq$$

$$\simeq \frac{1}{3} e_\alpha S_\alpha \tau_s v_p \frac{k_{\parallel}}{k} \left(4 - \frac{v_\alpha^2}{v_p^2}\right)\left(1 - \frac{v_p^2}{v_\alpha^2}\right)^{1/2} \tag{45}$$

The current $J_\alpha^{(3)}$ is defined by

$$J_\alpha^{(3)} = \pi e_\alpha \int_{w_1}^{w_2} dw \int_{z_r}^{\infty} dz \frac{z\, w^{1/2}}{(1+z^2)^2} \left[v_{\parallel p} (1+z^2)^{1/2} - w^{1/2} \right] f_\alpha^{(\beta)}(w,z) \tag{46}$$

where $w_{1,2}$ and the function $f_\alpha^{(3)}$ are given by eq. (30) and eqs. (38) - (40), respectively. In order to evaluate the integrals in eq. (46), we introduce the following transformation of variables:

$$x = \frac{w}{v_{\parallel p}^2} - x_0$$

$$y = \frac{v_{\parallel p}}{v_\alpha} \sqrt{z^2 - z_r^2} \tag{47}$$

where $x_0 = 1 + (k_{\parallel}^2/k^2)(v_\alpha^2/v_p^2 - 2)$. Note that $x \sim k_{\parallel}/k << 1$ and $y \sim v_\perp/v_\alpha \sim 0(1)$.

Then, retaining terms of $0(k_{\parallel}^2/k^2)$ in eq. (46) we get

$$J_\alpha^{(3)} \simeq \frac{\pi}{2} e_\alpha v_\alpha^2 \Big\{ v_\alpha^2 \int_{-\Delta}^{\Delta} dx \left(1+\frac{5}{2}x\right) \int_0^\infty dy\, y^3 f_\alpha^{(3)}(x,y) - $$
$$ - v_{\parallel p}^2 \int_{-\Delta}^{\Delta} dx \left[\frac{k_\parallel^2}{k^2}\left(\frac{v_\alpha^2}{v_p^2} - 3\right) + \frac{x^2}{4} + x \right] \int_0^\infty dy\, y f_\alpha^{(3)}(x,y) \Big\} \tag{48}$$

where $\Delta = 2(k_\parallel/k)(v_\alpha/v_p)(1-v_p^2/v_\alpha^2)^{1/2} \sim 0(k_\parallel/k)$. Here, the function $f_\alpha^{(3)}(x,y)$ is obtained by expressing eqs. (36) and (38) - (40) in the variables (x, y) and by taking into account only the leading terms in $k_\parallel/k$. The result is

$$f_\alpha^{(3)}(x,y) \simeq \frac{S_\alpha \tau_s}{4\pi v_\alpha^3} e^{-\lambda(x,y)} \left\{ \frac{v_\alpha^2}{v_f^2} + \frac{\pi(1+x)}{2\xi} \int_0^y dy' y'^2 e^{\lambda(x,y')} \eta[y_\alpha(x) - y'] \right\} \tag{49}$$

where

$$\lambda(x,y) \simeq \frac{\pi}{10\xi}(1+x) y^3 \left(y^2 + \frac{5}{3}\frac{v_p^2}{v_\alpha^2} \right) \equiv (1+x)\lambda_0(y) \tag{50}$$

and

$$y_\alpha^2(x) = \frac{v_{\parallel p}^2}{v_\alpha^2} \frac{(x+\Delta)(x-\Delta)}{(2+x)^2}$$
$$\xi \simeq \frac{\pi}{2} \frac{\tau_s e_\alpha^2 \omega^2 E_0^2}{m_\alpha^2 v_\alpha^5 k_\perp^3} \tag{51}$$

Assuming now $\xi >> 1$ we neglect the second term in eq. (49) and use the approximation

$$f_\alpha^{(3)}(x,y) \simeq \frac{S_\alpha \tau_s}{4\pi v_\alpha v_p^2} e^{-\lambda(x,y)} \simeq \frac{S_\alpha \tau_s}{4\pi v_\alpha v_p^2}[1 - x\lambda_0(y)] e^{-\lambda_0(y)} \simeq$$
$$\simeq \frac{S_\alpha \tau_s}{4\pi v_\alpha v_p^2}[1 - x\lambda_0(y)] \exp(-\delta y^5) \sum_{s=0}^\infty \frac{(-1)^s}{s!}\left(\frac{5}{3}\delta \frac{v_p^2}{v_\alpha^2} y^3 \right)^s \tag{52}$$

where $\delta = \pi/(10\xi)$.

Substituting the expansion (52) into eq. (48) and evaluating the corresponding integrals, we arrive at the expression

$$J_\alpha^{(3)} \simeq \frac{e_\alpha S_\alpha \tau_s v_p}{10\,\delta^{4/5}} \frac{k_\parallel}{k} \frac{v_\alpha^4}{v_p^4} \left(1 - \frac{v_p^2}{v_\alpha^2}\right)^{1/2} \dot{j}_3(\delta) \tag{53}$$

with

$$j_3(\delta) \simeq \Gamma\left(\frac{4}{5}\right) - \frac{4}{5}\left(1 - \frac{8}{3}\frac{v_p^2}{v_\alpha^2}\right)\Gamma\left(\frac{2}{5}\right)\delta^{2/5} + \frac{20}{9}\frac{v_p^2}{v_\alpha^2}\left(1 - \frac{15}{8}\frac{v_p^2}{v_\alpha^2}\right)\delta^{4/5} \tag{54}$$

where $\Gamma(x)$ is the gamma function. Note that because of previous approximations the expression (54) is valid to $O(\delta)$.

Making use of eq. (43) together with eqs. (44), (45), (53) and (54), we finally obtain the approximate expression for the alpha particle generated current as

$$J_\alpha \simeq \frac{e_\alpha S_\alpha \tau_s v_p}{10\,\delta^{4/5}} \frac{k_\parallel}{k} \frac{v_\alpha^4}{v_p^4} \left(1 - \frac{v_p^2}{v_\alpha^2}\right)^{1/2} j(\delta) \tag{55}$$

where

$$j(\delta) \simeq \Gamma\left(\frac{4}{5}\right) - \frac{4}{5}\left(1 - \frac{8}{3}\frac{v_p^2}{v_\alpha^2}\right)\Gamma\left(\frac{2}{5}\right)\delta^{2/5} - \frac{10}{9}\frac{v_p^2}{v_\alpha^2}\left(1 - \frac{15}{4}\frac{v_p^2}{v_\alpha^2}\right)\delta^{4/5} \tag{56}$$

It follows from eqs. (55) and (56) that in the limit $\delta << 1$ ($\xi >> 1$) the current is mainly driven by the tail particles, i.e. those in region 3. We also want to emphasize that the neglected effect of slowing-down across the quasi-linear diffusion characteristics would lead to an increased value of the alpha current as compared to the above result. This can be seen by considering the marginal characteristic passing through the origin in the $(v_\parallel, v_\perp)$-phase, see Fig. 1. Along this characteristic, both slowing-down and quasi-linear diffusion have the same direction. Particles that are located on the right-hand-side of the marginal characteristic will be, due to slowing-down, shifted to the left and thus, decreasing the resulting current. The opposite takes place for particles located on the left-hand-side of the characteristic. Since the number of the right-shifted particles is larger than the left-shifted, the net result would be a current increase. On the other hand, we have neglected the presence of trapped alpha particles. The assumption of passing particles only leads to an overestimate of the alpha-particle current.

We have mentioned in Section 1 that the alpha particles may strongly affect the efficiency of the LH-current drive if the RF-power absorbed by the alphas, P_α, is sufficiently large. The problem of determining P_α has been considered in Refs. [8, 9]. Here, let us estimate the absorbed power by using the alpha particle distribution function found in Section 3.

In the variables (w, z) introduced previously, the power P_α is expressed by

$$P_\alpha = \frac{m_\alpha}{2} \int d\vec{v}\, v^2\, C^{QL}(f_\alpha) = -\pi m_\alpha v_{\parallel p} \int_{w_1}^{w_2} dw\, w^{1/2} \int_{z_r}^{\infty} \frac{dz}{\sqrt{1+z^2}} \frac{\partial \Gamma_\alpha}{\partial z} \tag{57}$$

where Γ_α is the alpha particle flux due to quasi-linear diffusion and it is related to $C^{QL}(f_\alpha)$ through the relation

$$C^{QL}(f_\alpha) = \frac{(1+z^2)^{3/2}}{w^{1/2}\, z} \frac{\partial \Gamma_\alpha}{\partial z} \tag{58}$$

It follows from eqs. (26) - (28), (34) and (58) that

$$\Gamma_\alpha \simeq \frac{2\xi}{\pi \tau_s} \frac{v_\alpha^5}{v_{\parallel p}^2} \frac{(1+z^2)^2}{w z \sqrt{z^2 - z_r^2}} \frac{\partial f_\alpha}{\partial z} \simeq -\frac{v_{\parallel p}}{\tau_s} \frac{z^2}{(1+z^2)} f_\alpha \tag{59}$$

where corrections of $0(k_{\parallel}/k)$ as well as $0(v_c^3/v_\alpha^3)$ have been neglected.

Substituting now (59) into (57) and using the distribution function given by eqs. (38) - (40), we obtain in the limit $\xi >> 1$

$$P_\alpha \simeq \frac{m_\alpha S_\alpha v_\alpha^4}{5\delta^{4/5} v_p^2} \left(1 - \frac{v_p^2}{v_\alpha^2}\right)^{1/2} \Gamma\!\left(\frac{4}{5}\right) \tag{60}$$

where $\delta = \pi/(10\xi)$). This expression is similar to the result obtained in Ref. [9].

Writing eq. (60) in the form

$$P_\alpha \simeq P_f\, \delta^{-4/5}\, \frac{\sqrt{1-x}}{x} \tag{61}$$

where $P_f = \varepsilon_\alpha S_\alpha = \varepsilon_\alpha\, n/\tau_f$ is the fusion power, ε_α = 3.5 MeV, and $x = v_p^2/v_\alpha^2$, it is interesting to note that due to acceleration of alpha particles a situation may arise where locally $P_\alpha >> P_f$. Note that in a sufficiently hot and dense plasma ($n \gtrsim 10^{20}\ \mathrm{m^{-3}}$, $T \gtrsim 20\ \mathrm{keV}$) with J_e < $1\mathrm{MA/m^2}$, the power density absorbed by electrons is less than P_f.

Equations (55), (56) and (60) enable us to estimate the local efficiency, η_α^L, connected with alpha particles as

$$\eta_\alpha^L = \frac{J_\alpha}{P_\alpha} \simeq \frac{N_\parallel}{2}\, \frac{e_\alpha \tau_s}{m_\alpha c} \tag{62a}$$

which for practical use can be written as

$$\eta_\alpha^L \simeq 1.8\times 10^{-2}\, \frac{N_\parallel\, T_i^{3/2}}{n_{20}\, \Lambda} \quad \left[\frac{A\cdot m}{W}\right] \tag{62b}$$

where T is the plasma temperature in keV, and n_{20} is the plasma density in units 10^{20} $\mathrm{m^{-3}}$. Taking, for example, $N_\parallel \sim 2$, T = 20 keV, $\Lambda \sim 20$, and $n_{20} = 1$, we have $\eta_\alpha^L \cong 0.16$ A·m/W. This efficiency is rather low from the point of view of a reactor application.

5. Concluding remarks

The principal results obtained in this paper can be summarized as follows:

(i) A new equation describing the interaction between high-energy ions and LH-waves has been derived. It generalizes the corresponding equation of Refs. [10, 11] by taking into account effects associated with $k_\parallel \neq 0$ and it holds even when the wave amplitudes are small provided the wave spectrum is not too narrow.

(ii) By making appropriate simplifying assumptions, the analytical expression for the distribution function of alpha particles interacting with LH-waves in a fusion plasma has been determined. The new feature is the inclusion of finite $k_{\parallel}$-effects which gives rise to a parallel distortion of the alpha particle velocity distribution. We note, however, that the analysis neglects toroidal effects like e.g. the presence of trapped alpha particles. This issue will be the subject of a future study.

(iii) The generated alpha particle current as well as the RF-power absorbed by alpha particles have been estimated by using the obtained distribution function.

In order to briefly discuss the influence of the alpha generated current on the LH-current drive, let us compare the alpha efficiency, η_α^L, defined by eq. (62) with the local efficiency of LH-current drive, $\eta_e^L = J_e/P_e$, calculated in the absence of alpha particles. Making use of the expression, [2],

$$\eta_e^L \simeq \frac{0.77\, T}{n_{20}\Lambda}\,\tilde{\eta} \quad \left[\frac{Am}{W}\right] \tag{63}$$

where $\tilde{\eta}$ is the dimensionless efficiency, we find

$$\frac{\eta_\alpha^L}{\eta_e^L} \simeq 2.3\times 10^{-2}\,\frac{N_{\parallel} T^{1/2}}{\tilde{\eta}} \tag{64}$$

Taking again the parameters $N_{\parallel} = 2$, $T = 20$ keV and assuming $\tilde{\eta} \cong 3.5$ we obtain from $\eta_\alpha^L/\eta_e^L \cong 0.06$. Thus, we conclude that for a typical LH-current drive scenario the ratio η_α/η_e is small. However, if the alpha absorbed power, P_α, is of the order of the power absorbed by electrons, P_e, the LH-current drive efficiency, η_{LH}^L, becomes

$$\eta_{LH}^L \simeq \frac{J_e}{P_\alpha + P_e} \sim 0.5\,\eta_e^L \tag{65}$$

When $P_\alpha >> P_e$ (this may take place in a hot and dense plasma and/or when power densities absorbed by electrons and alphas are peaked at different flux surfaces) we find that

$$\eta_{LH}^{L} = \eta_{\alpha}^{L} + \frac{P_e}{P_\alpha}\eta_e^{L} << \eta_e^{L} \tag{66}$$

Thus, in this case alpha particles may significantly decrease the local efficiency of the current drive.

Acknowledgements

We gratefully acknowledge the encouragement and support of H. Wilhelmsson. Two of the Authors (V. Belikov and Ya. Kolesnichenko) would like to acknowledge the hospitality of the Institute for Electromagnetic Field Theory and Plasma Physics at Chalmers University of Technology.

References

[1] N.J. Fish, Rev. Mod. Phys. 59 (1987), 175.

[2] Ya.I. Kolesnichenko, V.V. Parail, G.V. Pereverzev, Voprosy Teorii Plasmy (Problems of Plasma Theory), Vol. 17, Moscow (1989), 3.

[3] C. Gormezano et al, Nucl. Fus. 21 (1981), 1047.

[4] J.J. Schuss et al, Nucl. Fus. 21 (1981), 427.

[5] M. Brambilla, Y.-P. Chen, Nucl. Fus. 23 (1983), 541.

[6] F.W. Perkins, Bull. Am. Phys. Soc. 27 (1982), 1101.

[7] K.L. Wong, M. Ono, Nucl. Fus. 24 (1984), 615.

[8] E. Barbato, F. Santini, (Techn. Com. IAEA Meeting on Alpha Particles, Kiev), 1 (1989), 672.

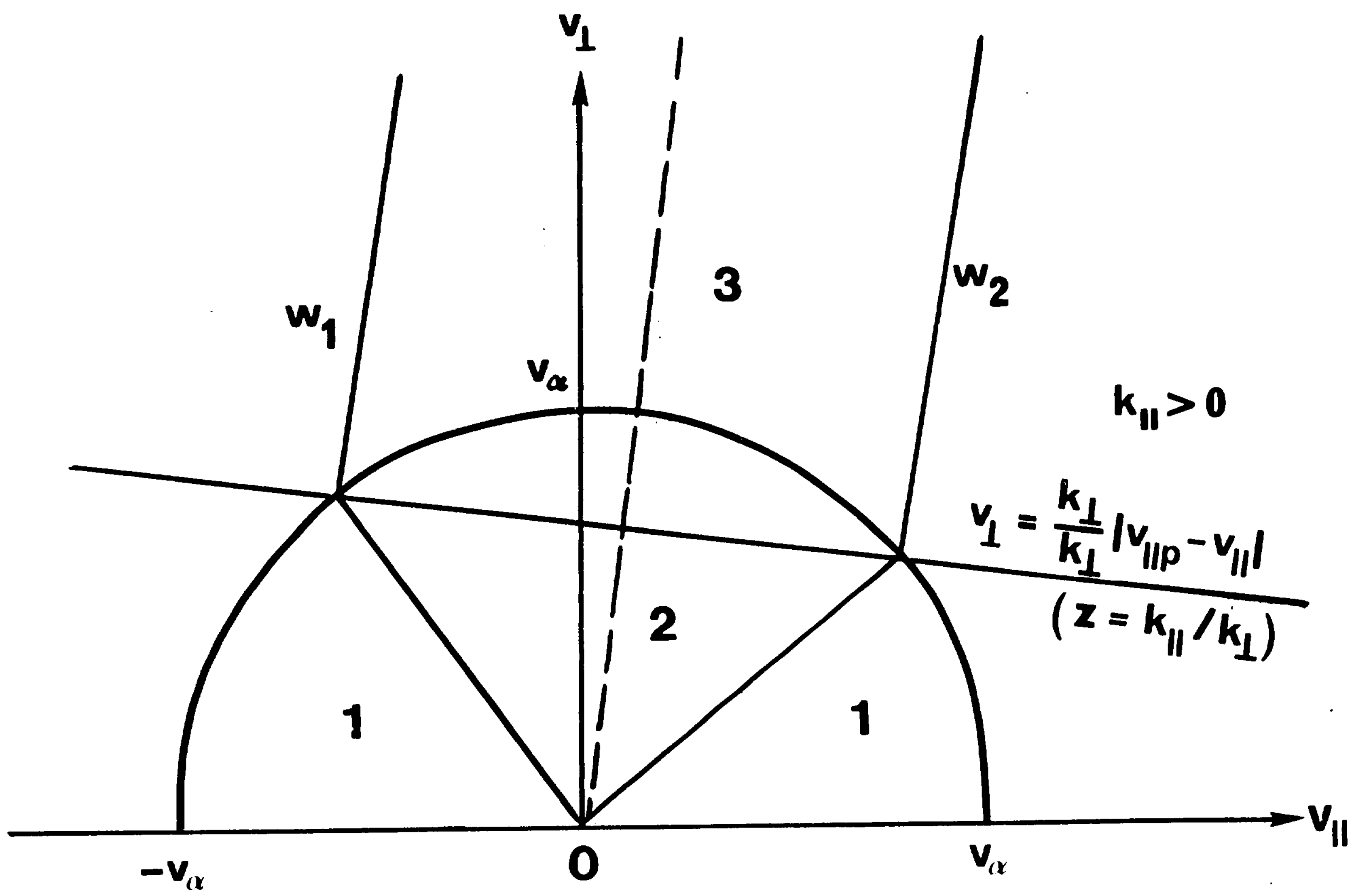
[9] V.S. Belikov, Ya.I. Kolesnichenko, O.A. Silivra, to be presented at the 17th EPS Conf. on Contr. Fusion and Plasma Heating, June 1990,. Amsterdam).

[10] C.F.F. Karney,Phys. Fluids 22 (1979), 2188.

[11] C.F. Kennel, F. Engelmann, Phys. Fluids 9 (1966), 2377.

[12] C.T. Dum, T.H. Dupree, Phys. Fluids 13 (1970), 2064.

[13] M. Lampe et al, Phys. Fluids 15 (1972), 662.



$v_\perp$
w_1
w_2
3
v_α
$k_\parallel > 0$
$v_\perp = \frac{k_\perp}{k_\perp} |v_{\parallel p} - v_\parallel|$
$(z = k_\parallel / k_\perp)$
2
1
1
$v_\parallel$
$-v_\alpha$
0
v_α

Fig. 1.