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Lepton Flavour Symmetry and the Neutrino Magnetic Moment

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Abstract

With the standard model gauge group and the three standard left-handed Weyl neutrinos, two minimal scenarios are investigated where an arbitrary non-abelian lepton flavour symmetry group G_H is responsible for a light neutrino with a large magnetic moment. In the first case, with scalar fields carrying lepton flavour, some finetuning is necessary to get a small enough neutrino mass for $\mu_\nu = O(10^{-11}\ \mu_B)$. In the second scenario, the introduction of heavy charged gauge singlet fermions with lepton flavour allows for a strictly massless neutrino to one-loop order. In both cases, the interference mechanism for small m_ν and large μ_ν is unique, independently of G_H. In explicit realizations of the two scenarios, the horizontal groups are found to be non-abelian extensions of a Zeldovich-Konopinski-Mahmoud lepton number symmetry. Only a discrete part of G_H is spontaneously broken leading to a light Dirac neutrino with a large magnetic moment.

1 Introduction

The possible variation of the solar neutrino flux in anticorrelation with the solar magnetic activity has revived the question whether a light neutrino ($m_\nu \lesssim 10$ eV) can naturally have a rather big magnetic moment $\mu_\nu = O(10^{-11}\ \mu_B)$. Referring to Ref. [1] for an up-to-date account of the experimental and theoretical situation, we only recall the fundamental theoretical problem here. Since both the mass and the magnetic moment operators change chirality, any mechanism for a big μ_ν tends to produce too big a neutrino mass. Although a more sophisticated treatment of solar dynamics may well lead to a reduction [2] of the naive estimate $\mu_\nu = O(10^{-11}\ \mu_B)$, we adopt the view that the seeming discrepancy between a large μ_ν and a small m_ν calls for an underlying symmetry beyond the standard model.

The original proposal of Voloshin [3] invoked a custodial $SU(2)_\nu$ symmetry for this purpose relating a left-handed gauge doublet neutrino to a left-handed gauge singlet antineutrino. More recently, such a symmetry has been implemented in a more economical way as a horizontal lepton flavour symmetry involving only the three standard gauge doublet Weyl neutrinos [4–9]. Such models exhibit two attractive features in comparison with Voloshin's original idea: there are no sterile singlet neutrinos with tight astrophysical and cosmological constraints [10] and, secondly, the low-energy gauge group need not be extended to account for the custodial neutrino symmetry.

For the present investigation we adopt again the standard model gauge group and an arbitrary non-abelian horizontal lepton flavour symmetry G_H which by definition commutes with the gauge group. The arbitrariness of G_H will be a central theme of our approach. As in the quark sector for the problem of quark masses and weak mixing angles, we have really no clues at all what this flavour symmetry should be like. Rather than looking for the needle in a stack of hay, we accept our ignorance about the underlying dynamics responsible for a possible flavour symmetry and undertake a general discussion independent of a specific group.

The corresponding analysis in the quark sector was performed some time ago for two generations [11]. The main ingredient will again be useful for the present investigation: given only the dimensions of irreducible representations (irreps) and the structure of the Clebsch-Gordan series of Kronecker products, the freedom of basis choice can be used to bring the relevant Clebsch-Gordan coefficients into certain standard forms without making any additional assumptions about the group in question. Since the structure of the Yukawa interaction and of the Higgs potential is governed by Clebsch-Gordan coefficients, a group independent approach becomes possible.

We distinguish two scenarios to enforce a small m_ν together with a big μ_ν at the one-loop level. In all cases, the non-abelian flavour group G_H acts irreducibly on the two neutrino flavours connected by a transition magnetic moment [12]. In the first scenario, only the scalar fields in the one-loop diagrams are affected by G_H, but not the internal fermion which can be identified with the τ lepton. In the second scenario, all the scalar fields are flavour blind so that the internal fermions must carry a non-singlet irrep of G_H. Since we consider only three sequential lepton generations, the internal fermions must be heavy gauge singlets. In both scenarios, the mechanism to achieve a constructive interference for μ_ν together with a destructive one for m_ν *is unique, independently of a specific choice for* G_H. In addition to the general analysis, we discuss one specific example of each scenario in detail. Once the essential part of the Yukawa and Higgs sectors is fixed, we determine the maximal symmetry G_H in each case and check for the completeness of the Lagrangian to guarantee natural predictions. If such a completeness test is ignored [6], we would be hesitant to accept the model as a natural explanation of a light neutrino with a large (transition) magnetic moment.

In Sect. 2, the two scenarios for enhancing μ_ν and suppressing m_ν are discussed in a general manner. With the given fermion content, we consider the relevant fermion bilinears to determine the minimal scalar sector in the two cases. Sect. 3 contains a detailed discussion of the case of lepton flavour carrying scalar fields with the τ lepton as internal fermion (scenario I). In the minimal realization of scenario I we determine the maximal G_H which turns out to be a non-abelian extension of a Zeldovich-Konopinski-Mahmoud (ZKM) lepton number symmetry [13]. A general feature of scenario I is exemplified for the specific realization: $m_\nu = 0$ cannot be achieved in a natural way. In Sect. 4 we consider scenario II where all scalar fields are G_H singlets requiring the presence of heavy charged fermions (gauge singlets). In contrast to the previous case, $m_\nu = 0$ emerges naturally. In the simplest realization of scenario II introduced earlier [5] we find again G_H to be a non-abelian version of a ZKM symmetry. The seeming discrepancy between large μ_ν and small m_ν is understood in a completely natural way in scenario II. Our conclusions are summarized in Sect. 5. App. A contains the general proof that for $m_e \neq m_\mu$ $m_\nu = 0$ cannot be obtained in a natural way in scenario I. Finally, in App. B we demonstrate the uniqueness of the interference mechanism of scenario II for arbitrary G_H.

2 Magnetic Moment vs. Mass

We consider the standard gauge group $SU(2)_L \times U(1)_Y$ with three generations of lepton doublets L_i and charged lepton singlets ℓ_{Ri}. The scalar sector is assumed to allow only massless neutrinos at tree level. In this case, the neutrino magnetic moment μ_ν arises at the one-loop level only from diagrams of the generic type shown in Fig. 1 with charged fermions f and charged scalars Φ. Since Majorana neutrinos cannot possess a magnetic moment, μ_ν is necessarily a transition magnetic moment $\mu_{\nu_e \nu_2}$ with $\nu_2 = \nu_\mu$ or ν_τ. More precisely, the transition is of the form $\nu_{Le} \to (\nu_{L2})^c$ to account for the change of chirality.

Because of this chirality change, the diagram of Fig. 1 also gives rise to a neutrino mass m_ν. If there are no other contributions to m_ν we expect

$$\mu_\nu \sim e\frac{m_\nu}{M^2} = \frac{2m_e m_\nu}{M^2}\mu_B \tag{2.1}$$

where M is the mass of the heavier particle in the loop. With $m_\nu \lesssim 10$ eV and $M > 35$ GeV [14] for a charged scalar boson Φ we would get

$$\mu_\nu \lesssim 10^{-14}\,\mu_B, \tag{2.2}$$

three orders of magnitude too small to be relevant for the solar neutrino problem. If μ_ν is to arise at the one-loop level (cf. Ref. [15] for an alternative scenario), a symmetry mechanism is needed to have $\mu_\nu \sim 10^{-11}\,\mu_B$ and $m_\nu \lesssim 10$ eV at the same time.

In Ref. [5], a simple mechanism was suggested for this purpose. If in addition to the diagram of Fig. 1 there is a similar diagram with f and Φ replaced by their charge conjugates, an appropriate choice of Yukawa couplings will give rise to a destructive interference for m_ν, but a constructive one for μ_ν. Without resorting to finetuning, such an appropriate choice of couplings must be due to a symmetry.

We shall assume that this symmetry commutes with the low-energy gauge group.[1] It is not difficult to convince oneself that the symmetry group G_H must be non-abelian and that the

[1] See Ref. [16] for an alternative approach.

L_i must carry at least a 2-dimensional irrep of G_H. The symmetry is thus a horizontal lepton flavour symmetry version [4-9] of the original proposal of Voloshin [3] where ν_{Le} was related to a right-handed neutral gauge singlet.

With L_i $(i = 1,2)$ a doublet and L_3 a singlet of G_H, the following two minimal scenarios emerge concerning the representation content of the fields participating in the one-loop diagram of Fig. 1.

Scenario I: f is a G_H singlet which requires Φ to be a doublet. f can be identified with the τ lepton implying $\nu_2 = \nu_\mu$.

Scenario II: Φ is a G_H singlet so that f must be a doublet. Since we assume only three sequential generations, the charged fermion f is a gauge singlet.

We shall perform a complete discussion of these two possibilities, but we disregard the more complicated case that both f and Φ are non-singlets of G_H. All fields are assumed to be colour singlets.[2]

2.1 Scenario I

The fermion content in this case is

$$\begin{aligned} &L_i \sim (2,-1,2) && \ell_{Ri} \sim (1,-2,d_R) \\ &f_L \in L_\tau \sim (2,-1,1) && f_R = \tau_R \sim (1,-2,1) \end{aligned} \tag{2.3}$$

indicating the respective representation of $SU(2) \times U(1)_Y \times G_H$ and leaving the G_H assignment d_R of ℓ_{Ri} open for the moment.

The fermion bilinears relevant for the one-loop diagram of Fig. 1 are

$$\bar{L}_i \tau_R \sim (2,-1,2), \qquad \bar{L}_i (L_\tau)^c \sim \begin{cases} (1,2,2) \\ (3,2,2). \end{cases} \tag{2.4}$$

To construct invariant Yukawa couplings we must introduce scalar fields

$$\varphi \sim (2,1,2) \qquad \text{and} \qquad \eta_s \sim (1,-2,2) \qquad \text{or} \qquad \eta_t \sim (3,-2,2). \tag{2.5}$$

In addition to two Higgs doublets φ_i $(i = 1,2)$ with the usual hypercharge $Y = 1$ we need either charged singlets $\eta_{si} = \eta_i^-$ or triplets $\eta_{ti} = (\eta_i^0, \eta_i^-, \eta_i^{--})$. The additional couplings in the triplet case are irrelevant for the problem at hand and we can restrict ourselves to the singlets η_i^- without loss of generality. The minimal Yukawa structure is then of the form

$$\bar{L} \tau_R \varphi, \qquad \bar{L} \tilde{L}_\tau \eta^- \tag{2.6}$$

with $\tilde{L}_\tau = i\sigma_2 (L_\tau)^c$.

We will show explicitly in Sect. 3 that with the structure introduced so far we can always construct the G_H singlet

$$\varepsilon_{ij} \varphi_i \eta_j^- \qquad (\varepsilon_{ij} = -\varepsilon_{ji},\ \varepsilon_{12} = 1) \tag{2.7}$$

[2] Although the introduction of coloured f, Φ seems rather far-fetched for an understanding of μ_ν, models of this type have been considered in the literature [7,8].

which allows for a cubic scalar interaction

$$\tilde{\Phi}_S^\dagger \varepsilon_{ij} \varphi_i \eta_j^- \tag{2.8}$$

with an additional G_H singlet $\Phi_S \sim (2,1,1)$. The antisymmetric coupling (2.8) ensures the destructive interference of the two diagrams in Fig. 2 for m_ν and a corresponding constructive interference for μ_ν after spontaneous symmetry breaking (SSB) with $\langle \Phi_S^0 \rangle \neq 0$.

An exact cancellation for $m_\nu = 0$ requires that the four charged mass eigenstates related to φ_i^+, η_i^+ $(i = 1,2)$ are pairwise degenerate as can be read off from Fig. 2 (see Sect. 3 for more details). It will turn out, however, that even with an arbitrary G_H and imposing CP invariance on the Lagrangian such a pairwise degeneracy requires some finetuning. In other words, already to one loop order $m_\nu = 0$ cannot be obtained naturally in this scenario. Nevertheless, we shall present a specific model in Sect. 3 where m_ν can be expected to be sufficiently small.

2.2 Scenario II

The fermion content is now

$$\begin{array}{ll} L_i \sim (2,-1,2) & \ell_{Ri} \sim (1,-2,d_R) \\ L_3 \sim (2,-1,1) & \ell_{R3} \sim (1,-2,1) \\ f_{Li} \sim (1,-2,2) & f_{Ri} \sim (1,-2,2) \end{array} \tag{2.9}$$

where f_{Li}, f_{Ri} will combine to yield two massive fermions f_1, f_2 which are degenerate at tree level. All scalar fields are assumed to be G_H singlets in this case.

The relevant fermion bilinears for Fig. 1 involving L_i $(i = 1,2)$ are of the generic form

$$\bar{L} f_R \sim (2,-1,1), \qquad \bar{L}(f_L)^c \sim (2,3,1) \tag{2.10}$$

requiring scalar fields

$$\Phi_R = \begin{pmatrix} \Phi_R^+ \\ \Phi_R^0 \end{pmatrix} \sim (2,1,1), \qquad \Phi_L = \begin{pmatrix} \Phi_L^- \\ \Phi_L^{--} \end{pmatrix} \sim (2,-3,1) \tag{2.11}$$

to construct invariant Yukawa couplings. Note that in addition to the scalar field Φ_R with the usual hypercharge $Y = 1$ scenario II needs a scalar gauge doublet Φ_L with $Y = 3$. Similar to scenario I, a mixing between Φ_R^+ and Φ_L^+ is needed to give rise to an interference between the two diagrams shown in Fig. 3. In this case, G_H acts non-trivially on the internal fermion line to achieve the desired interference. In contrast to the previous case, there is an exact cancellation between the two diagrams to yield $m_\nu = 0$ as long as $\langle \Phi_R^0 \rangle = 0$. This is due to the absence of mixing between light and heavy fermions and to the degeneracy of f_1, f_2 at tree level.

In the following two sections we shall discuss the two scenarios in more detail and present a specific example in each case. The strategy will be to investigate the structure of Yukawa couplings without specifying G_H a priori. The analysis will make extensive use of the existence of standard forms for Clebsch-Gordan coefficients [11] which are independent of the underlying group, but depend only on the type of Clebsch-Gordan series for a given product representation.

3 Scalar Fields with Lepton Flavour (Scenario I)

We recapitulate the minimal particle content of scenario I:

fermions $L_i \sim (2,-1,2)$, $\ell_{Ri} \sim (1,-2,d_R)$, $L_\tau \sim (2,-1,1)$, $\tau_R \sim (1,-2,1)$

scalars $\varphi_i \sim (2,1,2)$, $\eta_i^- \sim (1,-2,2)$, $\Phi_S \sim (2,1,1)$.

The bases of the various G_H doublets can always be chosen in such a way [11] that the couplings (2.6) take the special form

$$-\mathcal{L}_Y = h_1(\bar{L}_1\varphi_1 + \bar{L}_2\varphi_2)\tau_R + h_2(\bar{L}_1\eta_1^- + \bar{L}_2\eta_2^-)\tilde{L}_\tau + \dots \tag{3.1}$$

The maximal invariance group of $\mathcal{L}_Y$, which will of course be broken subsequently by further terms in $\mathcal{L}_Y$ and in the Higgs potential $\mathcal{L}_H$, is $U(2)\times U(1)\times U(1)$ with

$$L \to VL, \quad \varphi \to e^{i\alpha}V\varphi, \quad \eta^- \to e^{i\beta}V\eta^-, \quad L_\tau \to e^{i\beta}L_\tau, \quad \tau_R \to e^{-i\alpha}\tau_R, \quad V \in U(2). \tag{3.2}$$

Consequently, $\varepsilon_{ij}\varphi_i\eta_j^-$ transforms as a singlet with

$$\varepsilon_{ij}\varphi_i\eta_j^- \to e^{i(\alpha+\beta)}\det V\varepsilon_{ij}\varphi_i\eta_j^-. \tag{3.3}$$

Postulating the transformation property

$$\Phi_S \to e^{-i(\alpha+\beta)}\det V^*\Phi_S \tag{3.4}$$

for the G_H-singlet field Φ_S, we obtain the desired cubic coupling

$$\mathcal{L}_H^{(3)} = \mu\tilde{\Phi}_S^\dagger\varepsilon_{ij}\varphi_i\eta_j^- \tag{3.5}$$

in accordance with (2.8). The Lagrangians (3.1) and (3.5) give rise to the required interference pattern of the two diagrams of Fig. 2.

For the given scalar sector the interference mechanism (3.5) is unique. Other than the singlet (3.3), the additional irreps in the Kronecker product $2_\varphi \otimes 2_\eta$ depend on G_H. Using the standard forms for Clebsch-Gordan coefficients derived in Ref. [11] (see also App. A), it can be shown that the additional irreps cannot provide the necessary interference between the two diagrams of Fig. 2.

Up to this point, the discussion has been completely general and is valid for any G_H which contains a subgroup of $U(2)\times U(1)\times U(1)$ with the required irreps. The structure (3.1) and (3.5) applies in particular to models with a horizontal $SU(2)_H$ [4,6,9].

Turning now to the charged lepton masses, the most economical choice to generate m_τ is to use again the field Φ_S with a coupling

$$\bar{L}_\tau\tau_R\Phi_S \tag{3.6}$$

which, of course, reduces G_H to a subgroup of $U(2)\times U(1)\times U(1)$. There is a much bigger freedom to choose the Higgs representations which generate m_e, m_μ. We defer a complete discussion to App. A and concentrate here on a specific example which already exhibits the generic features.

In this example we take ℓ_{Ri} to be a G_H doublet ($d_R = 2$) and we assume that the product $\bar{L}\ell_R$ contains at least two singlets which are then necessarily inequivalent:

$$\ell_{Ri} \sim (1,-2,2), \qquad 2_L^* \otimes 2_{\ell_R} = 1 \oplus 1' \oplus \dots \tag{3.7}$$

Clearly, this choice implies that G_H does not contain $SU(2)_H$, the favourite choice of other authors [4,6,9]. With the Clebsch-Gordan decomposition (3.7) we may choose an appropriate basis for the G_H doublet ℓ_R such that the two singlets are of the form [11]

$$\bar{L}_1\ell_{R1} + \bar{L}_2\ell_{R2} \qquad \text{and} \qquad \bar{L}_1\ell_{R1} - \bar{L}_2\ell_{R2}. \tag{3.8}$$

Introducing additional G_H-singlet fields

$$\Phi_I, \Phi_{II} \sim (2,1,1), \tag{3.9}$$

the Yukawa Lagrangian takes the final form $(i,j=1,2)$

$$\begin{aligned} -\mathcal{L}_Y &= g_1\bar{L}_i\ell_{Ri}\Phi_I + g_2(\sigma_3)_{ij}\bar{L}_i\ell_{Rj}\Phi_{II} + g_3\bar{L}_\tau\tau_R\Phi_S + \\ &+ h_1\bar{L}_i\varphi_i\tau_R + h_2\bar{L}_i\eta_i^-\tilde{L}_\tau + h_3\bar{L}_\tau\varepsilon_{ij}\ell_{Ri}\varphi_j + h.c. \end{aligned} \tag{3.10}$$

The following comments are in order:

i) To generate the charged lepton masses, the neutral scalar fields Φ_I^0, Φ_{II}^0, Φ_S^0 develop non-vanishing VEVs. To avoid unnecessary complications due to mixing, the Higgs potential can always be chosen such that $\langle\varphi_i^0\rangle = 0$.

ii) One needs two different singlets Φ_I, Φ_{II} to get $m_e \neq m_\mu$ so that both fermion bilinears (3.8) must appear in $\mathcal{L}_Y$.

iii) In principle, Φ_S could be identified with either Φ_I or Φ_{II}, but for a reason to become apparent later we choose $\Phi_S \neq \Phi_{I,II}$.

iv) Both cases $h_3 = 0$ and $h_3 \neq 0$ will be considered.

The Higgs potential $\mathcal{L}_H = \mathcal{L}_H^{(2)} + \mathcal{L}_H^{(3)} + \mathcal{L}_H^{(4)}$ will contain the usual host of terms of which only the following are interesting for our purposes:

$$\mathcal{L}_H^{(3)} = \mu\tilde{\Phi}_S^\dagger\varepsilon_{ij}\varphi_i\eta_j^- + h.c., \qquad \mathcal{L}_{H,1}^{(4)} = \sum_{A=I,II}\lambda_A(\Phi_S^\dagger\Phi_A)^2 + h.c. \tag{3.11}$$

with $\mathcal{L}_H^{(4)} = \mathcal{L}_{H,1}^{(4)} + \ldots$ The quartic term $\mathcal{L}_{H,1}^{(4)}$ is introduced to avoid Goldstone bosons. In other words, it will serve to break a continuous part of G_H to a discrete subgroup to be determined shortly.

So far, we have not specified G_H except that it must possess doublet irreps and the Clebsch-Gordan series (3.7). In order to make sure that $\mathcal{L}_Y + \mathcal{L}_H$ is indeed the most general Lagrangian invariant under G_H we must now determine this maximal invariance group (always in addition to the gauge group $SU(2)\times U(1)_Y$, of course). The analysis is substantially made easier by the fact [11] that in the standard basis employed in (3.8) and (3.10) the (unitary) representation matrices for all G_H doublets are either diagonal or purely off-diagonal. Using this property, the horizontal invariance group of $\mathcal{L}_Y + \mathcal{L}_H^{(3)} + \mathcal{L}_{H,1}^{(4)}$ is found to be

$$G_H = G_{ZKM} \times \mathbf{Z}_4 \tag{3.12}$$

where G_{ZKM} is a non-abelian extension of a ZKM lepton number group $U(1)_{ZKM}$ [13]. It can be defined by the following composition rules:

Table 1: Representations of the generating elements of $G_{ZKM} \times \mathbf{Z}_4$. $U(1)_\tau$ is a symmetry for $h_3 = 0$ only.

	L	ℓ_R	L_τ	τ_R	Φ_I	Φ_{II}	Φ_S	φ	η
$g(\alpha)$	$\mathrm{diag}\,(e^{i\alpha}, e^{-i\alpha})$		1	1	1	1	1	$\mathrm{diag}\,(e^{i\alpha}, e^{-i\alpha})$	
T	$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$		1	1	1	-1	1	$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	
$\mathbf{Z}_4$	1	-1	$-i$	$-i$	-1	-1	1	i	$-i$
$U(1)_\tau$	1	1	$e^{i\beta}$	$e^{i\beta}$	1	1	1	$e^{-i\beta}$	$e^{i\beta}$

$$
\begin{aligned}
g(\alpha) \in U(1)_{ZKM}, \qquad & g(\alpha + 2\pi n) = g(\alpha) \qquad (n \in \mathbf{Z}), \\
T^2 = g(\pi), \qquad & T \circ g(\alpha) = g(-\alpha) \circ T.
\end{aligned}
\tag{3.13}
$$

A more concrete realization of its generating elements is via its faithful 2-dimensional irrep

$$
g(\alpha) \to \begin{pmatrix} e^{i\alpha} & 0 \\ 0 & e^{-i\alpha} \end{pmatrix}, \qquad T \to \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{3.14}
$$

In table 1 we display how the generating elements of $G_{ZKM} \times \mathbf{Z}_4$ are represented on the various fields of the model. For $h_3 = 0$ an additional $U(1)$ appears which is identified as a τ lepton symmetry $U(1)_\tau$. Since we have chosen the Higgs potential to yield $\langle \varphi_i^0 \rangle = 0$, $U(1)_\tau$ remains unbroken and we have $m_{\nu_\tau} = 0$ to all orders for $h_3 = 0$. Of course, for $h_3 \neq 0$ there is no conserved τ lepton number and the invariance group is given by (3.12).

It is now straightforward to verify that $\mathcal{L}_Y + \mathcal{L}_H^{(3)}$ is indeed the most general Lagrangian invariant with respect to (3.12) guaranteeing stability in higher orders. Furthermore, it is remarkable that the Lagrangian (3.10) automatically exhibits a conserved ZKM lepton number as long as $\langle \varphi_i^0 \rangle = 0$ holds. The transition magnetic moment $\mu_{\nu_e \nu_\mu}$ may thus be viewed as the proper magnetic moment μ_ν of the Dirac neutrino (up to a possible relative phase between ν_e and $\nu_\mu{}^c$)

$$
\nu = \nu_{Le} + (\nu_{L\mu})^c. \tag{3.15}
$$

Before we go on with the discussion of the model we would like to mention the conceivable ways of breaking G_{ZKM} which could arise in a wider context. One possibility is that G_{ZKM} is broken by $\mathcal{L}_H$ to a non-abelian discrete subgroup which would lead to a similar situation with a discrete version [17,18] of a ZKM neutrino. There is also the possibility that $\mathcal{L}_H$ breaks G_{ZKM} completely, but in such a way that m_ν, μ_ν are unaffected at the one-loop level. The magnetic moment in this case is really a transition magnetic moment with the two Majorana neutrinos making up a "quasi-ZKM neutrino" [19].

Coming back to the specific model under consideration, we observe as a crucial advantage of our horizontal group $G_H = G_{ZKM} \times \mathbf{Z}_4$ that only a discrete part is spontaneously broken in $G_H \to U(1)_{ZKM}$ (see table 1). It therefore avoids the usual pitfalls of spontaneously broken local or global G_H where the breaking scale must be large, at least in the TeV range to suppress the effects of flavour changing gauge bosons (local G_H) and usually much higher to suppress the couplings of Goldstone bosons (global G_H). This requirement is difficult to reconcile with a symmetry that should be effective at or below the Fermi scale to suppress m_ν and allow for a large enough μ_ν.

A generic problem of all models of scenario I is the appearance of G_H invariant couplings in $\mathcal{L}_H^{(4)}$ which lead to $m_\nu \neq 0$. As a consequence, it will turn out that in scenario I some amount of finetuning is unavoidable. In App. A we present a general proof that the requirement of non-degenerate e, μ is incompatible with $m_\nu = 0$ at the one-loop level unless some coupling constants are finetuned which are not restricted by the symmetries of the Lagrangian. In the present case, the culprits are terms of the form (assuming CP invariant couplings, cf. App. A)

$$(\Phi_I^\dagger\Phi_{II} + \Phi_{II}^\dagger\Phi_I)(\varphi_1^\dagger\varphi_1 - \varphi_2^\dagger\varphi_2), \qquad (\Phi_I^\dagger\Phi_{II} + \Phi_{II}^\dagger\Phi_I)(\eta_1^+\eta_1^- - \eta_2^+\eta_2^-) \tag{3.16}$$

which are G_H invariant and must therefore be included in $\mathcal{L}_H^{(4)}$. From the Yukawa Lagrangian (3.10) we infer that $m_e \neq m_\mu$ demands

$$\langle\Phi_I^0\rangle^* \cdot \langle\Phi_{II}^0\rangle + \langle\Phi_{II}^0\rangle^* \cdot \langle\Phi_I^0\rangle \neq 0. \tag{3.17}$$

Therefore, the quartic terms (3.16) contribute to the charged φ and η mass matrices unless their coupling constants are finetuned to zero. We will now show that without such a finetuning $m_\nu \neq 0$ is inevitable already at the one loop level.

The charged φ, η mass matrices are of the form

$$\begin{aligned} -\mathcal{L}_m(\varphi,\eta) &= (\varphi_1^-,\eta_2^-)\begin{pmatrix} A+\Delta A & B^* \\ B & C+\Delta C \end{pmatrix}\begin{pmatrix} \varphi_1^+ \\ \eta_2^+ \end{pmatrix} + \\ &\quad + (\varphi_2^-,\eta_1^-)\begin{pmatrix} A-\Delta A & -B^* \\ -B & C-\Delta C \end{pmatrix}\begin{pmatrix} \varphi_2^+ \\ \eta_1^+ \end{pmatrix} \end{aligned} \tag{3.18}$$

where $B = \mu\langle\Phi_S^0\rangle$ and ΔA, ΔC are due to (3.16), (3.17).

Let us first consider $\Delta A = \Delta C = 0$. In this limit, the two mass matrices in (3.18) have the same eigenvalues M_1^2, M_2^2 and we have indeed four pairwise degenerate charged scalar particles. The diagonalizing matrices are simply related by a change of sign of the Cabibbo-like mixing angle $\Theta_{\varphi\eta}$. For the realistic case $m_\tau^2/M_i^2 \ll 1$, one obtains approximately from the diagrams of Fig. 2

$$\begin{aligned} |\mu_\nu| &\simeq \frac{em_\tau}{16\pi^2}|h_1h_2\sin 2\Theta_{\varphi\eta}| \cdot |(\ln\frac{M_1^2}{m_\tau^2} - 1)/M_1^2 - (\ln\frac{M_2^2}{m_\tau^2} - 1)/M_2^2| = \\ &= 1.2\cdot 10^{-9}\mu_B|h_1h_2\sin 2\Theta_{\varphi\eta}|\cdot|(\ln\frac{M_1^2}{m_\tau^2}-1)(100\text{ GeV}/M_1)^2-(\ln\frac{M_2^2}{m_\tau^2}-1)(100\text{ GeV}/M_2)^2|. \end{aligned} \tag{3.19}$$

Thus, $\mu_\nu \sim 10^{-11}\,\mu_B$ can be achieved without difficulties as long as at least one charged scalar mass is of the order of 100 GeV or less.

In the limit $\Delta A = \Delta C = 0$ under consideration, the two diagrams of Fig. 2 cancel exactly to produce $m_\nu = 0$. As is evident from (3.16), this limit cannot be enforced by a horizontal symmetry. For $\Delta A\cdot\Delta C \neq 0$ and anticipating $|\Delta M_i^2|/M_i^2 \ll 1$, one finds four different eigenvalues $M_1^2 \pm \Delta M_1^2$, $M_2^2 \pm \Delta M_2^2$ for the charged scalar bosons to first order in ΔM_i^2. For the neutrino mass one gets approximately

$$m_\nu \simeq \frac{m_\tau}{16\pi^2}|h_1h_2\sin 2\Theta_{\varphi\eta}|\cdot\left|\frac{\Delta M_1^2}{M_1^2} - \frac{\Delta M_2^2}{M_2^2} + \frac{\Delta M_1^2 - \Delta M_2^2}{M_1^2 - M_2^2}\ln\frac{M_2^2}{M_1^2}\right|. \tag{3.20}$$

To get a feeling for the relation between m_ν and μ_ν, let us consider the limit $M_2/M_1 \gg 1$. In this case (compare with Eq. (2.1))

$$|\mu_\nu| \simeq \frac{em_\nu}{|\Delta M_1^2|}\left|\ln\frac{M_1^2}{m_\tau^2}-1\right| \tag{3.21}$$

or

$$|\Delta M_1^2| \lesssim \left|\ln\frac{M_1^2}{m_\tau^2}-1\right|\cdot\frac{10^{-11}\mu_B}{|\mu_\nu|} \simeq 7\ \mathrm{GeV}^2\cdot\frac{10^{-11}\mu_B}{|\mu_\nu|} \tag{3.22}$$

for $m_\nu \lesssim 10$ eV and the numerical value in (3.22) applies for $M_1 = 100$ GeV. Consequently, some finetuning is certainly necessary to get $|\Delta M_i^2|/M_i^2 \ll 1$ for the coexistence of $\mu_\nu \sim 10^{-11}\ \mu_B$ with $m_\nu \lesssim 10$ eV. We emphasize once more (see App. A) that this problem is common to all models with G_H-non-singlet scalar fields [4,6,7,8]. In the case of spontaneously broken local or global parts of G_H the problem is, of course, much more acute because of the large breaking scale which makes the conditions $|\Delta M_i^2| \lesssim O(10\ \mathrm{GeV}^2)$ more and more unnatural.

On the other hand, in the model under discussion it is not completely unnatural to assume that the quantities $|\Delta A|$, $|\Delta C|$ responsible for the mass splitting ΔM_i^2 are substantially smaller than the dominant matrix elements A, $|B|$, C. The reason is that ΔA, ΔC are induced by the VEVs of Φ_I, Φ_{II} related to the light masses m_e, m_μ. In contrast, A, B, C receive contributions from Φ_S which in turn gives rise to m_τ. Moreover, unrestricted bare mass terms appear in A and C. Thus, if one is ready to accept small VEVs of Φ_I, Φ_{II} compared to $\langle\Phi_S^0\rangle$ one is automatically led to $|\Delta M_i^2| \ll M_i^2$.

A final comment concerns the economy of the Higgs sector. It is legitimate to ask why one could not identify Φ_S with either Φ_I or Φ_{II}. It turns out that the only change in G_H is a replacement of $\mathbf{Z}_4$ by $\mathbf{Z}_2$. However, this modification entails the appearance of a second term in $\mathcal{L}_H^{(3)}$ in (3.11) which necessarily breaks the pairwise degeneracy of charged scalars. Since all charged lepton masses are now due to the same fields Φ_I, Φ_{II}, any argument explaining $|\Delta M_i^2| \ll M_i^2$ is tantamount to finetuning.

4 Heavy Fermions with Lepton Flavour (Scenario II)

This section is devoted to a general discussion of the cancellation mechanism due to heavy gauge singlet fermions $f_{L,R}$ with charge -1 described in Sect. 2. With the assumptions of Sect. 2, only the two lepton families in a G_H doublet play a rôle in this scheme. It does not matter for the following discussion if we put L_μ or L_τ into a G_H doublet with L_e.

The gauge singlet fermions f have an explicit mass term

$$-\mathcal{L}_m = m_f(\overline{f_{L1}}f_{R1}+\overline{f_{L2}}f_{R2})+h.c. \tag{4.1}$$

Consequently, the G_H irreps of f_L and f_R must be equivalent. Moreover, because of (4.1) their representation matrices must actually be identical. The scalar fields Φ_R, Φ_L appearing in the diagrams of Fig. 3 are assumed to be G_H singlets as indicated in Eq. (2.11). In group-theoretical notation, this can be expressed as

$$\begin{aligned} 2_L^* \otimes 2_f &= 1_R^* \oplus \dots \\ 2_L^* \otimes 2_f^* &= 1_L^* \oplus \dots \end{aligned} \tag{4.2}$$

The Yukawa Lagrangian responsible for μ_ν has the general form

$$-\mathcal{L}_Y^f = \bar{L}_i \Gamma_{ij} f_{Rj} \Phi_R + \bar{L}_i \Delta_{ij} (f_{Lj})^c \Phi_L + h.c. \tag{4.3}$$

with Φ_R, Φ_L transforming as 1_R, 1_L, respectively under G_H.

We show in App. B that with the assumption (4.2) there is always a basis for the fields in $\mathcal{L}_Y^f$ such that the coupling matrices Γ, Δ have the simple form

$$\Gamma = h_1 \mathbf{1}, \qquad \Delta = h_2 \begin{pmatrix} 0 & 1 \\ \varepsilon & 0 \end{pmatrix} \tag{4.4}$$

with $\varepsilon = \pm 1$. It is straightforward to demonstrate that $\varepsilon = -1$ leads to the desired constructive interference for μ_ν and destructive interference for m_ν whereas $\varepsilon = 1$ does the opposite. Thus, we have arrived at the important result that the Yukawa couplings responsible for the interference mechanism explained in Sect. 2 are unique. We want to emphasize that Eqs. (4.4) are valid for any group fulfilling (4.2) and that no further input concerning G_H has been used in the proof of App. B.

We now turn to the Yukawa sector giving mass to the charged leptons. As for scenario I, this can be done in different ways, but in contrast to scenario I, already the simplest realization allows for the exact result $m_\nu = 0$ at the one-loop level without any finetuning [5]. We shall therefore concentrate on this special case where in complete analogy to Eq. (3.7) we assume that ℓ_R is a two-dimensional irrep of G_H and we have two different G_H-singlet fields Φ_I, Φ_{II} (Eq. (3.9)). Using Eq. (3.8), we can write down the complete Yukawa Lagrangian as

$$-\mathcal{L}_Y = g_1 \bar{L}_i \ell_{Ri} \Phi_I + g_2 (\sigma_3)_{ij} \bar{L}_i \ell_{Rj} \Phi_{II} + h_1 \bar{L}_i f_{Ri} \Phi_R + h_2 \varepsilon_{ij} \bar{L}_i (f_{Lj})^c \Phi_L + h.c. \tag{4.5}$$

neglecting the third generation altogether.[3]

It is useful in this case to determine the maximal invariance group of the model before considering the Higgs potential. For definiteness, we denote as maximal invariance group of a Lagrangian $\mathcal{L}$ the largest symmetry group which is represented faithfully on the space of fields occurring in $\mathcal{L}$. Proceeding as in Sect. 3, the maximal invariance group of $\mathcal{L}_m + \mathcal{L}_Y$ can be written in two isomorphic ways:

$$SU(2)\times U(1)_Y \times G_{ZKM} \times U(1)_\ell \times U(1)_f / (\mathbf{Z}_2^g \times \mathbf{Z}_2^h) \cong SU(2)\times U(1)_Y \times EL \times U(1)_\ell \times U(1)_f / (\mathbf{Z}_2^g \times \mathbf{Z}_2^h). \tag{4.6}$$

The non-abelian extension EL of $U(1)_{ZKM}$ was already defined in Ref. [5]. Its defining relations are identical to those of G_{ZKM} in (3.13) except that the generating element S of EL, which replaces T in Eq. (3.13), fulfils

$$S^2 = e. \tag{4.7}$$

The horizontal $U(1)$ factors are represented as

$$\begin{array}{lll} U(1)_\ell: & \ell_R \to e^{i\gamma} \ell_R, & \Phi_{I,II} \to e^{-i\gamma} \Phi_{I,II} \\ U(1)_f: & f_{L,R} \to e^{i\delta} f_{L,R}, & \Phi_R \to e^{-i\delta} \Phi_R,\ \Phi_L \to e^{i\delta} \Phi_L \end{array} \qquad \gamma, \delta \in \mathbf{R} \tag{4.8}$$

with all other fields transforming trivially. To avoid double-counting of the symmetries of $\mathcal{L}_m + \mathcal{L}_Y$ and in accordance with the definition of the maximal invariance group, two $\mathbf{Z}_2$ groups were

[3] We recall that 2 stands for either μ or τ.

Table 2: Representations of the generating elements of EL and G_{ZKM} for the model of scenario II ($\mathcal{L}_m + \mathcal{L}_Y$ in Eqs. (4.1) and (4.5)).

	L ℓ_R f	Φ_I	Φ_{II}	Φ_R	Φ_L
$g(\alpha)$	diag $(e^{i\alpha}, e^{-i\alpha})$	1	1	1	1
S	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	1	−1	1	−1
T	$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	1	−1	1	1

factored out in (4.6). They are defined by their respective generating elements as

$$\begin{aligned} Z_2^g: &\quad e^{2\pi i T_3} \circ g_Y(\pi) = e^{2\pi i Q} \\ Z_2^h: &\quad g_Y(\pi) \circ g(\pi) \circ g_\ell(\pi) \circ g_f(\pi) \end{aligned} \tag{4.9}$$

where $g(\alpha), g_Y(\beta), g_\ell(\gamma), g_f(\delta)$ denote elements of $U(1)_{ZKM}$, $U(1)_Y$, $U(1)_\ell$, $U(1)_f$, respectively, T_3 is the usual (diagonal) generator of $SU(2)$ and Q is the charge operator in units of e. Thus, Z_2^g involves a pure global gauge transformation whereas Z_2^h combines a hypercharge with a horizontal transformation. Factoring out $Z_2^g \times Z_2^h$ in the maximal invariance group (4.6) is equivalent to the statement that both Z_2^g [20] and Z_2^h are realized trivially on all fields in $\mathcal{L}_m + \mathcal{L}_Y$.

We have gone to some length in discussing the invariance group of our model to clarify a possible misconception appearing in the literature [18]. The isomorphism claimed in Eq. (4.6) is given explicitly by

$$S \leftrightarrow T \circ g_Y(\frac{\pi}{2}) \circ g(\frac{3\pi}{2}) \circ g_\ell(\frac{\pi}{2}) \circ g_f(\frac{\pi}{2}) \tag{4.10}$$

in the previous notation. This relation defines an isomorphism because Z_2^h in (4.9) is represented trivially. It is therefore completely equivalent to use either EL and S or G_{ZKM} and the Voloshin symmetry [3] T, as long as $\mathcal{L}_m + \mathcal{L}_Y$ is concerned. Thus, contrary to what seems to be suggested in Ref. [18], T is of course contained in the invariance group given in Ref. [5]. The representations of the generating elements of EL and G_{ZKM} can be found in table 2.

As in Sect. 3, SSB is achieved by VEVs

$$v_I = \langle \Phi_I^0 \rangle, \qquad v_{II} = \langle \Phi_{II}^0 \rangle. \tag{4.11}$$

Again, both v_I and v_{II} must be non-zero to get $m_e \neq m_2$ ($2 = \mu$ or τ). If $\langle \Phi_R^0 \rangle$ were also non-vanishing, ℓ_R and f_R would mix already at tree level leading to $m_\nu \neq 0$ at one-loop order. To avoid finetuning, we assume $\langle \Phi_R^0 \rangle = 0$ which can always be enforced by a large enough mass term for Φ_R in the Higgs potential.

To generate a neutrino magnetic moment we need mixing between Φ_R^+ and Φ_L^+ (see Fig. 3). The gauge structure of the model allows for two different kinds of mixing terms in $\mathcal{L}_H$:

$$V_1 = \lambda_1 \tilde{\Phi}_I^\dagger \Phi_L \tilde{\Phi}_{II}^\dagger \Phi_R + \lambda_2 \tilde{\Phi}_{II}^\dagger \Phi_L \tilde{\Phi}_I^\dagger \Phi_R + \lambda_3 \tilde{\Phi}_I^\dagger \Phi_{II} \tilde{\Phi}_R^\dagger \Phi_L + h.c. \tag{4.12}$$

leading to

$$(\lambda_1 + \lambda_2) v_I v_{II} \Phi_L^- \Phi_R^+ + h.c. \tag{4.13}$$

or

$$V_2 = \lambda_4 \tilde{\Phi}_I^\dagger \Phi_L \tilde{\Phi}_I^\dagger \Phi_R + \lambda_5 \tilde{\Phi}_{II}^\dagger \Phi_L \tilde{\Phi}_{II}^\dagger \Phi_R + h.c. \tag{4.14}$$

giving

$$(\lambda_4 v_I^2 + \lambda_5 v_{II}^2) \Phi_L^- \Phi_R^+ + h.c. \tag{4.15}$$

The last term in (4.12) is added for completeness only to enumerate all quartic Higgs couplings with four different fields. Both V_1 and V_2 leave $U(1)_f$ unbroken but they break $U(1)_\ell$ explicitly to a $\mathbf{Z}_2$ which is, however, already contained in the remainder of the symmetry group (4.6). This explicit breaking of $U(1)_\ell$ prevents the appearance of a Goldstone boson through SSB via (4.11).

What other terms can we have in the Higgs potential? In order to prevent mixing between ℓ_R and f_R in the Lagrangian, we demand that at least a discrete subgroup

$$\mathbf{Z}_2^f : \quad (f_R, f_L, \Phi_R, \Phi_L) \to -(f_R, f_L, \Phi_R, \Phi_L) \tag{4.16}$$

of $U(1)_f$ remains unbroken. Then, the gauge structure allows for only two more terms W_1, W_2 in addition to V_1, V_2 which can reduce the symmetry group (4.6):

$$\begin{aligned} W_1 &= \lambda_6 \Phi_I^\dagger \Phi_R \Phi_I^\dagger \Phi_R + \lambda_7 \Phi_{II}^\dagger \Phi_R \Phi_{II}^\dagger \Phi_R + h.c. \\ W_2 &= \lambda_8 \Phi_I^\dagger \Phi_R \Phi_{II}^\dagger \Phi_R + h.c. \end{aligned} \tag{4.17}$$

All other gauge invariant Higgs couplings conserve (4.6).

It is important to realize that not all of V_1, V_2, W_1, W_2 can appear in the Higgs potential at the same time. A straightforward analysis shows that $V_1 + V_2$ or $W_1 + W_2$ would break the horizontal part of (4.6) to an abelian subgroup. In this case, the basic interference mechanism of Sect. 2 would not be guaranteed by a symmetry anymore because $\mathcal{L}_m + \mathcal{L}_Y$ would not be the most general Lagrangian with such an abelian symmetry. Keeping in mind that we need at least either V_1 or V_2 to induce $\Phi_R - \Phi_L$ mixing, we arrive at six essentially different possibilities for $\mathcal{L}_H$ disregarding the trivial additional terms invariant under (4.6):

$$V_1, \quad V_1 + W_1, \quad V_1 + W_2, \quad V_2, \quad V_2 + W_1, \quad V_2 + W_2. \tag{4.18}$$

For the main purpose of guaranteeing the interference mechanism of Sect. 2 all six cases are equally good. It may be useful nevertheless to include a short discussion of their differences. If W_1 or W_2 appears in $\mathcal{L}_H$, $U(1)_f$ is broken to $\mathbf{Z}_2^f$ (Eq. (4.16)). Otherwise, $U(1)_f$ remains unbroken. This suggests the notation

$$G_f = \begin{cases} U(1)_f & W_i \notin \mathcal{L}_H \quad (i = 1,2) \\ \mathbf{Z}_2^f & W_1 \text{ or } W_2 \in \mathcal{L}_H. \end{cases} \tag{4.19}$$

The specific model of Ref. [5] corresponds to either $V_1 \in \mathcal{L}_H$ or $V_1 + W_1 \in \mathcal{L}_H$. In this case, T is explicitly broken by $\mathcal{L}_H$ and the maximal invariance group of $\mathcal{L}_m + \mathcal{L}_Y + \mathcal{L}_H$ is determined as

$$SU(2) \times U(1)_Y \times EL \times G_f / \mathbf{Z}_2^g. \tag{4.20}$$

As already remarked, the remainder $\mathbf{Z}_2^\ell$ of $U(1)_\ell$ is already contained in (4.20) since it can be traded for $\mathbf{Z}_2^h$ in (4.6). It is important, but straightforward to verify [5] that $\mathcal{L}_m + \mathcal{L}_Y + \mathcal{L}_H$ is indeed the most general Lagrangian (with the given fields) invariant under (4.20).

Similarly, the cases $V_2 \in \mathcal{L}_H$ and $V_2 + W_1 \in \mathcal{L}_H$ are characterized by the maximal symmetry group

$$SU(2) \times U(1)_Y \times G_{ZKM} \times G_f/\mathbf{Z}_2^g. \tag{4.21}$$

Finally, the cases $V_1 + W_2 \in \mathcal{L}_H$ or $V_2 + W_2 \in \mathcal{L}_H$ correspond to an invariance group of the form (4.20) where EL is replaced by still another non-abelian extension EL' of $U(1)_{ZKM}$ with $S'^4 = e$ instead of (4.7).

Independently of the specific form of $\mathcal{L}_H$, $m_\nu = 0$ is an exact relation at the one-loop level without any finetuning. This is in sharp contrast to models of scenario I. The exact cancellation of the two diagrams in Fig. 3 for m_ν can be attributed [18] to the fact that T remains operative in Fig. 3. Interestingly enough, this is valid even for the model of Ref. [5] where the invariance group (4.20) does not even contain T. The reason is that the mixing term (4.13) is T invariant nevertheless. The Yukawa couplings are T invariant in any case because of (4.6).

We recall from Ref. [5] that $\mu_\nu = O(10^{-11}\ \mu_B)$ can easily be achieved if m_f and the charged Higgs masses appearing in the diagrams of Fig. 3 are all of order M_W, the natural scale of the model. A non-zero m_ν appears first at the two-loop level. Both the contributions due to scalar exchange only [5] and an additional two-loop contribution involving both W and Higgs exchange [21] are suppressed by factors

$$\frac{m_2^2 - m_e^2}{M_H^2} \sim \frac{m_2^2 - m_e^2}{M_W^2}. \tag{4.22}$$

An order-of-magnitude estimate [5] leads to $m_\nu \lesssim 1$ eV even for $\nu_2 = \nu_\tau$ and to a correspondingly smaller value for $\nu_2 = \nu_\mu$. As in the example of scenario I discussed in Sect. 3, we have again a light Dirac neutrino with big μ_ν because the symmetry group of the vacuum

$$U(1)_{em} \times U(1)_{ZKM} \times G_f \tag{4.23}$$

contains a conserved lepton number.

Finally, as in Sect. 3 one could again be tempted to reduce the number of scalar multiplets by identifying Φ_R with Φ_I or Φ_{II}. It is easy to check that such an identification would introduce additional terms in $\mathcal{L}_m + \mathcal{L}_Y$ destroying the cancellation mechanism for m_ν described in Sect. 2.

5 Summary and Conclusions

We have discussed possibilities of reconciling a large magnetic moment $\mu_\nu = O(10^{-11}\ \mu_B)$ with a small neutrino mass $m_\nu \lesssim 10$ eV in a natural way. Our analysis was based on the standard model gauge group $SU(2) \times U(1)_Y$ with neutrinos appearing only in the left-handed gauge doublets of the three standard lepton families. The neutrinos are massless at tree level. Consequently, neutrino masses are calculable quantities arising at one-loop or higher orders in perturbation theory. Even then, there is usually a clash between the two requirements of a large magnetic moment and a small neutrino mass. If the magnetic moment is to arise to one-loop order, there must at least be two amplitudes interfering constructively for μ_ν, but destructively for m_ν. Such an interference can only be natural if due to a symmetry.

The main assumption of this paper is that this symmetry is a horizontal lepton flavour symmetry. We have argued that G_H must be non-abelian, but we have kept G_H arbitrary otherwise for the general discussion. We have confined the analysis to the case where two of the left-handed lepton gauge doublets L_i $(i = 1, 2)$ form a G_H doublet whereas L_3 is a singlet. Consequently, the magnetic moment arising at the one-loop level induces transitions of the type $\nu_{L1} \leftrightarrow (\nu_{L2})^c$.

Two minimal scenarios were distinguished. In scenario I, the internal fermion f in the general one-loop diagram of Fig. 1 is a G_H singlet and can be identified with the τ lepton. A simple analysis of fermion bilinears revealed that two kinds of G_H-doublet scalar fields are needed in this case, namely gauge doublets and gauge singlets (or triplets). The necessary interference mechanism for the diagrams of Fig. 2 turned out to be unique, independently of G_H. To get a complete cancellation for m_ν, the charged scalars participating in the diagrams of Fig. 2 would have to be pairwise degenerate. We presented a general proof (Sect. 3 and App. A) that this pairwise degeneracy is incompatible with $m_e \neq m_\mu$ in a natural way. Therefore, for realistic m_e, m_μ some finetuning is unavoidable to ensure $m_\nu \lesssim 10$ eV.

In the simplest version of scenario I G_H-singlet scalar fields generate the charged lepton masses. To obtain $\mu_\nu = O(10^{-11}\, \mu_B)$, at least one charged scalar mass must be of $O(100\,\mathrm{GeV})$ or less. The necessary finetuning is expressed by the condition $\Delta M^2 \lesssim 10\,\mathrm{GeV}^2$ for the mass splitting of the charged scalars.

In scenario II, the rôle of lepton flavour carrying fields in the generic loop diagram of Fig. 1 is attributed to heavy charged gauge singlet fermions $f_{L,R}$ (G_H doublets). All scalar fields are G_H singlets in this case. In particular, scenario II requires gauge doublet scalar fields with both $Y = 1$ and $Y = -3$ for the diagrams of Fig. 3. The interference mechanism encoded in the Yukawa couplings is again unique (App. B). Analogous to the special example of scenario I, we concentrated again on the simplest version [5] with G_H-singlet scalar fields being responsible for the charged lepton masses. In contrast to scenario I, however, $m_\nu = 0$ emerges naturally to one-loop order because SSB does not disturb the cancellation mechanism. At the same time, a magnetic moment $\mu_\nu = O(10^{-11}\, \mu_B)$ is naturally obtained if the charged scalar masses and the mass of the charged fermion f are all of the same order of magnitude as M_W.

In both examples of scenarios I and II, the maximal (horizontal) invariance groups were found to be non-abelian extensions of a ZKM lepton number symmetry $U(1)_{ZKM}$. Thus, the two Weyl neutrinos connected by the magnetic moment transition actually coalesce into a Dirac neutrino with a proper magnetic moment μ_ν in both cases. As also noted by other authors [8,18], it is not necessary to have the full $SU(2)_\nu$ [3] to forbid the mass term

$$\frac{1}{2}\nu_{Li}^T C^{-1} m_{ij} \nu_{Lj} \qquad (i,j = 1,2) \tag{5.1}$$

while allowing the magnetic moment interaction

$$\frac{1}{4}\nu_{Li}^T C^{-1} \sigma_{\mu\nu} \mu_{ij} \nu_{Lj} F^{\mu\nu} \qquad (\mu_{ij} = -\mu_{ji}). \tag{5.2}$$

In both models considered in Sects. 3 and 4, the conserved ZKM lepton number guarantees $m_{11} = m_{22} = 0$ to all orders, whereas a single discrete element T is responsible for $m_{12} = m_{21} = 0$. To allow for a splitting of the charged lepton masses, T must be broken leading to $m_{12} = m_{21} \neq 0$ eventually. In the model of scenario I T is broken spontaneously, while it may be broken explicitly by the Higgs potential in the example [5] of scenario II. Interestingly enough, it is in the latter case that one obtains $m_\nu = 0$ at the one-loop level because the explicit breaking of T becomes effective only in higher orders.[4] In both models considered, only a discrete part of G_H is spontaneously broken. Consequently, there are neither horizontal gauge bosons nor Goldstone bosons to worry about and the breaking scale of G_H is naturally identified with the Fermi scale.

A horizontal lepton flavour symmetry provides a natural explanation for a light neutrino with a large magnetic moment. With scalar fields carrying lepton flavour, some finetuning is needed to obtain a small enough neutrino mass for $\mu_\nu = O(10^{-11}\ \mu_B)$. In contrast, the introduction of heavy charged gauge singlet fermions with non-trivial lepton flavour allows naturally for a strictly massless neutrino at the one-loop level.

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[4]Note that a horizontal symmetry can never yield $m_\nu = 0$ to all orders if $\mu_\nu \neq 0$ [17].

App. A: One-Loop Contributions to m_ν in Scenario I

In Sect. 3 we found in a special model that $m_\nu = 0$ could not be enforced naturally at the one-loop level. We will now show that this is actually a general feature of scenario I.

Our strategy will be to investigate the conditions for having $m_\nu = 0$ naturally to one-loop order and to show that these conditions would imply $m_e = m_\mu$. In addition to the gauge and horizontal symmetries, we will also assume a CP invariant $\mathcal{L}_Y + \mathcal{L}_H$. Clearly, if $m_\nu = 0$ cannot be obtained naturally in a theory with CP invariant couplings, it is a fortiori impossible if CP violating couplings are added. We can restrict the discussion to the standard CP transformations diagonal in generation space (both for fermions and scalars). Since we consider all possible G_H, a generalized CP transformation [22] can be shown to be equivalent to the standard one for the following analysis.

The analysis proceeds by investigating the Kronecker products $\bar{L}\ell_R$ for the possible G_H assignments for ℓ_R. Recall that $L_i\ (i = 1,2)$ must transform as a G_H doublet for the interference mechanism of Sect. 2 to be operative. We discuss first the cases where the ℓ_{Ri} make up a doublet 2_{ℓ_R} of G_H and consider the reducible case $\ell_{Ri} \sim 1 \oplus 1'$ at the end. As in all of this paper, extensive use will be made of standard forms of Clebsch-Gordan coefficients [11].

1. $2_L^* \otimes 2_{\ell_R} = 4$

   The invariant Yukawa interaction involves a quartet scalar field $\Phi \sim (2,1,4)$. With the matrix notation

$$\Phi = \begin{pmatrix} \Phi_1 & \Phi_2 \\ \Phi_3 & \Phi_4 \end{pmatrix}, \qquad \Phi \xrightarrow{G_H} V_L \Phi V_R^\dagger \tag{A.1}$$

   the Yukawa coupling can be written in the form [11]

$$\bar{L}_i \Phi_{ij} \ell_{Rj} + h.c., \qquad L \xrightarrow{G_H} V_L L, \qquad \ell_R \xrightarrow{G_H} V_R \ell_R. \tag{A.2}$$

   Recalling the scalar fields $\varphi \sim (2,1,2)$ and $\eta^- \sim (1,-2,2)$ of Sect. 3, we obtain a G_H invariant quartic term

$$(\varphi^\dagger \vec{\tau} \varphi) \cdot \mathrm{tr}(\Phi^\dagger \vec{\tau} \Phi) \tag{A.3}$$

   where the Pauli matrices act on the representation indices. There is a similar term with φ replaced by η^-. The VEVs $v_i = \langle \Phi_i^0 \rangle$ will contribute to both the charged φ and η mass matrices and the charged lepton masses. Comparing (A.3) with Eq. (3.18), we find that pairwise degeneracy of the charged φ, η mass eigenstates requires

$$|v_1|^2 + |v_2|^2 = |v_3|^2 + |v_4|^2, \qquad v_1^* v_3 + v_2^* v_4 = 0. \tag{A.4}$$

   On the other hand, (A.2) gives rise to a charged lepton mass matrix

$$M_\ell \sim \begin{pmatrix} v_1 & v_2 \\ v_3 & v_4 \end{pmatrix} \tag{A.5}$$

   which is proportional to a unitary matrix in view of (A.4) implying $m_e = m_\mu$. Thus, with a single scalar quartet field $m_\nu = 0$ and $m_e \neq m_\mu$ are incompatible, unless the coupling constant in front of (A.3) is finetuned to zero.

2. $2_L^* \otimes 2_{\ell_R} = 1 \oplus 3$

With a G_H-singlet scalar field Φ and a G_H triplet χ_i, the Yukawa couplings can be written in the standard form [11]

$$g_s \bar{L}_i \ell_{Ri} \Phi + g_t (\bar{L} \vec{\tau} \ell_R) \cdot \vec{\chi} + h.c. \tag{A.6}$$

with real g_s, g_t assuming a CP invariant $\mathcal{L}_Y$. Note that (A.6) can always be obtained by a suitable choice of bases, irrespective of whether G_H contains $SU(2)$ or not.

In this case, we can write down G_H invariant quartic terms

$$\lambda_1 (\varphi^\dagger \vec{\tau} \varphi) \cdot (\Phi^\dagger \vec{\chi}) + i\lambda_2 \varphi^\dagger \tau_i \varphi \varepsilon_{ijk} \chi_j^\dagger \chi_k + h.c., \qquad \lambda_1, \lambda_2 \in \mathbf{R}. \tag{A.7}$$

From now on we suppress the recurring statement that similar terms exist with φ replaced by η. Without finetuning, pairwise degeneracy of the charged scalars requires in this case

$$\begin{aligned} &\varepsilon_{ijk} v_j^* v_k = 0 \qquad && v_s = \langle \Phi^0 \rangle \\ &v_s^* v_i + v_s v_i^* = 0 \qquad && v_i = \langle \chi_i^0 \rangle . \end{aligned} \tag{A.8}$$

The fermion mass matrix due to (A.6) is

$$M_\ell = \begin{pmatrix} g_s v_s + g_t v_3 & g_t (v_1 - i v_2) \\ g_t (v_1 + i v_2) & g_s v_s - g_t v_3 \end{pmatrix}. \tag{A.9}$$

A straightforward application of (A.8) leads once again to the conclusion that M_ℓ is proportional to a unitary matrix and, consequently, $m_e = m_\mu$.

3. $2_L^* \otimes 2_{\ell_R} = 2 \oplus 2'$

With an appropriate basis choice for the G_H doublets Φ_i, χ_i the Yukawa Lagrangian assumes the standard form [11]

$$g_1 (\bar{L}_1 \ell_{R1} \Phi_1 + \bar{L}_2 \ell_{R2} \Phi_2) + g_2 (\bar{L}_1 \ell_{R2} \chi_1 + \bar{L}_2 \ell_{R1} \chi_2) + h.c., \qquad g_1, g_2 \in \mathbf{R}. \tag{A.10}$$

The invariant Higgs potential $\mathcal{L}_H^{(4)}$ includes terms of the form

$$\begin{aligned} &(\varphi_1^\dagger \varphi_1 - \varphi_2^\dagger \varphi_2)(\Phi_1^\dagger \Phi_1 - \Phi_2^\dagger \Phi_2) \\ &(\varphi_1^\dagger \varphi_1 - \varphi_2^\dagger \varphi_2)(\chi_1^\dagger \chi_1 - \chi_2^\dagger \chi_2) \\ &\Phi_1^\dagger \chi_2 \varphi_2^\dagger \varphi_1 + \Phi_2^\dagger \chi_1 \varphi_1^\dagger \varphi_2 + h.c. \end{aligned} \tag{A.11}$$

and the fermion mass matrix is given by

$$M_\ell = \begin{pmatrix} g_1 v_1 & g_2 w_1 \\ g_2 w_2 & g_1 v_2 \end{pmatrix}, \qquad v_i = \langle \Phi_i^0 \rangle, \qquad w_i = \langle \chi_i^0 \rangle. \tag{A.12}$$

Without finetuning, pairwise degeneracy of the charged scalars requires

$$|v_1| = |v_2|, \qquad |w_1| = |w_2|, \qquad v_1^* w_2 + v_2 w_1^* = 0 \tag{A.13}$$

because of (A.11) implying again $m_e = m_\mu$ in view of (A.12).

4. $2^*_L \otimes 2_{\ell_R} = 1 \oplus 1' \oplus 1'' \oplus 1'''$

   This case was already discussed in Sect. 3 (see Eq. (3.7)).

5. $2^*_L \otimes 2_{\ell_R} = 1 \oplus 1' \oplus 2$

   In view of the previous case, it remains to analyse the case where a scalar G_H singlet Φ and a G_H doublet χ_i participate in the Yukawa interaction

$$g_s \bar{L}_i \ell_{Ri} \Phi + g_d(\bar{L}_1 \ell_{R2} \chi_1 + \bar{L}_2 \ell_{R1} \chi_2) + h.c., \qquad g_s, g_d \in \mathbf{R} \tag{A.14}$$

   employing a standard basis choice [11]. The quartic invariants of interest are

$$\begin{aligned} &(\varphi_1^\dagger \varphi_1 - \varphi_2^\dagger \varphi_2)(\chi_1^\dagger \chi_1 - \chi_2^\dagger \chi_2) \\ &\Phi^\dagger(\chi_1 \varphi_1^\dagger \varphi_2 + \chi_2 \varphi_2^\dagger \varphi_1) + h.c. \end{aligned} \tag{A.15}$$

   requiring ($v = \langle \Phi^0 \rangle$, $w_i = \langle \chi_i^0 \rangle$)

$$|w_1| = |w_2|, \qquad vw_1^* + v^* w_2 = 0 \tag{A.16}$$

   for $m_\nu = 0$. As in all previous cases, the fermion mass matrix

$$M_\ell = \begin{pmatrix} g_s v & g_d w_1 \\ g_d w_2 & g_s v \end{pmatrix} \tag{A.17}$$

   yields $m_e = m_\mu$ because of (A.16).

6. $2^*_L \otimes (1_{\ell_R} \oplus 1'_{\ell_R}) = 2 \oplus 2'$

   For a reducible G_H representation ℓ_R the Yukawa interaction is given by

$$g_1 \bar{L}_i \ell_{R1} \Phi_i + g_2 \bar{L}_i \ell_{R2} \chi_i + h.c., \qquad g_1, g_2 \in \mathbf{R} \tag{A.18}$$

   with G_H doublets Φ_i, χ_i. The quartic invariants of relevance are in this case

$$(\varphi^\dagger \vec{\tau} \varphi) \cdot (\Phi^\dagger \vec{\tau} \Phi), \qquad (\varphi^\dagger \vec{\tau} \varphi) \cdot (\chi^\dagger \vec{\tau} \chi). \tag{A.19}$$

   To ensure $m_\nu = 0$ in a natural way we must have

$$|v_1| = |v_2|, \qquad v_1 v_2^* = 0 \Longrightarrow v_1 = v_2 = 0, \qquad v_i = \langle \Phi_i^0 \rangle \tag{A.20}$$

   and likewise for $\langle \chi_i^0 \rangle$. Consequently, the Yukawa couplings (A.18) would not contribute at all to the lepton mass matrix leading to $m_e = m_\mu = 0$.

This concludes the discussion of all possible Yukawa couplings relevant for the charged lepton masses. The analysis was based on the assumption that at the one-loop level $m_\nu = 0$ should hold for arbitrary values of coupling constants which are not constrained by the gauge and global symmetries (naturalness criterion). On the other hand, no assumption had to be made concerning the VEVs of the various scalar fields. Even for an ingeniously constructed Higgs potential yielding the required VEVs to ensure $m_\nu = 0$ one cannot avoid $m_e = m_\mu$. It was shown in Ref. [17] that a horizontal symmetry enforcing $m_\nu = 0$ with $\mu_\nu \neq 0$ will always lead to degenerate electron and muon. The preceding arguments show that this general incompatibility between $m_\nu = 0$ and $m_e \neq m_\mu$ already exists at the one-loop level in scenario I.

App. B: The Yukawa Lagrangian $\mathcal{L}_Y^f$ (Scenario II)

In the following we will show that under assumption (4.2) there is always a basis where the 2×2 coupling matrices Γ, Δ of $\mathcal{L}_Y^f$ (Eq. (4.3)) have the form

$$\Gamma = h_1 \mathbf{1}, \qquad \Delta = h_2 \begin{pmatrix} 0 & 1 \\ \varepsilon & 0 \end{pmatrix} \tag{B.1}$$

with $\varepsilon = \pm 1$.

We denote the irreps of G_H in the following way:

$$2_L : e^{i\alpha} U, \quad 2_f : e^{i\beta} V, \quad 1_R : e^{i\varphi_R}, \quad 1_L : e^{i\varphi_L} \text{ with } U, V \in SU(2). \tag{B.2}$$

Then invariance of the Lagrangian (4.3) under G_H leads to the conditions

$$U^\dagger \Gamma V e^{i(-\alpha+\beta+\varphi_R)} = \Gamma, \qquad U^\dagger \Delta V^* e^{i(-\alpha-\beta+\varphi_L)} = \Delta. \tag{B.3}$$

One of the two coupling matrices can be disposed of by using a result of Ref. [11]: If a tensor product of two two-dimensional irreps contains a singlet ($2 \otimes 2' = 1 \oplus \ldots$), one can always find a basis where the Clebsch-Gordan coefficients pertaining to the singlet form a 2×2 unit matrix apart from a factor. Thus, without loss of generality we can choose $\Gamma = h_1 \mathbf{1}$ and therefore we obtain

$$V = U e^{i(\alpha-\beta-\varphi_R)}. \tag{B.4}$$

In the next step we use this relation to derive the invariance condition for Δ:

$$U^\dagger \Delta U^* e^{i\varphi} = \Delta \qquad \text{with } \varphi = -2\alpha + \varphi_R + \varphi_L. \tag{B.5}$$

Multiplying Eq. (B.5) with its hermitian conjugate we get

$$U^\dagger \Delta \Delta^\dagger U = \Delta \Delta^\dagger \tag{B.6}$$

which is valid for all matrices U of the irrep 2_L. Therefore, from Schur's lemma we deduce that

$$\Delta = h_2 D \quad \text{with } D \in U(2). \tag{B.7}$$

We still have the freedom to perform basis transformations of the kind

$$L = W L', \qquad f_{R,L} = W f'_{R,L} \tag{B.8}$$

with unitary W leaving Γ unchanged. Such a transformation changes D to $W^\dagger D W^*$. Observing that for $W \in SU(2)$ one can write

$$W^* = \tau^\dagger W \tau \quad \text{with} \quad \tau = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{B.9}$$

we get

$$W^\dagger D W^* = W^\dagger D \tau^\dagger W \tau = \begin{pmatrix} e^{i\theta_1} & 0 \\ 0 & e^{i\theta_2} \end{pmatrix} \tau \tag{B.10}$$

where we have chosen W to diagonalize the unitary matrix $D\tau^\dagger$. Finally, performing again a basis transformation (B.8) with a diagonal phase matrix we can achieve

$$D = \begin{pmatrix} 0 & 1 \\ -d & 0 \end{pmatrix}, \qquad |d| = 1 \tag{B.11}$$

in the new basis. Having simplified the range of possible coupling matrices Δ we can go back to the condition (B.5). Using once more relation (B.9) we can cast it into the form

$$U^\dagger \begin{pmatrix} 1 & 0 \\ 0 & d \end{pmatrix} U e^{i\varphi} = \begin{pmatrix} 1 & 0 \\ 0 & d \end{pmatrix}. \tag{B.12}$$

Now we have to distinguish two cases. In the first one, we assume $e^{i\varphi} = 1$ for all elements of G_H. Then we get $d = 1$ on account of the irreducibility of 2_L which corresponds to $\varepsilon = -1$ in Eq. (B.1). In the other case there is at least one φ' with $e^{i\varphi'} \neq 1$. With

$$U' = \begin{pmatrix} a & b \\ -b^* & a^* \end{pmatrix} \tag{B.13}$$

being the G_H transformation belonging to the same group element, we derive from Eq. (B.12) that

$$a = 0, \qquad e^{i\varphi'} = d = -1. \tag{B.14}$$

This corresponds to $\varepsilon = 1$ in Eq. (B.1) which concludes the proof. In the case $\varepsilon = -1$ the only restriction on the group representations is given by $e^{i\varphi} = 1$. On the other hand, for $\varepsilon = 1$ the matrices $U \in SU(2)$ must be diagonal for $e^{i\varphi} = 1$ and purely off-diagonal for $e^{i\varphi} = -1$. Other values for $e^{i\varphi}$ are not allowed.

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Figure Captions

Fig. 1: Generic one-loop diagram for both m_ν and μ_ν (with the photon attached to either the fermion f or the scalar Φ).

Fig. 2: Interference mechanism for scenario I. The mixing of scalars φ, η is due to the cubic coupling (2.8). For μ_ν the photon must be attached to either the scalar or the fermion line.

Fig. 3: Interference mechanism for scenario II.

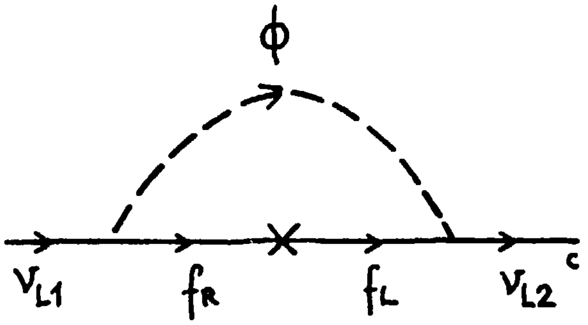

FIG. 1

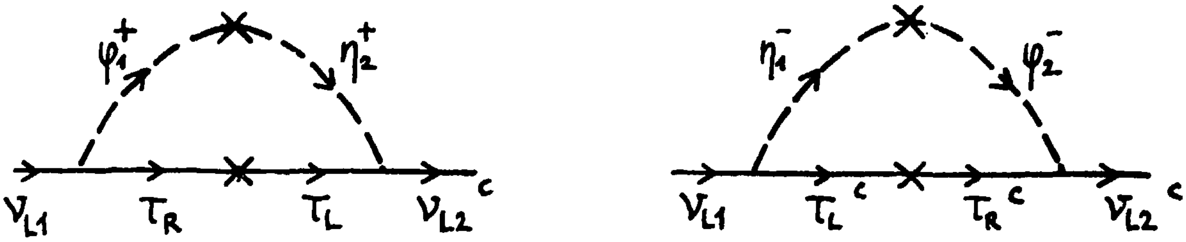

FIG. 2

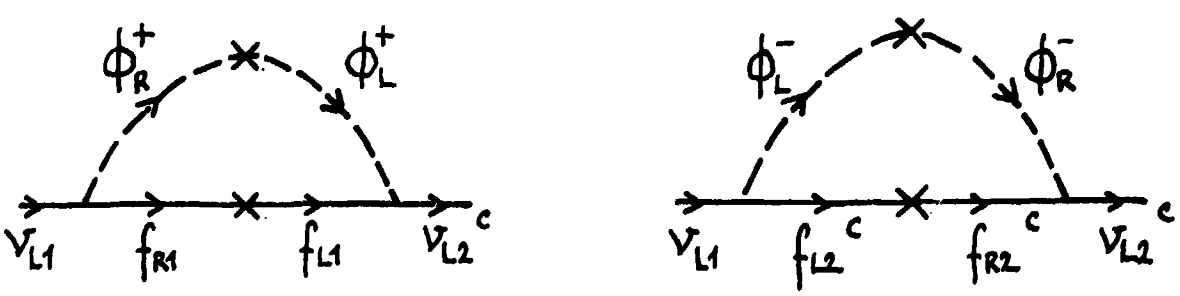

FIG. 3