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Microscopic Theory of Nuclear Collective Dynamics

- Inter-relationship between Quantum Chaos and Nuclear Spectroscopy-

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Abstract. A recent development of the INS-TSUKUBA joint research project on large-amplitude collective motion is summarized by putting special emphasis on an inter-relationship between quantum chaos and nuclear spectroscopy. Aiming at introducing various concepts used in this lecture, we start with recapitulating the semi-classical theory of nuclear collective dynamics formulated within the time-dependent Hartree-Fock (TDHF) theory. The central part of the semi-classical theory is provided by the self-consistent collective coordinate (SCC) method which has been developed to properly take account of the non-linear dynamics specific for the finite many-body quantum system. A decisive role of the level crossing dynamics on the order-to-chaos transition of collective motion is discussed in detail. Extending the basic idea of the semi-classical theory, we discuss a full quantum theory of nuclear collective dynamics which allows us to properly define a concept of the quantum integrability as well as the quantum chaoticity for each eigenfunction. The lecture is arranged so as to clearly show the similar structure between the semi-classical and quantum theories of nuclear collective dynamics. Using numerical calculations, we illustrate what the quantum chaos for each eigenfunction means and relate it to the usual definition of quantum chaos for nearest neighbor level spacing statistics based on the random matrix theory.

1. Introduction

1.1 Fundamental Theory of Nuclear Matter Physics

An exploration on the transition mechanism between the ***Quantum Integrable*** and ***Quantum Chaotic Motions*** is one of the most fascinating subject in a variety of quantum many-body systems. The most important step in this study might be to properly define the concept of the quantum integrability. Possible ways in defining the quantum integrability are to use the quantum group (q-deformation) based on the Yang-Baxter equation or to employ the group theoreti-

cal property of the Hamiltonian, which are discussed by the other authors of this volume[1]. In this lecture, we will discuss another way of dynamically defining the quantum chaoticity, which has been introduced in the context of developing the theory of large-amplitude nuclear collective motion under the INS-TSUKUBA joint research project.

Let us start with discussing why the project treats the large-amplitude collective motion in both the semi-classical and quantum systems, simultaneously. The nucleus is known to be a ***finite many-body quantum system*** whose eigenstates ought to be understood by developing an appropriate quantum theory of many-body problems. Nevertheless, it is not so simple to develop such a quantum many-body theory which is capable of describing various states situating at far from the "vacuum" like a stable mean-field or thermodynamical state of equilibration, i.e. the states in the large-amplitude region. In the nuclear physics, however, it is becoming more and more clear that some of the excited states are well understood by introducing a suitable mean-field. In the past decade, the most exciting progress in the nuclear physics has been an establishment of the "rotating (cranked Nilsson) shell model" as well as the "super-deformed shell model"[2]. It is astonishing that there exist many stable mean-fields, i.e. various vacua with different symmetries in the nuclear excited states under extreme condition of rapid rotation, characterizing various "phases" in the finite many-body quantum system. The existence of the various stable mean-fields in the nucleus enables us to explore the nuclear collective dynamics in terms of the mean-field dynamics, i.e. the single-particle dynamics within the time-dependent Hartree-Fock (TDHF) theory (which is formally equivalent to the classical dynamics), indicating a possibility of understanding the full quantum system in terms of a "classical level" dynamics with geometrical and topological arguments in the TDHF phase space. It has been also established that the TDHF theory is directly related to the full quantum theory via the boson expansion theory. This theoretical relation suggests us that the nucleus may provide us with a unique opportunity to construct the full quantum theory of many-body problems under instruction from what is developed in the semi-classical theory. From the above argument, it is expected that the nucleus provides us with a unique possibility of studying an interesting question on an inter-relationship between the ***classical chaos*** (within the symplectic manifold obtained under the time-dependent mean-field theory) and ***quantum chaos*** (within the full quantum theory).

Before discussing on the quantum chaoticity, it is preferable to summarize the present status of the nuclear physics. The nuclear physics is one of the most rapidly expanding fields of contemporary physical science, searching for new states of matter under various extreme conditions, i.e. under fast rotation, large isospin, high temperature, high density and etc. In accordance with recent diverse experimental observations studied by means of improved experimental facilities, it is becoming an inevitable theoretical subject to develop a ***fundamental theory of nuclear matter physics*** which gives us a unified understanding of those phenomena ranging from low-lying collective excited states up to high-energy heavy-ion collisions in the

so-called VUU or BUU regime. What should be stressed is an importance of developing the basic theoretical framework like the equilibrium or non-equilibrium statistical physics which are providing us with a unified understanding of various phenomena in the solid state physics as well as the condensed matter physics.

In order to make clear the current basic problems in nuclear matter physics, let us classify a variety of phenomena into three characteristic regimes; The first regime is characterized by *local phenomena* which are understood by introducing an appropriate stable mean-field with one local minimum, special coupling scheme, group theoretical model space or collective subspace with a fixed symmetry and etc. In the second regime, there are ***medium- or large-amplitude collective phenomena*** which are usually described by a suitable mean-field with many local minima with different symmetries. The shape coexisting model based on the diabatic single-particle states and the two-center shell model for the nuclear fission process are well-known examples. The third regime consists of ***statistical or probabilistic phenomena*** which are explained by means of thermodynamical concepts like transport equation, equation of states, temperature, stochastic equations and etc. The giant dipole resonances on top of highly excited states observed by high-energy gamma transitions discussed by introducing the temperature dependent random phase approximation, heavy-ion deep inelastic collision described by Fokker-Planck equation or multi-dimensional fission process by Langivan equation, rotational damping discussed by the motional narrowing mechanism and the neutron resonance states by the random matrix theory are typical examples.

1.2 Basic Problems in Nuclear Matter Physics

In order to get a unified understanding of various phenomena classified into the above-stated three regimes, one has to treat the following two basic problems in nuclear matter physics; Problem I: Dynamical relation between regimes 1 and 2. In this case, we have to answer how does the stable mean-field, i.e. the vacuum with fixed symmetry (more generally, the group theoretical model space or the coupling scheme) become unstable and how does a new mean-field (new vacuum) with a different symmetry get stable. This problem is strongly related to an important question on the breaking of the symmetry in the finite system and an appearance of collective motion in association with restoring the broken symmetry. It is also very interesting question to study a dynamics of the "phase transition" or more generally a dynamical relation between two vacua with different symmetries. Problem II: Dynamical relation between regimes 2 and 3. In this case, we have to study how the statistical or probabilistic features arise as a consequence of a deterministic dynamics and how a system in a non-equilibrium state reaches a state of equilibration. In other words, we encounter an essential problem why the irreversible dynamics results from the reversible dynamics.

Now-a-days these basic problems are becoming rather common in many fields of theoretical physics in connection with a) *the order-to-chaos transition mechanism in quantum systems*, b) *geometrical or topological structure of the symplectic manifold* and c) *dynamics on the symmetry breaking*. Among others, there has been a long history in the theoretical nuclear physics for exploring these subjects from its early stage, i.e. from the early 1950's. Aiming at studying them, there have been many works mainly based on the *adiabatic* or *diabatic* single-particle potential models, i.e. within the mean-field theory with fixed symmetry put in by hand. It is widely considered that i) a potential barrier in between two local minima for the collective motion, ii) the dynamics of barrier penetration in fission process or exotic decay, iii) the dissipation process of collective motion, iv) an appearance of super-deformation, v) particle alignment mechanism to the rotation axis and etc. ought to be understood by the microscopic dynamics of single-particle modes of motion, i.e. by the level crossing dynamics of single-particle states. For example, the importance of level slippage in the nuclear fission process was first pointed out by Hill-Wheeler[3] and recently by Swiatecki[4]. The diabatic level crossing followed by two-body collisions was discussed by Nørenberg[5] in describing the dissipating collective motion in heavy-ion deep inelastic collisions and a concept of successive level crossing was introduced by Barranco-Broglia-Bertsch[6] in understanding the exotic decay process. However, an essential role of the level crossing dynamics on the nuclear collective dynamics has been still left unresolved because of a difficulty of going beyond the adiabatic approximation.

Recently, it has been stated by Swiatecki[4] that the level crossing in the single-particle dynamics affects on the property of the collective motion and is strongly related to the "order-to-chaos" transition mechanism: If the nucleonic motion in the nuclear potential is ***integrable***, we get a diabatic collective potential. If it is ***non-integrable***, we get an adiabatic collective potential. It was also stated by Swiatecki[4] that the order-to-chaos transition in nucleonic motion should be studied by paying careful attention to both the dynamics of symmetry breaking in the single-particle potential and the inclusion of residual interaction among single-particle states. In conventional approaches, however, the symmetry of the nuclear potential (specified by shape parameters) is imposed artificially from the outset and the residual interactions (pairing interaction is usually emploied) among adiabatic or diabatic single-particle states are put in by hand. If one intends to explore the order-to-chaos transition dynamics of nucleonic motion in the single-particle potential whose symmetry might change during a large-amplitude collective motion like the fission process, one has to explore the self-consistent dynamical relation between nuclear potential and residual interaction, by going beyond the adiabatic or diabatic approximations.

However, it has been considered to be one of the most difficult problems in the microscopic theory of nuclear collective motion to go beyond the adiabatic approximation. If one intends to get over the adiabatic approximation, one has to be eventually confronted with a difficult problem of properly treating the non-linear dynamics between the collective and single-particle

modes of motion, which is coming from the self-consistent property inherent to such a self-sustained finite many-body quantum system as the nucleus. As is well known, the collective coordinate (e.g. deformation δ) is expressed as a coherent sum of particle-hole ($a^{\dagger}_{\mu}b^{\dagger}_{i}$) operators. On the other hand, the single-particle orbits in turn should adjust their feature self-consistently in accordance with the time-dependent mean-field expressing the collective motion. Namely, the single-particle operators $a^{\dagger}_{\mu}$ and $b^{\dagger}_{i}$ change their structure depending on δ. Consequently, there appears a strong non-linear dependence between the dynamical change in the symmetry of single-particle potential and the nucleonic motion inside the nuclear potential, which is schematically expressed by

$$\hat{\delta} = \Sigma_{\mu i}\, \chi_{\mu i}\, a^{\dagger}_{\mu}(\delta)\, b^{\dagger}_{i}(\delta) + \cdots . \tag{1.1}$$

In 1980, the self-consistent collective coordinate (SCC) method[7] was proposed to properly treat the above stated non-linear mean-field dynamics, without introducing any adiabatic assumptions. What we will discuss in this lecture is a further development of the SCC method, aiming at getting a unified understanding of various phenomena exhibited by the nucleus.

In §2, we will discuss how to define the concept of classical integrability in the context of nuclear collective dynamics within the TDHF theory. We will also discuss the dynamical condition on an onset of classical chaos by paying special attention on its relation to the level crossing dynamics. We will briefly discuss the relation between the level crossing point and the weak stability point, the latter of which is regarded to play an essential role in understanding why the probabilistic feature comes out as a result of deterministic dynamics. In §3, we will discuss the quantum theory of nuclear collective dynamics which has been developed by extending the basic idea discussed in §2. The quantum theory allows us to properly introduce a concept of quantum integrability as well as the quantum chaoticity for individual eigenfunctions. Special attention will be paied to show how the quantum theory of nuclear collective dynamics has been developed under instruction of the single-particle dynamics formulated within the TDHF theory. By using numerical calculation in §4, we will illustrate what the quantum chaos for each eigenfunction means and relate it to the usual discussion of the quantum chaos for the nearest neighbor level spacing statistics based on the random matrix theory.

2. Semi-Classical Theory of Nuclear Collective Dynamics

2.1 Time-Dependent Hartree-Fock Theory

Let us start with discussing why we employ the TDHF theory. As is discussed in §1, there are plenty of experimental evidences to support a variety of stable mean-fields which are well realized in nuclear system. Since the TDHF theory gives a theoretical foundation for i) the static mean-field (Hartree-Fock (-Bogoliubov) theory), ii) the random phase approximation (RPA)

describing small-amplitude collective motions, iii) the constrained (cranked) Hartree-Fock (-Bogoliubov) theory expressing the nuclear collective rotation which is a typical example of large-amplitude collective motion, it is natural to consider the TDHF equation to be the basic equation for exploring the basic dynamics organizing the various phenomena in nuclei. On top of that, with the aid of high speed electric computer, it has been shown numerically that the TDHF equation can well simulate the dissipating nuclear collective dynamics in the heavy-ion deep inelastic collision, fission and fusion processes. Moreover, the TDHF equation gives a foundation of Vlasov equation which are currently considered to be a starting point to derive the VUU or BUU equations, used in describing the high-energy heavy-ion collisions.

Aiming at treating the non-linear dynamics between the collective and single-particle modes of motion specifically for the nuclear system stated in the previous section, we start with the TDHF equation

$$\delta < f(t)|\hat{H} - i\frac{\partial}{\partial t}|f(t)>= 0. \tag{2.1}$$

Here and hereafter, we use a convention $\hbar = 1$. In Eq. (2.1), $|f(t)>$ denotes a time-dependent single Slater determinant state given by

$$|f(t)\rangle = \exp\left\{i\hat{F}(t)\right\}|\phi_0\rangle, \qquad \hat{F}(t) = \sum_{\mu,i}\left\{f_{\mu i}(t)\, a_\mu^\dagger b_i^\dagger + f^*_{\mu i}(t)\, b_i a_\mu\right\}, \tag{2.2}$$

where $|\phi_0>$ denotes a certain Hartree-Fock stationary state satisfying

$$\delta\langle\phi_0|\hat{H}|\phi_0\rangle = 0. \tag{2.3}$$

In Eq.(2.2), $a_\mu^\dagger$ and $b_i^\dagger$ represent particle and hole operators with respect to $|\phi_0>$

$$\begin{aligned} a_\mu|\phi_0\rangle &= 0\ ,\ m = 1,\ldots, M\ , \\ b_i|\phi_0\rangle &= 0\ ,\ i = 1,\ldots, N\ . \end{aligned} \tag{2.4}$$

In Eq. (2.4), N (M) denotes a number of single-hole (-particle) states. Instead of 2MN which expresses a number of particle-hole amplitudes $\{ f_{\mu i}^*, f_{\mu i} \}$, we hereafter use a number 2K (=2MN). Here, it should be noticed that $\{ f_{\mu i}^*, f_{\mu i} \}$ are not canonical variables, i.e. they do not satisfy a set of canonical equations of motion;

$$i\dot{f}_{\mu i} \neq \frac{\partial H}{\partial f^*_{\mu i}}\ , \qquad H = \langle f|\hat{H}|f\rangle\ , \quad |f\rangle = |f(t)\rangle\ . \tag{2.5}$$

Instead of $\{ f_{\mu i}^*, f_{\mu i} \}$, one may introduce a set of 2K canonical variables $\{C_{\mu i}^*, C_{\mu i}\}$ through variable transformation;

$$\begin{aligned} &\left\{f_{\mu i}, f_{\mu i}^*\right\} \Leftrightarrow \left\{C_{\mu i}, C_{\mu i}^*\right\}, \\ &C_{\mu i} = C_{\mu i}(f, f^*), \qquad f_{\mu i} = f_{\mu i}(C, C^*). \end{aligned} \tag{2.6}$$

With the aid of $\{C_{\mu i}^*, C_{\mu i}\}$, one may introduce the following one-body operators,

$$\hat{O}_{\mu i}^{\dagger} \equiv \left\{\frac{\partial}{\partial C_{\mu i}} e^{i\hat{F}}\right\} \cdot e^{-i\hat{F}}, \qquad \hat{O}_{\mu i} \equiv -\left\{\frac{\partial}{\partial C_{\mu i}^*} e^{i\hat{F}}\right\} \cdot e^{-i\hat{F}}. \tag{2.7}$$

The transformation in Eq. (2.6) is determined by requiring the canonical variables condition given by

$$\begin{aligned} &\langle f | \hat{O}_{\mu i}^{\dagger} | f \rangle \left(\equiv \langle f | \frac{\partial}{\partial C_{\mu i}} | f \rangle \right) = \frac{1}{2} C_{\mu i}^* - i \frac{\partial}{\partial C_{\mu i}} S , \\ &\langle f | \hat{O}_{\mu i} | f \rangle \left(\equiv -\langle f | \frac{\partial}{\partial C_{\mu i}^*} | f \rangle \right) = \frac{1}{2} C_{\mu i} + i \frac{\partial}{\partial C_{\mu i}^*} S , \end{aligned} \tag{2.8}$$

where $S^*=S$ denotes an arbitrary real function of $\{C_{\mu i}^*, C_{\mu i}\}$, expressing a generating function to characterize the transformation in Eq. (2.6). From Eq. (2.8), we derive the following "weak" boson-like commutation relations,

$$\begin{aligned} &\langle f | \left[\hat{O}_{\mu i}, \hat{O}_{\nu j}^{\dagger}\right] | f \rangle = \delta_{\mu\nu} \cdot \delta_{ij} , \\ &\langle f | \left[\hat{O}_{\mu i}, \hat{O}_{\nu j}\right] | f \rangle = 0 , \end{aligned} \tag{2.9}$$

which is independent of any choice of S. For simplicity, we take S=0. The solution of Eq. (2.8) is known to be expressed as[8]

$$C_{\mu i} = \left\{ \frac{\sin\sqrt{f\, f^{\dagger}}}{\sqrt{f\, f^{\dagger}}} f \right\}_{\mu i}, \qquad C_{\mu i}^* = \left\{ f^{\dagger} \frac{\sin\sqrt{f\ f^{\dagger}}}{\sqrt{f\ f^{\dagger}}} \right\}_{i\mu}. \tag{2.10}$$

By means of $\{C_{\mu i}^*, C_{\mu i}\}$ thus defined, the TDHF equation (2.1) is expressed as

$$\delta\langle f|\sum_{\mu,i}\left\{i\dot{C}_{\mu i}\frac{\partial}{\partial C_{\mu i}}+i\dot{C}^{*}_{\mu i}\frac{\partial}{\partial C^{*}_{\mu i}}\right\}-\hat{H}|f\rangle=0. \tag{2.11}$$

With the aid of one-body operators defined in Eq. (2.7), Eq. (2.11) is rewritten as

$$\delta\langle f|\sum_{\mu,i}\left\{i\dot{C}_{\mu i}\hat{O}^{\dagger}_{\mu i}-i\dot{C}^{*}_{\mu i}\hat{O}_{\mu i}\right\}-\hat{H}|f\rangle=0\,. \tag{2.12}$$

Taking the variations in Eq. (2.12) with respect to $\{C_{vj}^{*},C_{vj}\}$ for $|f\rangle$, we get

$$\langle f|[\,\hat{O}^{\dagger}_{vj}\,,\,(\sum_{\mu,i}\left\{i\dot{C}_{\mu i}\hat{O}^{\dagger}_{\mu i}-i\dot{C}^{*}_{\mu i}\hat{O}_{\mu i}\right\}-\hat{H}\;)\,]|f\rangle=0\,,$$

$$\langle f|[\,\hat{O}_{vj}\,,\,(\sum_{\mu,i}\left\{i\dot{C}_{\mu i}\hat{O}^{\dagger}_{\mu i}-i\dot{C}^{*}_{\mu i}\hat{O}_{\mu i}\right\}-\hat{H}\;)\,]|f\rangle=0\,. \tag{2.13}$$

By using the weak boson-like commutation relations in Eq. (2.9), Eq. (2.13) reduces into a set of classical Hamilton's equation of motion

$$i\dot{C}_{\mu i}=\frac{\partial H}{\partial C^{*}_{\mu i}}=\left\{C_{\mu i}\,,\,H\right\}_{P.B.}\,,$$

$$i\dot{C}^{*}_{\mu i}=-\frac{\partial H}{\partial C_{\mu i}}=\left\{C^{*}_{\mu i}\,,\,H\right\}_{P.B.}\,, \tag{2.14}$$

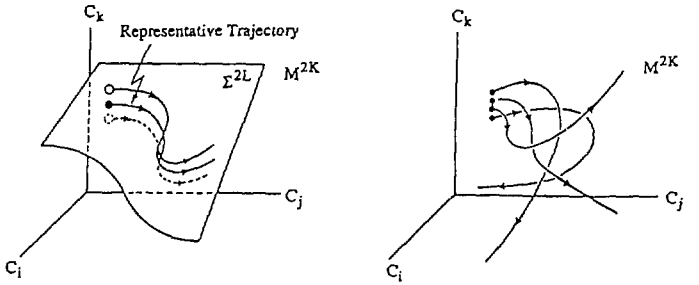
where

$$\{A\,,\,B\}_{P.B.}=\sum_{\mu i}\left(\frac{\partial A}{\partial C_{\mu i}}\frac{\partial B}{\partial C^{*}_{\mu i}}-\frac{\partial B}{\partial C_{\mu i}}\frac{\partial A}{\partial C^{*}_{\mu i}}\right)\,. \tag{2.15}$$

Equation (2.14) clearly shows a symplectic structure of the TDHF manifold M^{2K} : $\{C_{\mu i}^{*},C_{\mu i}\}$. It is also clear that the TDHF equation (2.1) is equivalent to the classical canonical equations of motion in Eq. (2.14).

2.2 Self-consistent Collective Coordinate (SCC) Method

In the previous subsection, we have studied that the TDHF equation (2.1) defines a trajectory within the 2K-dimensional symplectic manifold (TDHF phase space) under a specific initial condition. For the classical dynamical system organized by Eq. (2.14), one may imagine two



a) collective regime b) chaotic regime

FIGURE 1. Schematic figures representing collective and chaotic behavior of trajectories in the TDHF manifold M^{2K}. Sheet in a) represents the collective submanifold Σ^{2L}.

extreme cases; If an ensemble of trajectories with slightly different initial conditions develops almost the same way forming a bundle, the system is classified to be in the ***collective regime*** as shown in Fig.1a. In this case, one could introduce a ***collective submanifold*** Σ^{2L} (representing an approximate integral surface) to which many trajectories are accumulating. On the other hand, the system is classified to be in the ***chaotic regime***, if many trajectories with almost the same initial conditions develop quite differently, having lost their initial focussing (Fig.1b). In the latter case, one could not introduce any submanifold within M^{2K}. From the above discussion, one may expect that the transition mechanism between "regular" and "chaotic" motions may be studied by extracting the collective submanifold Σ^{2L} out of M^{2K}.

In the collective regime where many trajectories are accumulating on a certain submanifold Σ^{2L}, there must be an optimal coordinate system called the ***dynamical canonical coordinate*** (DCC) system[9] where the minimum number of relevant (collective) variables (η_a, η_a^*: a=1,...,L<<K) is required in describing Σ^{2L}. In this case, one may introduce the ***representative trajectory*** of the bundle, which is running on the collective submanifold Σ^{2L} (which is nothing but the integral surface of the representative trajectory). The remaining canonical variables (ξ_α, ξ_α^* : α=L+1,...,K) in the DCC system are called the irrelevant (intrinsic) variables. The coordinate system $\{C_{\mu i}, C_{\mu i}^*\}$ used in describing Eq. (2.1) is called the ***initial canonical coordinate*** (ICC) system, because this coordinate system has nothing to do with a specific correlation in the Hamiltonian $H(C,C^*)$.

The extraction of Σ^{2L} is achieved by determining a canonical transformation between the ICC and DCC systems. Since the transformation should be canonical, there must hold the following canonical variable conditions;

$$\sum_j \left\{ C_j \frac{\partial C_j^*}{\partial \eta_a^*} - C_j^* \frac{\partial C_j}{\partial \eta_a^*} \right\} = \eta_a + 2i \frac{\partial \mathcal{S}}{\partial \eta_a^*}, \quad a = 1,...,L,$$

$$\sum_j \left\{ C_j \frac{\partial C_j^*}{\partial \xi_\alpha^*} - C_j^* \frac{\partial C_j}{\partial \xi_\alpha^*} \right\} = \xi_\alpha + 2i \frac{\partial \mathbf{S}}{\partial \xi_\alpha^*}, \quad \alpha = L+1,\ldots,K . \tag{2.16}$$

Here and hereafter, the double indexes (μi) specifying the particle-hole states are simply expressed by the single indexes (j). Equation (2.16) has the same structure as Eq. (2.8) and $\mathbf{S}$ denotes a generating function of the canonical transformation between the ICC and DCC systems. In order to fix the transformation, it is decisive to introduce the following expansion scheme: Since $\{\eta_a,\eta^*_a\}$ are supposed to describe large-amplitude collective motions inside Σ^{2L}, whereas $\{\xi_\alpha,\xi^*_\alpha\}$ describe small fluctuations around Σ^{2L}, it is reasonable to introduce a power series expansion with respect to irrelevant variables. For example,

$$C_j = \sum_n C_j^{(n)} = [C_j] + \sum_\alpha \left\{ \xi_\alpha \left[\frac{\partial C_j}{\partial \xi_\alpha} \right] + \xi_\alpha^* \left[\frac{\partial C_j}{\partial \xi_\alpha^*} \right] \right\} + \cdots, \tag{2.17}$$

$$H = \sum_n \mathcal{H}^{(n)} = [\,H\,] + \sum_\alpha \left\{ \xi_\alpha \left[\frac{\partial H}{\partial \xi_\alpha} \right] + \xi_\alpha^* \left[\frac{\partial H}{\partial \xi_\alpha^*} \right] \right\} + \cdots, \tag{2.18}$$

$$\mathbf{S} = \sum_n \mathbf{S}^{(n)} = [\,\mathbf{S}\,] + \sum_\alpha \left\{ \xi_\alpha \left[\frac{\partial \mathbf{S}}{\partial \xi_\alpha} \right] + \xi_\alpha^* \left[\frac{\partial \mathbf{S}}{\partial \xi_\alpha^*} \right] \right\} + \cdots, \tag{2.19}$$

where the symbol [g] for any function $g(\eta,\eta^*;\ \xi,\xi^*)$ denotes an expansion function on the expansion surface Σ^{2L}, i.e. $[g] = g(\eta,\eta^*;\ \xi=0,\xi^*=0)$. In Eqs. (2.17,18,19), n denotes a power of irrelevant variables contained in each term. Since the lowest order Hamiltonian $\mathcal{H}^{(0)} = [H]$ contains only the relevant variables, it is identified to be the collective Hamiltonian $\mathcal{H}_{coll} = \mathcal{H}^{(0)}$.

Now, let us define Σ^{2L} within M^{2K}. This task is achieved by determining the functional forms of $\{[C_j],[C^*_j]\}$ with respect to $\{\eta_a,\eta^*_a\}$, because we have the following diffeomorphic mapping after getting $\{[C_j],[C^*_j]\}$;

$$\textbf{Diffeomorphic Mapping}\ ;\ M^{2L}:\{\eta_a,\eta_a^*\} \rightarrow \Sigma^{2L}:\{[C_j],[C_j^*]\}\ \text{embedded in}\ \Sigma^{2K}, \tag{2.20}$$

which defines Σ^{2L} in M^{2K} as is shown in Fig.2. According to the self-consistent collective coordinate (SCC) method[7], the basic equations for defining Σ^{2L} within M^{2K} are given by the following two requirements

Requirement 1: There should hold the lowest order ***canonical variable condition;***

$$(\mathrm{I}) \quad \sum_j \left\{ [C_j] \frac{\partial [C_j^*]}{\partial \eta_a^*} - [C_j^*] \frac{\partial [C_j]}{\partial \eta_a^*} \right\} = \eta_a + 2i \frac{\partial [S]}{\partial \eta_a^*}, \tag{2.21}$$

which is derived from Eq.(2.16).

Requirement 2: There should hold the ***equation of collective submanifold;***

$$(\mathrm{II}) \quad \left[\frac{\partial H}{\partial C_j^*} \right] = \sum_a \left\{ \frac{\partial [H]}{\partial \eta_a^*} \frac{\partial [C_j]}{\partial \eta_a} - \frac{\partial [H]}{\partial \eta_a} \frac{\partial [C_j]}{\partial \eta_a^*} \right\}. \tag{2.22}$$

This equation is originally derived by Marumori[10] under the name of "the invariance principle of the time-dependent Schrödinger equation". The physical meaning of Eq. (2.22) is obtained by following consideration: As is discussed in the beginning of this subsection, the representative trajectory of the bundle in Fig. 1a should satisfy both the equation of motion in the ICC system given by

$$i[\dot{C}_j] = [\partial H / \partial C_j^*], \tag{2.23a}$$

and the equation of motion in the DCC system given by

$$i\dot{\eta}_a = \partial [H] / \partial \eta_a^*. \tag{2.23b}$$

These equations contain the symbol [ ], because the representative trajectory is always travelling on Σ^{2L}. Since the integral surface of the representative trajectory is time independent object, we need the time independent equation to characterize Σ^{2L}. Eliminating the time derivative terms in Eq. (2.23a) by means of Eq. (2.23b), we get a time independent equation (2.22) which is then called the equation of collective submanifold.

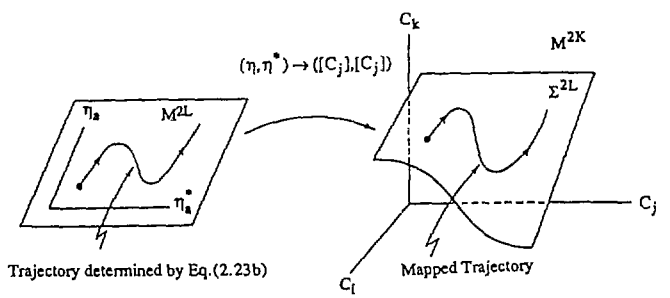


FIGURE 2. Diffeomorphic mapping defining the collective submanifold Σ^{2L} in M^{2K}.

At this point, we briefly discuss how to solve the basic equations (I) and (II) of the SCC method. In order to illustrate the method as simple as possible, we treat a case with L=1, i.e. a case with the single pair of relevant variables denoted by $\{\eta,\eta^*\}$ which characterizes the collective submanifold $\Sigma^{2L=2}$. Aiming at solving the basic equations, we assume that $\{[C_j],[C^*_j]\}$ are analytic functions of relevant variables $\{\eta,\eta^*\}$, i.e. $\{[C_j],[C^*_j]\}$ are expanded in power series of $\{\eta,\eta^*\}$;

$$[C_j] = \sum_{n=1}[C_j]^{(n)} \quad ; \quad [C_j]^{(n)} = \sum_{\substack{r,s \\ (r+s=n)}} \mathfrak{R}_j^{rs}\left(\eta^*\right)^r(\eta)^s \,, \tag{2.24}$$

which is called the (η,η^*)-expansion. Let us consider the representative trajectory whose initial condition is expressed by using the collective RPA mode with the lowest energy eigenvalue alone. In other words, we bear in mind such a large-amplitude collective motion whose small amplitude limit is expressed by the lowest-energy collective RPA mode denoted by $j=1$. (note: Since a set of RPA modes and that of particle-hole states are related with each other through the linear unitary transformation, we use the same indexes (j) to specify the RPA modes.) In this case, one may choose the following boundary condition for the collective submanifold Σ^{2L};

$$[C_j]^{(1)} = \delta_{j,1}\cdot\eta \quad \text{,i.e.} \quad \mathfrak{R}_j^{10} = 0 \text{ and } \mathfrak{R}_j^{01} = \delta_{j,1} \,, \text{ and } \mathfrak{S}^{(0)} = 0 \,, \tag{2.25}$$

which is only one physical input of the SCC method in specifying the collectivity under consideration. Assuming that the basic equations (I) and (II) are satisfied order by order in the (η,η^*)-expansion and substituting the lowest order coefficients $\{[C_j]^{(1)}, [C_j^*]^{(1)}\}$ in Eq.(2.25) for them, we get a set of necessary and sufficient conditions to determine the expansion coefficients $\mathfrak{R}_j^{rs}$ with $r+s=2$. By repeating the same procedure, we get $\{[C_j], [C^*_j]\}$ up to the desired order.

2.3 Validity of the Collective Submanifold

By means of $\{[C_j], [C^*_j]\}$ thus determined, we get the collective submanifold Σ^{2L} on which the representative trajectory is running. As is easily understood, it is always possible to define the submanifold for a given trajectory by using the SCC method. However, it is important to study whether the submanifold thus defined is embedded in the collective region or in the chaotic region shown in Fig. 1. Namely, we have to ask whether or not the trajectory starting from the vicinity of the collective submanifold Σ^{2L}(, i.e. the trajectory having almost the same initial condition as that of the representative trajectory) always stays in a tiny region around Σ^{2L}. This problem is studied by both the ***separability*** and ***stability*** conditions[8]. Before discussing

these two conditions, let us calculate the variation of the total Hamiltonian toward the irrelevant directions on Σ^{2L}, i.e. we consider $[\partial H/\partial\xi_\alpha]$ which is expressed as

$$\left[\frac{\partial H}{\partial \xi_\alpha}\right] = \Sigma_j \left\{ \left[\frac{\partial H}{\partial C_j}\right]\left[\frac{\partial C_j}{\partial \xi_\alpha}\right] + \left[\frac{\partial H}{\partial C_j^*}\right]\left[\frac{\partial C_j^*}{\partial \xi_\alpha}\right] \right\}. \tag{2.26}$$

Substituting the basic equation (II) in Eq. (2.22) for Eq.(2.26), we get

$$\left[\frac{\partial H}{\partial \xi_\alpha}\right] = \Sigma_\alpha \left\{ \begin{array}{l} \dfrac{\partial[H]}{\partial \eta_\alpha} \Sigma_j \left(\dfrac{\partial[C_j^*]}{\partial \eta_\alpha^*}\left[\dfrac{\partial C_j}{\partial \xi_\alpha}\right] - \left[\dfrac{\partial C_j^*}{\partial \xi_\alpha}\right]\dfrac{\partial[C_j]}{\partial \eta_\alpha^*} \right) \\ -\dfrac{\partial[H]}{\partial \eta_\alpha^*} \Sigma_j \left(\dfrac{\partial[C_j^*]}{\partial \eta_\alpha}\left[\dfrac{\partial C_j}{\partial \xi_\alpha}\right] - \left[\dfrac{\partial C_j^*}{\partial \xi_\alpha}\right]\dfrac{\partial[C_j]}{\partial \eta_\alpha} \right) \end{array} \right\}. \tag{2.27}$$

By using the first order canonical variables condition expressed as

$$\begin{aligned} &\Sigma_j \left(\frac{\partial[C_j^*]}{\partial \eta_\alpha^*}\left[\frac{\partial C_j}{\partial \xi_\alpha}\right] - \left[\frac{\partial C_j^*}{\partial \xi_\alpha}\right]\frac{\partial[C_j]}{\partial \eta_\alpha^*} \right) = 0 , \\ &\Sigma_j \left(\frac{\partial[C_j^*]}{\partial \eta_\alpha}\left[\frac{\partial C_j}{\partial \xi_\alpha}\right] - \left[\frac{\partial C_j^*}{\partial \xi_\alpha}\right]\frac{\partial[C_j]}{\partial \eta_\alpha} \right) = 0 , \end{aligned} \tag{2.28}$$

which is derived from the first order expansion of Eq.(2.16) with respect to irrelevant variables, we get

$$\left[\frac{\partial H}{\partial \xi_\alpha}\right] = 0 , \quad \left[\frac{\partial H}{\partial \xi_\alpha^*}\right] = 0 , \tag{2.29}$$

i.e.,

$$\mathfrak{H}^{(1)} = 0 . \tag{2.30}$$

From Eq.(2.30), it is clear that the collective submanifold Σ^{2L} is defined so as to satisfy the ***stationary condition*** with respect to the variations towards the irrelevant directions. At this point, it should be remembered that the Hartree-Fock state just corresponds to the minimum

point of the total Hamiltonian (potential energy) in the coordinate space: { Re(C_j), Re(C_j^*)}, satisfying the condition

$$\frac{\partial H(\mathrm{Re}(C),\ \mathrm{Re}(C^*))}{\partial \mathrm{Re}(C)} = 0\,, \tag{2.31}$$

whereas the collective submanifold corresponds to the minimum ***surface*** of the total Hamiltonian in the phase space {C_j, C^*_j}. In this sense, the SCC method is a natural extension of the Hartree-Fock theory from the stationary ***point*** to the stationary 2L-dimensional ***surface***.

A fulfillment of the stationary condition (2.30) does not necessarily means the stability of the collective submanifold Σ^{2L}. The stability condition is known[8] to be given by calculating the small-amplitude normal modes toward the irrelevant directions (perpendicular to the collective submanifold) by using the RPA-type equation defined at each point on Σ^{2L}, i.e.

$$\left[\frac{\partial^2 H}{\partial \xi_\alpha \partial \xi^*_\beta}\right] = \omega_\alpha(\eta,\eta^*)\,\delta_{\alpha,\beta}\,, \qquad \left[\frac{\partial^2 H}{\partial \xi_\alpha \partial \xi_\beta}\right] = 0. \tag{2.32}$$

Here it should be noticed that the second order Hamiltonian $\mathcal{H}^{(2)}$ is responsible for the stability of Σ^{2L}. By means of $\omega_\alpha(\eta,\eta^*)$, the ***stability condition*** of Σ^{2L} at each point of (η,η^*) is given as follows;

i) If $\omega_\alpha(\eta,\eta^*) > 0$, Σ^{2L} is stable. In this case, many trajectories starting from the vicinity of Σ^{2L} are expected to stay in a tiny region near Σ^{2L}. In this case, the collective submanifold is embedded in the "collective" regime.

ii) If $\omega_\alpha(\eta,\eta^*) \le 0$, Σ^{2L} is unstable. In this case, the representative trajectory under discussion is running on the ridge-line of the total Hamiltonian and trajectories starting from the vicinity of Σ^{2L} develop quite differently, showing the stochastic behavior.

The ***separability condition*** between the relevant and irrelevant degrees of freedom is given by $\left\{\mathcal{H}_{\mathrm{coll}}\,,\ \omega_\alpha(\eta,\eta^*)\right\}_{\mathrm{P.B.}} = \left\{\mathcal{H}_{\mathrm{coll}},\ \mathcal{H}^{(2)}\right\}_{\mathrm{P.B.}} = 0$. The full explanation of the separability condition will be discussed in Eq.(2.39) in the next subsection.

With the aid of these two conditions, one may study whether the collective submanifold has physical significance or not. The number of relevant variables L necessary for describing the representative trajectory is defined in such a way that the corresponding collective submanifold Σ^{2L} with dimension 2L should be the smallest submanifold satisfying both the separability and stability conditions.

2.4 DCC System with Kinematical Choice

In subsection 2.2, we have discussed how to determine the functional form of $\{C_j^{(0)}, C_j^{*(0)}\} = \{[C_j],\ [C^*_j]\}$ with respect to $\{\eta_a, \eta^*_a\}$. In order to get the DCC system, we have to determine

the higher order terms $\{C_j^{(n)}, C_j^{*(n)};\ n=1,2,\ldots\}$ as well as $\{\mathfrak{S}^{(n)};\ n=1,2,\ldots\}$. It has been shown[9] that these higher order terms can be defined in such a kinematical way that the higher order canonical variable conditions

$$\sum_j \sum_{\substack{n_1,n_2 \\ (n_1+n_2=n)}} \left\{ C_j^{(n_1)} \frac{\partial C_j^{(n_2)*}}{\partial \eta_a^*} - C_j^{(n_1)*} \frac{\partial C_j^{(n_2)*}}{\partial \eta_a^*} \right\} = \delta_{n,0}\, \eta_a + 2i \frac{\partial \mathfrak{S}^{(n)}}{\partial \eta_a^*}, \tag{2.33}$$

$$\sum_j \sum_{\substack{n_1,n_2 \\ (n_1+n_2=n)}} \left\{ C_j^{(n_1)} \frac{\partial C_j^{(n_2)*}}{\partial \xi_\alpha^*} - C_j^{(n_1)*} \frac{\partial C_j^{(n_2)*}}{\partial \xi_\alpha^*} \right\} = \delta_{n,2}\, \xi_\alpha + 2i \frac{\partial \mathfrak{S}^{(n)}}{\partial \xi_\alpha^*}, \tag{2.34}$$

which are obtained from Eq. (2.16) by means of a power expansion with respect to the irrelevant variables, are satisfied order by order. Here, it should be mentioned that the number of unknown variables contained in $C_j^{(n)}, C_j^{*(n)}$ and $\mathfrak{S}^{(n)}$ are much larger than that of conditions derived from Eqs. (2.33) and (2.34). In this sense, the DCC system defined so as to satisfy the canonical variable condition alone is called the DCC system with kinematical choice.

2.5 DCC System with Maximum Integrable Form

In subsection 2.4, we have discussed how to get the DCC system with kinematical choice. The irrelevant part of the DCC system thus defined has no special dynamical meaning, even though the separation between the relevant and irrelevant variables, i.e. the extraction of collective submanifold Σ^{2L} performed by the SCC method has definite physical significance. Namely, the canonical coordinate system thus defined is not unique, because the separation defined by the SCC method is invariant under any canonical transformation among either relevant variables or among irrelevant variables. By using this freedom of choice, we may ask ***whether or not the system is integrable*** [12].

In order to define the DCC system with definite dynamical meaning, let us first determine the canonical transformation among relevant variables;

$$\text{Canonical Transformation} \quad \{\eta_a, \eta_a^*\} \Rightarrow \{\eta_a', \eta_a'^*\}. \tag{2.35}$$

This transformation is obtained by imposing

Requirement 3: The resultant collective Hamiltonian $\mathfrak{H}'_{\text{coll}}$ after the transformation in Eq. (2.35) depends on collective action variables alone.

With the aid of canonical transformation in Eq.(2.35) thus determined, the Hamiltonian is transformed as

$$\mathfrak{H}_{coll}(\eta,\eta^*)+\mathfrak{H}^{(2)} \Rightarrow \mathfrak{H}'_{coll}(N_1,\cdots,N_L)+\mathfrak{H}'^{(2)} \ , \tag{2.36}$$

where N_a means a relevant action variable given by

$$N_a = \eta_a^*\eta_a = \eta_a^{'*}\eta_a' . \tag{2.37}$$

This canonical transformation always exists provided the perturbative treatment of the Birkhoff-Gustavson normal-form expansion method[11] is applicable. Secondly, we determine a canonical transformation among irrelevant variables, i.e.

$$\text{Canonical Transformation} \quad \{N_a,\xi_\alpha,\xi_\alpha^*\} \Rightarrow \{\tilde{N}_a,\tilde{\xi}_\alpha,\tilde{\xi}_\alpha^*\} \ . \tag{2.38}$$

This transformation is fixed by imposing

Requirement 4: There holds the ***separability condition*** given by

$$\{\mathfrak{H}'_{coll.}, \mathfrak{H}'^{(2)}\}_{P.B.} = 0 , \tag{2.39}$$

where $\mathfrak{H}''_{coll}$ and $\mathfrak{H}''^{(2)}$ denote the resultant collective and intrinsic Hamiltonians obtained after the transformation (2.38), respectively.

Corresponding to the canonical transformation (2.38), the Hamiltonian is transformed as

$$\mathfrak{H}'_{coll}(N)+\mathfrak{H}'^{(2)} \Rightarrow \mathfrak{H}''_{coll}(\tilde{N})+\mathfrak{H}''^{(2)} \ , \quad \mathfrak{H}''^{(2)} = \Sigma_\alpha \omega_\alpha(\tilde{N})\,\tilde{n}_\alpha \ , \tag{2.40}$$

where

$$\tilde{N}_a = N_a \ , \ \tilde{n}_\alpha = \tilde{\xi}_\alpha^*\tilde{\xi}_\alpha \ . \tag{2.41}$$

It is obvious from the above discussion that the last coordinate system $\{\tilde{N}_a,\tilde{\xi}_\alpha,\tilde{\xi}_\alpha^*\}$ is defined such that both the collective and intrinsic Hamiltonians are expressed by only the collective (relevant) and intrinsic (irrelevant) action variables. Since the resultant Hamiltonian clearly shows an integrability of the system, the canonical coordinate system $\{\tilde{N}_a,\tilde{\xi}_\alpha,\tilde{\xi}_\alpha^*\}$ is called the DCC system with ***maximum integrable-form representation***[12].

Here, it should be mentioned that any canonical transformation can't meet Requirement 4 if there hold ***resonance conditions***. In the language of single-particle dynamics, the resonance conditions are satisfied at the level crossing points. Consequently, an essentially non-integrable interaction manifests by itself at the level-crossing point. In other words, a violation of the separability condition means an appearance of non-integrable motion, indicating an onset of classical chaos. As is easily understood, the resonance points just correspond to the "weak stability

points". According to Prigogine[13], when there appear many weak stability points, the ***physics of being***, i.e. the dynamics described by a single trajectory organized by the Hamilton equation of motion should be replaced by the ***physics of becoming***, i.e. the dynamics described by a distribution function following the Liouville equation. Consequently, the level crossing dynamics may play a decisive role in studying an important question why statistical or probabilistic features result from deterministic dynamics[9]. Since this interesting subject is not directly related to the present lecture, it will be discussed elsewhere.

2.6 SCC Method in TDHF Theory

In the preceding four subsections from 2.2 to 2.5, we have discussed the "classical" theory of nuclear collective dynamics within the classical dynamics by using the symplectic structure of the TDHF-manifold. However, this does not necessarily means that we are dealing with the ***classical limit*** of the quantum system, because all the equations discussed in the classical theory are restated within the single-particle dynamics. Let us reformulate the SCC method within the TDHF theory. This reformulation clearly shows that the SCC method gives a foundation of the single-particle states in the nuclear potential with a specific symmetry, e.g. the Routhian in the cranked Nilsson model.

By using the canonical variables $\{C_j, C^*_j\}$, the TDHF equation (2.1) is expressed as

$$\delta\left\langle C,C^*\right|\left(i\frac{\partial}{\partial t} - \hat{H}\right)\left|C,C^*\right\rangle = 0 \ , \tag{2.42}$$

where

$$|C,C^*\rangle = |f(t)\rangle \ . \tag{2.43}$$

According to the discussion in § 2.2, the TDHF equation on the collective submanifold Σ^{2L} (which is equivalent to Eq. (2.23a) describing the representative trajectory on Σ^{2L}) is given by

$$\delta\left\langle [C],[C^*]\right|\left(i\sum_a\left\{\dot{\eta}_a\frac{\partial}{\partial\eta_a}+\dot{\eta}^*_a\frac{\partial}{\partial\eta^*_a}\right\} - \hat{H}\right)\left|[C],[C^*]\right\rangle = 0 \ , \tag{2.44}$$

because the time dependence of $\{[C],[C^*]\}$ is supposed to be described by the collective variables $\{\eta_a, \eta^*_a\}$ alone. In Eq. (2.44),

$$\left|[C],[C^*]\right\rangle = |[f]\rangle = \exp\left\{i[\hat{F}]\right\}|\phi_0\rangle \ , \ [\hat{F}] = \sum_{\mu,i}\left\{[f_{\mu i}]a^\dagger_\mu b^\dagger_i + [f^*_{\mu i}]b_i a_\mu\right\} . \tag{2.45}$$

Let us introduce the following one-body collective operators defined by

$$\hat{O}_a^\dagger = \left(\frac{\partial}{\partial \eta_a} e^{i[\hat{F}]}\right) \cdot e^{-i[\hat{F}]} \ , \quad \hat{O}_a = -\left(\frac{\partial}{\partial \eta_a^*} e^{i[\hat{F}]}\right) \cdot e^{-i[\hat{F}]} \ , \tag{2.46}$$

which have the same structure as those defined in Eq. (2.7). Apparently, these collective operators change their microscopic structure depending on the amplitudes of collective variables $\{\eta_a, \eta_a^*\}$. Substituting Eq. (2.46) for Eq. (2.44), we get

$$\delta\left\langle [C],[C^*]\right| \left(i\sum_a \left\{\dot{\eta}_a \hat{O}_a^\dagger - \dot{\eta}_a^* \hat{O}_a\right\} - \hat{H}\right) \left|[C],[C^*]\right\rangle = 0. \tag{2.47}$$

Now, let us discuss how to express the canonical variable condition (2.21) within the TDHF theory. Similar to Eq. (2.8), the collective variables $\{\eta_a, \eta_a^*\}$ are certified to be of canonical, provided there hold the following relations;

$$\left\langle [C],\ [C^*]\right| \hat{O}_a^\dagger \left|[C],\ [C^*]\right\rangle = \frac{1}{2}\eta_a^* - i\frac{\partial[S]}{\partial \eta_a}. \tag{2.48}$$

Equation (2.48) represents the ***canonical variable condition*** and is equivalent to Eq. (2.21). From Eq. (2.48), we get the following "weak" boson-like commutation relation

$$\left\langle [C],[C^*]\right| [\hat{O}_b\ ,\ \hat{O}_a^\dagger] \left|[C],[C^*]\right\rangle = \delta_{ab}\ ,$$

$$\left\langle [C],[C^*]\right| [\hat{O}_b\ ,\ \hat{O}_a] \left|[C],[C^*]\right\rangle = 0. \tag{2.49}$$

It should be noticed that these relations hold at any value of (η, η^*), whereas the conventional RPA modes satisfy the boson-like commutation relation at one point of the stationary Hartree-Fock state $|\phi_0>$. This desirable property of the collective operators assures that they are ***global*** operators, whereas the RPA modes are ***local*** operators. Taking the variation of $|[C],[C^*]>$ in Eq. (2.47) with respect to η_b and η^*_b, we get

$$\left\langle [C],[C^*]\right| [\hat{O}_b, \left(i\sum_a \left\{\dot{\eta}_a \hat{O}_a^\dagger - \dot{\eta}_a^* \hat{O}_a\right\} - \hat{H}\right)] \left|[C],[C^*]\right\rangle = 0\ ,$$

$$\left\langle [C],[C^*]\right| [\hat{O}_b^\dagger, \left(i\sum_a \left\{\dot{\eta}_a \hat{O}_a^\dagger - \dot{\eta}_a^* \hat{O}_a\right\} - \hat{H}\right)] \left|[C],[C^*]\right\rangle = 0\ . \tag{2.50}$$

By using Eq. (2.49), Eq. (2.50) reduces into "classical" equation of motion

$$i\dot{\eta}_a = \left\langle [C],[C^*] \right| \ [\hat{O}_a, \hat{H}] \ \left| [C],[C^*] \right\rangle = \frac{\partial [H]}{\partial \eta_a^*},$$

$$i\dot{\eta}_a^* = \left\langle [C],[C^*] \right| \ [\hat{O}_a^\dagger, \hat{H}] \ \left| [C],[C^*] \right\rangle = -\frac{\partial [H]}{\partial \eta_a} \ , \tag{2.51}$$

which is equivalent to Eq.(2.23b). Here,

$$[H] \equiv \mathcal{H}^{(0)} = \left\langle [C],[C^*] \right| \ \hat{H} \ \left| [C],[C^*] \right\rangle \ . \tag{2.52}$$

Substituting Eq. (2.51) for Eq. (2.47), we get

$$\delta \left\langle [C],[C^*] \right| \left(\sum_a \left\{ \frac{\partial [H]}{\partial \eta_a} \hat{O}_a^\dagger + \frac{\partial [H]}{\partial \eta_a^*} \hat{O}_a \right\} - \hat{H} \right) \left| [C],[C^*] \right\rangle = 0 \ , \tag{2.53}$$

which represents the ***equation of collective submanifold*** and is equivalent to Eq. (2.22). If one takes a variation in Eq. (2.53) toward the relevant direction, one gets the equation of collective motion in Eq. (2.51). If one takes a variation toward the irrelevant direction, on the other hand, one obtains a Hartree-Fock type stationary condition equivalent to the maximal-decoupling condition (2.30).

Here, it is worthwhile to point out that Eq. (2.53) is nothing but a generalization of the constrained Hartree-Fock equation given by

$$\delta \langle \omega | \ \omega \hat{J} - \hat{H} \ | \omega \rangle = 0, \quad \delta \langle q | \ q \hat{Q} - \hat{H} \ | q \rangle = 0 \ . \tag{2.54}$$

In the conventional approach, the constraining (collective) operators in Eq. (2.54) have fixed structure and are put in by hand. On the contrary, the microscopic structure of the collective operators in Eq. (2.46) changes depending on the amplitude of the collective motion and they are determined dynamically by means of the SCC method, which properly takes account of the non-linear, microscopic single-particle dynamics in the TDHF theory. Namely, the SCC method can extract the dynamical collective coordinates, on which the system under discussion develops by itself.

It should be also noticed that the state |[C], [C*]> in Eq. (2.53) satisfies the Hartree-Fock stationary condition. Namely, the auxiliary Hamiltonian in the generalized moving frame defined by

$$\hat{H}_{\text{moving}} \ = \ \hat{H} - \left\{ \ \omega \hat{J} \ \right\}_{\text{generalize}} \ , \tag{2.55}$$

with the generalized cranking term

$$\left\{ \omega\hat{J} \right\}_{\text{generalize}} = \sum_a \left\{ \frac{\partial[H]}{\partial\eta_a}\hat{O}_a^\dagger + \frac{\partial[H]}{\partial\dot{\eta}_a}\hat{O}_a \right\}, \tag{2.56}$$

has following property; the Hartree-Fock Hamiltonian (, i.e. the single-particle part of the auxiliary Hamiltonian in Eq. (2.55) with respect to the single Slater determinant state |[C], [C*]>) has no particle-hole components with respect to|[C], [C*]>. This is due to the fact that there holds the stationary relation (2.53). In this case, one may introduce a set of single-particle states at any point on |[C], [C*]>, i.e. at any value of (η, η^*). This (η,η^*)-dependent single-particle states may give a microscopic foundation of the single-particle states (Routhian) in the nuclear potential with arbitrary symmetry, e.g. Nilsson model, rotating shell model and etc.

3. Quantum Theory of Nuclear Collective Dynamics

3.1 Quantum Integrability

In §2, we have discussed that the extraction of an approximate integral surface Σ^{2L} (corresponding to the KAM torus) for a given trajectory, and the investigation on the stability and separability conditions of Σ^{2L} provide us with a powerful method in understanding the microscopic dynamics of nuclear collective motion within the mean-field approximation. It has been also pointed out that the order-to-chaos transition mechanism in classical dynamical system plays a decisive role in understanding the single-particle dynamics of the TDHF theory.

In contrast with the classical chaos, the investigation on the quantum chaos is seemed to be still in its immature stage. However, there are some attempts to properly define the concept of quantum integrability, a counter concept of quantum chaos, on the basis of both the quantum group approach and group theoretical consideration[1]. On the other hand, there is a well establish method[14] for describing statistical aspects of an ensemble of eigenstates in terms of the random matrix theory. As is well known, the nearest neighbor level spacing statistics of neutron resonance states is well described by the gaussian orthogonal ensemble (GOE)[15]. As an argument relevant to the quantum chaos, a following empirical correspondence is frequently stated[16]: If the classical system is integrable, the corresponding quantized system shows the Poisson distribution in the statistics of the nearest level spacing. If it is non-integrable or chaotic, on the other hand, its quantum correspondent shows the Wigner distribution. On the basis of this empirical rule, it is widely accepted that the order-to-chaos transition mechanism in quantum system can be studied in terms of a transition from the Poisson to the Wigner distributions. Here, it should be noticed that the above discussion treats a statistical aspects of an ***ensemble*** of eigenstates, whereas the classical order-to-chaos transition is discussed by

using the dynamical properties exhibited by *individual* trajectories. Moreover, the discussion based on GOE introduces some probabilistic assumptions from the beginning.

In this section, we will define the quantum integrability and quantum chaoticity for individual eigenstates on the basis of the quantum theory of nuclear collective dynamics[17] which has been developed by extending the basic idea of the classical theory discussed in §2.

3.2 Basic Idea of Quantum Theory

In the classical theory, an introduction of the DCC system associated with Σ^{2L} plays a central role in characterizing the integrability of the system. In a case of the quantum system, we will discuss an importance of introducing the concept of ***dynamical representation*** (DR)[17] associated with a collective subspace under discussion.

Let us start with the boson system described by K-kinds of boson operators

$$\{B_j, B_j^\dagger;\ j=1,\ldots,K\} \quad . \tag{3.1}$$

Since the even-number many-fermion system is known to be transformed into the boson system by means of the boson expansion theory[18], it is general enough to formulate a quantum theory of nuclear collective dynamics within the boson system. The representation using $(B,B^\dagger)$ will be called an ***initial representation*** (IR). The IR is a kinematical representation and has nothing to do with a specific correlation characteristic to the Hamiltonian $H(B^\dagger,B)$. The boson Hilbert space $\mathcal{V}^{2K}$ is spanned by

$$|n_1,\ldots,n_K\rangle \equiv \prod_{j=1}^{K} \frac{1}{\sqrt{n_j}} (B_j^\dagger)^{n_j} |0\rangle \ , \tag{3.2}$$

where $|0\rangle$ denotes the vacuum of the boson operators $\{B^\dagger,B\}$, i.e. $B_j|0\rangle = 0$. The dynamics of the system is governed by the Heisenberg equation of motion

$$i\dot{B}_j^\dagger(t) = \left[B_j^\dagger(t),\ H(B^\dagger,B) \right] \ , \tag{3.3}$$

where the time-dependent operator $B^\dagger_j(t)$ is defined as

$$B_j^\dagger(t) \equiv e^{iH(B^\dagger,B)t} B_j^\dagger e^{-iH(B^\dagger,B)t} \ . \tag{3.4}$$

Let us consider an ideal situation where a set of collective excited states constitutes an invariant subspace (, i.e. an ideal collective subspace) $\mathcal{V}^{2L}$ (L<<K) of the Hamiltonian. In this case, there

must be an optimal representation called the DR where the minimum number of relevant operators is required in describing $\mathcal{T}^{2L}$. Let

$$\{\alpha_j, \alpha_j^\dagger;\ j = 1,\ldots,L\} \ , \tag{3.5a}$$

be a set of relevant (collective) operators necessary for spanning $\mathcal{T}^{2L}$ and let

$$\{\beta_j, \beta_j^\dagger;\ j = L+1,\ldots,K\} \ , \tag{3.5b}$$

be the remaining operators called the irrelevant (intrinsic) operators in the DR. In the DR, the dynamics are still governed by $H(B^\dagger,B)$. Namely, there holds

$$\begin{aligned} i\dot{\alpha}_j^\dagger(t) &= [\alpha_j^\dagger(t)\ , H(B^\dagger,B)]\ ; \qquad j = 1,..L, \\ i\dot{\beta}_j^\dagger(t) &= [\beta_j^\dagger(t)\ , H(B^\dagger,B)]\ ; \qquad j = L+1,...K, \end{aligned} \tag{3.6}$$

where the time-dependent operators in the DR are given by

$$\alpha_j^\dagger(t) \equiv e^{iH(B^\dagger,B)t}\alpha_j^\dagger e^{-iH(B^\dagger,B)t} \ , \quad \beta_j^\dagger(t) \equiv e^{iH(B^\dagger,B)t}\beta_j^\dagger e^{-iH(B^\dagger,B)t} \ . \tag{3.7}$$

In the DR, the Hamiltonian is expressed as

$$\begin{aligned} H(B^\dagger,B) &= e^{F(\alpha^\dagger,\alpha;\beta^\dagger,\beta)} H(\alpha^\dagger,\alpha;\beta^\dagger,\beta) e^{-F(\alpha^\dagger,\alpha;\beta^\dagger,\beta)} \\ &= \mathfrak{H}_{coll}(\alpha^\dagger,\alpha) + \mathfrak{H}_{irr}(\beta^\dagger,\beta) + \mathfrak{H}_{coupl}(\alpha^\dagger,\alpha,\beta^\dagger,\beta) \ . \end{aligned} \tag{3.8}$$

Here $\exp\{F(B^\dagger,B)\} = \exp\{F(\alpha^\dagger,\alpha,\beta^\dagger,\beta)\}$ denotes the non-linear unitary transformation between IR and DR;

$$\alpha_j^\dagger = e^{-F(B^\dagger,B)} B_j^\dagger e^{F(B^\dagger,B)} \ , \ j = 1,\ldots,L\ ; \ \beta_j^\dagger = e^{-F(B^\dagger,B)} B_j^\dagger e^{F(B^\dagger,B)} \ , \ j = L+1,\ldots K \ . \tag{3.9}$$

The dynamical condition specifying the invariant subspace is obviously given by the ***complete decoupling condition*** between the relevant and irrelevant operators;

$$\text{(A - I)} \quad \mathfrak{H}_{coupl}(\alpha^\dagger,\alpha,\beta^\dagger,\beta) = 0 \ . \tag{3.10}$$

With the aid of condition (3.10), the time-dependent operators in Eq. (3.7) are reduced into

$$\alpha_j^\dagger(t) = e^{i\mathcal{H}_{coll}t}\alpha_j^\dagger e^{-i\mathcal{H}_{coll}t} \quad , \quad \beta_j^\dagger(t) = e^{i\mathcal{H}_{irr}t}\beta_j^\dagger e^{-i\mathcal{H}_{irr}t}. \tag{3.11}$$

satisfying

(A - II)

$$i\dot{\alpha}_j^\dagger(t) = [\alpha_j^\dagger(t), \mathcal{H}_{coll}(\alpha^\dagger,\alpha)] \; ; \quad j = 1,...,L \quad ,$$

$$i\dot{\beta}_j^\dagger(t) = [\beta_j^\dagger(t), \mathcal{H}_{irr}(\beta^\dagger,\beta)] \; ; \; j = L+1,...,K. \tag{3.12}$$

Namely, the relevant and irrelevant operators are governed separately by $\mathcal{H}_{coll}$ and $\mathcal{H}_{irr}$, which satisfy the following commutability condition

(A - III) $$[\mathcal{H}_{coll}(\alpha^\dagger,\alpha), \mathcal{H}_{irr}(\beta^\dagger,\beta)] = 0 \, . \tag{3.13}$$

Equations (A-I), (A-II) and (A-III) constitute a set of basic relations characterizing the invariant subspace $\mathcal{V}^{2L}$ of the Hamiltonian. From the above discussion, it is clear that an introduction of the DR for a given invariant subspace $\mathcal{V}^{2L}$ is decisive so as to manifest an invariant property of the Hamiltonian.

3.3 Basic Equations for Determining the Dynamical Representation

Let us discuss the general case where the complete decoupling condition (3.10) does not necessarily hold. Namely, we intend to extract a set of collective states which form an approximate invariant subspace of the Hamiltonian. To this aim, we start with the resultant DR where the collective subspace D^{2L}: $\{|m_1,m_2,...m_L\rangle\}$ is expressed as

$$|m_1,m_2,...,m_L\rangle \; = \; \prod_{j=1}^{L} \frac{1}{\sqrt{m_j!}} \left(\alpha_j^\dagger\right)^{m_j} |0\rangle \; , \tag{3.14}$$

where $|0\rangle = \exp\{-F(B^\dagger, B)\}|0)$. In order to get the DR, i.e. to determine a generating function $F(B^\dagger,B) = -F^\dagger(B^\dagger,B)$ of the unitary transformation in Eq. (3.9), we have to discuss how to characterize $F(B^\dagger,B)$.

We start with introducing the following new notation for the vacuum state;

$$|0\rangle \; = \; |0\rangle\rangle_R \otimes |0\rangle\rangle_I \; , \tag{3.15}$$

which satisfy $\alpha_j|0>>_R=0$ and $\beta_j|0>>_I=0$. In this notation, the total Hilbert space is expressed as a product space whose basis states are represented as

$$|m_1,m_2,\ldots,m_L;n_{L+1},n_{L+2},\ldots,n_K> = |m_1,m_2,\ldots,m_L>>_R \otimes |n_{L+1},n_{L+2},\ldots,n_K>>_I$$

$$= \prod_{j=1}^{L} \frac{1}{\sqrt{m_j}} \left(\alpha_j^\dagger\right)^{m_j} |\,0\,>>_R \otimes \prod_{j=L+1}^{K} \frac{1}{\sqrt{n_j}} \left(\beta_j^\dagger\right)^{n_j} |\,0\,>>_I \quad . \tag{3.16}$$

With the aid of this notation, the Hamiltonian is decomposed into

$$H(B^\dagger,B) = e^{F(\alpha^\dagger,\alpha;\beta^\dagger,\beta)} H(\alpha^\dagger,\alpha;\beta^\dagger,\beta)\, e^{-F(\alpha^\dagger,\alpha;\beta^\dagger,\beta)}$$

$$= \mathcal{H}_{coll}\,(\alpha^\dagger,\alpha) + \Delta\mathcal{H} \quad , \tag{3.17}$$

where

$$\mathcal{H}_{coll}\,(\alpha^\dagger,\alpha) = {}_I<<0|\, H(B^\dagger,B)\,|0>>_I \quad . \tag{3.18}$$

It is obvious that $\mathcal{H}_{coll}$ satisfies

$$\left[\mathcal{H}_{coll}\,,\,\beta_j^\dagger\right] = 0 \,. \tag{3.19}$$

In order to get the basic equations specifying the most optimum representation, we have to employ some expansion scheme; Since the relevant operators are supposed to describe the large-amplitude collective motion within D^{2L} whereas the irrelevant operators describe the small-fluctuation around D^{2L}, it is reasonable to introduce a power series expansion with respect to the irrelevant boson operators. For example, $\Delta\mathcal{H}$ is expanded as

$$\Delta\mathcal{H} = \sum_{n=1} \mathcal{H}(n),$$

$$\mathcal{H}(1) = \sum_{j=L+1} \left\{ \beta_j^\dagger {}_I\langle\langle 0|[\beta_j,\, H(B^\dagger,B)]|0\rangle\rangle_I + \text{h.c.} \right\},$$

$$\mathcal{H}(2) = \sum_{j,k=L+1} \beta_j^\dagger {}_I\langle\langle 0|[\beta_j,[H(B,B),\, \beta^\dagger{}_k]]|0\rangle\rangle_I \beta_k$$

$$+ \sum_{j,k=L+1} \mathcal{N}_{jk}^{-1} \left\{ \beta_j^\dagger \beta_k^\dagger {}_I\langle\langle 0|[\beta_k\beta_j, H(B^\dagger,B)]|0\rangle\rangle_I + \text{h.c.} \right\}, \text{ and etc.} \tag{3.20}$$

Here, the normalization constant is given by

$$\mathcal{N}_{jk}=_I\langle\langle 0|\beta_k\beta_j\beta_j^\dagger\beta_k^\dagger|0\rangle\rangle_I \ . \tag{3.21}$$

Within the first order expansion in $\Delta\mathcal{H}$, the time-dependent operators in Eq. (3.7) are expressed as

$$\begin{aligned}\alpha_j^\dagger(t) \ &\equiv e^{iH(B^\dagger,B)t}\alpha_j^\dagger e^{-iH(B^\dagger,B)t} \\ &\approx e^{i\mathcal{H}_{coll}\ t}\alpha_j^\dagger e^{-i\mathcal{H}_{coll}\ t} \\ &\quad +[\int_0^1 d\lambda\, e^{\lambda(i\mathcal{H}_{coll}\ t)}i\Delta\mathcal{H}\ t\, e^{-\lambda(i\mathcal{H}_{coll}\ t)},\ e^{i\mathcal{H}_{coll}\ t}\alpha_j^\dagger e^{-i\mathcal{H}_{coll}\ t}] \ , \end{aligned} \tag{3.22a}$$

$$\begin{aligned}\beta_j^\dagger(t) \ &\equiv e^{iH(B^\dagger,B)t}\beta_j^\dagger e^{-iH(B^\dagger,B)t} \\ &\approx \beta_j^\dagger \ +[\int_0^1 d\lambda\, e^{\lambda(i\mathcal{H}_{coll}\ t)}i\Delta\mathcal{H}\ t\, e^{-\lambda(i\mathcal{H}_{coll}\ t)},\ \beta_j^\dagger] \ . \end{aligned} \tag{3.22b}$$

If there holds the ***maximal-decoupling condition***

$$\text{(B - I)} \qquad \mathcal{H}(1)=0 \ , \tag{3.23}$$

the relevant time-dependent operators in Eq. (3.22a) satisfy the following equation of motion given as

$$\text{(B - IIa)} \qquad i\frac{d}{dt}\alpha_j^\dagger(t)=[\alpha_j^\dagger(t),\ \mathcal{H}_{coll}] \ , \tag{3.24}$$

within the approximation of neglecting the second order effects with respect to the irrelevant operators. Further more, the irrelevant time-dependent operator in Eq. (3.22b) satisfy

$$\text{(B - IIb)} \qquad i\frac{d}{dt}\beta_j^\dagger(t)=[\beta_j^\dagger(t),\ \mathcal{H}(2)] \ , \tag{3.25}$$

provided that there holds the ***separability condition*** expressed as

$$\text{(B - III)} \qquad [\mathcal{H}_{coll},\ \mathcal{H}(2)]=0 \ . \tag{3.26}$$

By comparing Eqs. (B-I),(B-II) and (B-III) with Eqs. (A-I),(A-II) and (A-III), it is easily seen that the maximal-decoupling condition (3.23) and the separability condition (3.26) just correspond to the complete-decoupling condition (3.10) and the commutability condition (3.13), respectively. Furthermore, they are quantum analogue of Eq. (2.30) and Eq. (2.39), respectively. Equations (B-I), (B-II) and (B-III) constitute the basic equations of quantum theory of nuclear collective dynamics[17].

3.4 Dynamical Representation and Validity of Collective Subspace D^{2L}

It has been shown[17] that the maximal-decoupling condition (B-I) gives necessary conditions for determining the functional form of $F(B^\dagger,B)$ which specifies the unitary transformation between IR and DR. Here it should be noticed that the canonical variable condition (2.16) has played an important role in the classical theory discussed in §2, together with the equation of collective submanifold given by Eq. (2.22). In the quantum theory, however, there are no equations corresponding to the canonical variable condition, because the unitarity of the transformation is automatically imposed by Eq. (3.9).

According to the quantum theory of nuclear collective dynamics[17], the generating function $F(B^\dagger,B)$ specifying the DR is determined so as to satisfy the maximal decoupling condition (B-I), under the boundary condition that the relevant and irrelevant operators are reduced to

$$\lim(\alpha_j^\dagger) = B_j^\dagger \ , \quad \text{for } j = 1,...,L \ ; \ \lim(\beta_j^\dagger) = B_j^\dagger \ , \quad \text{for } j = L+1,...,K \ , \tag{3.27}$$

in the small-amplitude region, respectively. This boundary condition is only one physical input of the theory in specifying the collectivity under discussion. A detailed discussion how to get the unitary transformation is found in Ref. 17. The usefulness and feasibility of the DR has been demonstrated[19] by applying it to the pairing problem and the phase transition from normal to super phases is well described by the quantum theory of collective dynamics.

We are now at the position to discuss the validity of D^{2L} which is spanned by the basis states defined by Eq. (3.14) or Eq. (3.16) in terms of the relevant operators thus obtained. After getting an explicit expression of $F(B^\dagger,B)$, we have a concrete forms of $\mathcal{H}_{coll}$ and $\Delta\mathcal{H}$. With the aid of an explicit expression of $\mathcal{H}_{coll}$, we have a set of collective excited states by diagonalizing $\mathcal{H}_{coll}$ within D^{2L};

$$\mathcal{H}_{coll}|\mu\rangle\rangle_R = E_\mu|\mu\rangle\rangle_R, \quad |\mu\rangle\rangle_R \in D^{2L}:\{|m_1,m_2,...,m_L\rangle\rangle_R\}. \tag{3.28}$$

With the aid of the collective excited states $|\mu>>_R$ thus introduced, the ***separability condition*** (3.26) is expressed as

$$ {}_R\langle\langle\mu|\,[\mathcal{H}_{coll},\mathcal{H}(2)]\,|\mu'\rangle\rangle_R = 0, \quad \mu \neq \mu'. \tag{3.29} $$

Equation (3.29) states that the collective states $|\mu>>_R$ defined by $\mathcal{H}_{coll}$ should not be disturbed under the effect of $\mathcal{H}(2)$ and gives the first condition to evaluate the validity of collective states.

If there holds the separability condition (3.29), it is easily derived that $\mathcal{H}(2)$ has only diagonal terms with respect to collective states, i.e., $\mathcal{H}(2)$ is expressed as

$$ \mathcal{H}(2) = \sum_{\mu} \mathcal{H}_{\mu}(2) \ , $$

$$ \mathcal{H}_{\mu}(2) = \sum_{j,k\geq L+1} \mathcal{N}^{-1}_{jk} \left\{ \beta^{\dagger}_j \beta^{\dagger}_k |\mu\rangle\langle\mu| \cdot f^{jk}_{\mu} + \text{h.c.} \right\} + \sum_{j,k\geq L+1} \beta^{\dagger}_j |\mu\rangle \cdot g^{jk}_{\mu} \cdot \langle\mu|\beta_k \ . \tag{3.30} $$

Here,

$$ |\mu\rangle = |\mu\rangle\rangle_R \otimes |0\rangle\rangle_I \ , \quad f^{jk}_{\mu} = \langle\mu|[\beta_k\beta_j, H(B,B)]|\mu\rangle \ , \quad g^{jk}_{\mu} = \langle\mu|[\beta_j,[H(B,B),\beta^{\dagger}_k]]|\mu\rangle. \tag{3.31} $$

Introducing a general coherent state for the irrelevant part of the wave function given by

$$ |\mu,\gamma\rangle = |\mu\rangle\rangle_R \otimes e^{\sum_{j=L+1}\left(\gamma_j\beta^{\dagger}_j - \gamma^{*}_j\beta_j\right)}|0\rangle\rangle_I \ , \tag{3.32} $$

we have

$$ \langle\mu,\gamma|\,\mathcal{H}_{\mu}(2)\,|\mu,\gamma\rangle = \frac{1}{2} e^{-\sum_j \gamma^{*}_j\gamma_j} (\gamma^{*},\gamma) S_{\mu} \begin{pmatrix} \gamma \\ \gamma^{*} \end{pmatrix} , \tag{3.33} $$

where

$$ S_{\mu} \equiv \begin{pmatrix} g^{jk}_{\mu} & f^{jk}_{\mu} \\ f^{jk^{*}}_{\mu} & g^{jk}_{\mu} \end{pmatrix} \ . \tag{3.34} $$

Even though the matrix S_μ in Eq. (3.34) has the same structure as the usual stability matrix appearing in the RPA theory, it should be noticed that S_μ has only components between irrelevant degrees of freedom. With the aid of the stability matrix S_μ in Eq. (3.34), we may study whether or not the collective state $|\mu>>_R$ defined by Eq. (3.28), or more generally, the

collective subspace D^{2L} is stable with respect to small variations toward the irrelevant directions. Namely, the ***stability condition*** is given by

$$S_\mu \geq 0 \ , \quad \text{for each collective state } |\mu\rangle\rangle_R \ , \tag{3.35}$$

which is the second condition to evaluate the validity of collective states.

At the end of this subsection, it should be stated that the number of relevant operators 2L necessary for describing the collective states under consideration is determined in such a way that the corresponding subspace D^{2L} should be the smallest subspace where both the stability and separability conditions are satisfied.

3.5 How to Evaluate Stochasticity of Individual Eigenfunctions

In the previous subsection, we have discussed how to estimate the validity of D^{2L}. Namely, our discussion is limited to the collective states $|\mu\rangle\rangle_R \otimes |n=0\rangle\rangle_I$ with no intrinsic excitations. In order to introduce the chaoticity of individual eigenfunctions, we have to treat the eigenstates which are situating at any part of the boson Hilbert space. In this subsection, we will discuss how to evaluate the property of an arbitrary eigenstate by extending the separability and stability conditions to the whole Hilbert space.

mn-Representation

Aiming at extending the separability and stability conditions given by Eqs.(3.29) and (3.35) into the whole Hilbert space, we will hereafter consider the simplest case with L=1 and K=2, i.e. the system described by one relevant $\alpha^\dagger$ and one irrelevant $\beta^\dagger$ boson operators. The extension to the general case is trivial. In this case, the physical boson space $\mathcal{T}$ corresponding to the fermion space is described as

$$\mathcal{T} : \{|m,n\rangle \, ; \, m+n \leq N\} \ , \tag{3.36}$$

where

$$|m,n\rangle = \frac{1}{\sqrt{m!}} \frac{1}{\sqrt{n!}} \left(\alpha^\dagger\right)^m \left(\beta^\dagger\right)^n |0\rangle \ . \tag{3.37}$$

The projection operator I onto $\mathcal{T}$ is expressed as

$$I = \sum_{\substack{m,n \\ (m+n \leq N)}} |m,n\rangle\langle m,n| \ . \tag{3.38}$$

The representation using the basis states {|m,n>} is called the mn-representation. As is easily recognized, the mn-representation has a sense in every part of the physical boson space, provided the complete decoupling condition (3.10) is satisfied.

μn-Representation

Let us divide the physical boson space $\mathcal{V}$ into multi-layer of subspace $\{\mathcal{D}_n\ ;\ n=0,1,...,N\}$. Here $\mathcal{D}_n$ consists of a set of basis states having the same irrelevant boson number n, i.e.

$$\mathcal{D}_n: \{\,|m,n\rangle\ ;\ m=0,1,...,N-n\,\}\ . \tag{3.39}$$

Here, $\mathcal{D}_{n=0}$ just corresponds to the collective subspace D^{2L} discussed in the previous subsection. The projection operator $\mathcal{P}(n)$ onto $\mathcal{D}_n$ is given by

$$\mathcal{P}(n) = \sum_{m=0}^{N-n} |m,n\rangle\langle m,n|, \quad \mathcal{I} = \sum_{n=0}^{N} \mathcal{P}(n)\ . \tag{3.40}$$

By using the projection operator $\mathcal{P}(n)$, $\Delta\mathcal{H}$ in Eq.(3.17) is divided into two parts

$$\mathcal{I}\Delta\mathcal{H}\mathcal{I} = \Delta\mathcal{H}^{dia} + \Delta\mathcal{H}^{off-dia}\ ,$$

$$\Delta\mathcal{H}^{dia} = \sum_n \mathcal{P}(n)\Delta\mathcal{H}\mathcal{P}(n)\ ,\quad \Delta\mathcal{H}^{off-dia} = \sum_{\substack{n,n' \\ (n\neq n')}} \mathcal{P}(n)\Delta\mathcal{H}\mathcal{P}(n')\ . \tag{3.41}$$

Here, we briefly discuss characteristic roles of each part of the Hamiltonian. As is clearly understood from Eq. (3.19), $\mathcal{H}_{coll}$ commutes with any irrelevant boson operators by definition. Namely, it does not violate the irrelevant quantum number n. It is also the case for the part $\Delta\mathcal{H}^{dia}$ in Eq. (3.41), because it interacts only between two states with the same irrelevant boson number n. Namely, $\mathcal{H}_{coll}$ and $\Delta\mathcal{H}^{dia}$ conserve the irrelevant quantum number n. On the other hand, $\Delta\mathcal{H}^{off-dia}$ plays a role to violate the irrelevant quantum number *n*.

Let us extend the concept of collectivity μ defined in Eq. (3.28) to the whole physical boson space. This task is achieved by diagonalizing $H(B^\dagger,B)$ within the subspace $\mathcal{D}_n$ with arbitrary n,

$$\mathcal{P}(n)H(B^\dagger,B)\mathcal{P}(n)|\mu,n\rangle = \mathcal{P}(n)(\mathcal{H}_{coll} + \Delta\mathcal{H}^{dia})\mathcal{P}(n)|\mu,n\rangle = E_\mu^{(n)}|\mu,n\rangle\ . \tag{3.42}$$

Here, the quantum number μ coincides with the definition in Eq. (3.28) in a case with n=0 and specifies the relevant part of the states in order of their energy eigenvalues;

$$\mu = 0,1,\ldots,N-n \qquad \text{with } E^{(n)}_{\mu=0} \leq E^{(n)}_{\mu=1} \leq \ldots, \leq E^{(n)}_{\mu=N-n} \ . \tag{3.43}$$

With the aid of a set of basis states $|\mu,n\rangle$ thus obtained, the subspace $\mathcal{D}_n$ is alternatively expressed as

$$\mathcal{D}_n : \{\, |\mu,n\rangle \ ; \mu = 0,1,\ldots,N-n \,\} \ . \tag{3.44}$$

The representation using the basis states $|\mu,n\rangle$ is called the μn-representation. Here, it should be mentioned that this representation is the best one may choose in incorporating the dynamical correlations among the relevant bosons into the basis states under the restriction of conserving the number of quantum numbers. It should be also noticed that the μn-representation is a quantum analogue of the canonical coordinate system discussed in Eq. (2.35).

μν-Representation

Since the relevant parts of the basis states $|\mu,n\rangle$ and $|\mu,n'\rangle$ with $n \neq n'$ are expected to have almost the same structure, one may divide the physical boson space $\mathcal{V}$ into a set of vertical-layers of subspaces $\{\mathcal{R}_\mu \ ; \mu=0,1,\ldots,N\}$

$$\mathcal{R}_\mu : \{\, |\mu,n\rangle \ ; n = 0,1,\ldots,N-\mu \,\} \ . \tag{3.45}$$

The projection operator onto $\mathcal{R}_\mu$ is expressed as

$$\mathcal{P}(\mu) = \sum_{n=0}^{N-\mu} |\mu,n\rangle\langle\mu,n| \ , \quad \mathcal{I} = \sum_{\mu=1}^{N} \mathcal{P}(\mu). \tag{3.46}$$

By means of $\mathcal{P}(\mu)$, the Hamiltonian is expressed as

$$\mathcal{I}\, H(B^\dagger,B)\, \mathcal{I} = \sum_{\mu=0}^{N} \sum_{n=0}^{N-\mu} |\mu,n\rangle E^{(n)}_\mu \langle\mu,n| + \Delta\mathcal{H}^{off\text{-}dia} \ , \tag{3.47}$$

where $\Delta\mathcal{H}^{off\text{-}dia}$ splits into two parts

$$\Delta\mathcal{H}^{off\text{-}dia} = \mathcal{H}^{[stab]} + \mathcal{H}^{[sepa]} \ ,$$

$$\mathcal{H}^{[stab]} = \sum_\mu \mathcal{H}^{[stab]}_\mu = \sum_\mu \mathcal{P}(\mu)\Delta\mathcal{H}\,\mathcal{P}(\mu) \ ,$$

$$\mathfrak{H}^{[sepa]} = \sum_{\mu,\mu'(\mu\neq\mu')} \mathfrak{H}^{[sepa]}_{\mu,\mu'} = \sum_{\mu,\mu'(\mu\neq\mu')} \mathcal{P}(\mu)\Delta\mathfrak{H}\mathcal{P}(\mu') . \tag{3.48}$$

In the expression of Eq.(3.48), it is directly recognized that a part $\mathfrak{H}^{[stab]}$ conserves the relevant quantum number μ with violation of the irrelevant quantum number n. For a case with n=0, $\mathfrak{H}^{[stab]}$ acts to induce an instability of the collective subspace $D^{2L}=\mathcal{D}_{n=0}$ (see Eq. (3.35) where the diagonal component of the second order Hamiltonian $\mathfrak{H}_{\mu}(2)$ is discussed). On the other hand, a part $\mathfrak{H}^{[sepa]}$ violates both the relevant quantum number μ and irrelevant quantum number n. From the above discussion, it becomes clear that the dynamical effects coming from $\mathfrak{H}^{[stab]}$ can be incorporated into the basis vectors by diagonalizing $H(B^{\dagger},B)$ within the subspace $\mathcal{R}_{\mu}$, i.e.

$$\mathcal{P}(\mu)H(B^{\dagger},B)\mathcal{P}(\mu)\,|\mu,\nu\rangle = E_{\mu,\nu}|\mu,\nu\rangle \ , \quad |\mu,\nu\rangle \in \mathcal{R}_{\mu}. \tag{3.49}$$

Since there holds

$$\mathcal{P}(\mu)H(B^{\dagger},B)\mathcal{P}(\mu) = \mathcal{P}_{\mu}\left\{\mathfrak{H}_{coll} + \Delta\mathfrak{H}^{dia} + \mathfrak{H}^{[stab]}_{\mu}\right\}\mathcal{P}_{\mu} , \tag{3.50}$$

the dynamical correlations contained in $H(B^{\dagger},B)$ except for a part $\mathfrak{H}^{[sepa]}$ are successfully incorporated into the basis states $|\mu,\nu\rangle$. By using $|\mu,\nu\rangle$, the subspace $\mathcal{R}_{\mu}$ is spanned as

$$\mathcal{R}_{\mu}: \ \{|\mu,\nu\rangle;\nu = 0,1,\ldots,N-\mu\}. \tag{3.51}$$

The representation using the basis states $|\mu,\nu\rangle$ is called the $\mu\nu$-representation. As is easily understood from the above prescription, the $\mu\nu$-representation is the best representation which one may choose in incorporating the correlations into the basis states as much as possible under a restriction of conserving the number of quantum numbers. In this sense, the $\mu\nu$-representation is nothing but a quantum analogue of the DCC system with the ***maximum integrable form representation*** , whose validity is evaluated by means of separability condition (2.39) alone. In the same way, the validity of $\mu\nu$-representation is studied by only the separability condition given by

$$\mathfrak{H}^{[sepa]}_{\mu,\mu'} = 0 \ , \ (\mu\neq\mu') \ , \tag{3.52}$$

because the $\mu\nu$-representation is defined so as to satisfy the stability condition

$$\langle\mu\nu|\,\mathfrak{H}^{[stab]}_{\mu}\,|\mu\nu'\rangle = 0 \ , \ \text{for all } \mu \text{ with } \nu\neq\nu'. \tag{3.53}$$

Equations (3.52) and (3.53) are natural extension of the separability and stability conditions for the collective subspace D^{2L} given by Eqs.(3.29) and (3.35), respectively, to the whole Hilbert space.

Definition of Quantum Chaos

Let $\{|i>\}$ be a set of exact eigenstates of the total Hamiltonian $H(B,B^{\dagger})$

$$\mathbf{1}\, H(B^{\dagger},B)\, \mathbf{1}\, |i\rangle = E_i\, |i\rangle\,, \quad |i\rangle \in \mathbf{1} : \{|m,n\rangle\,;\, m+n \le N\}\,. \tag{3.54}$$

From the above discussion, it is clear that the exact eigenstate $|i>$ is no more specified by one of the (μ,ν)-basis states when the separability condition (3.52) is violated. Namely the exact eigenstate is not specified by K-kinds of quantum numbers any more but by smaller number of quantum numbers in accordance with the violation of the separability condition. In other words, there occurs a ***dissolution of quantum numbers*** when the separability condition does not hold. In the classical theory, a dissolution of integrability is induced by the violation of the separability condition (2.39) and simply means an onset of chaotic motion. In the quantum system, therefore, it is reasonable to state that a dissolution of quantum numbers may indicate an onset of "quantum chaos". In accordance with the above discussion of the quantum chaos, we may classify the exact eigenstates $\{|i>\}$ into three characteristic cases with the aid of the (μ,ν)-basis states;

Quantum integrable motion ; If one finds one of the (μ,ν)-basis states for a given eigenstate $|i>$ satisfying $|<\mu,\nu|i>|^2 \approx 1$, then $|i>$ is classified to be in "quantum integrable motion" because it is specified by K-kinds of quantum numbers (μ,ν) associated with the DR.

Quantum KAM motion; If $|i>$ is described perturbatively starting from the most appropriate (μ,ν)-basis state, then it is specified to be in "quantum KAM motion" because it is still characterized by K-kinds of asymptotic quantum numbers.

Quantum chaotic motion ; If $|i>$ is not described perturbatively starting from any (μ,ν)-basis states, then it is regarded to be in "quantum chaotic motion" because $|i>$ is not characterized by K-kinds of quantum numbers any more.

4. Application of the Quantum Theory

4.1 Fractional Parentage Plots

In the previous section, we have defined the concepts of quantum integrability and quantum chaoticity for each eigenstate by introducing the $\mu\nu$-representation. This representation is nothing but the quantum analogue of the DCC system with the maximum integrable-form representation, because their validity are simply evaluated by the separability condition alone. We are

now at the position to illustrate what the quantum chaos for individual eigenstates means. Since the above-derived definition of the quantum chaos does not depend on the probabilistic assumption whereas the usual definition of quantum chaos based on the GOE does, it is also instructive to discuss the relation between them. To these aims, let us apply the quantum theory of nuclear collective dynamics developed in §3 to a simple soluble three-level SU(3) model Hamiltonian by Li, Klein and Dreizler [20] with slight modification given as

$$\hat{H} = \sum_{i=0}^{2}\varepsilon_i\hat{K}_{ii} + \frac{V_1}{2}(\hat{K}_{10}\hat{K}_{10} + \text{h.c.}) + \frac{V_2}{2}(\hat{K}_{20}\hat{K}_{20} + \text{h.c.}) \quad , \quad \hat{K}_{ij} \equiv \sum_{m=1}^{N} C_{im}^{\dagger}C_{jm} \quad , \quad (4.1)$$

where $\{ C^{\dagger}_{jm}, C_{jm}\}$ obey fermion anti-commutation relations. There are three single-particle levels with energies $\varepsilon_0<\varepsilon_1<\varepsilon_2$ and each level has N-fold degeneracy. Hereafter, we treat the even number N-particle system where the lowest level with i=0 is completely occupied. By employing the generalized Holstein-Primakoff boson expansion theory[18], the fermion Hamiltonian in Eq.(4.1) is transformed into the boson Hamiltonian

$$\begin{aligned} H(B,B^{\dagger}) = {} & (\varepsilon_1 - \varepsilon_0)B_1^{\dagger}B_1 + (\varepsilon_2 - \varepsilon_0)B_2^{\dagger}B_2 \\ & + \frac{V_1}{2}(B_1^{\dagger}\sqrt{N - B_1^{\dagger}B_1 - B_2^{\dagger}B_2}\,B_1^{\dagger}\sqrt{N - B_1^{\dagger}B_1 - B_2^{\dagger}B_2} + \text{h.c.}) \\ & + \frac{V_2}{2}(B_2^{\dagger}\sqrt{N - B_1^{\dagger}B_1 - B_2^{\dagger}B_2}\,B_2^{\dagger}\sqrt{N - B_1^{\dagger}B_1 - B_2^{\dagger}B_2} + \text{h.c.}) \, . \end{aligned} \quad (4.2)$$

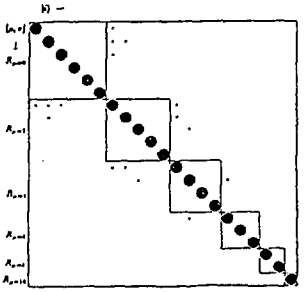
The "physical" boson space corresponding to the fermion space is given by

$$|m,n) = \frac{1}{\sqrt{m!}}\frac{1}{\sqrt{n!}}(B_1^{\dagger})^m(B_2^{\dagger})^n|0) \quad , \quad 0 \le m + n \le N \, . \quad (4.3)$$

In this paper, we concentrate ourselves to a set of "collective" excited states whose small-amplitude motion near the stationary state $|\,0\,)$ is described by a single kind of boson operators $\{B_1^{\dagger}, B_1\}$. Since the maximal-decoupling condition (3.23) is satisfied by the Hamiltonian in Eq.(4.2), the representation using the boson operators $\{B_i^{\dagger}, B_i\ ;\ i= 1,2\}$ just corresponds to the DR for the present case. Namely, a set of basis states in Eq.(4.3) gives the (m,n)-basis states defined in Eq.(3.16).

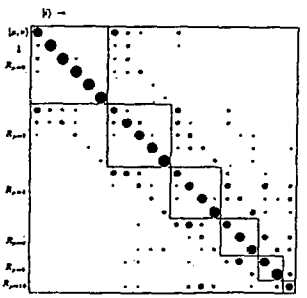
According to the theory discussed in §3, the chaoticity of each eigenstate is visualized by expanding it in terms of the (μ,ν)-basis states;

$$|i) = \sum_{\mu\nu} a^{i}_{\mu\nu}|\mu,\nu) \quad . \quad (4.4)$$

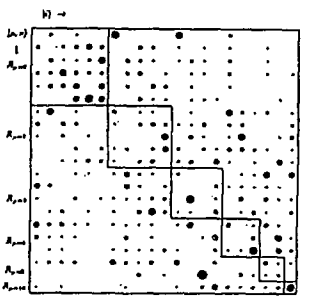


a) Quantum integrable motion.
V1=V2=-0.022

The μν-representation of {| i >} is shown in Fig.3, where the size of a solid circle is proportional to the square of expansion coefficients $|a_{\mu\nu}{}^i|^2$. The parameters used in calculating Fig.3 are N=10, $\varepsilon_1-\varepsilon_0=1$ and $\varepsilon_2-\varepsilon_0=\sqrt{2}$. According to the special symmetry of the interaction in Eq. (4.1), the Hamiltonian conserves the evenness or oddness of the population in the relevant and irrelevant boson numbers. In our numerical calculation, we consider the subspace with



b) Quantum KAM motion.
V1=-0.11, V2=-0.33



c) Quantum chaotic motions
V1=V2=-0.33

FIGURE 3. Fractional parentage plots for the case with N=10.

even number operators for both the relevant and irrelevant bosons. If the strength of the interaction is relatively weak, every eigenstate {|i >} belongs to the quantum integrable motion, because it is characterized by one of the (μ,ν)-basis states. In this case (Fig.3a), the system may be classified to be in "quantum integrable motion". If the force strength becomes large (Fig.3c), the expansion coefficients $a_{\mu\nu}{}^i$ for each eigenstate |i> are distributed over a wide range of the (μν)-basis states, indicating the dissolution of quantum numbers associated with the DR. In this case, the system may be classified to be in "quantum chaotic motion". In the intermediate case (Fig.3b), the expansion coefficients spread out over many (μν)-basis states. Since the diagonal components still remain the largest, the system in Fig.3b may be classified to be in "quantum KAM motion". These figures are called the ***fractional parentage plots*** (FPP). Since the FPP clearly shows the chaoticity of individual eigenfunctions, it is regarded as

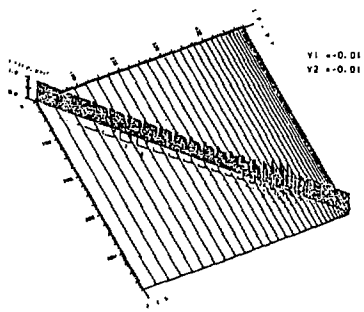
a quantum analogue of the *Poincaré* section map which has been used to clearly show the chaoticity of individual trajectories.

It has been shown that the properties of the wave function visualized in the FPP are well understood in terms of both the stability and separability conditions. A detailed discussion is found elsewhere[21].

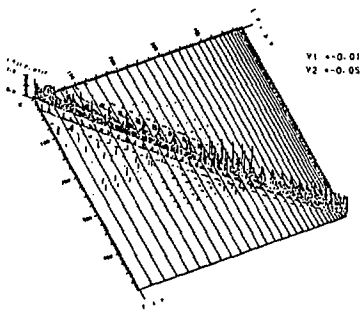
4.2 Nearest Neighbor Level Spacing

Let us discuss the relation between the present definition of quantum chaos for each eigenstate and the usual definition of quantum chaos for an ensemble of eigen-states. Since the quantum chaos defined at the end §3 is given for individual eigenfunctions, it is quite interesting to discuss how the nearest neighbor level spacing statistics relates to the quantum chaos defined for each eigenfunction.

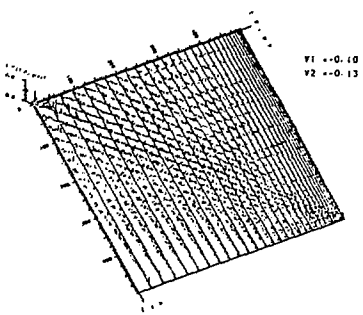
In order to get the nearest neighbor level spacing statistics, we calculated the Hamiltonian in Eq. (4.1) with N=60, $\varepsilon_1-\varepsilon_0 = 1$, $\varepsilon_2-\varepsilon_0 = 2$. The final dimension of the Hamiltonian matrix to be diagonalized is N_d=496 which might be sufficiently large for discussing a variety of statistical aspects of an ensemble of eigenstates. The FPP showing the properties of each eigenfunction is shown in Fig. 4. In this figure, the squares of expansion coefficients are shown by the lengths of vertical segments. As is easily seen from Fig.4, three different parameter sets used in our calculation just correspond to the quantum integrable, intermediate and quantum stochastic cases,



a) Case A. V1=-0.01, V2=-0.01

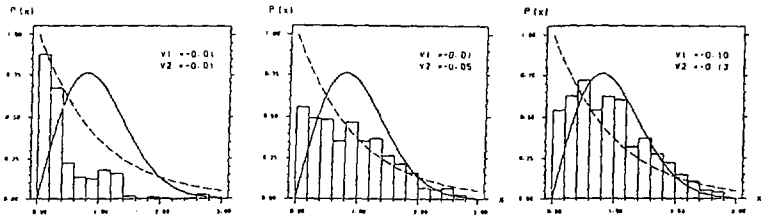


b) Case B. V1=-0.01, V2=-0.05



c) Case C. V1=-0.10, V2=-0.13

FIGURE 4. Fractional parentage plots for the case with N=60.

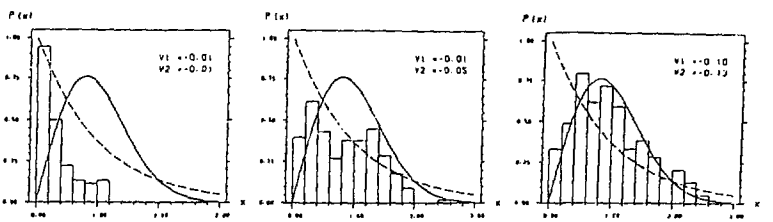


a) Case A b) Case B c) Case C

FIGURE 5. Nearest neighbor level spacing statistics for all eigenstates. Solid and dashed curves show the Wigner and Poisson distributions.

respectively. In Case A in Fig.5a, the level statistics gives a strong peak at the origin, indicating an "integrability" of the system. With increased effects of V_1- and V_2-interactions, the peak at the origin becomes less pronounced. Namely, the Poisson type distribution in CaseA is disturbed toward the Wigner type distribution when the system shows "quantum KAM" character as in Case B. In Case C where almost all eigenstates are classified to be in quantum chaotic motion, the peak at the origin disappears. From these numerical calculations, we may conclude that our definition of quantum chaos seems to be compatible with the usual discussion based on the GOE.

As is discussed previously, the random matrix theory relies on some probabilistic assumptions. It is derived under the assumption that the matrix elements should be independent random variables and the resultant ensemble should be invariant under any base transformation, i.e. should be representation independent. Consequently, the boundary effects specific for such a



a) Case A b) Case B c) Case C

FIGURE 6. Nearest neighbor level spacing statistics for 200 intermediate eigenstates.

finite system as the nucleus are not properly considered in the random matrix theory. In order to eliminate the boundary effects, we divided the total eigenstates N_d=496 into three groups specified by their energy eigenvalues: The lowest 200, the intermediate 200 and the highest 96. By extracting 200 intermediate eigenstates, we have calculated the nearest neighbor level spacing statistics shown in Fig.6 where the transition from Poisson to Wigner distributions is much more pronounced than the case shown in Fig.5. From the above numerical calculation, it is concluded that an appearance of the Wigner type distribution function does not directly correspond to (even though it is compatible with) the dissolution of quantum numbers associated with the DR. In other words, the dissolution of quantum numbers indicating the individual chaoticity does not necessarily mean an appearance of probabilistic feature which is the basic assumption of the random matrix theory.

4.3 Simulated Eγ-Eγ Plot

Let us discuss how to get an experimental spectroscopic indication on an onset of quantum chaos besides the nearest neighbor level spacing. A detailed information on the wave functions is obtained by evaluating many kinds of transition probabilities between many eigenstates. According to the special symmetry of the interaction in Eq. (4.1), we consider "collective" transition probabilities given by

$$T_{i,j} \equiv \left| \langle i | B_1 B_1 | j \rangle \right|^2 \quad . \tag{4.5}$$

With the aid of numerical calculation for $T_{i,j}$, we get interesting information how the collectivity undergoes a change as the stochasticity of the eigenfunction develops. Especially, we find many numerical evidence on bifurcation, trident, multi-forking and termination of collective band structure. However, it is not easy to deduce some general conclusion out of numerous data $T_{i,j}$, whose number is $O(N_d{}^2)$. Aiming at extracting some physical information from the numerous experimental data, there has been proposed the Eγ-Eγ plot[22] which has aroused many theoretical discussions on the rotational damping mechanism. Namely, we get some statistical aspect of the rotational motion out of two successive transitions. By means of two successive matrix elements and energy eigenvalues, we may introduce the following quantity:

$$I_{ijk} = (E_i - E_j)T_{i,j} - (E_j - E_k)T_{j,k} \quad ,$$

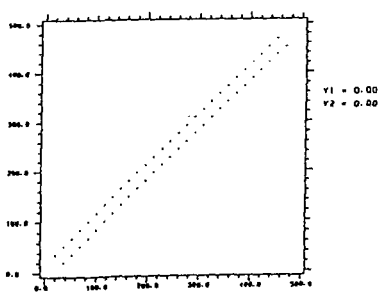
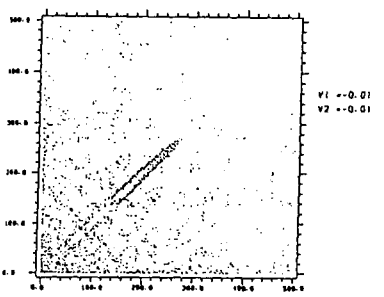


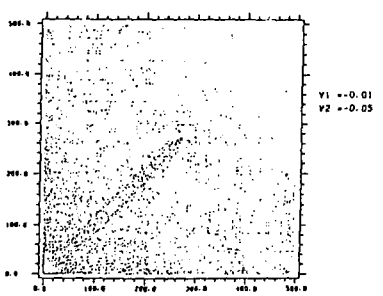
FIGURE 7.Simulated Eγ-Eγ Plots for ideal integrable case..

$$\text{with } E_i > E_j > E_k \ . \tag{4.6}$$

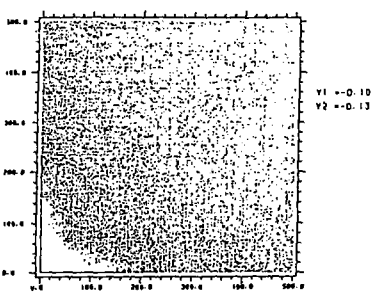
Plotting a couple of sequential values (I_{ijk}, I_{jkl}) as well as (I_{jkl}, I_{ijk}) in the xy-plane, we may evaluate statistical features of the system in terms of local information obtained from two successive transition probabilities. From the above discussion, it is quite interesting to study whether or not the stochasticity occurring in the individual eigenfunctions induces some irregularity in the (I_{ijk}, I_{jkl})-plot. We start with a specific situation where all the eigenstates are classified to be in "ideal integrable" motion, i.e. a case with no interaction V_1=V_2=0. In this case, we get two straight dotted lines in the xy-plane shown in Fig.7. By switching on the effects coming from V_1- and V_2-interactions, we get various (I_{ijk}, I_{jkl})-plots illustrated in Fig.8. As is seen from Fig.8a which corresponds to Case A in Fig.4, there exists a pronounced regularity when all the eigenstates are classified to be in quantum regular motion. On the other hand, it is hard to see any regularity in Fig.8c which represents Case C in Fig.4. In between these two extreme cases, we find a remnant of regularity in Fig.8b. It is striking that there still remains an information on the stochasticity of individual eigenfunctions after accumulating numerous local informations between two eigenstates connected by the collective operator. As far as the present model Hamiltonian is concerned, it is clearly seen that the (I_{ijk},I_{jkl})-plot gives an important information on the dissolution of quantum numbers. By looking at the more detailed information corresponding to the multi-coincidence experiments, we get more global information



a) Case A



b) Case B.



c). Case C

FIGURE 8. Simulated Eγ-Eγ Plots for three characteristic cases in Fig.4.

which may provide us with a new indication on the onset of the quantum chaos. Here it should be noticed that the motional narrowing mechanism[23] was studied numerically by calculating a strength function for the system with two degrees of freedom very similar to the present model Hamiltonian. It is a very interesting problem to explore whether the rotational damping is induced by the dissolution of quantum numbers specifying the collective rotational motion or by the motional narrowing mechanism which involves the order-to-chaos transition in the intrinsic system.

Following the present numerical calculation, we plan to extend our numerical calcu lation to a more complex Hamiltonian with many local minima. Since we may expect many level crossing points in between two local minima, this subject certainly involves an interesting question on a specific role of level crossing dynamics on an onset of quantum chaos in a nuclear system as well as an inter-relation between the quantum- and classical-chaos .

§5 Summary and Concluding Remarks

As discussed in §1, we are at the early stage in developing the fundamental theory of nuclear matter physics which provides us with a unified understanding of a variety of phenomena exhibited by the nucleus. The first inevitable task in understanding an inter-relationship between various phenomena relating to different local minima might be achieved by properly defining the global collective coordinates which have a sense in the wide area of the TDHF phase space. Since the dynamical evolution of the nuclear system is organized by the microscopic dynamics of the nucleons, the above task is performed by dynamically extracting the dynamical collective coordinates on which the nuclear system evolves by itself, rather than introducing the phenomenological collective coordinates from the outset. This task has been explored by the SCC method where the total system is dynamically divided into the relevant (collective) and irrelevant parts. Basing on the SCC method in §2, we have discussed how important it is to develop the microscopic theory of nuclear dynamics within the single-particle dynamics by going beyond the adiabatic or diabatic approximation. In this context, the role of the level crossing (appearing in the irrelevant degrees of freedom) on the collective dynamics, i.e. the "classical" order-to-chaos transition mechanism in the single-particle dynamics should be further developed by referring to a variety of phenomena stated in §1 so as to establish the geometrical or topological understanding of nuclear collective dynamics.

After clarifying the dynamics of the single trajectory in the TDHF phase space, there arises an interesting question how to formulate the dynamical evolution of an ensemble of trajectories, which may provide us with an understanding how the statistical or probabilistic feature appears as a consequence of the deterministic dynamics. This point has been shortly discussed at the end of §2 simply because of the limitation of available space. This question is related to the second problem of the fundamental theory in the nuclear matter physics discussed in §1 and

should be further explored in understanding the microscopic dynamics in the highly excited states, fission process, heavy-ion deep inelastic collision and etc.

In §3, we have discussed the quantum theory of nuclear collective dynamics under instruction of the single-particle dynamics developed in §2. It has been clarified that in both the classical and quantum theories, the separability and stability conditions play a decisive role in characterizing the order-to-chaos transition mechanism. In this respect, the further experimental and theoretical studies are required to explore whether the quantum chaos defined in this lecture is physical object or not. It is also an interesting theoretical subject to develop the full quantum mechanical nuclear master equation which may give a microscopic foundation of the motional narrowing mechanism in rotational damping phenomena.

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